Phase space with internal symmetries and Chiral Kinetic Theory

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Brookhaven National Laboratory

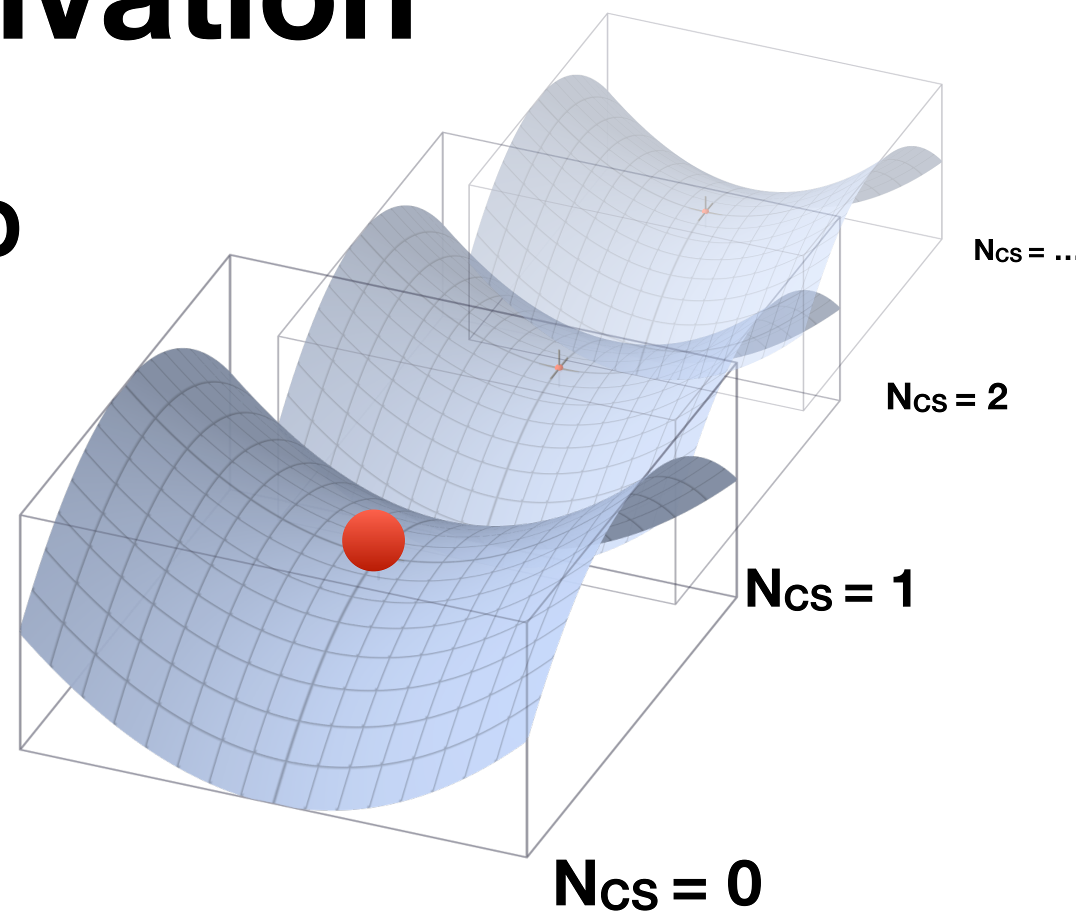
with Raju Venugopalan

PRD 99, 056003 (arXiv:1901.10492); PRD 97, 051901 (arXiv:1701.03331)
PRD 96, 016023 (arXiv:1702.01233)

The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, Tsinghua University, Bejing, China — April 9th, 2019

Motivation

- **Topological structure of QCD**



sources: (top) wikipedia O. Alexndrov; (bottom) Kharzeev, Warringa, Fukushima

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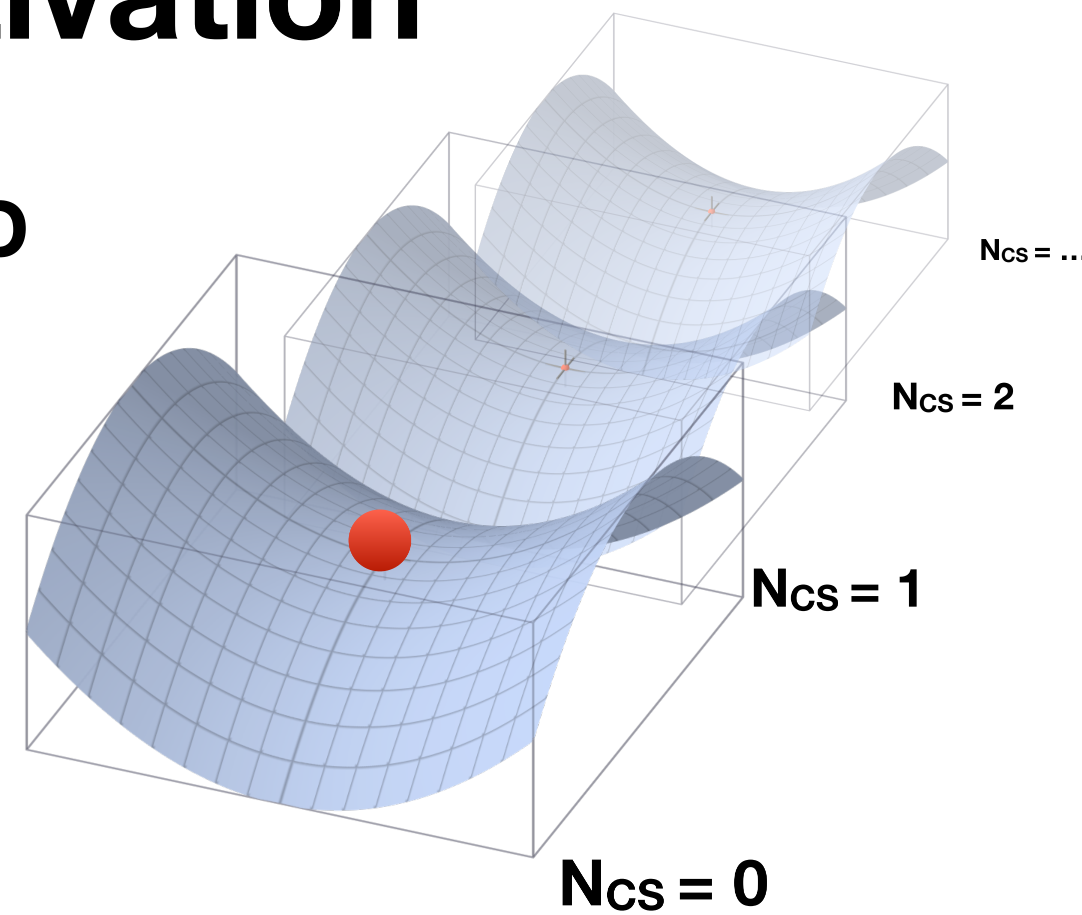
$$n_{CS}(t, \mathbf{x}) \equiv \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}(t, \mathbf{x})$$

$$N_{CS}(t) \equiv \int d^3x \, n_{CS}(t, \mathbf{x})$$



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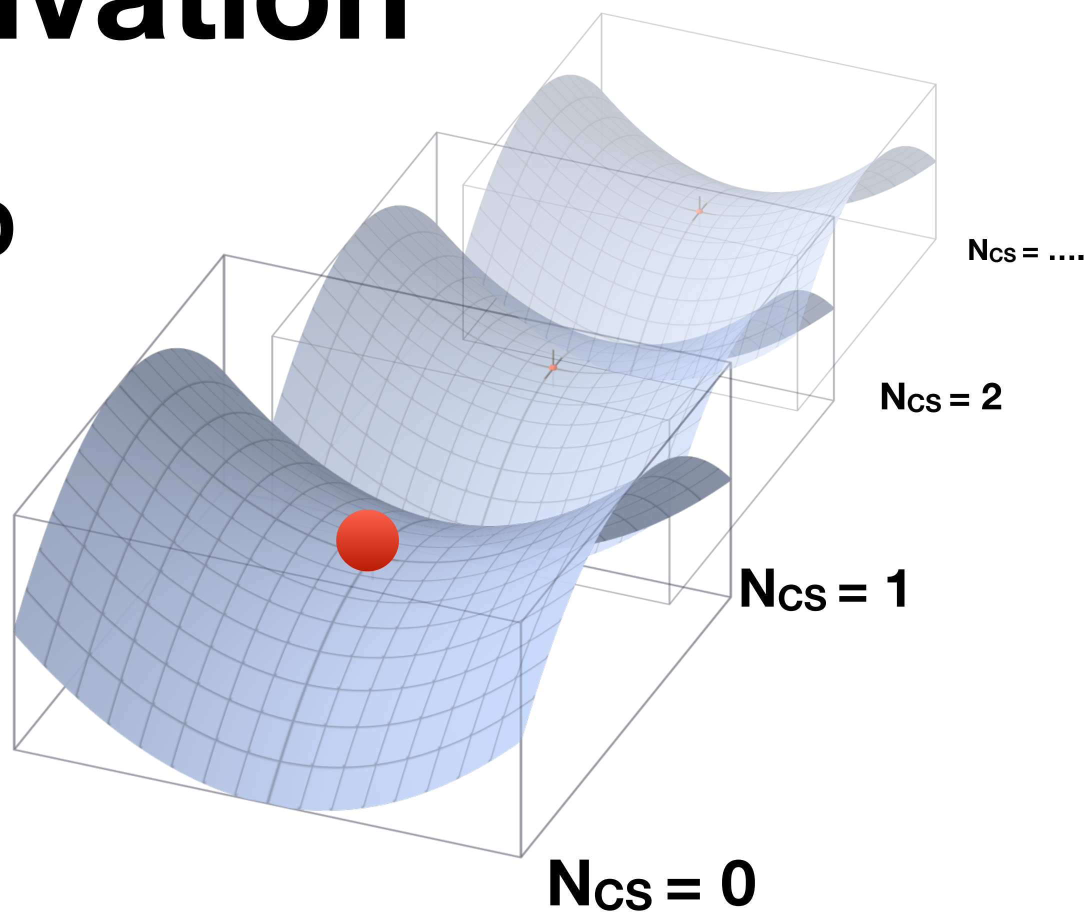

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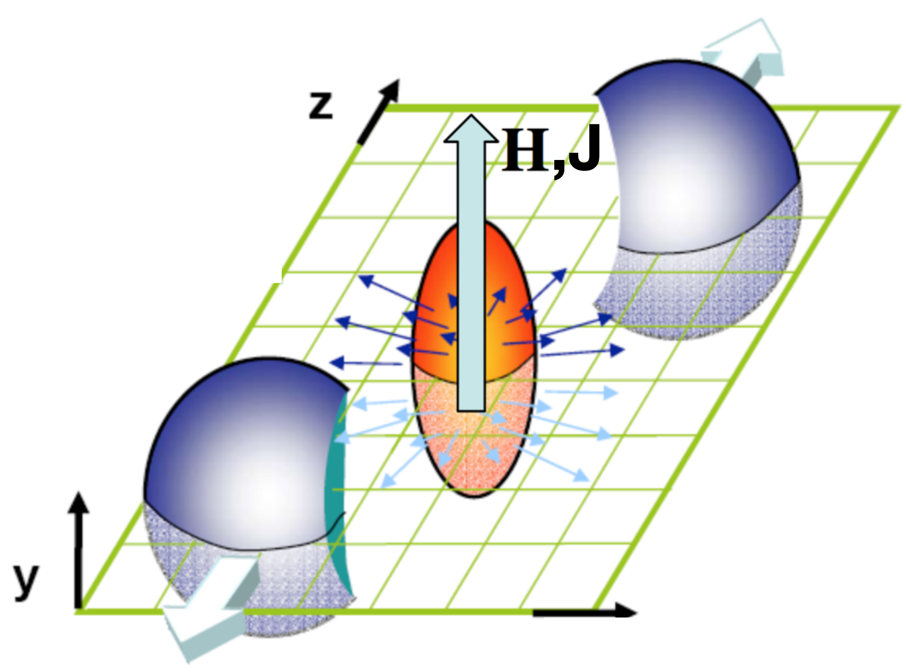

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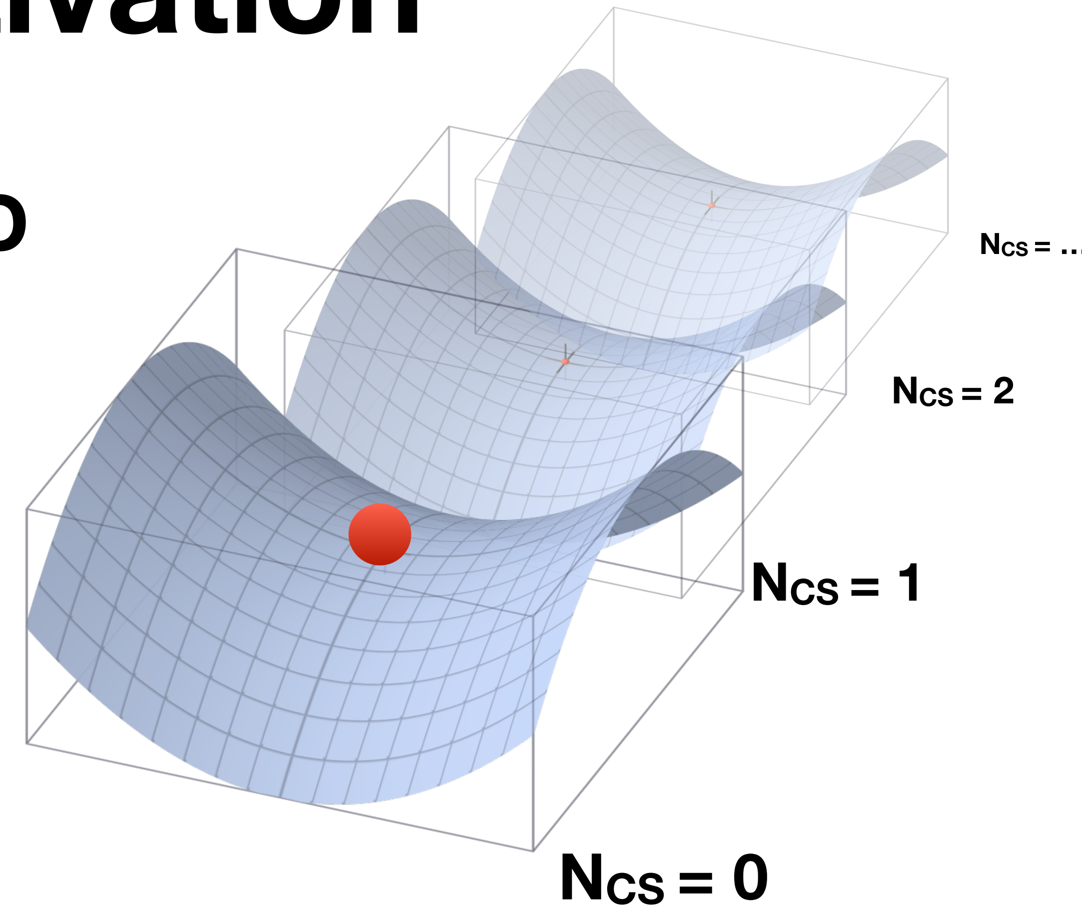

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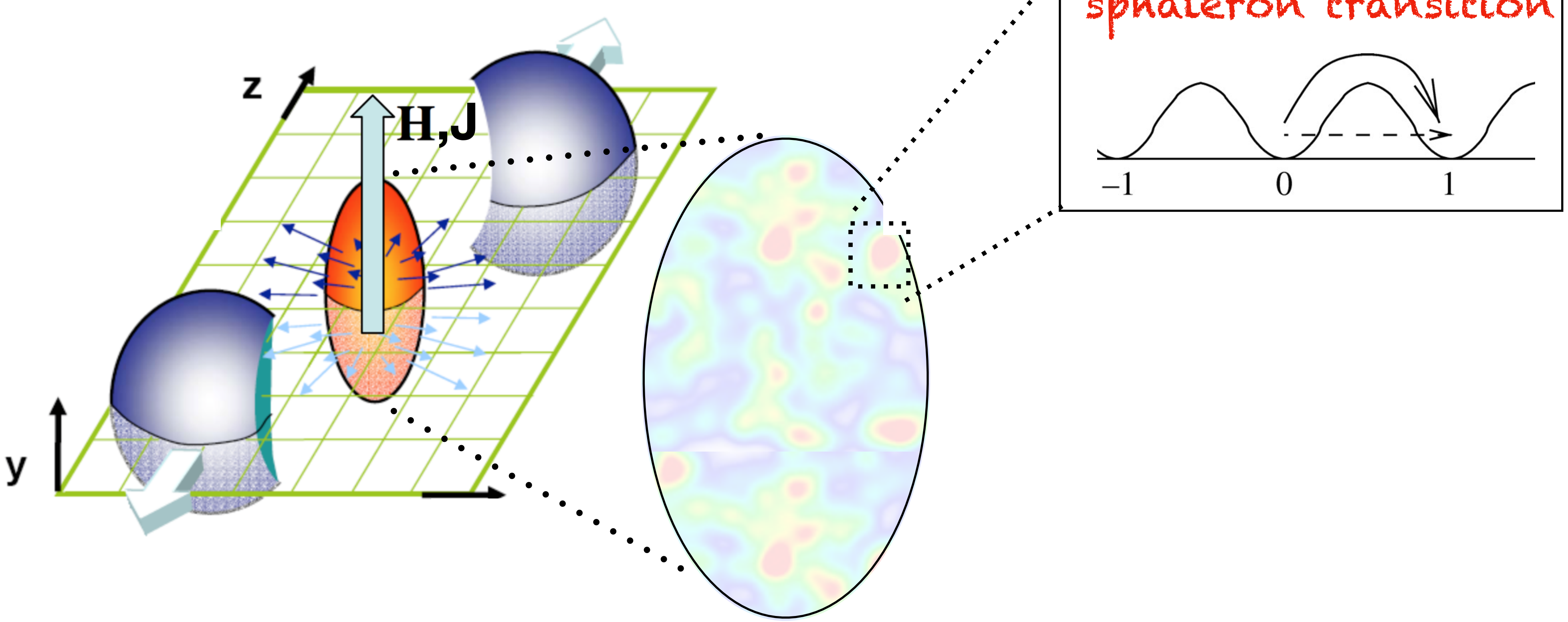

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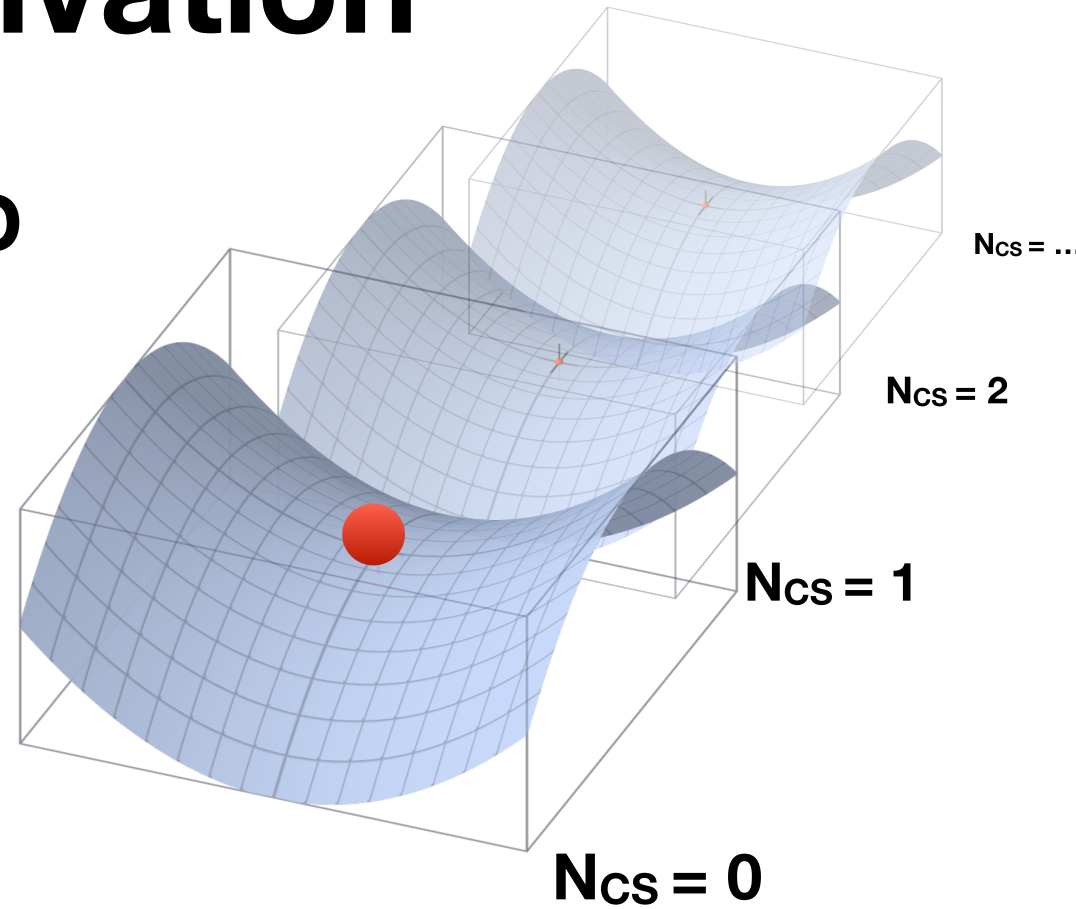

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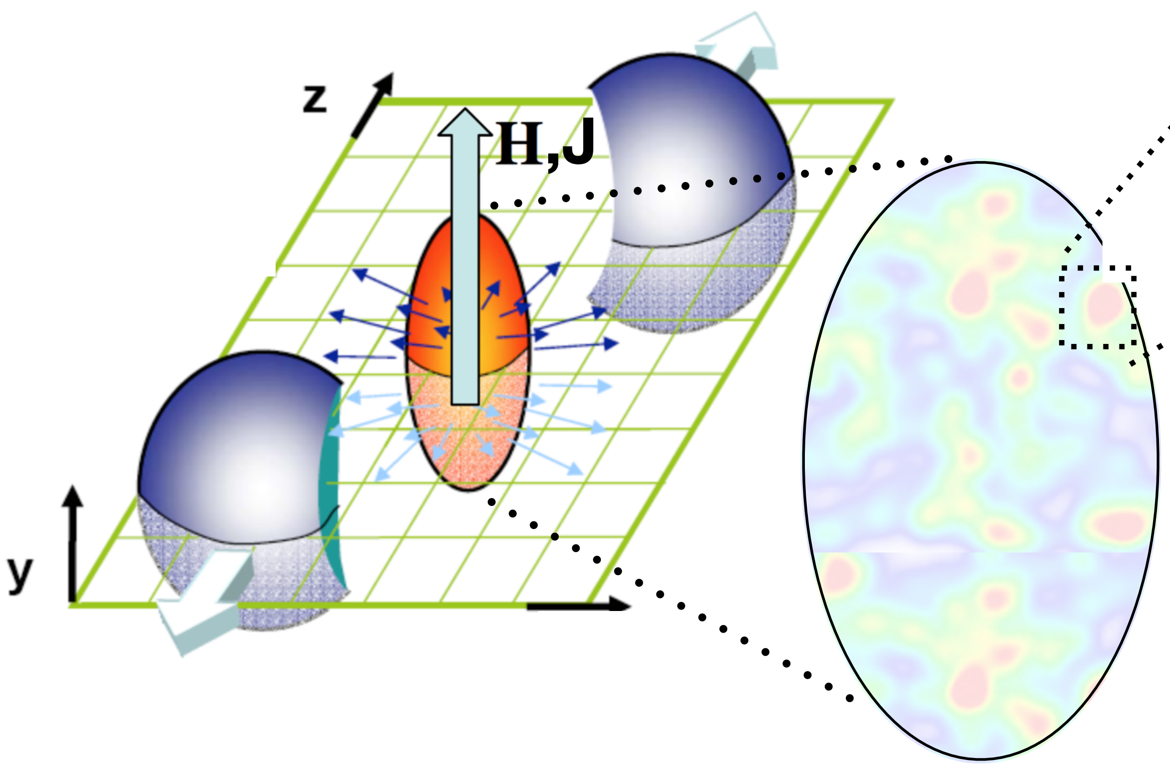
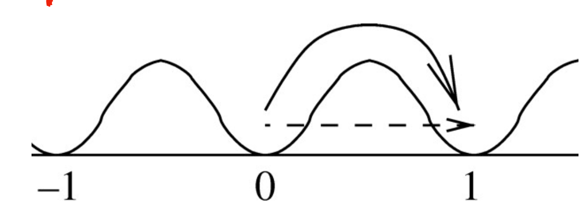


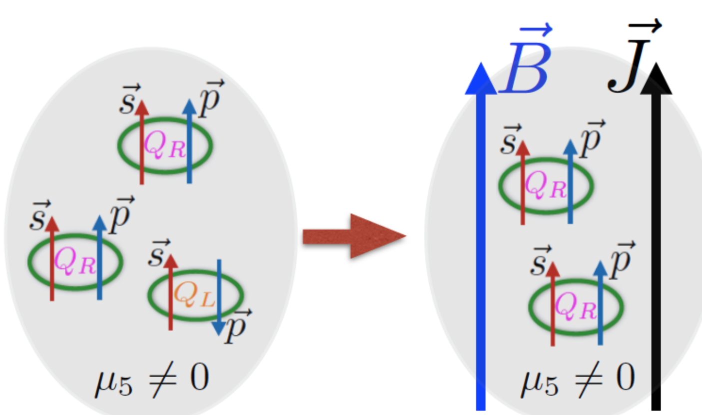
"sphaleron transition"



+

"chiral anomaly"

$$\langle \partial_\mu J^\mu_5(t,\mathbf{x})\rangle = -\frac{g^2 N_f}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}(t,\mathbf{x})$$



Fukushima, Kharzeev, McLerran, Warringa ~ 2008

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Dirac and Weyl Semi-metals

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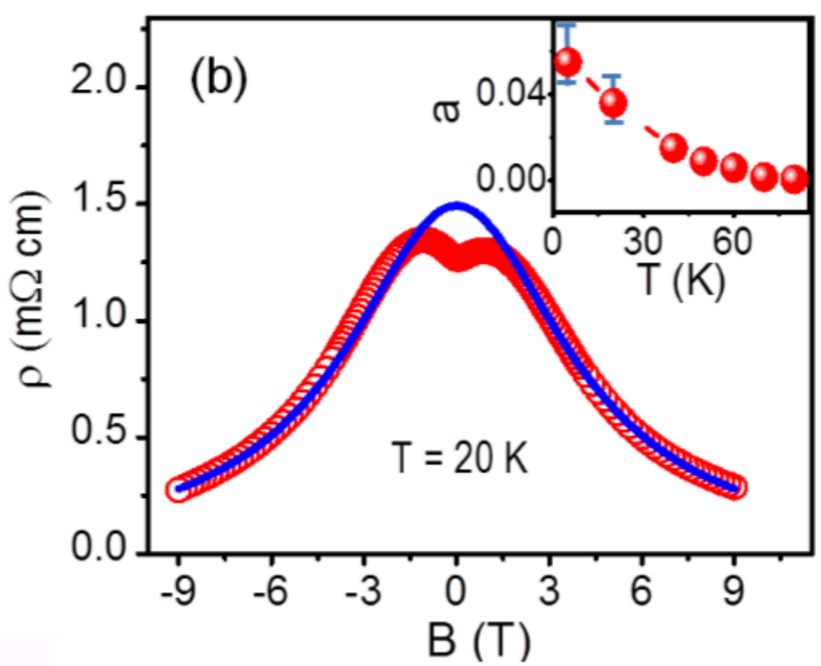
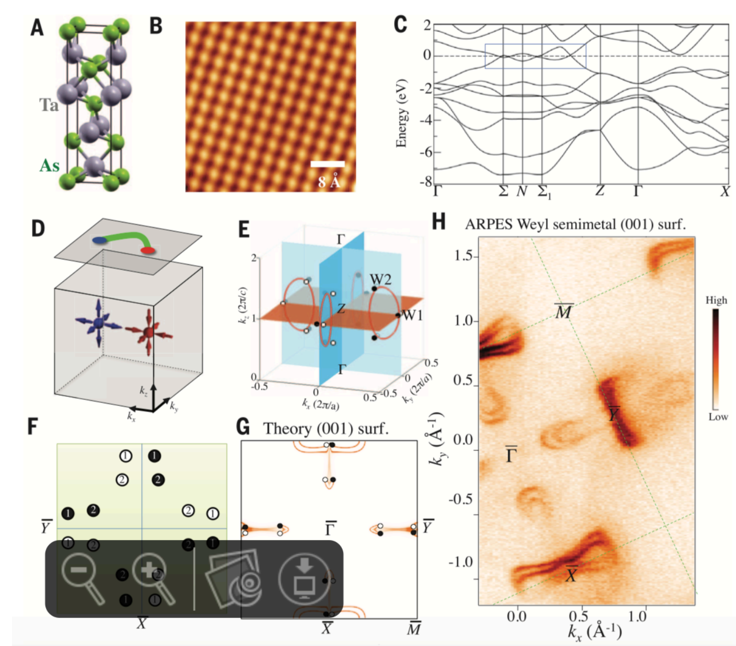


Li et al, Nature Physics volume 12, pages 550–554 (2016)



Xu et al., *Science* 07 Aug

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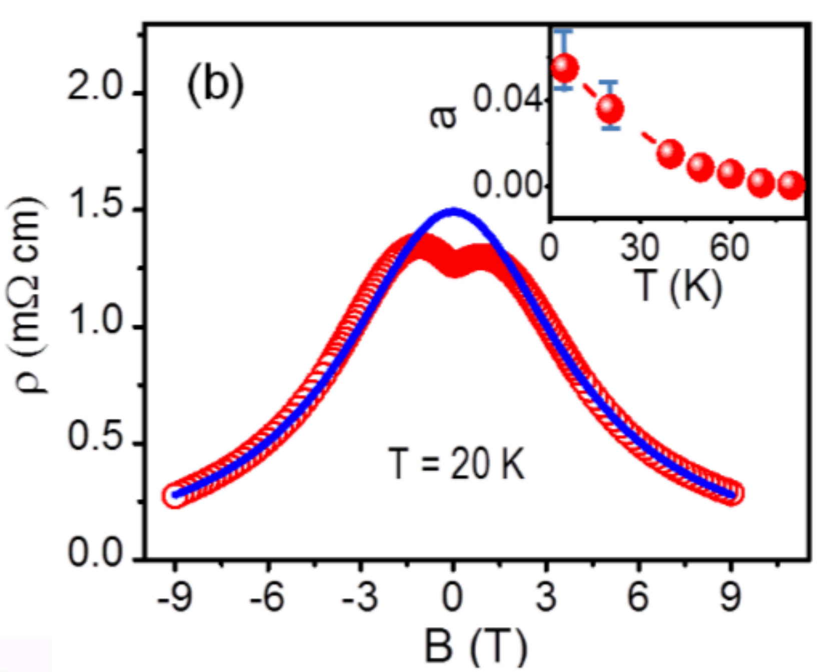
- **More Chiral Effects:** *novel electronic properties*

Dirac and Weyl Semi-metals **pseudo-chiral effects**

ZrTe *$Mo_xW_{1-x}Te_2$* *TaA* *Graphene*



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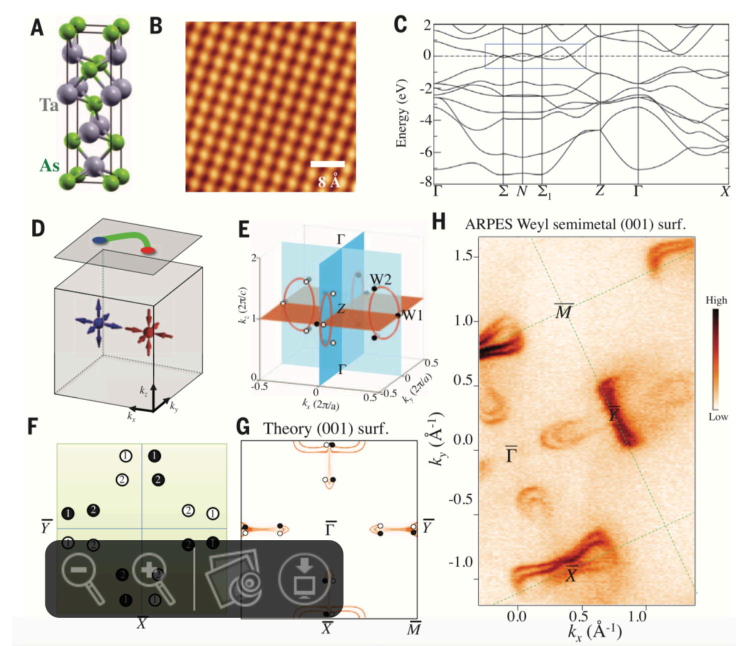

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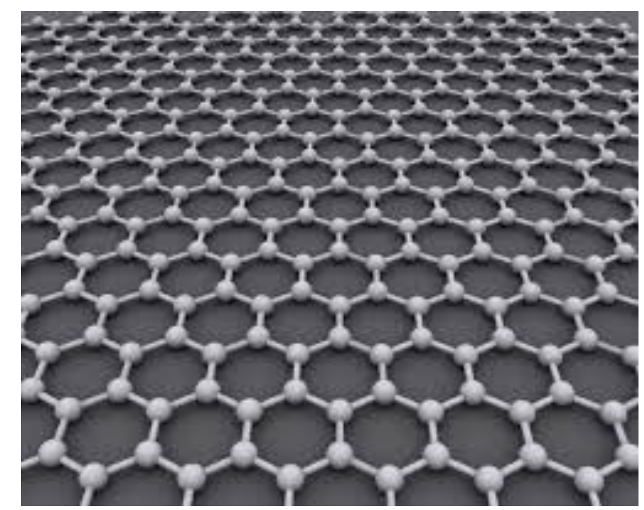

source: wikipedia

Mizher, Raya, Villavicenci 2016

Outline

1. Chiral Fluids
2. World-line approach
3. Quantum Phase space with internal symmetries
4. Chiral Kinetic Theory

1. Chiral Fluids: Theory

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— Anomalous Hydrodynamics —

- **Son & Surowka (2009)**

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu$$

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Derive anomalous hydrodynamics from QFT!
Effective action approach

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- **Microscopic to macroscopic:**
  — fluctuations and collision terms

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D'Hoker & Gagne

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D'Hoker & Gagne

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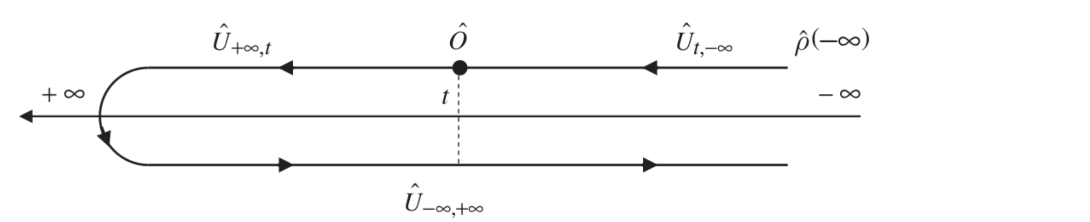

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D'Hoker & Gagne

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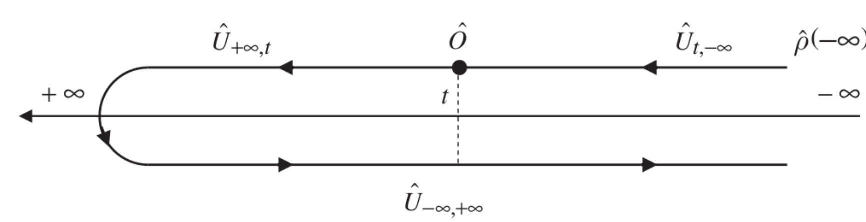

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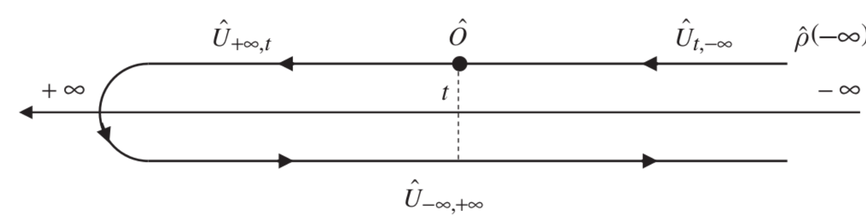

"Wigner distribution"

$$\chi_A(x_i^+, x_i^-, \lambda_i^+, \lambda_i^-, \lambda^{\dagger}{}_i^{+}, \lambda^{\dagger}{}_i^{-}) \equiv \int \frac{d^4\bar p_i}{(2\pi)^4} W_A^\chi(\bar x_i, \bar p_i, \bar\lambda_i, \bar\lambda_i^\dagger)\, e^{i(\bar p_i\cdot\tilde x_i + \frac{1}{2}\bar\lambda_i^\dagger\cdot\tilde\lambda_i + \frac{1}{2}\bar\lambda_i\cdot\tilde\lambda_i^\dagger)}$$

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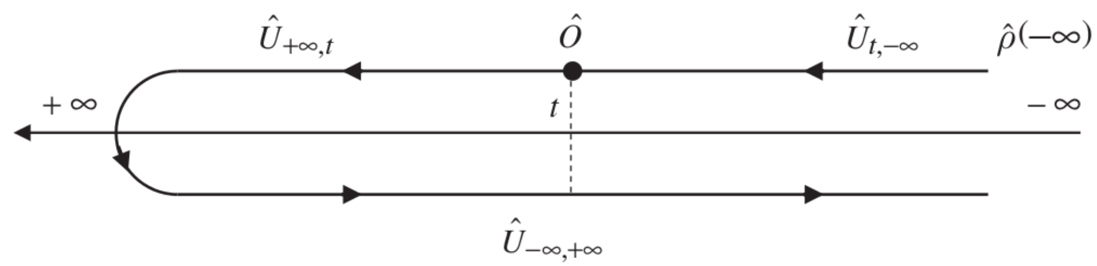
review: Polkovnikov 2009



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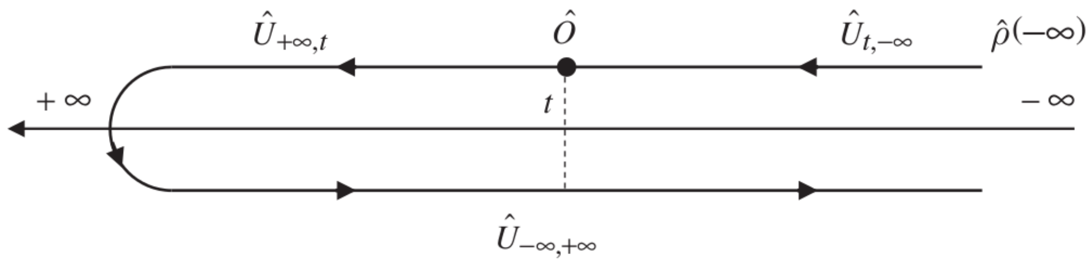

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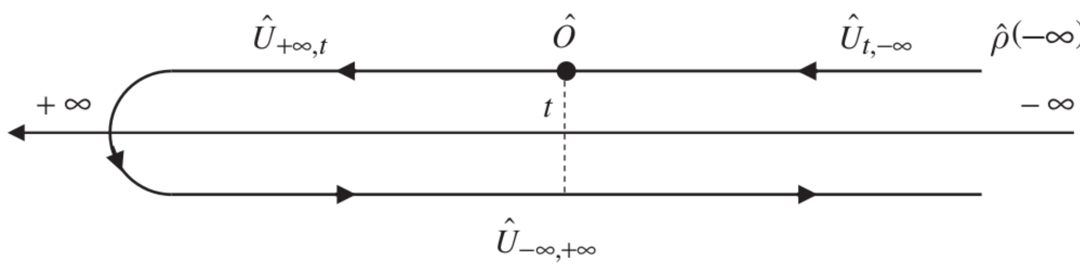

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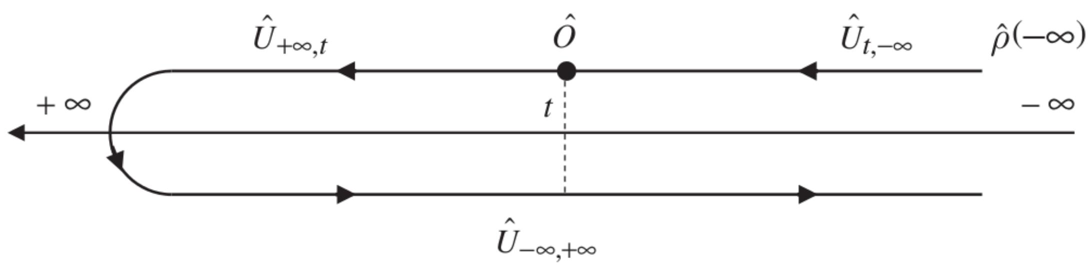
$$\Gamma_{\mathcal{C}} \approx \int d^4\bar{x}_i d^4\bar{p}_i d\bar{\lambda}_i d\bar{\lambda}_i^\dagger \; W_A^\chi(\bar{x}_i, \bar{p}_i, \bar{\lambda}_i, \bar{\lambda}_i^\dagger)$$
$$\times \int_{\mathcal{C}} \mathcal{D}x\mathcal{D}p\mathcal{D}\lambda\mathcal{D}\lambda^\dagger\mathcal{D}\epsilon D\phi \; \exp\left\{iS_0 + i\int d\tau \left(\left[\dot{\bar{p}} - \frac{\partial H}{\partial \tilde{x}}\right]\tilde{x} \right.\right.$$
$$\left.\left. - \left[\dot{\bar{x}} + \frac{\partial H}{\partial \tilde{p}}\right]\tilde{p} + \left[i\dot{\bar{\lambda}} - \frac{\partial H}{\partial \tilde{\lambda}^\dagger}\right]\tilde{\lambda}^\dagger - \left[i\dot{\bar{\lambda}}^\dagger + \frac{\partial H}{\partial \tilde{\lambda}}\right]\tilde{\lambda}\right)\right\}$$

2. Worldline approach

- **semi-classical phase space: Truncated Wigner Approximation**

review: Polkovnikov 2009



$$\bar{x} = \frac{x^+ + x^-}{2} \quad \text{"classical"}$$

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- **yields (quantum-) Liouville equation**

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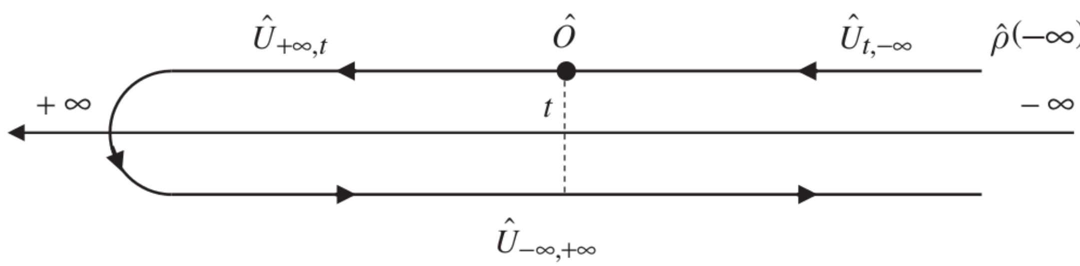

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Grassmann coordinates for color

2. Worldline approach

Spin and color as Grassmann coordinates

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- **color** $W^{\chi}_{A}(x, P, \lambda, \lambda^{\dagger})$

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- **Spin via anti-commuting variables** (Berezin and Marinov 1976)

$$W_A^\chi(x, P, \lambda, \lambda^\dagger) \longrightarrow W_A^\chi(x, P, \lambda, \lambda^\dagger, \underline{\psi})$$

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SK derivation in worldline formalism, see *PRD 99, 056003 (arXiv:1901.10492)*

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$$
\begin{aligned}
\dot{x}^\mu &= \epsilon P^\mu \,, \\
\dot{P}^\mu &= \epsilon g F^{a,\mu\nu} Q^a P_\nu - \frac{i\epsilon g}{2} \psi^\alpha (D^\mu F_{\alpha\beta})^a Q^a \psi^\beta \,, \\
\dot{\psi}^\mu &= \epsilon g F^{a,\mu\nu} Q^a \, \psi_\nu \,, \\
\dot{\lambda}_a^\dagger &= -igv^\mu t_{ab}^c A_\mu^c \lambda_b^\dagger - \frac{\epsilon g}{2} \psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c^\dagger \psi^\nu \,, \\
\dot{\lambda}_a &= igv^\mu t_{ab}^c A_\mu^c \lambda_b^\dagger + \frac{\epsilon g}{2} \psi^\mu F_{\mu\nu}^b t_{ac}^b \lambda_c \psi^\nu \,,
\end{aligned}
$$

$$
\dot{Q}^a = -igv^\mu f^{abc} A_\mu^b Q^c - \frac{g\epsilon}{2} f^{abc} \psi^\mu F_{\mu\nu}^b \psi^\nu Q^c
$$

= Wong's equation (1970)

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3. Quantum Phase Space

<u>Closer look:</u> phase space for spin and chirality

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$$\Gamma[A,B] \equiv \mathrm{tr} \int d^4x_i^+ d^4x_i^- d^4\psi_i^+ d^4\psi_i^- \, \zeta^{A,B}(x_i^+, x_i^-, \psi_i^+, \psi_i^-)$$

$$\times \int_{x_i^+}^{x_i^-} \mathcal{D}x\mathcal{D}p \int_{\psi_i^+}^{\psi_i^-} \mathcal{D}\psi \int \mathcal{D}\epsilon\mathcal{D}\chi e^{iSc[A,B]} \,,$$

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← unpolarized part

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unpolarized part

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Closer look: **chiral anomaly**

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- **Anomaly from phase of fermion determinant**

  Alvarez-Gaume & Witten, Nucl. Phys B234 (1984) 269

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<u>Closer look:</u> chiral anomaly

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- **Can be explicitly computed in worldline formulation**

$$\Gamma[A,B] = \Gamma[A] + \int d^4y \, \frac{\delta\Gamma[A,B]}{\delta B_\mu(y)}\Big|_{B=0} B_\mu(y) \qquad \delta\Gamma/i\delta B_\mu(y) \equiv \langle J_{5,\mu}(y)\rangle$$

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detailed derivation: arxiv:1702.01233 or arxiv:1901.10492 in real-time formulation

4. Chiral Kinetic Theory

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- **practical approach to CKT: color and spin, via moments**

$$f(x,P), f^a(x,P)\, \Sigma_\mu(x,P),\ \tilde{\Sigma}_\mu(x,P),\ \text{and}\ \tilde{\Sigma}^a_\mu(x,P)$$

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exact spin structure

$$f_A(x,P,Q,S) = f_A(x,P,Q)\left[i\Sigma_\mu(x,P,Q)S^{\mu\nu}v_\nu - \frac{i}{6}\epsilon_{\mu\nu\alpha\beta}v^\mu S^{\nu\alpha}S^{\beta\lambda}v_\lambda\right]$$

polarized — unpolarized

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polarized — unpolarized

exact color structure

$$f(x,P,Q) = f(x,P)\left[1 + \frac{2}{A_R d^2} d^{abc} Q^a Q^b Q^c\right] + 2 f^a(x,P) Q^a$$

singlet — octet

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singlet — octet

- **many body generalization Pauli-Lubanski vector (BMT equation)**

$$\dot{\Sigma}_\mu(x,P,Q) = \frac{g}{P^0}F^a_{\mu\nu}Q^a\,\Sigma^\nu(x,P,Q) + \frac{2g}{P^0}\Sigma_\alpha(x,P,Q)F^{a,\alpha\beta}Q^a v_\beta\, v_\mu$$

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$$f_A(x,P,Q,S) = f_A(x,P,Q)\left[i\Sigma_\mu(x,P,Q)S^{\mu\nu}v_\nu - \frac{i}{6}\epsilon_{\mu\nu\alpha\beta}v^\mu S^{\nu\alpha}S^{\beta\lambda}v_\lambda\right]$$

polarized unpolarized

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singlet octet

- **many body generalization Pauli-Lubanski vector (BMT equation)**

$$\dot\Sigma_\mu(x,P,Q) = \frac{g}{P^0}F^a_{\mu\nu}Q^a\,\Sigma^\nu(x,P,Q) + \frac{2g}{P^0}\Sigma_\alpha(x,P,Q)F^{a,\alpha\beta}Q^a v_\beta\, v_\mu$$

- **currents etc generalized phase space averages**

$$\langle J^\mu_{L/R}(x)\rangle \equiv e\int d^4P\, dS\,\epsilon\,[P^\mu + S^{\mu\nu}\partial_\nu]\,f(x,P,S)$$

4. Chiral Kinetic Theory

- **practical approach to CKT: color and spin, via moments**

$$f(x,P), f^a(x,P)\, \Sigma_\mu(x,P),\ \tilde{\Sigma}_\mu(x,P), \text{ and } \tilde{\Sigma}^a_\mu(x,P)$$

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- **anomaly: axial current requires ‘proper derivation’ from worldlines in TWA**

4. Chiral Kinetic Theory

NM, R. Venugopalan in preparation

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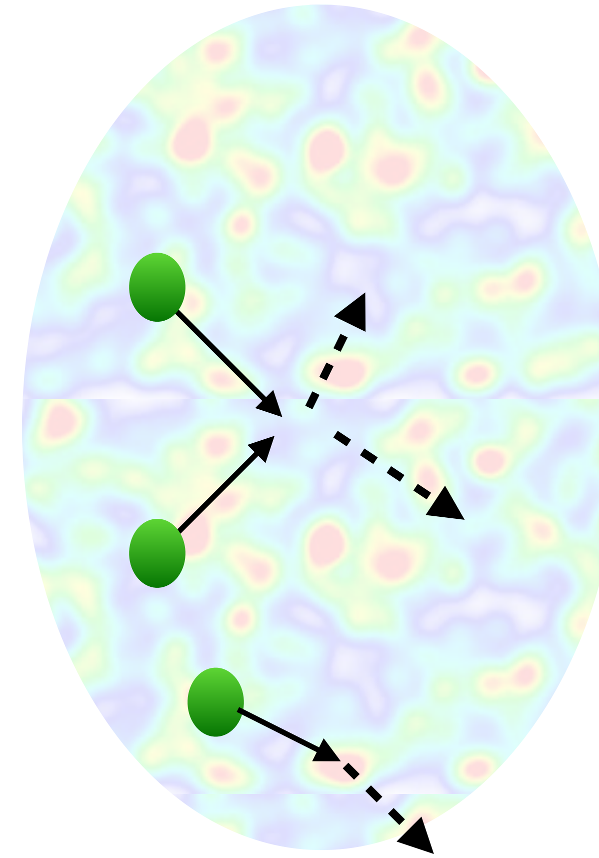
$$C(t,t') \equiv \langle N_{CS}(t)\, N_{CS}(t') \rangle$$

- **typical scale ~ g^2T**
  —> average distributions



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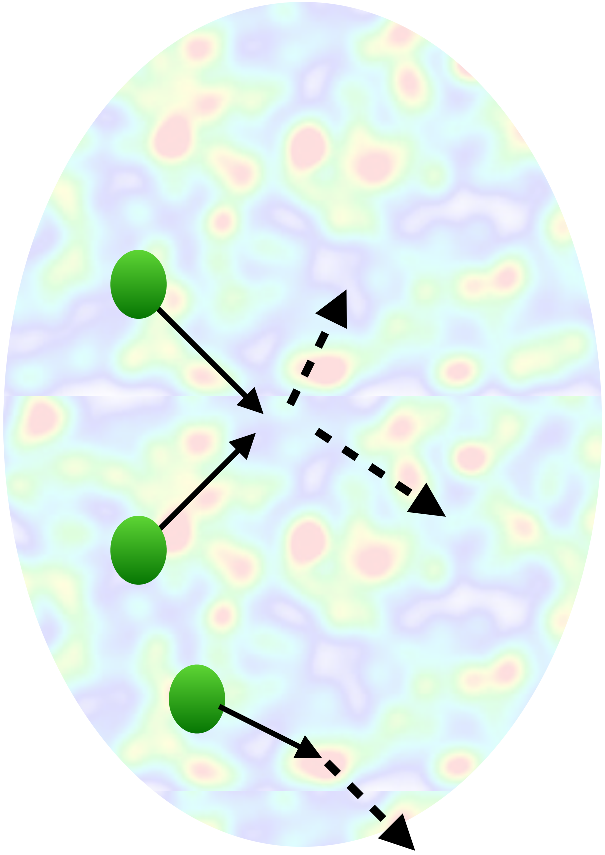

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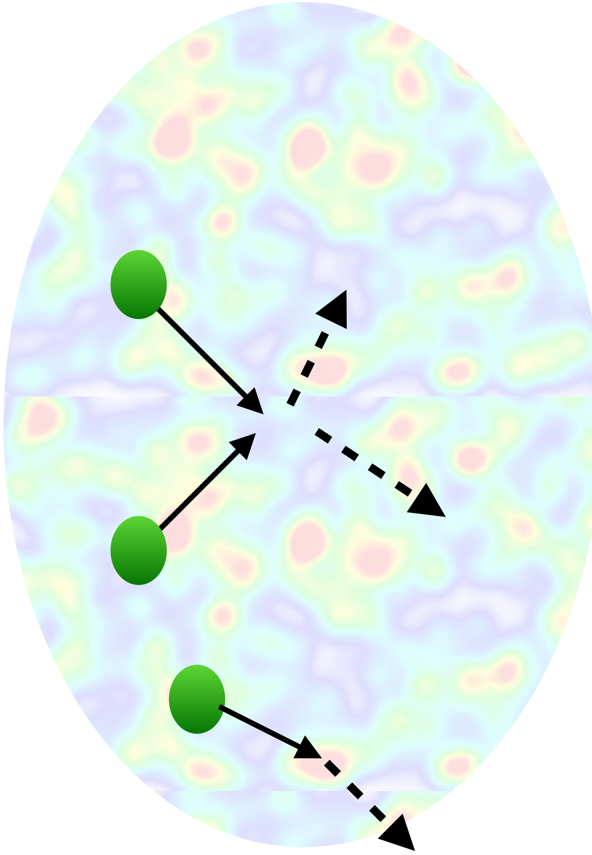

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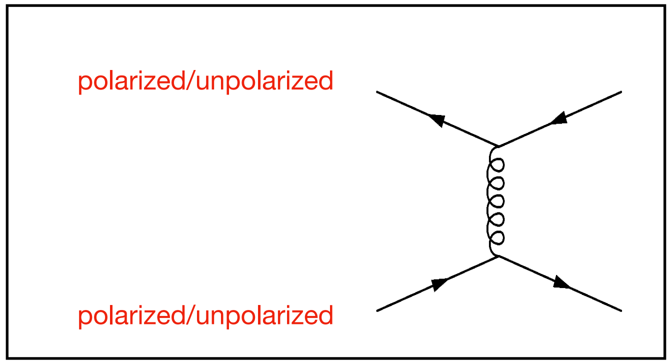
- **Collision terms**



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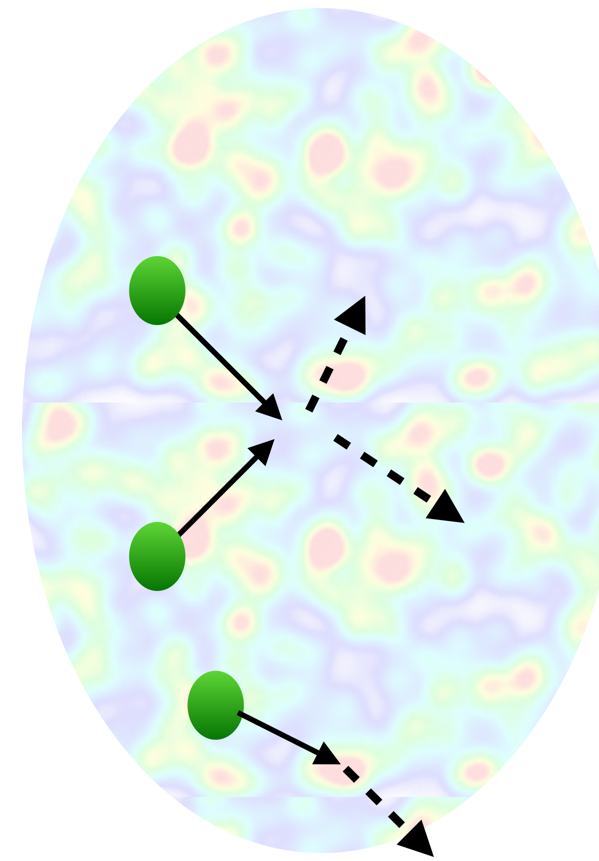

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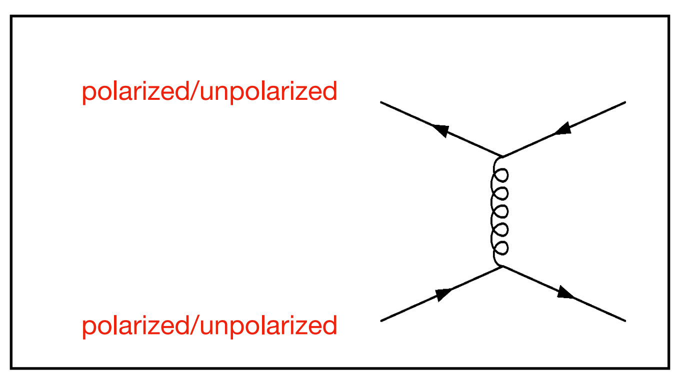

- **Simpler: (no spin) from world-lines: Bödeker's transport**

$$v^\mu D^{ab}_\mu[A]\, J^{b,i}(x,v) = -m_D^2 F^a_{j0}(x) v^j v^i + \xi^{a,i}_{(0)}(x,v)$$
$$+ N g^2 T \log\left(\frac{gT}{\mu}\right) \int \frac{d\Omega'_v}{4\pi}\, \tilde{I}^{ij}(v,v')\, J^{a,j}(x,v')$$

NM, R. Venugopalan in preparation

Summary

- **Worldline approach ab-initio:**
  Compute (!) kinetic theory from QFT
- **Closed Grassmann for internal symmetries**
- **Generalized Quantum Phase Space, measure, Wigner distribution, Liouville equation**
- **Chiral anomaly manifest**
- **May be useful to constrain anomalous hydrodynamics**

Backup: structure of phase space: color

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- **Color bilinears**

$$Q^a \equiv \lambda_c^\dagger t^a_{cd} \lambda_d$$

$$\{Q^a, Q^b\} = \lambda^\dagger [t^a, t^b] \lambda = i f^{abc} Q^c$$

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- **Color measure**

$$\int dQ = 0\,,$$

$$\int dQ\, Q^a = 0\,,$$

$$\int dQ\, Q^a Q^b = \frac{1}{2}\delta^{ab}\,,$$

$$\int dQ\, Q^a Q^b Q^c = \frac{A_R}{2} d^{abc}$$

$$f(x,P) \equiv \int dQ\, f(x,P,Q)\,,$$

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- **One unique form of phase space distribution**

$$f(x,P,Q) = f(x,P)\left[1 + \frac{2}{A_R d^2} d^{abc} Q^a Q^b Q^c\right] + 2 f^a(x,P) Q^a$$

$$d^2 \equiv d^{abc} d^{abc} = N_c^2 - 4$$

Backup: anomaly

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$$\langle J^{\mu}_{L/R}(x)\rangle \equiv e \int d^4P\, dS\, \epsilon\, [P^{\mu} + S^{\mu\nu}\partial_{\nu}]\, f(x,P,S)$$

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… is classically conserved. What is missing?

Backup: anomaly

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- **derivation from worldline SK path integral**

$$\Gamma[A,B] \equiv \mathrm{tr} \int d^4x_i^+ d^4x_i^- d^4\psi_i^+ d^4\psi_i^- \, \zeta^{A,B}(x_i^+, x_i^-, \psi_i^+, \psi_i^-) \int_{x_i^+}^{x_i^-} \mathcal{D}x\mathcal{D}p \int_{\psi_i^+}^{\psi_i^-} \mathcal{D}\psi \int \mathcal{D}\epsilon\mathcal{D}\chi e^{iS_{\mathcal{C}}[A,B]}$$

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variational axial-vector gauge field

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$$\Gamma[A,B] = \Gamma[A] + \int d^4y \, \frac{\delta\Gamma[A,B]}{\delta B_\mu(y)}\Big|_{B=0} B_\mu(y)$$

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$$+ \frac{\delta\zeta^{A,B}(x_i^+, x_i^-, \psi_i^+, \psi_i^-)}{\delta B_\mu(y)} \int_{x_i^+}^{x_i^-} \mathcal{D}x\mathcal{D}p \int_{\psi_i^+}^{\psi_i^-} \mathcal{D}\psi \int \mathcal{D}\epsilon\mathcal{D}\chi \, e^{iS_{\mathcal{C}}[A,B]}\Bigg]\Big|_{B=0}$$

initial density matrix (“spectrum”)

Backup: anomaly

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(*) by analytic continuation. We did not realize then it could be written in SK form / density matrix

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$$\zeta \equiv \zeta^{(0)} + \zeta^{(1)}$$

$$\zeta^{(0)} \equiv \begin{pmatrix} \zeta_R^A[x_i^+, x_i^-, \psi_i^+, \psi_i^-] & 0 \\ 0 & \zeta_L^A[x_i^+, x_i^-, \psi_i^+, \psi_i^-] \end{pmatrix}$$

$$\zeta^{(1)} \equiv 2\,\mathbb{I}_{2\times 2}\left[\partial_\mu B_\mu(\bar{x}_i) - \{\partial_\mu, B_\nu(\bar{x}_i)\}\bar{\psi}^\nu \bar{\psi}^\mu\right] \times \delta(x_i^+ - x_i^-)\,\delta(\psi_i^+ - \psi_i^-)\,,$$

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- **it gives the well known anomaly relation**

$$\langle \partial_\mu J_5^\mu(y) \rangle = -\frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}(y)$$

(*) by analytic continuation. We did not realize then it could be written in SK form / density matrix

Backup: anomaly and (in-)compressibility of semi-classical phase space

- Liouville's equation implies incompressibility of (semi-classical) phase space

$$0 = \frac{d}{d\tau} W(x, P, \psi, \lambda, \lambda^\dagger) = \Big(\dot{x}_\mu \frac{\partial}{\partial \bar{x}_\mu} + \dot{P}_\mu \frac{\partial}{\partial P} + \dot{\psi}_\mu \frac{\partial}{\partial \psi_\mu} + \dot{\lambda}_a \frac{\partial}{\partial \lambda_a} + \dot{\lambda}^\dagger_a \frac{\partial}{\partial \lambda^\dagger_a}\Big) W(x, P, \psi, \lambda, \lambda^\dagger)$$

- canonical phase space variables: phase space incompressible at this order (reverse not true)

- higher orders: Moyal equation, quantum phase space compressible

$$\frac{dW^\chi_A}{d\tau} = -2 H_W \sin\left[\frac{\Lambda}{2}\right] W^\chi_A = \{W^\chi_A, H_W\} + O(\hbar^2)$$

- compressibility on semi-classical level: understand as Jacobian to semi-classical phase space measure

Does this have to do anything with the anomaly?

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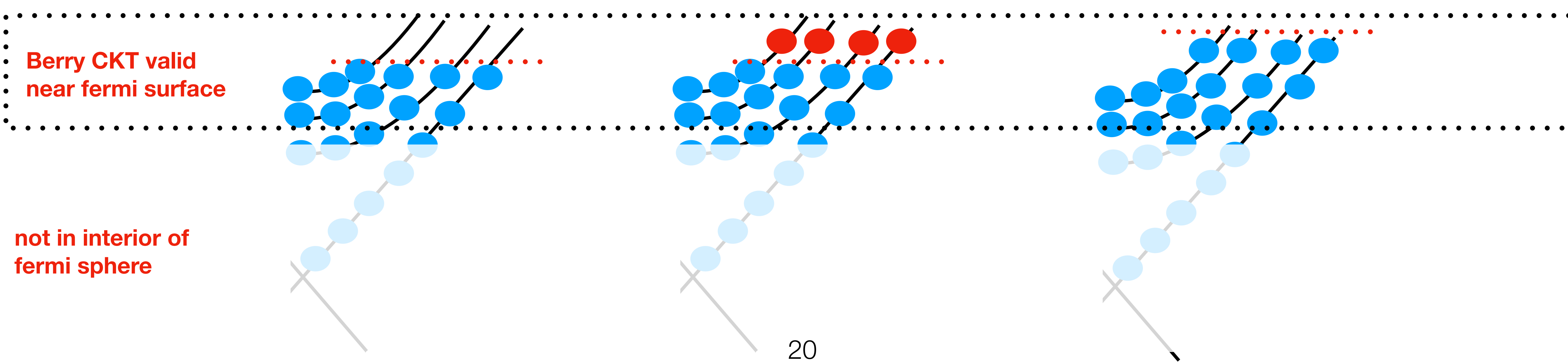

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- **Xiao, Shi, Niu make this semi-classical effective theory "many body"**

$$\dot{\mathbf{x}} = \frac{1}{\hbar}\frac{\epsilon_n(\mathbf{p})}{\partial \mathbf{p}} - \dot{\mathbf{k}} \times \boldsymbol{\Omega}_n(\mathbf{p}),$$
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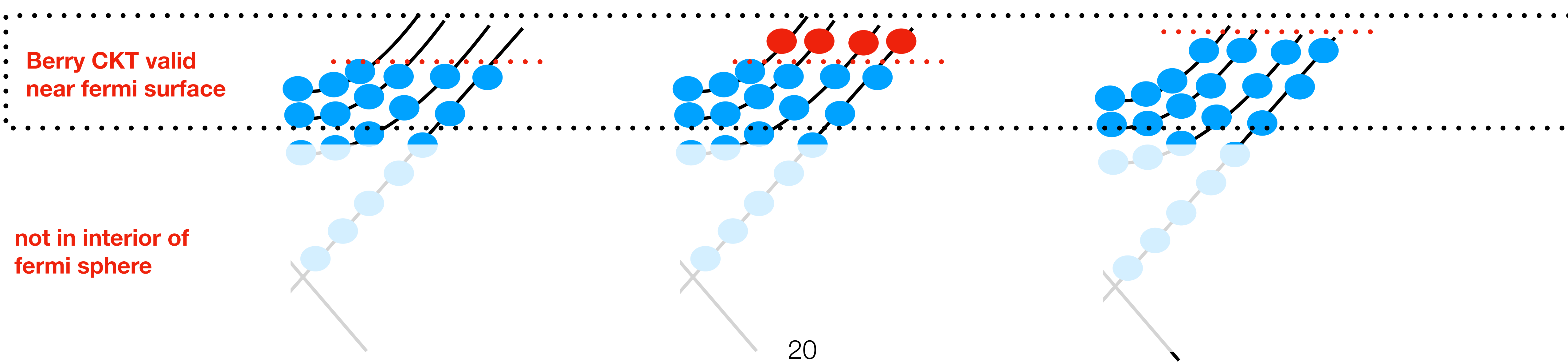

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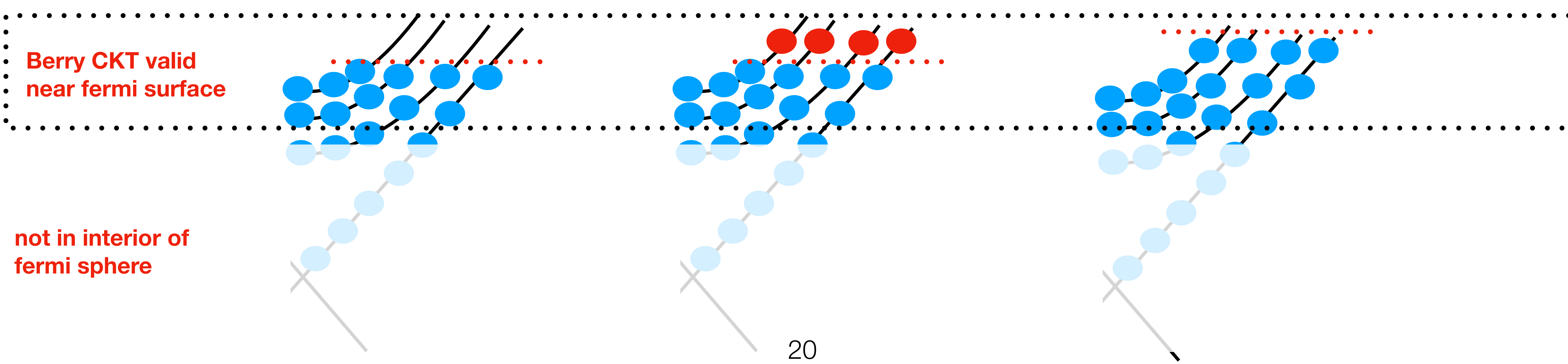
$$\Delta V \equiv \frac{\Delta V_0}{1 + e\mathbf{B}\cdot\mathbf{\Omega}}$$



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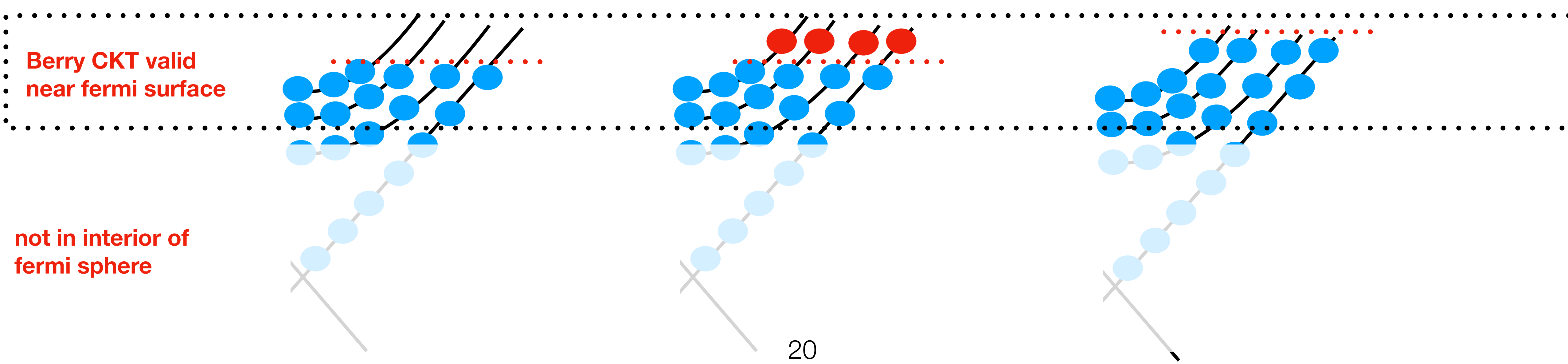
$$n_e = \int^{\mu} \frac{d^3p}{(2\pi)^3}\left[1 + \frac{e\mathbf{B}\cdot\mathbf{\Omega}}{\hbar}\right]$$



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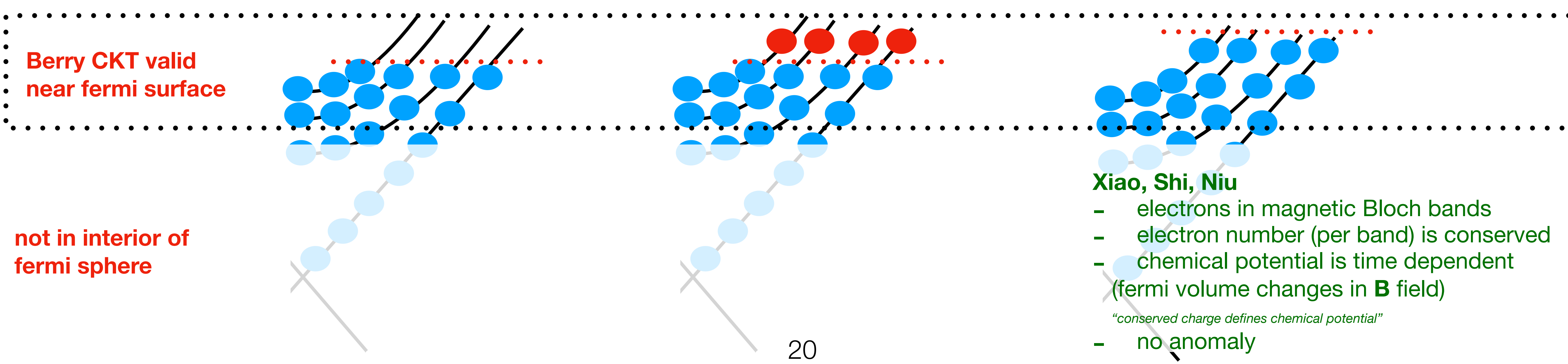

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