



Chiral crossover and phase transition in (2+1)-flavor QCD

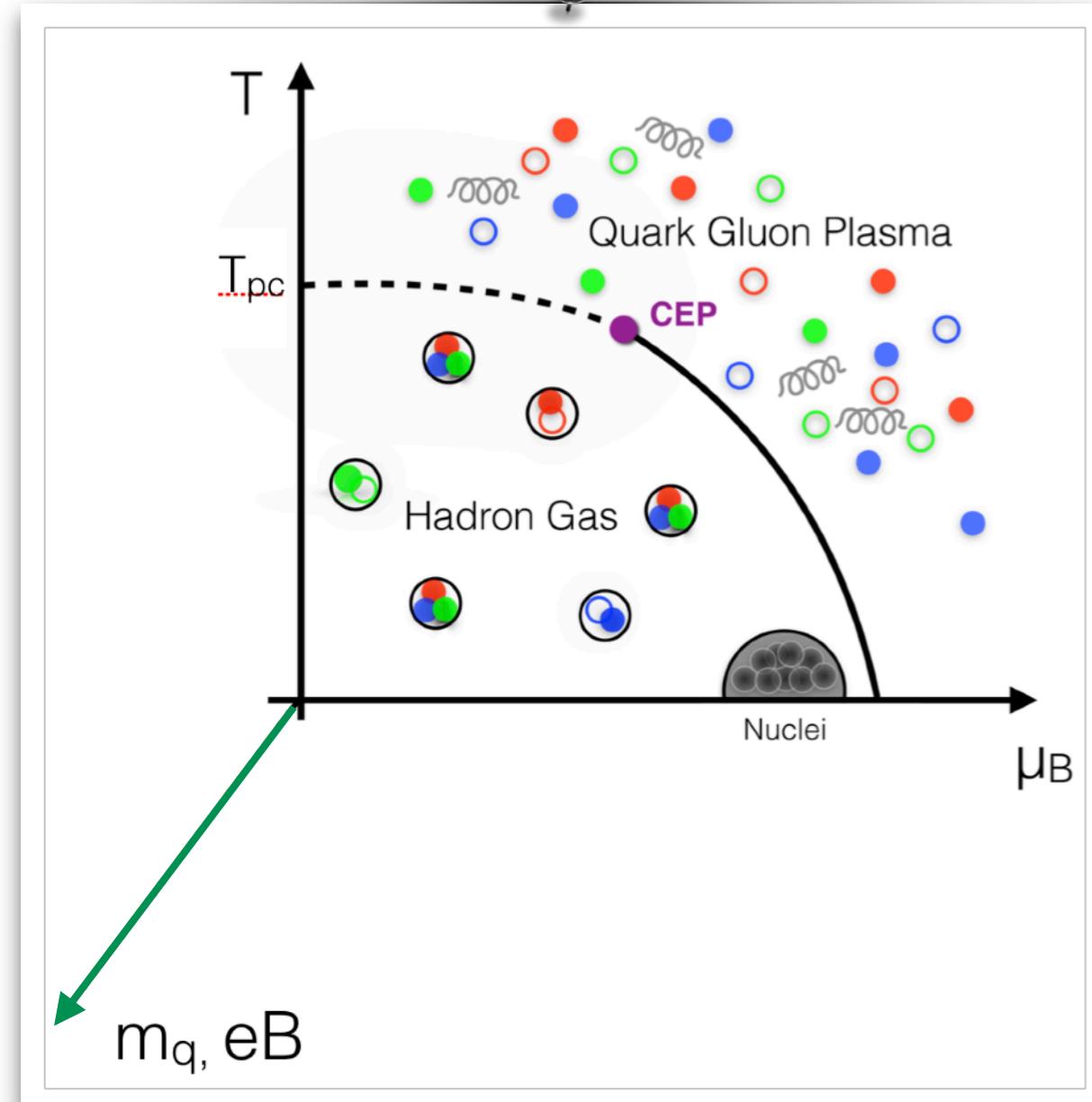
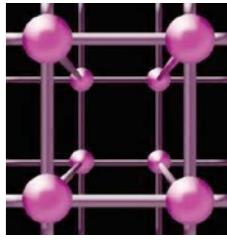
Heng-Tong Ding (丁亨通)

Central China Normal University

The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

8-12 April, 2019 @ Tsinghua University

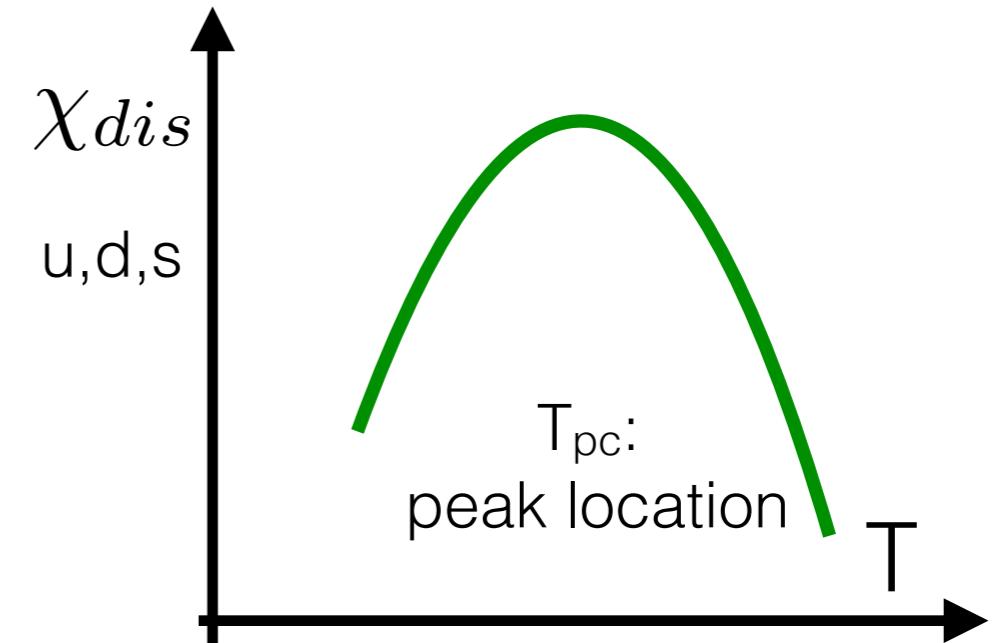
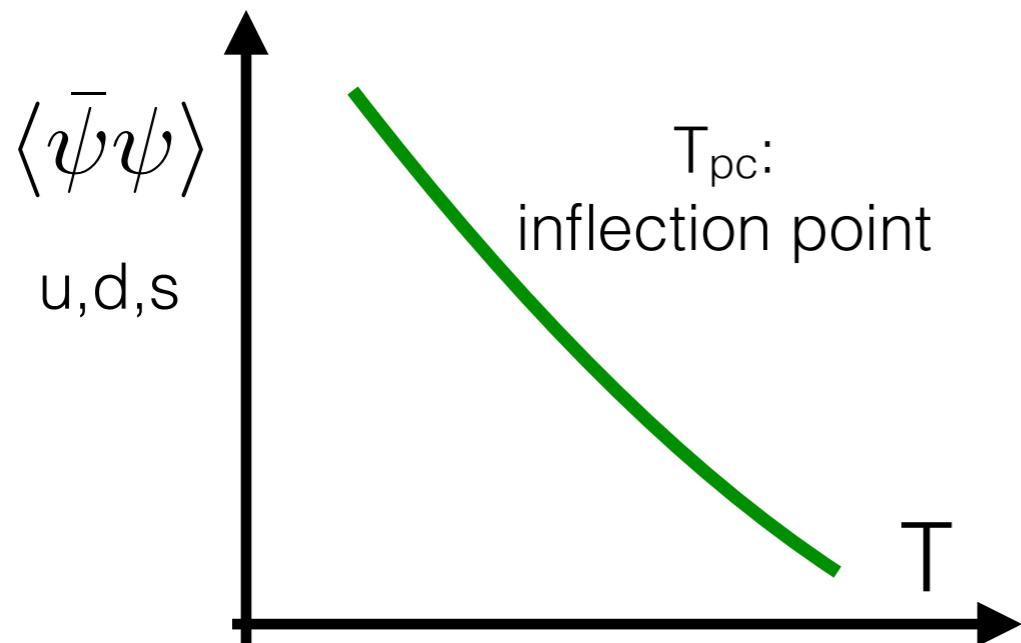
Outline: QCD phase structure



- Chiral crossover at zero and small μ_B
A. Bazavov, HTD, P. Hegde et al.
[HotQCD], arXiv:1812.08235
- Chiral phase transition temperature
HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801
- QCD transition in external B
Xiao-Dan Wang (汪晓丹) et al.,
work in progress & arXiv: 1904.01276

Crossover transition temperature T_{pc} in the real world

- Crossover nature of the transition



- Chiral phase transition: most likely 2nd order, 3d O(4)

Ejiri et al., PRD 80(2009)094505,
HotQCD, arXiv:1903.04801

...

- A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T, \mu_B) \sim m^{1/\delta-1} f_\chi$

Scaling behavior of chiral observables

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Taylor expansions:

$$\Sigma(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\Sigma(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

$$\chi(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\chi(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

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$$\begin{aligned} \partial_T \chi^\Sigma(T) \\ \partial_T C_0^\chi(T) \\ C_2^\chi(T) \end{aligned}$$

$$\sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\begin{aligned} \partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T) \end{aligned}$$

$$\sim m^{1/\delta-2/\beta\delta} f''_G(z)$$

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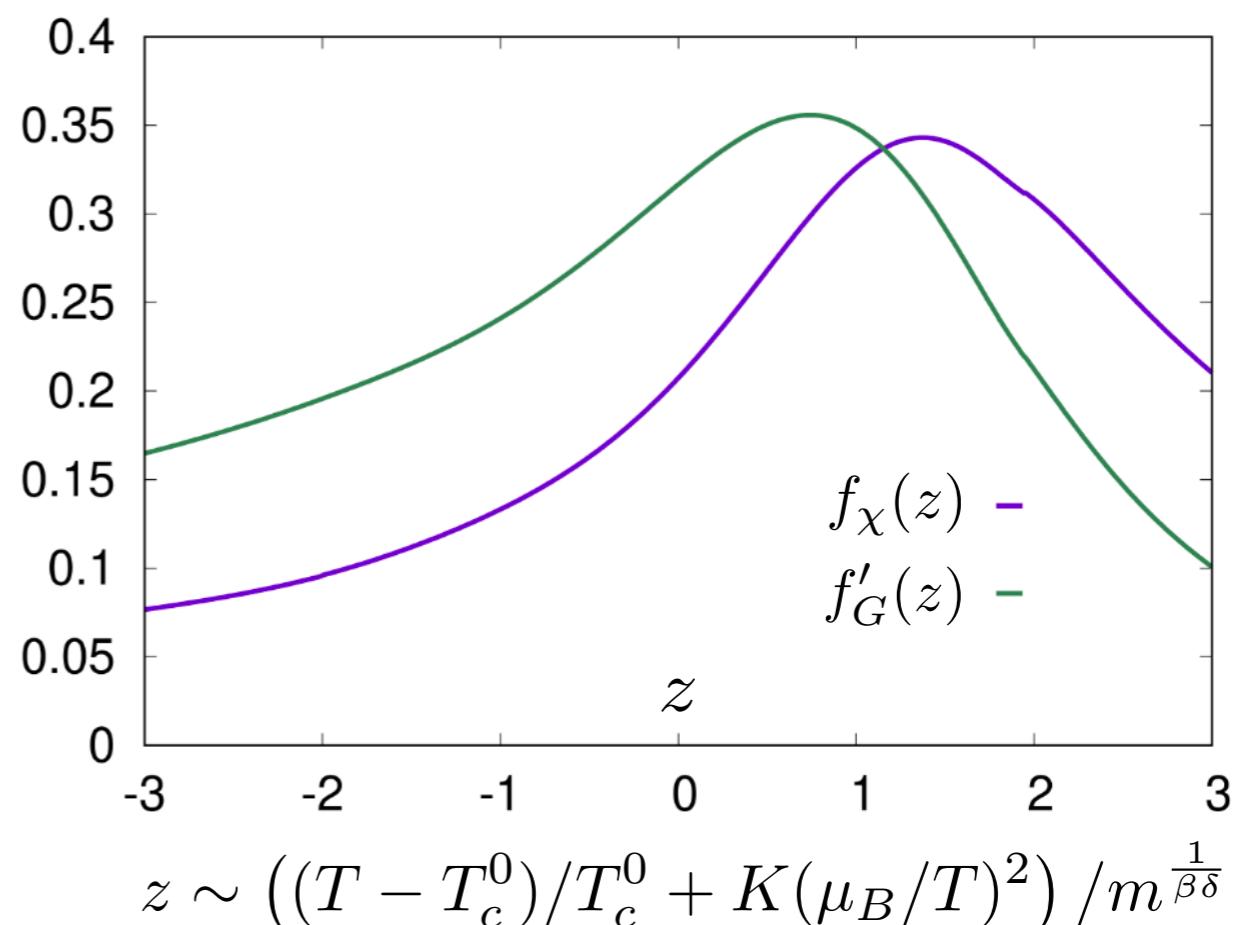
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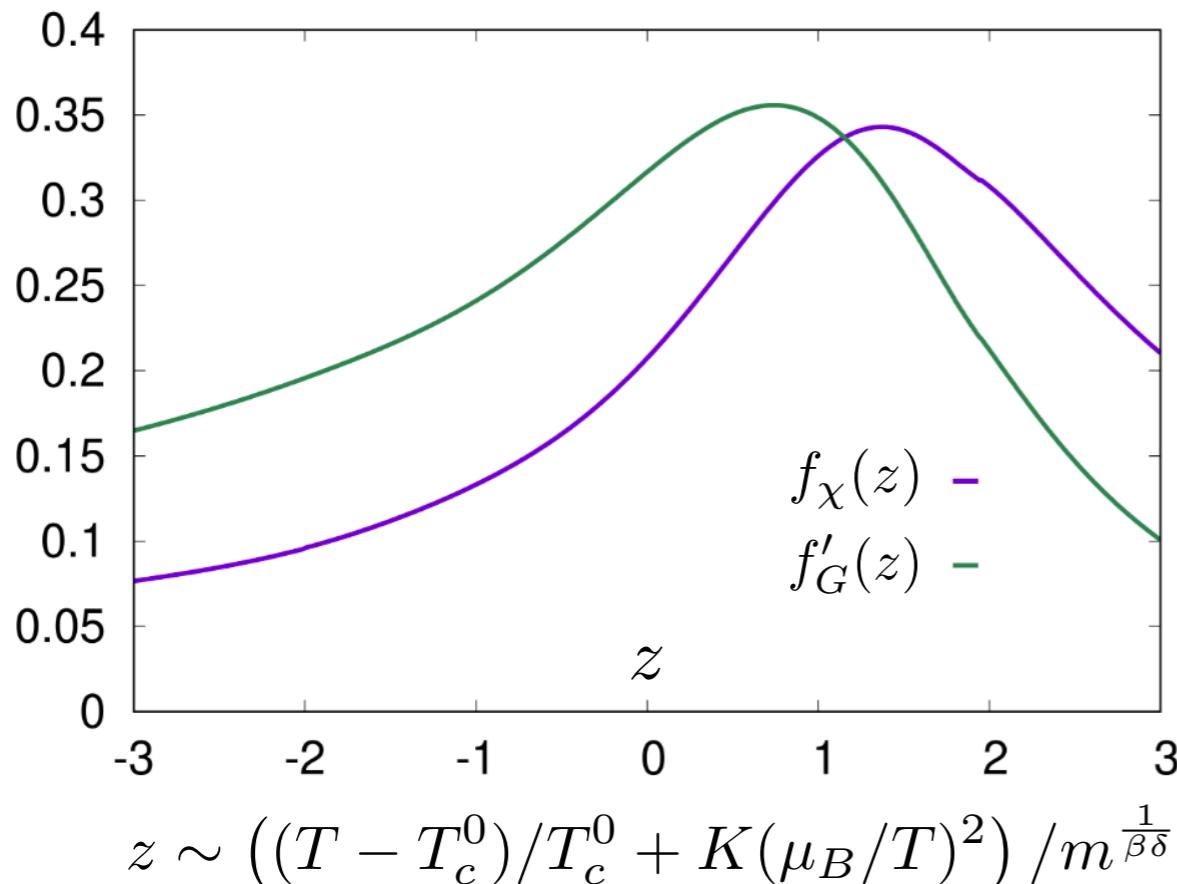
$$\sim m^{1/\delta-1-\beta\delta} f'_\chi(z)$$

$$\begin{aligned} \partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T) \end{aligned}$$

$$\sim m^{1/\delta-2/\beta\delta} f''_G(z)$$



Well-defined notation of chiral crossover transition temperature



$$\begin{aligned}\partial_T \chi^\Sigma(T) \\ \partial_T C_0^\chi(T) \\ C_2^\chi(T)\end{aligned}$$

$$\sim m^{1/\delta - 1 - 1/\beta\delta} f'_\chi(z)$$

$$\begin{aligned}\partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T)\end{aligned}$$

$$\sim m^{1/\delta - 2/\beta\delta} f''_G(z)$$

- 5 conditions to extract T_c : maxima of f_χ and f'_G

$$\partial_T \chi^\Sigma(T) = 0$$

$$\partial_T C_0^\chi(T) = 0$$

$$C_2^\chi(T) = 0$$

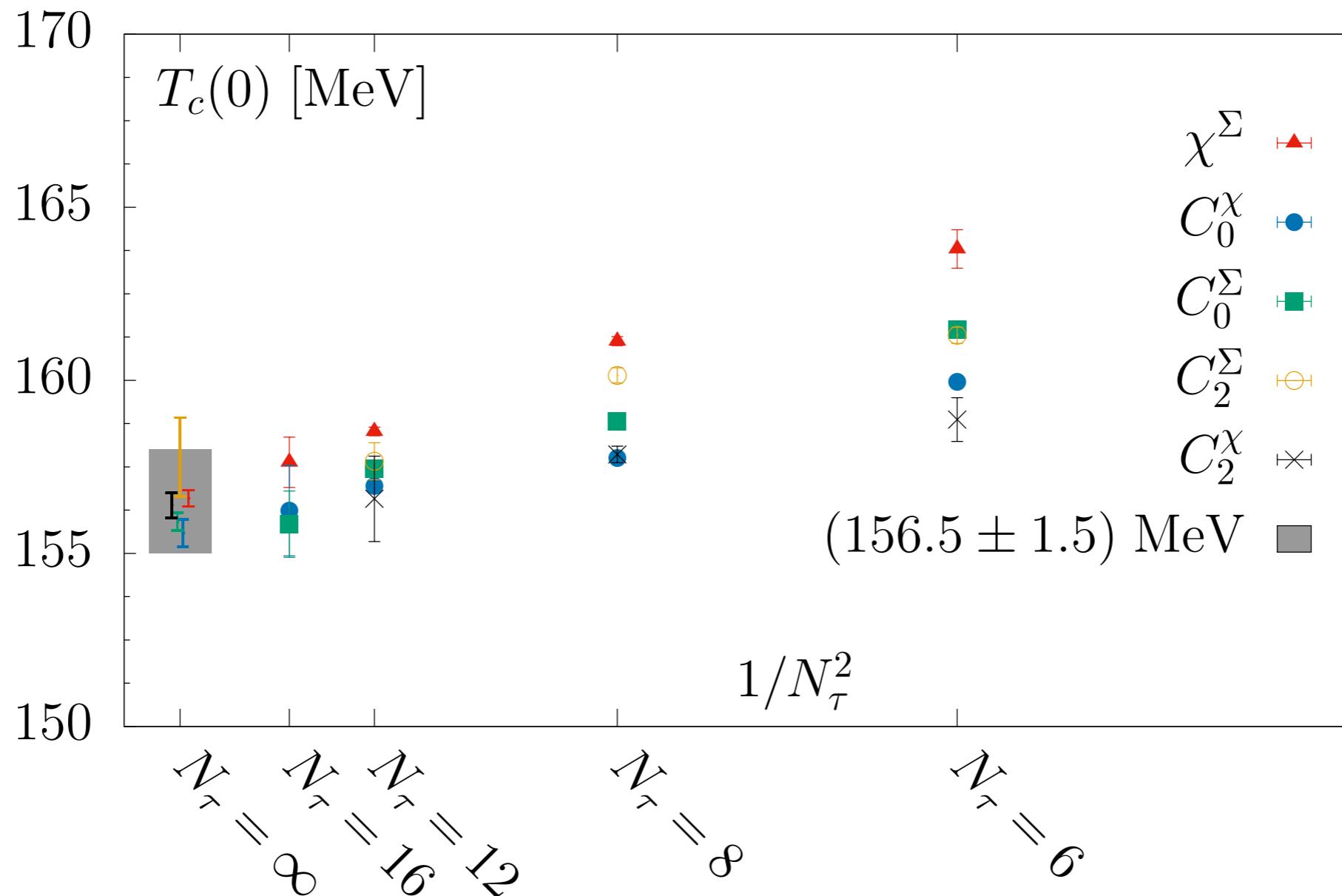
$$\partial_T^2 C_0^\Sigma(T) = 0$$

$$\partial_T C_2^\Sigma(T) = 0$$

- $m=0$: all these susceptibilities diverge at a unique T

- $m \neq 0$: non-unique temperatures, crossover

QCD transition with $m_\pi = 140$ MeV at $\mu_B=0$

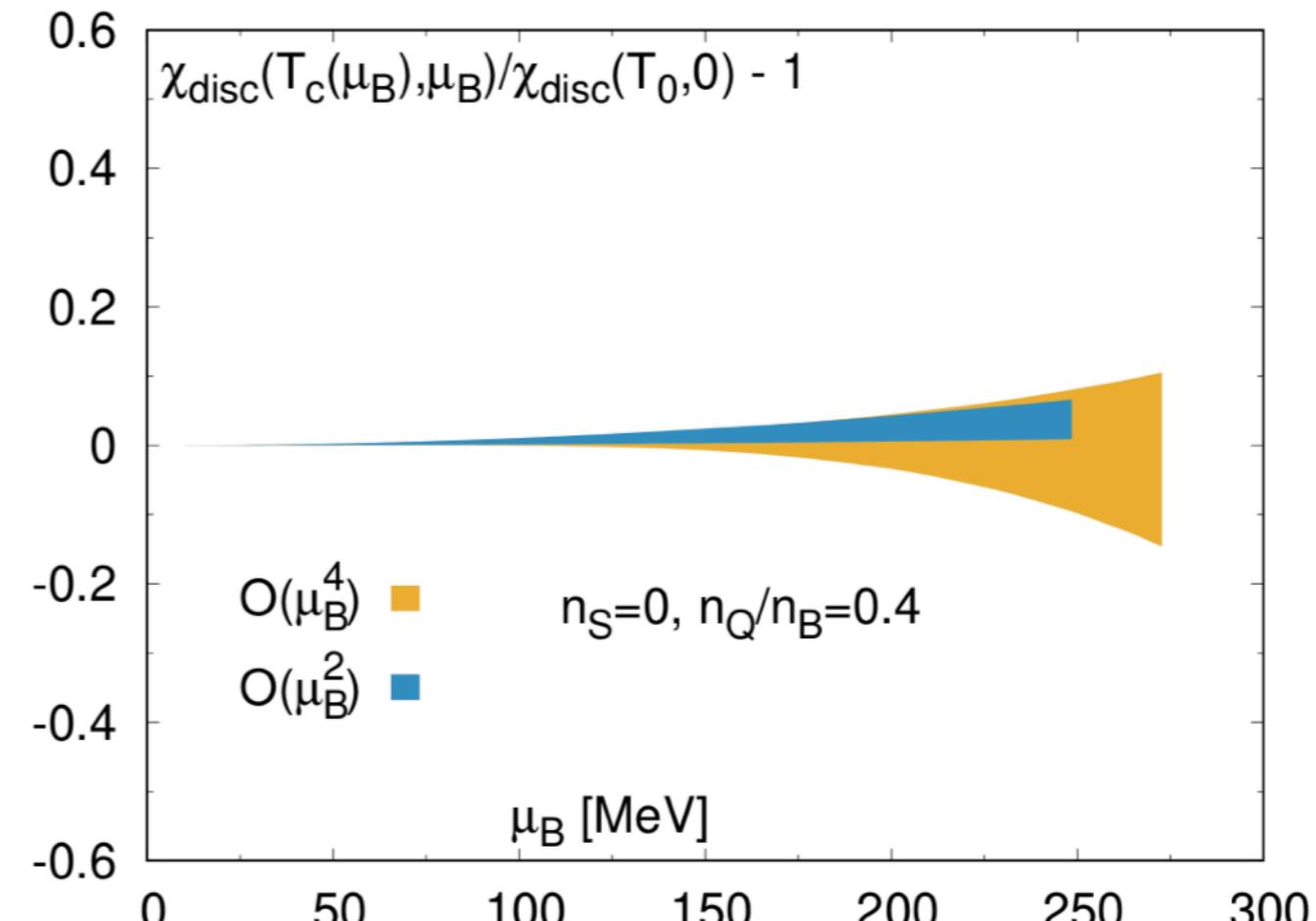
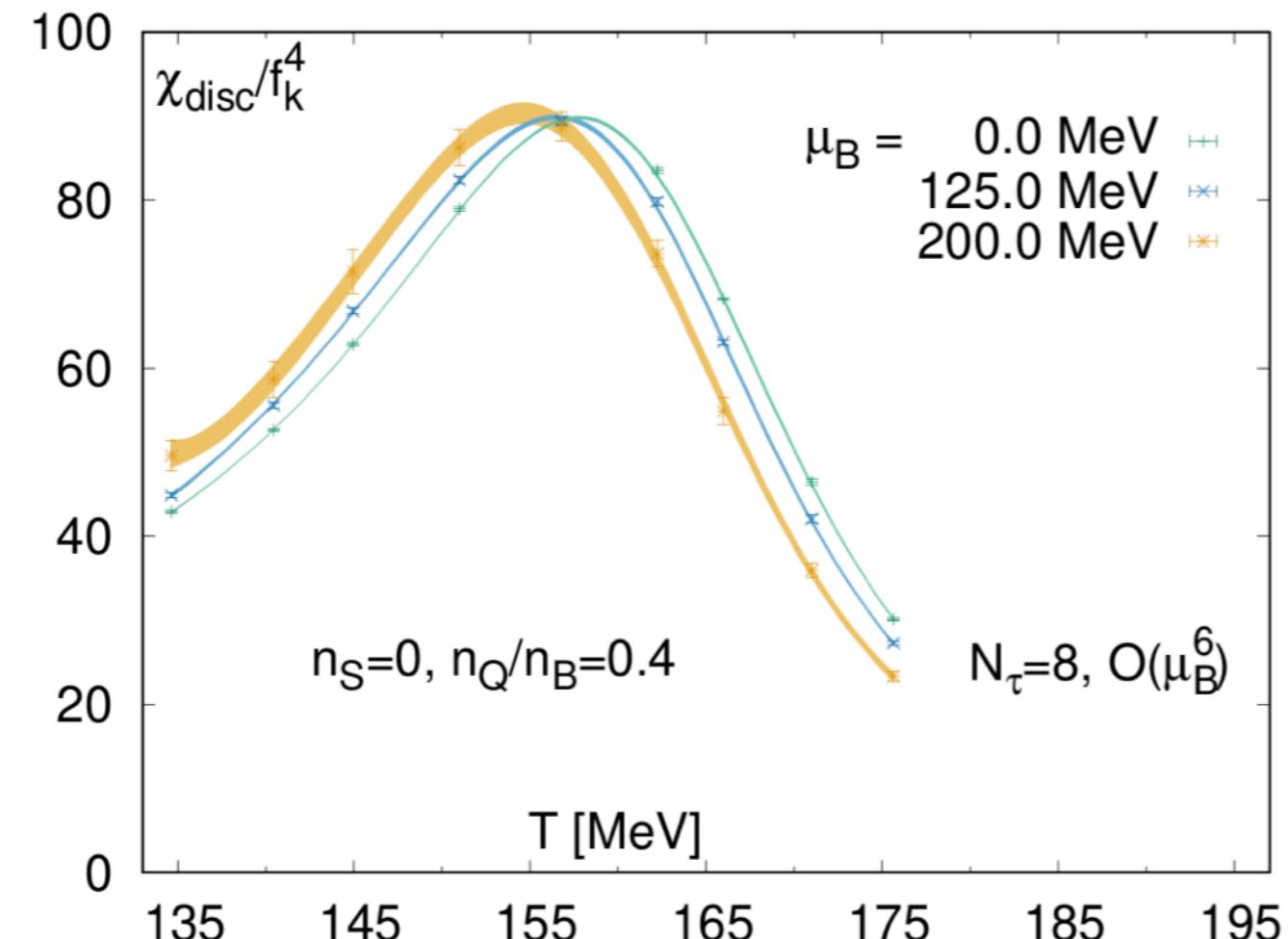


Higher precision in the continuum limit:

T_{pc} = 156.5(1.5) MeV

HotQCD, arXiv:1812.08235

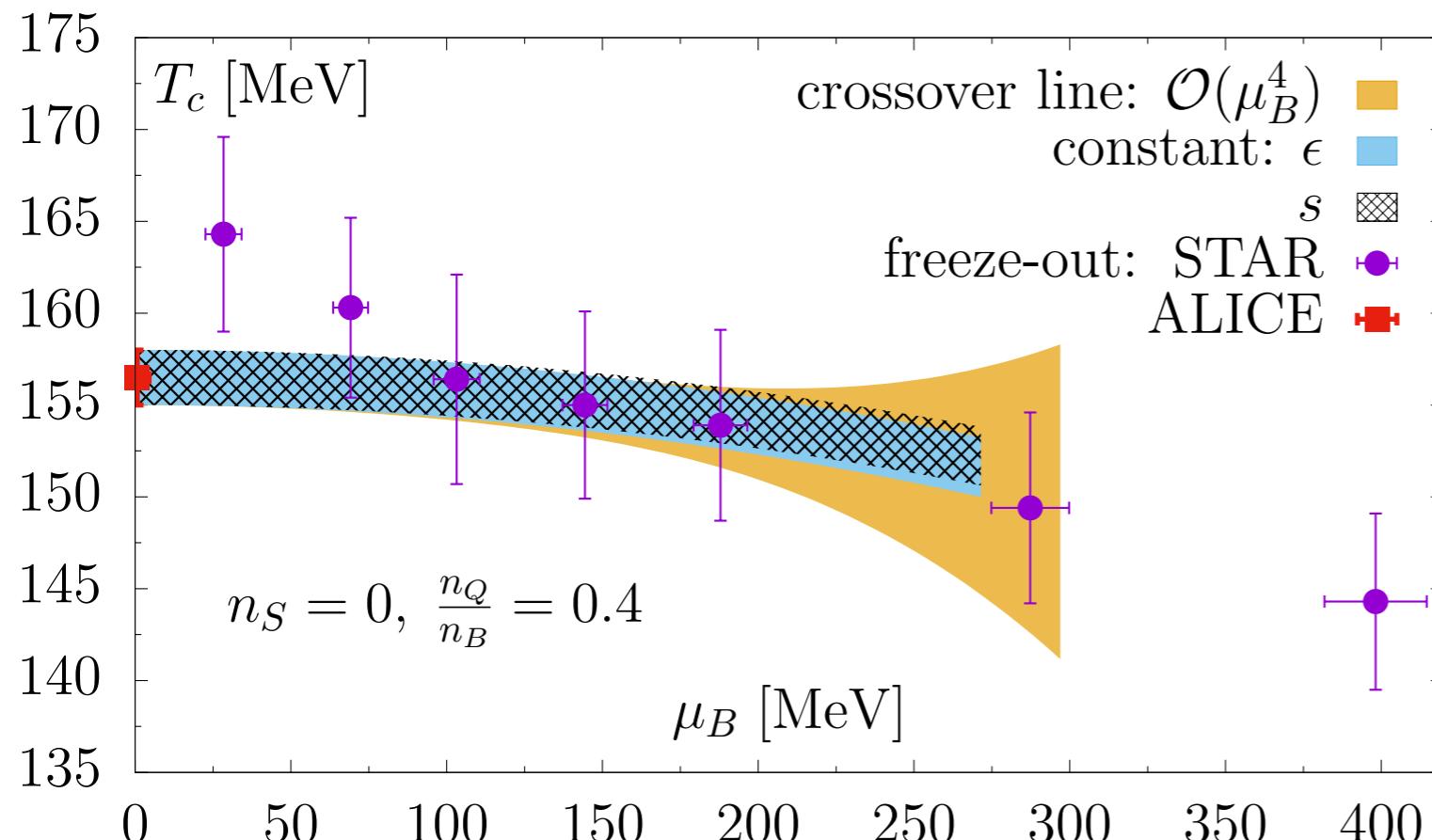
Order Parameter Susceptibility at $\mu_B=0$



No indication of a stronger phase transition at larger μ_B

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0) \left(1 - \kappa_2 \left(\frac{\mu_B}{T} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T} \right)^4 + \mathcal{O} \left(\frac{\mu_B}{T} \right)^6 \right)$$



A. Bazaovo, HTD et al. [HotQCD], arXiv:1812.08235

curvature of crossover line

$$\kappa_2 = 0.0123 \pm 0.003$$

$$\kappa_4 = 0.000131 \pm 0.0041$$

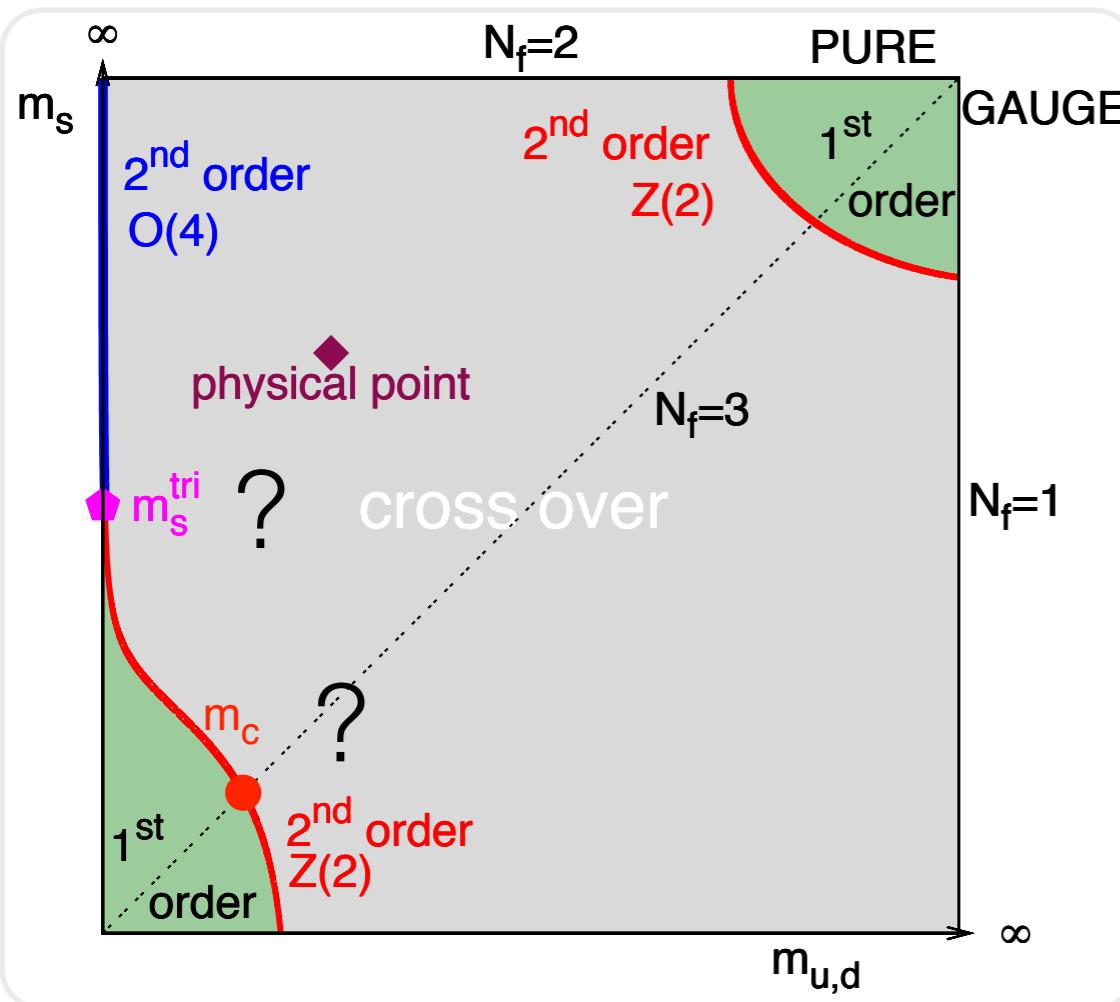
curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

QCD phase diagram in the quark mass plane

Columbia plot:

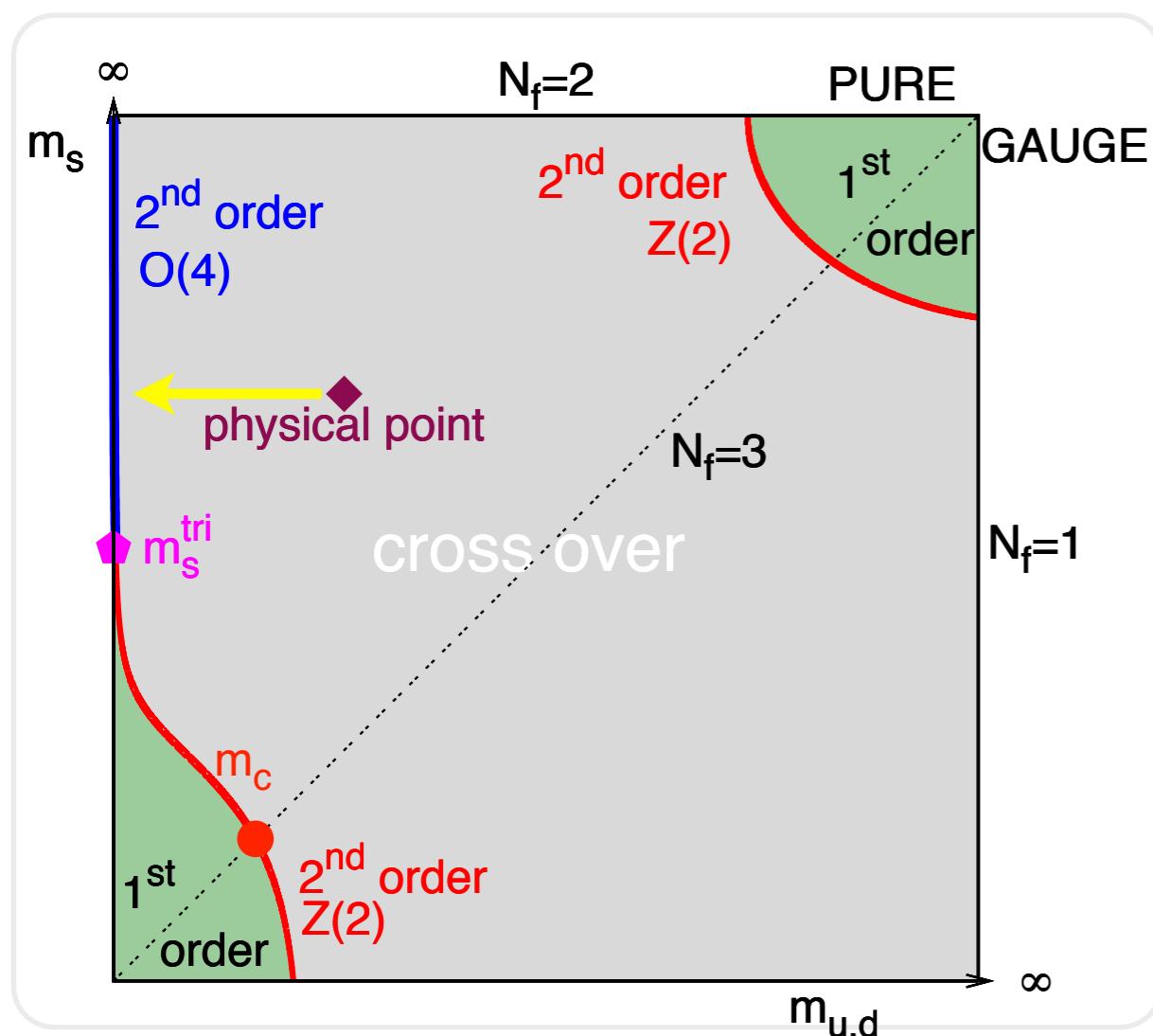


- At physical point: cross over,
 $T_{pc} = 156.5(1.5)$ MeV
HotQCD, arXiv:1812.08235
- $N_f=2(+1)$: $U_A(1)$ remains broken at $T_{\chi SB}$
JLQCD '13, '14, '15, HotQCD '13, '14
- Critical lines of second order transition
 - $N_f=2$: O(4) universality class Pisarski & Wilczek PRD '84, Kogut & Sinclair, PRD '06
 - $N_f=3$: Ising universality class Karsch, Laermann, Schmidt PLB '04, ...

Towards the chiral limit:

- $N_f=2+1$ QCD: m_s^{tri} ? m_s^{phy}
- Fundamental scale of QCD: chiral T_c ?

Towards chiral limit of (2+1)-flavor QCD



- HISQ/tree action

- **N_f=2+1:**

$$m_u = m_d \rightarrow 0$$

$$m_s = m_s^{\text{phy}}$$

$N_t = 6, 8, 12$

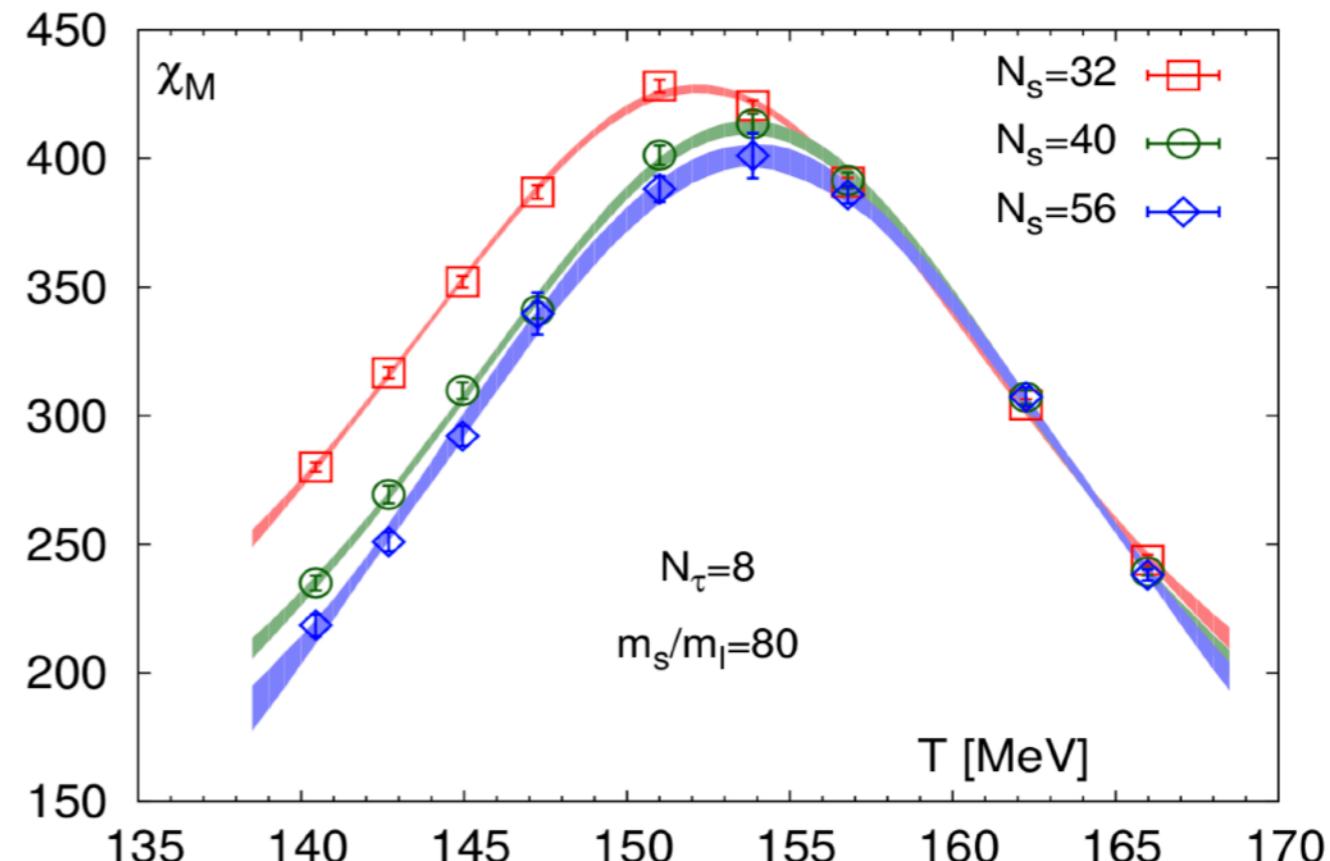
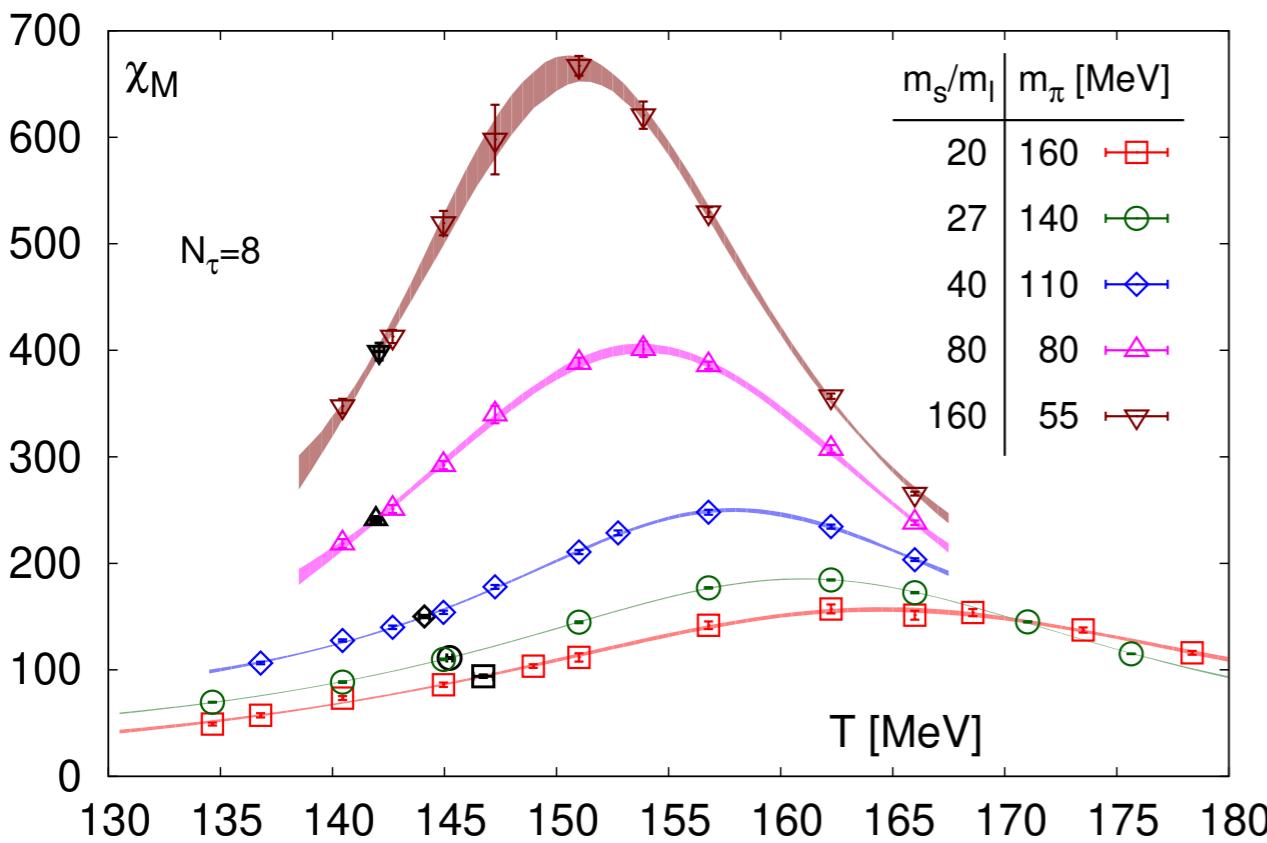
$m_s^{\text{phy}} / m_l = 20, 27, 40, 60, 80, 160$

$$m_\pi \approx 160, 140, 110, 90, 80, 55 \text{ MeV}$$

$7 \geq N_s/N_t \geq 4 \Leftrightarrow 5 \gtrsim m_\pi L \gtrsim 3$

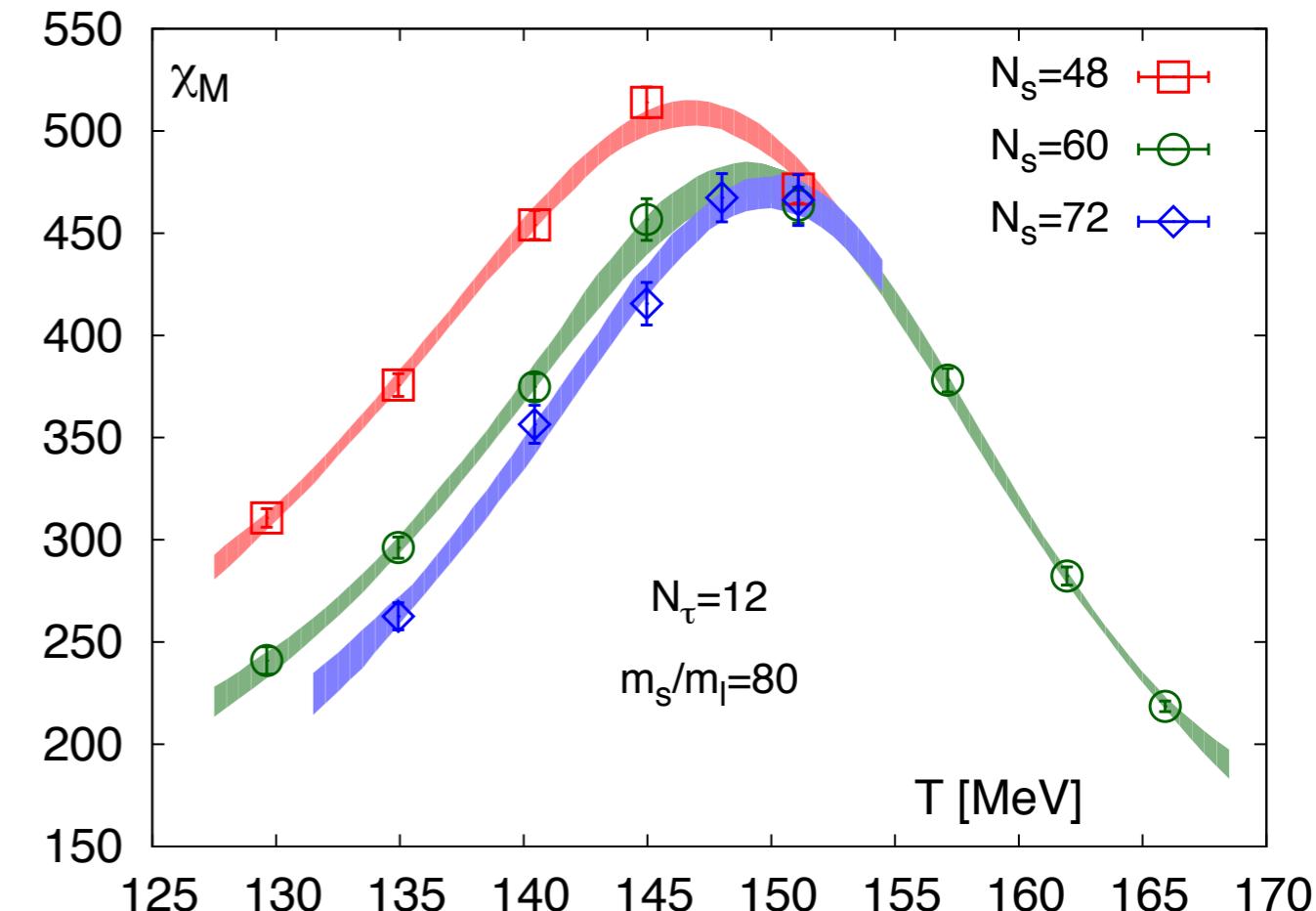
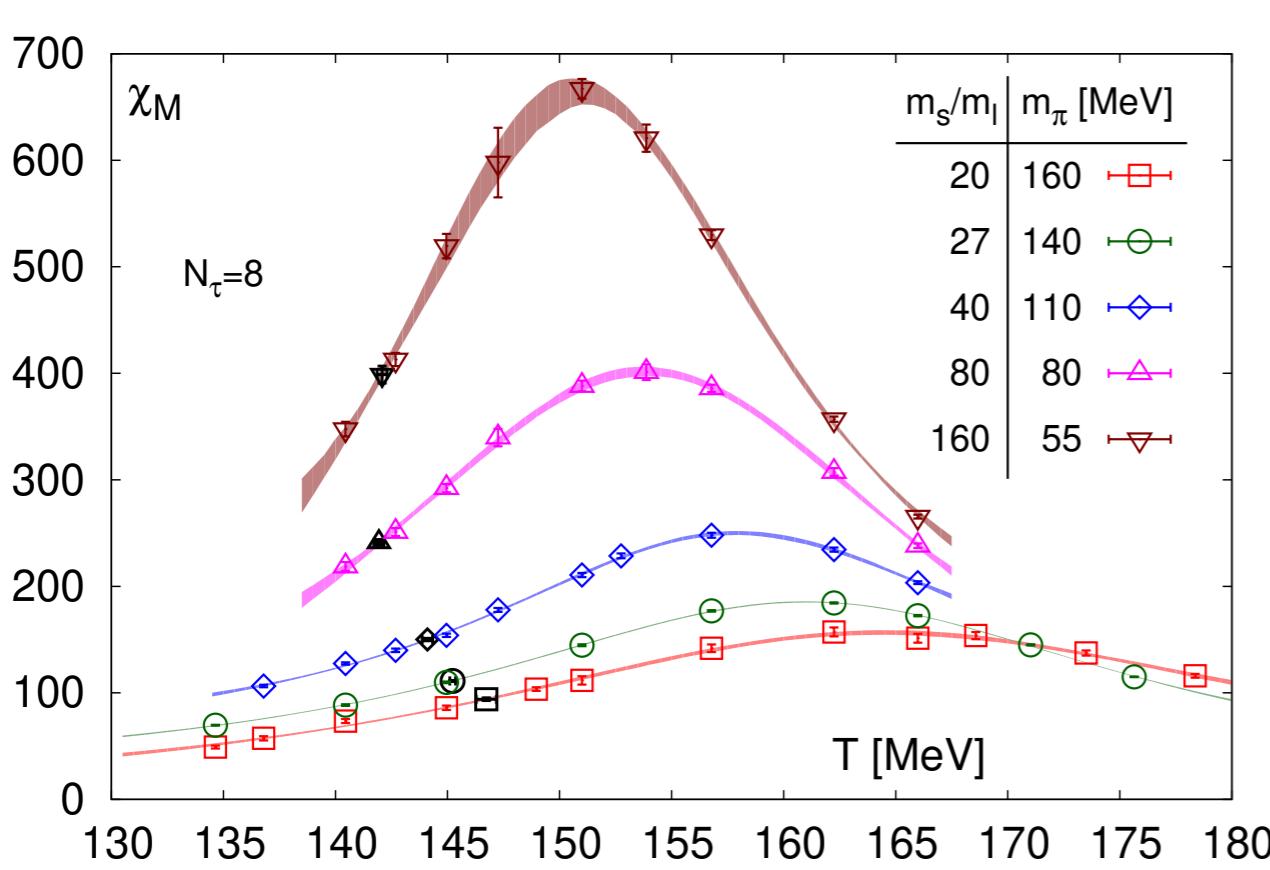
This allows us to perform infinite volume, continuum and then chiral extrapolation!

Quark mass and volume dependences of chiral susceptibility



- ✿ Susceptibility increases as $m_l^{1/\delta-1} + \text{const}$, here $\delta \approx 4.8$
- ✿ Peak height of susceptibility slightly changes with Volume
- ✿ Consistent with a continuous phase transition with $O(N)$ universality class in the chiral limit of m_l

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chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m, T) = h^{1+1/\delta} f_s(z), \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

$f_s(z)$: universal scaling function, $O(N)$ etc.

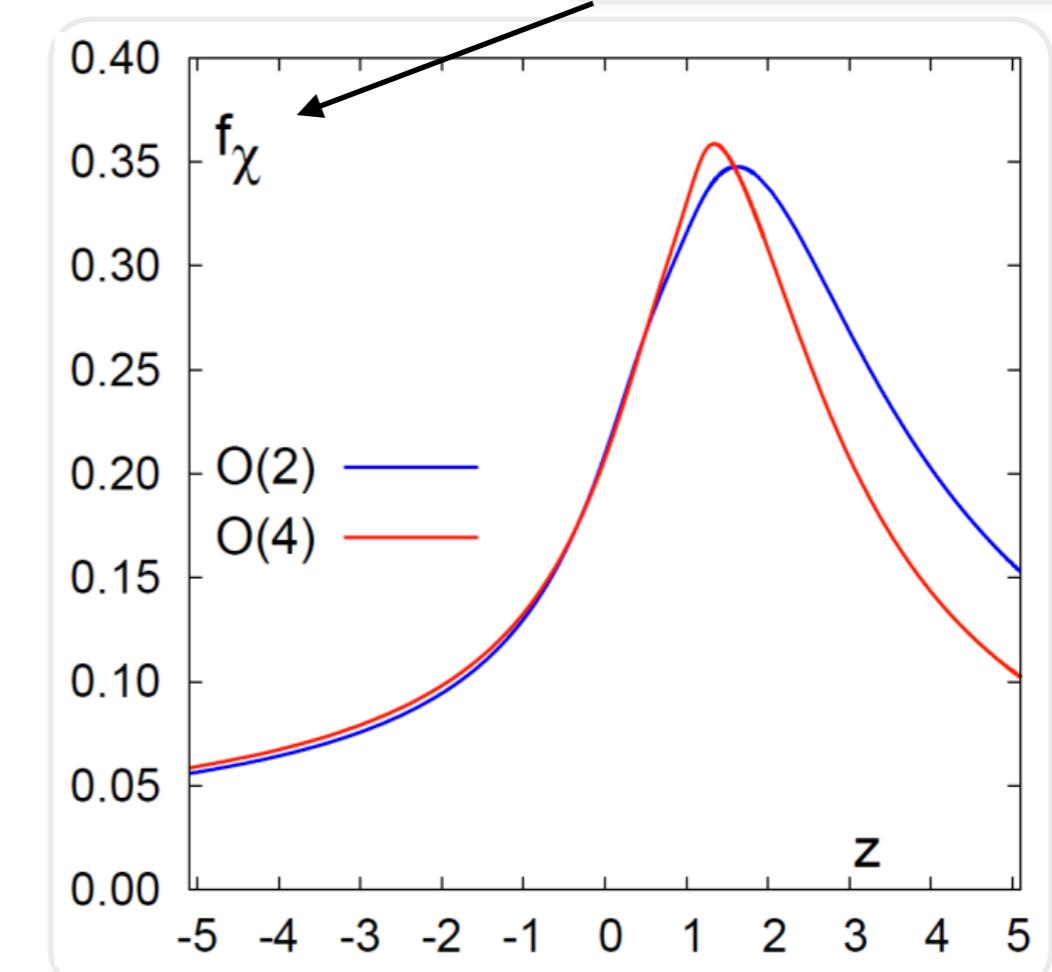
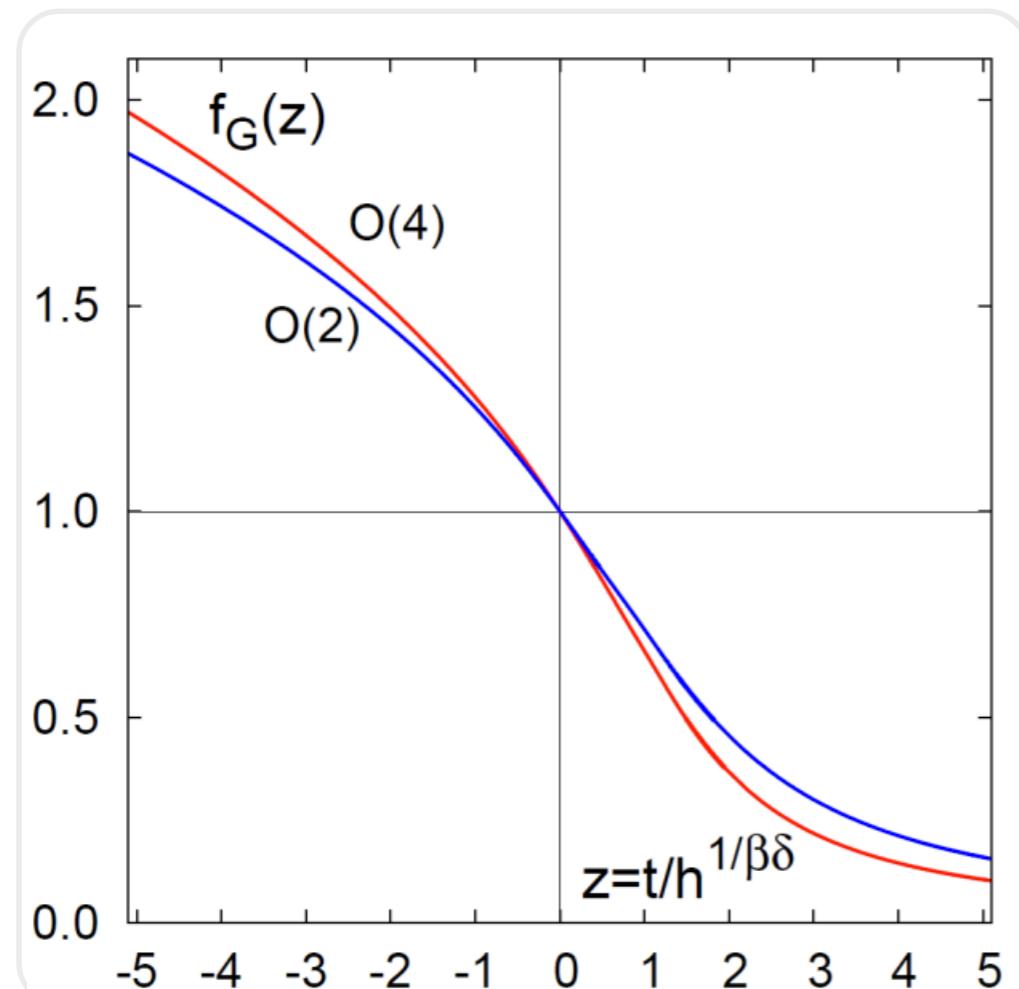
$$h = \frac{1}{h_0} \frac{m_l}{m_s}$$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t, h)/\partial H = h^{1/\delta} f_G(z)$$

$$\chi_M = \partial M / \partial H = \frac{h^{1/\delta}}{H} \frac{1}{\delta} \left(f_G(z) - \frac{z}{\beta} \frac{df_G(z)}{dz} \right)$$



Chiral phase transition temperature T_c^0

$$M = -\partial f_s(t, h)/\partial H = h^{1/\delta} f_G(z)$$

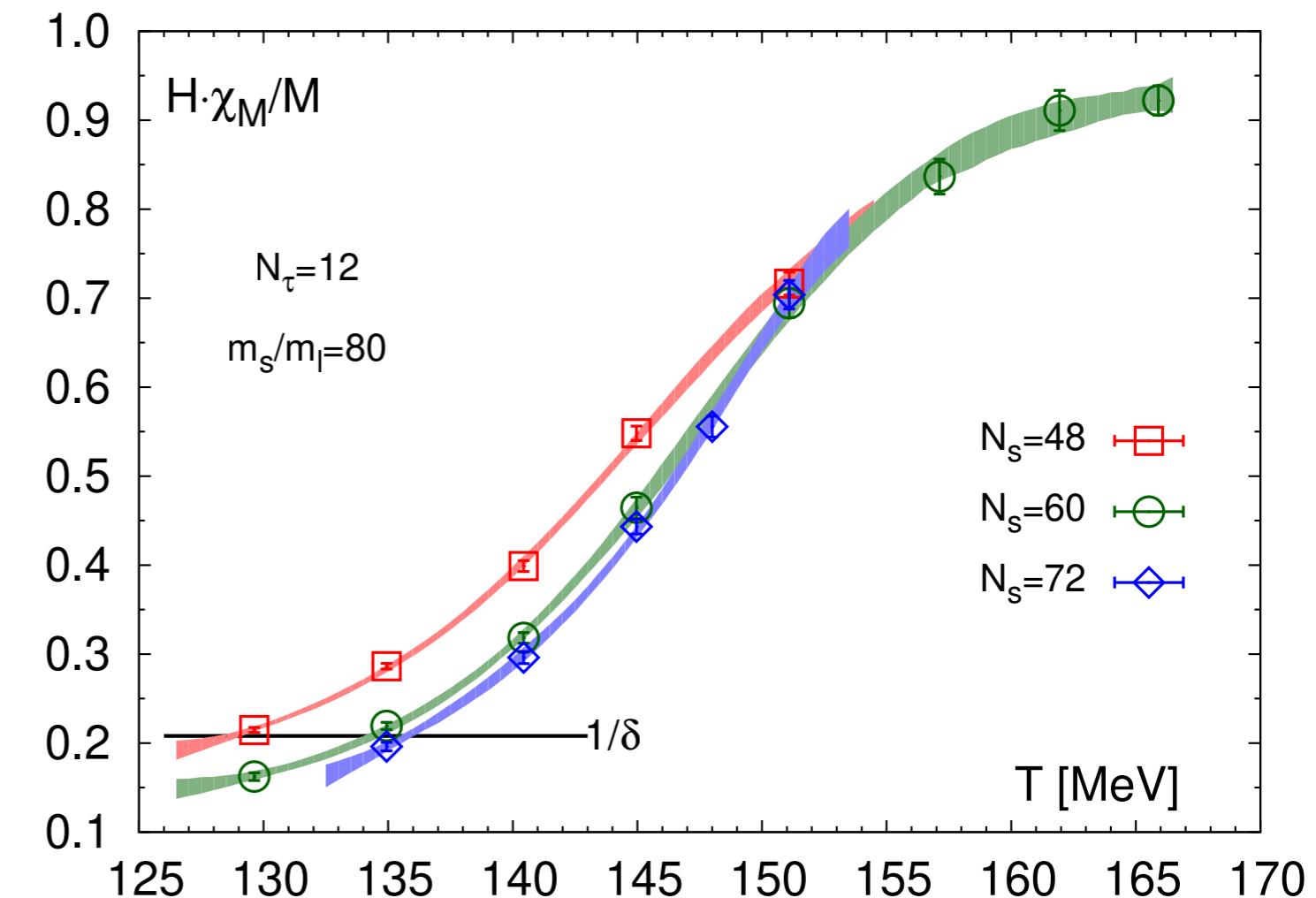
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$H \chi_M / M \rightarrow 1/\delta @ T_c^0$

$H: m_l/m_s$

$M: \text{chiral condensate}$

$\chi_M: \text{chiral susceptibility}$



A novel approach to estimate T_c^0

- Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}} \right)$$

$$z = \frac{1}{t_0} \frac{T - T_c^0}{T_c^0} \left(\frac{H}{h_0} \right)^{-1/\beta\delta} = z_0 \frac{T - T_c^0}{T_c^0} H^{-1/\beta\delta}$$

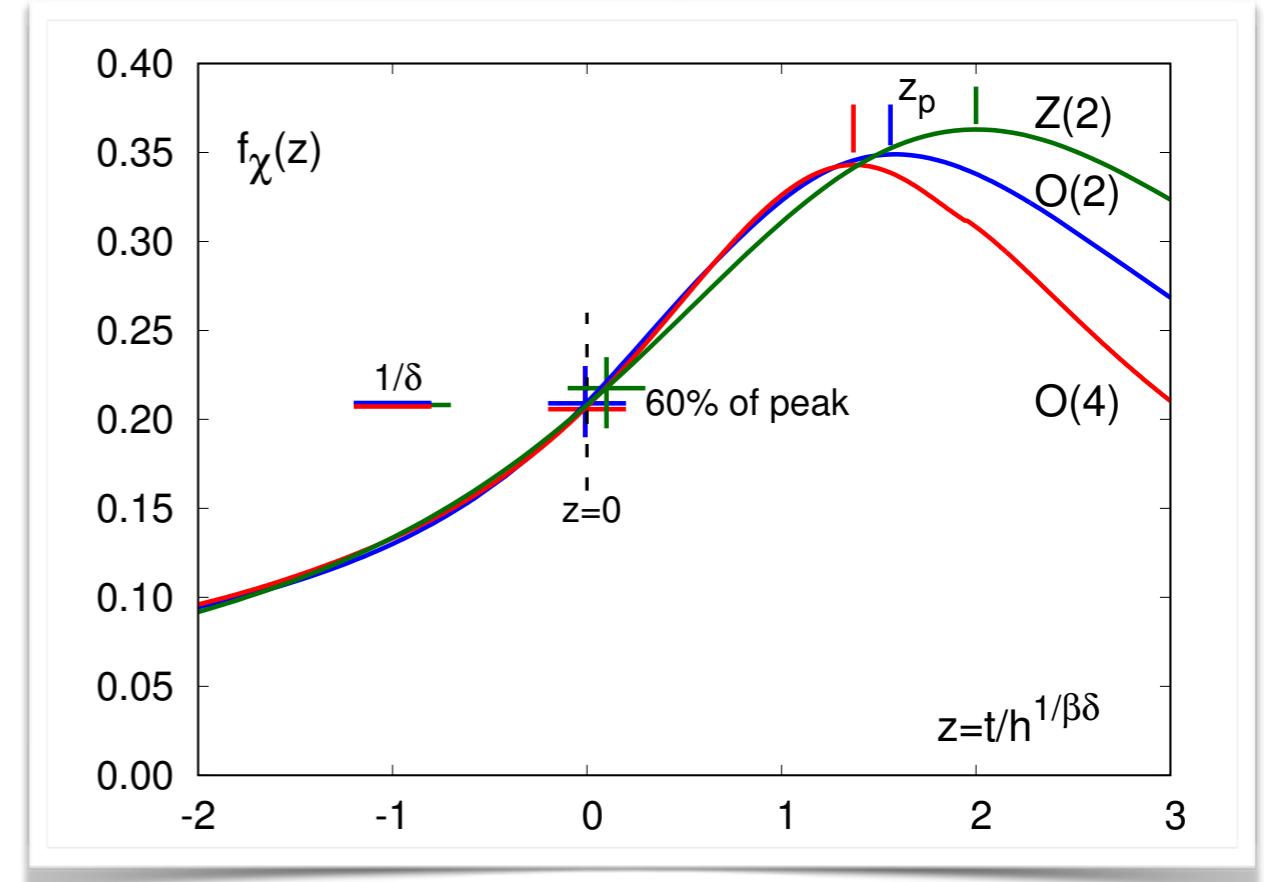
- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(\textcolor{violet}{T}_\delta, H, L)}{M(\textcolor{violet}{T}_\delta, H, L)} = \frac{1}{\delta} \quad \longleftrightarrow \quad z(T_\delta) = 0$$

$$\chi_M(\textcolor{blue}{T}_{60}, H) = 0.6\chi_M^{max} \quad \longleftrightarrow \quad z(T_{60}) \approx 0$$

small quark mass dependence

small variations among universality classes



z_p : peak location of the susceptibility

z_{60} : location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

Things need to be taken care of

- Thermodynamic limit
- Continuum limit
- Chiral limit

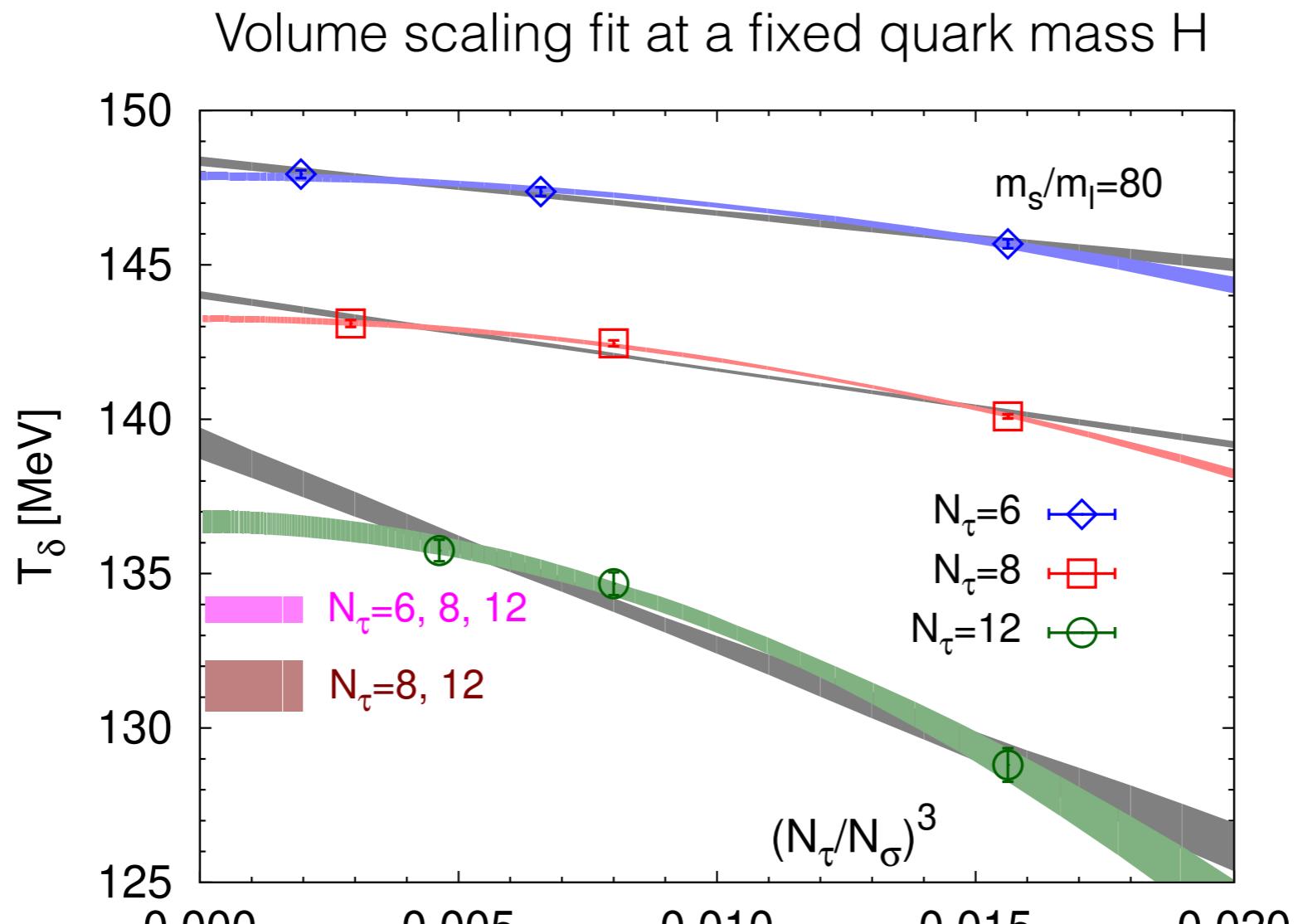
$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0} \right) H^{1/\beta\delta} \right) + c_X H^{1-1/\delta+1/\beta\delta}$$

Singular

Regular

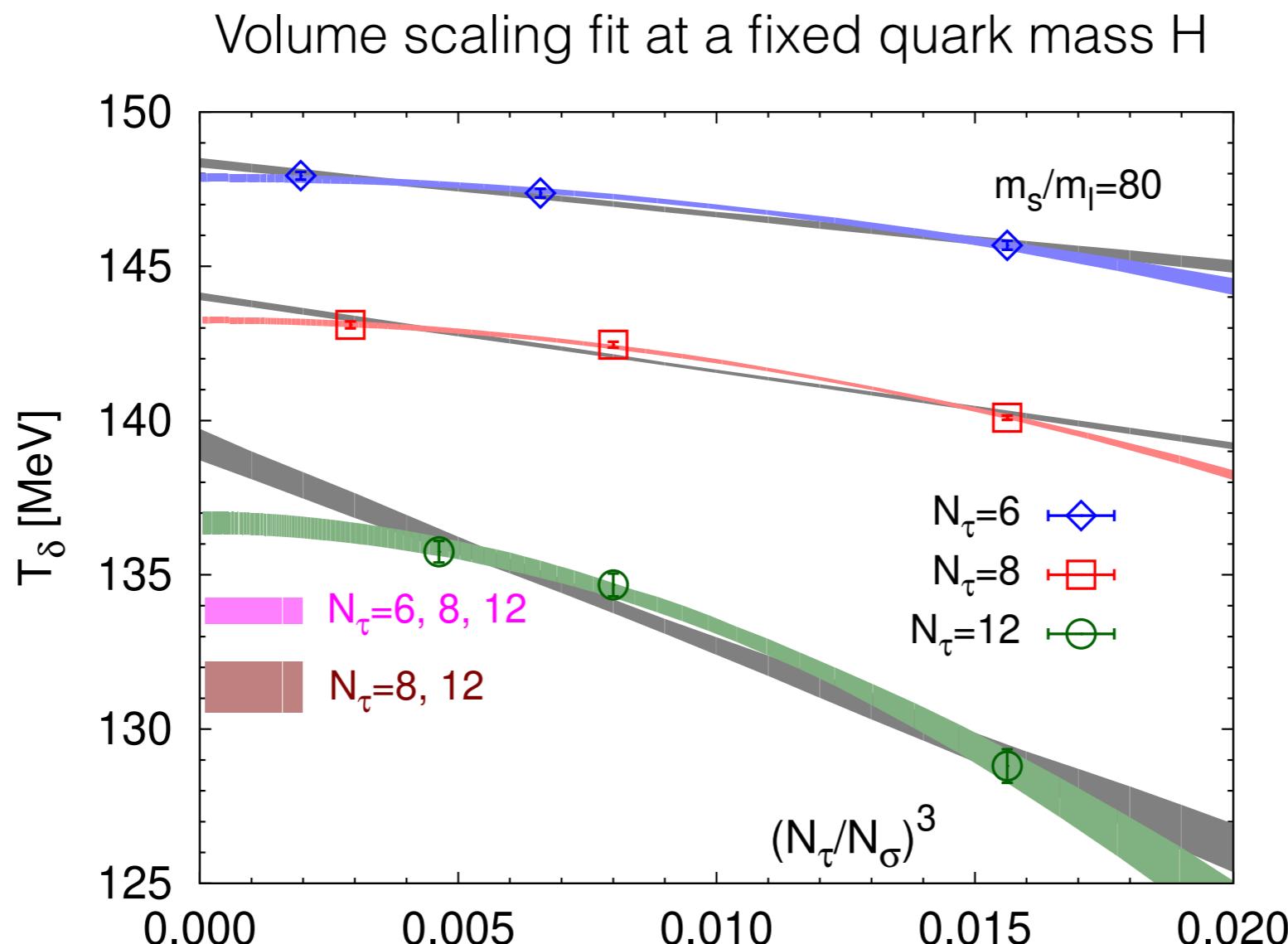
X=60,δ

T_δ : Infinite V limit \rightarrow continuum limit \rightarrow chiral limit



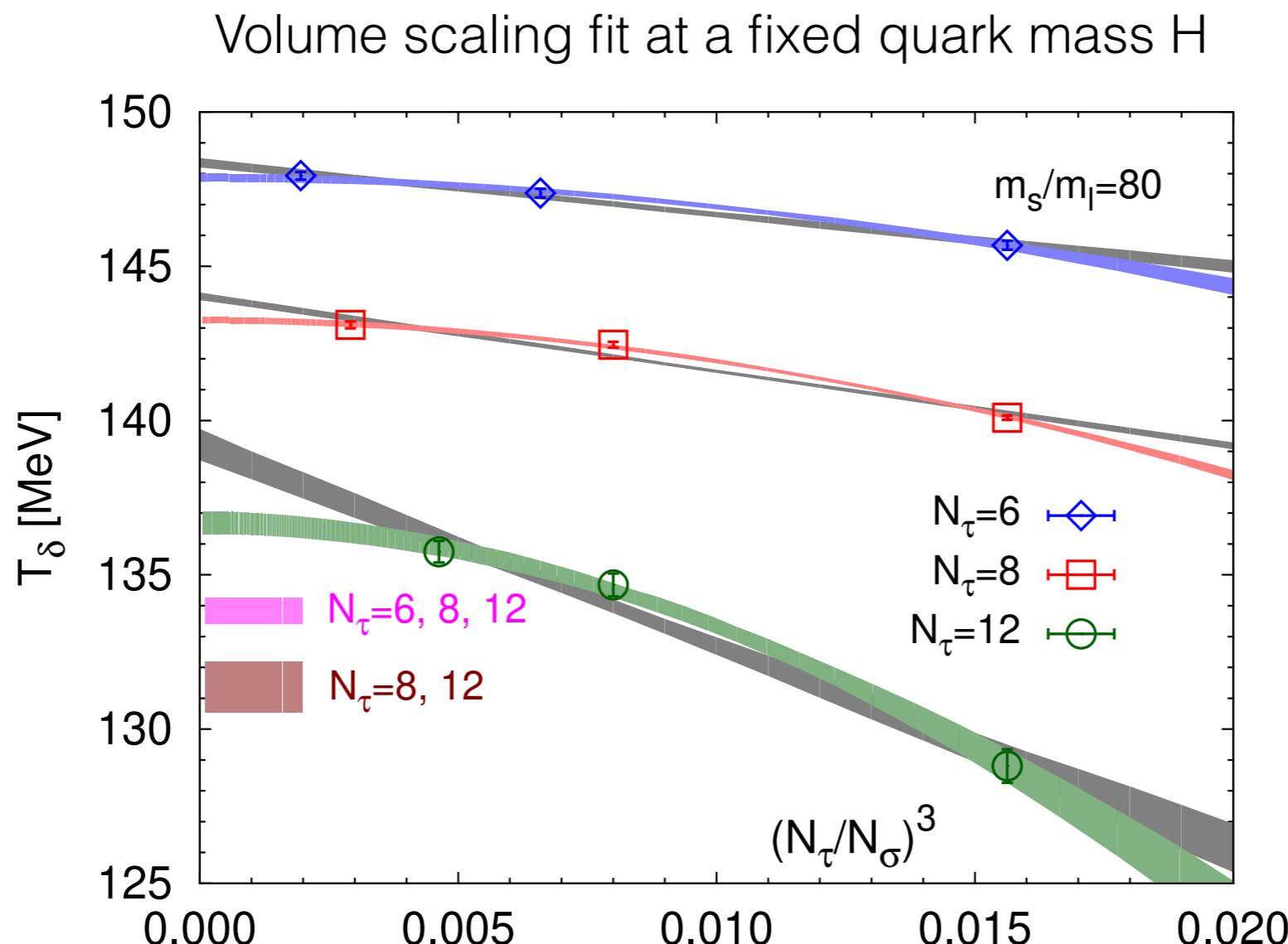
	$T_\delta(H, V, a) [\text{MeV}]$ with $N_t=6,8\&12$	$T_\delta(H, V, a) [\text{MeV}]$ with $N_t=8\&12$
$V \rightarrow \infty, a \rightarrow 0, H = 1/80$	133.8(4)	131.4(8)

T_δ : Infinite V limit \rightarrow continuum limit \rightarrow chiral limit



	$T_\delta(H, V, a)$ [MeV] with $N_t=6,8\&12$	$T_\delta(H, V, a)$ [MeV] with $N_t=8\&12$
$V \rightarrow \infty, a \rightarrow 0, H = 1/80$	133.8(4)	131.4(8)
$V \rightarrow \infty, a \rightarrow 0, H = 1/40$	136.9(5)	135.5(8)

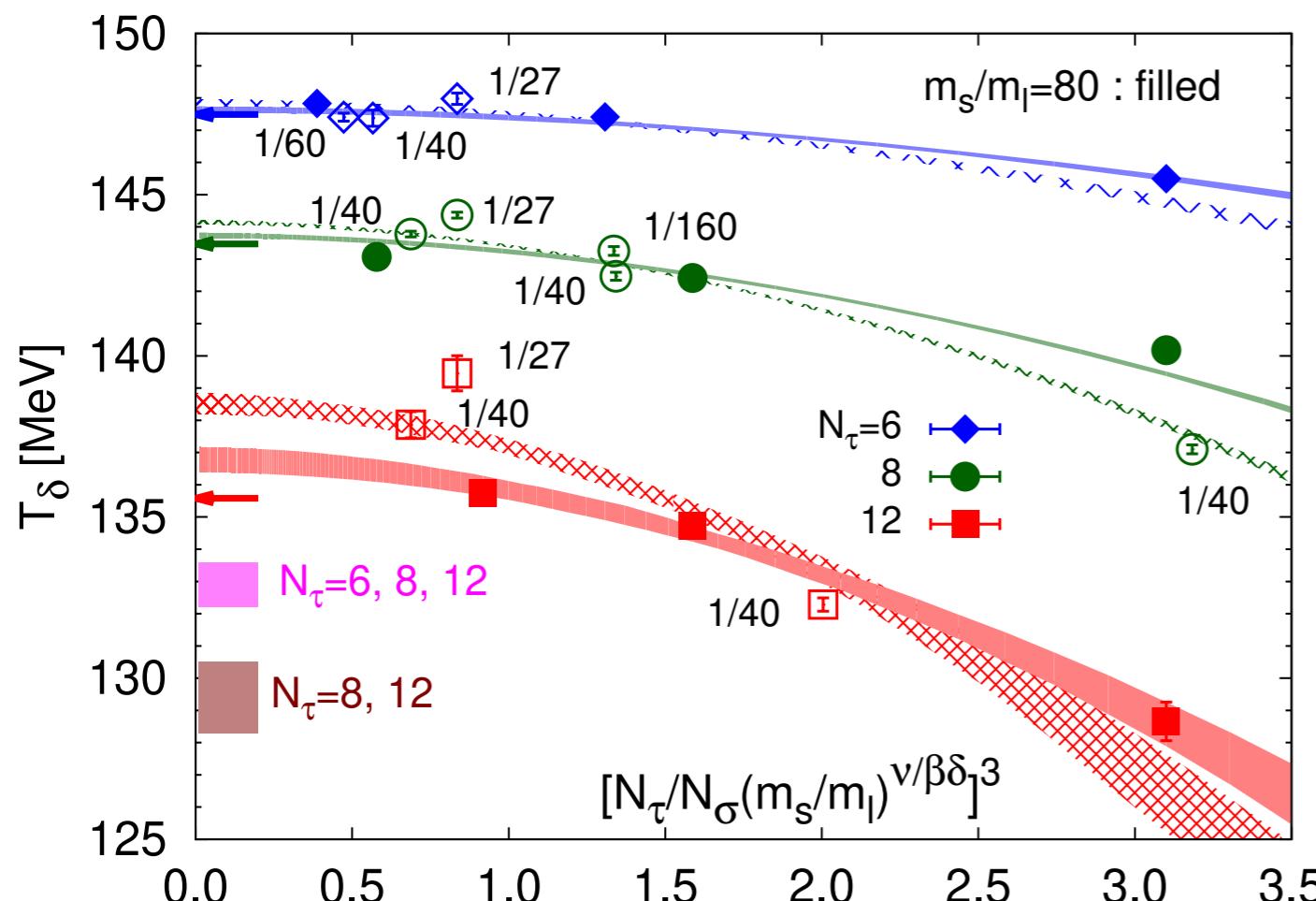
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$V \rightarrow \infty, a \rightarrow 0, H = 1/40$	136.9(5)	135.5(8)
$V \rightarrow \infty, a \rightarrow 0, H \rightarrow 0$	132.8(1.4)	130.6(2.4)

T_δ : Infinite V limit \rightarrow chiral limit \rightarrow continuum limit

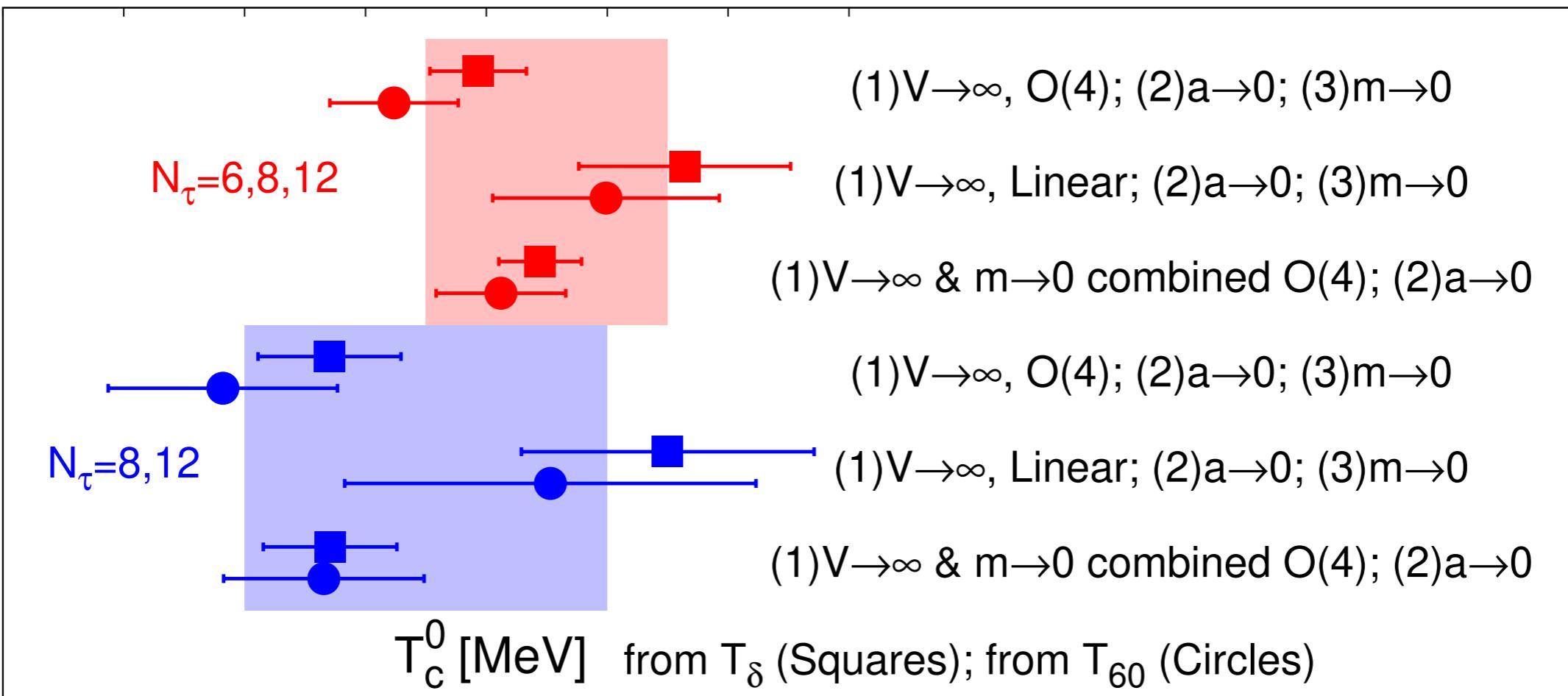
Joint volume scaling fit with all quark masses



$T_\delta(H, V, a)$ [MeV]	$N_t=6,8&12$	$N_t=8&12$
$V \rightarrow \infty, H \rightarrow 0, a \rightarrow 0$	132.9(6)	128.6(1.1)
$V \rightarrow \infty, a \rightarrow 0, H \rightarrow 0$	132.8(1.4)	130.6(2.4)

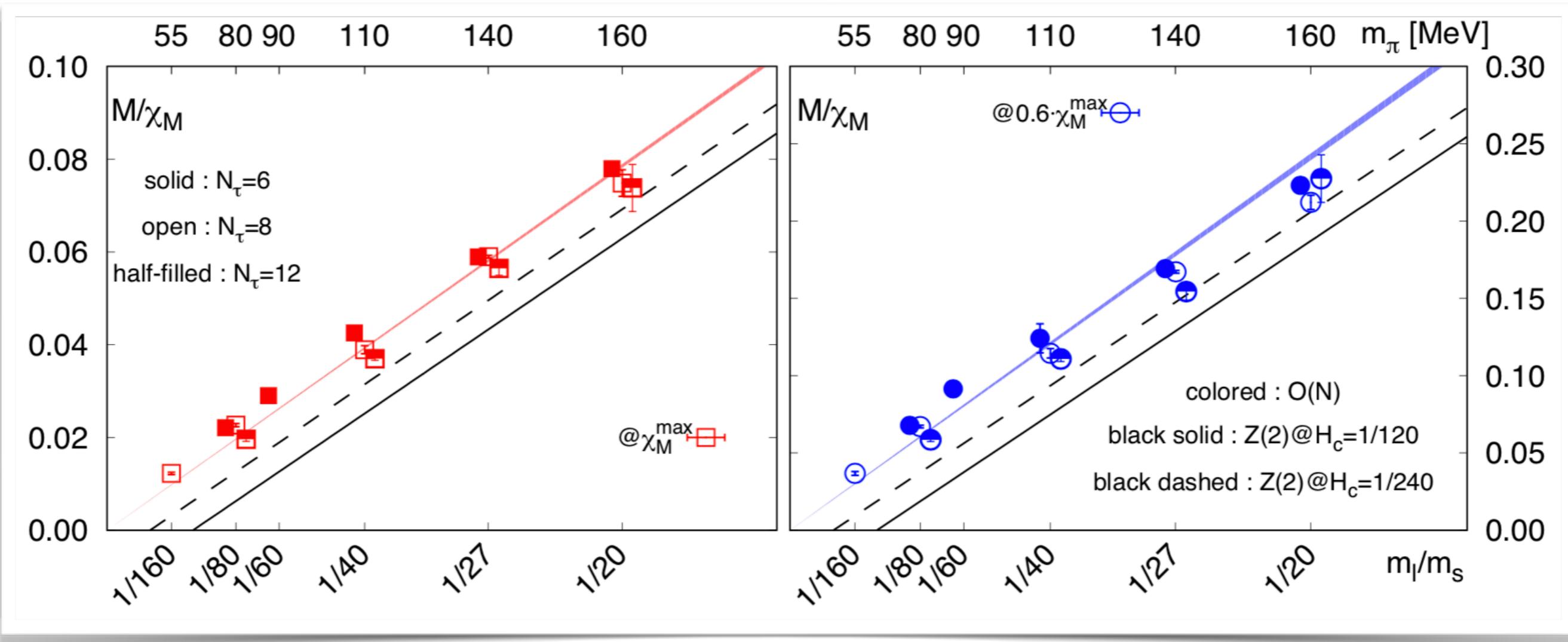
Chiral and continuum limits are Interchangeable

$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$



- T_{60} and T_δ give consistent results
- About 25 MeV lower than T_{pc} at the physical point!

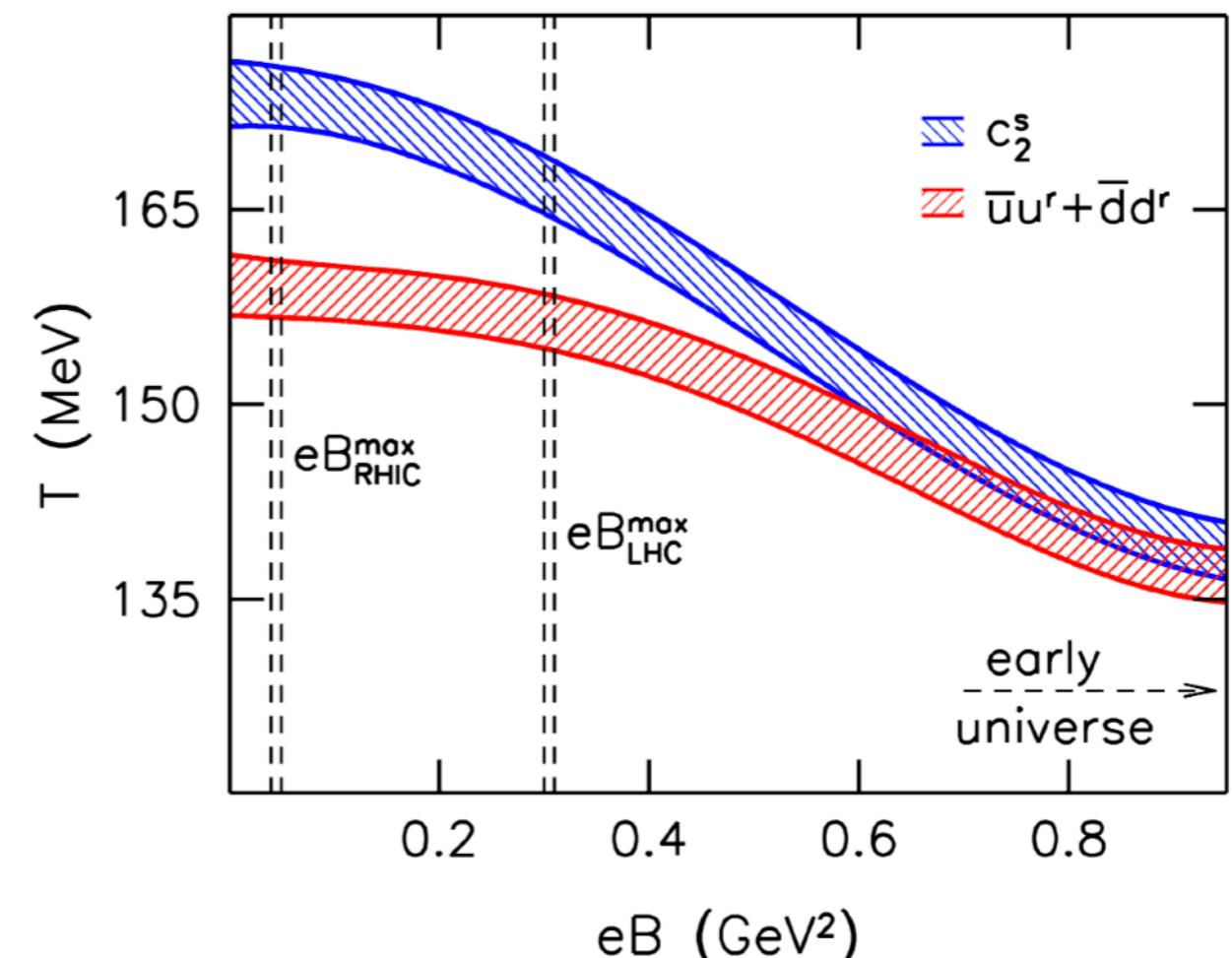
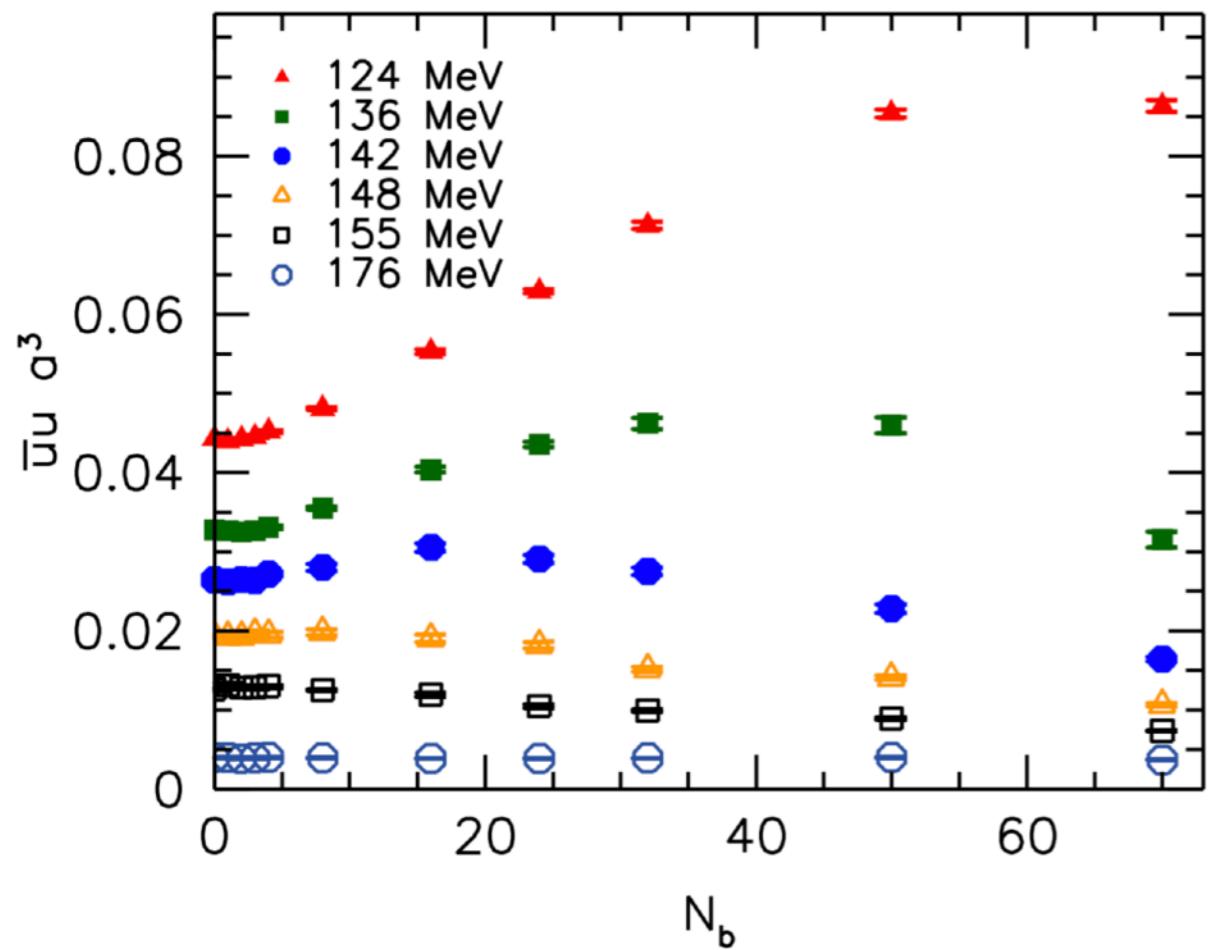
Consistency of QCD chiral phase transition with O(N) universality class



S.-T. Li(李胜泰), Lattice 2018, A. Lahiri, QM 2018

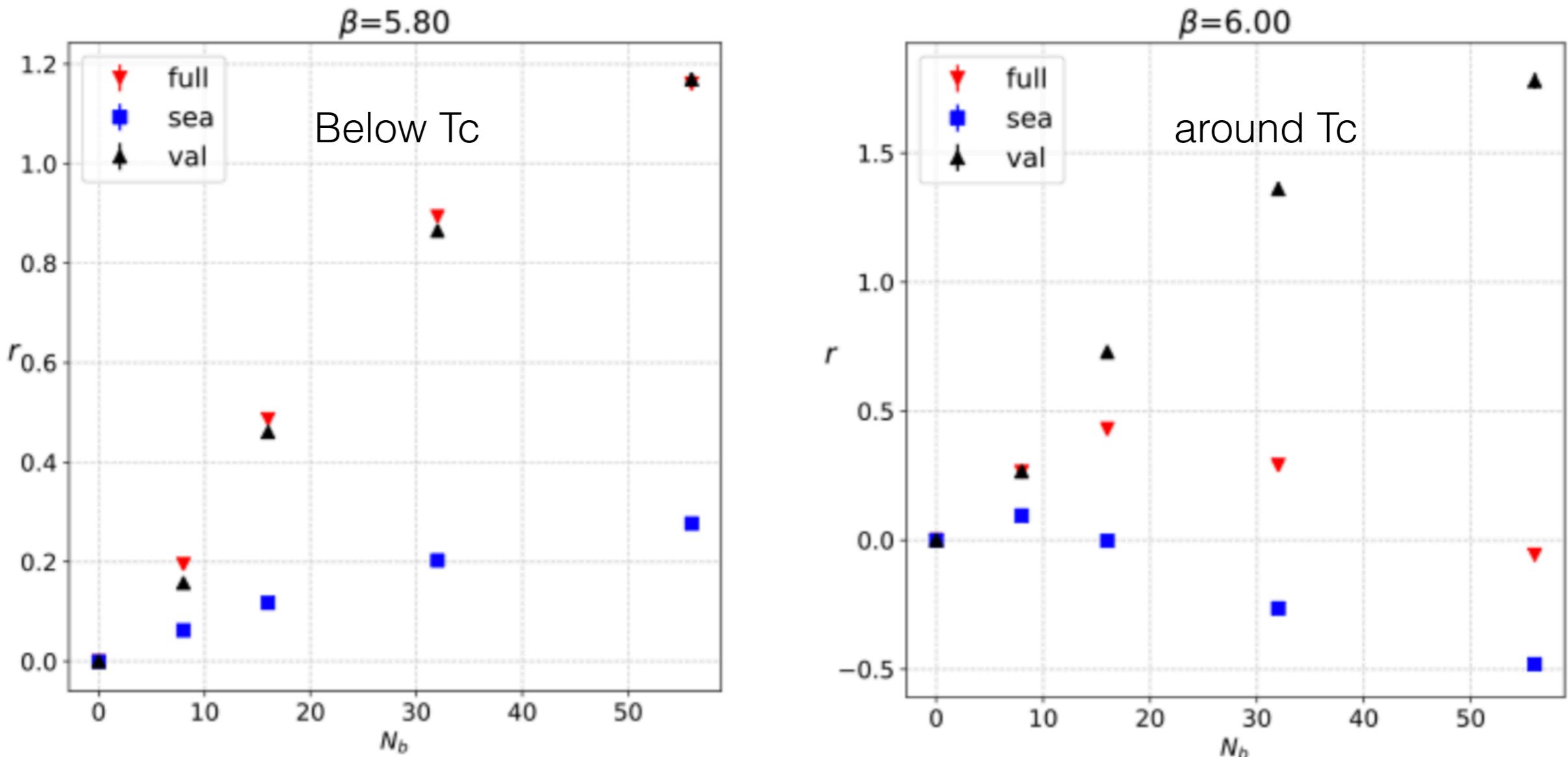
$$M/\chi_M = \frac{m_l - m_l^{critical}}{m_s^{phys}} \frac{f_M}{f_{\chi_M}}$$

Inverse magnetic catalyses v.s. $T_c(B)$



Bali et al., JHEP02(2012)044

Effects of dynamical quarks

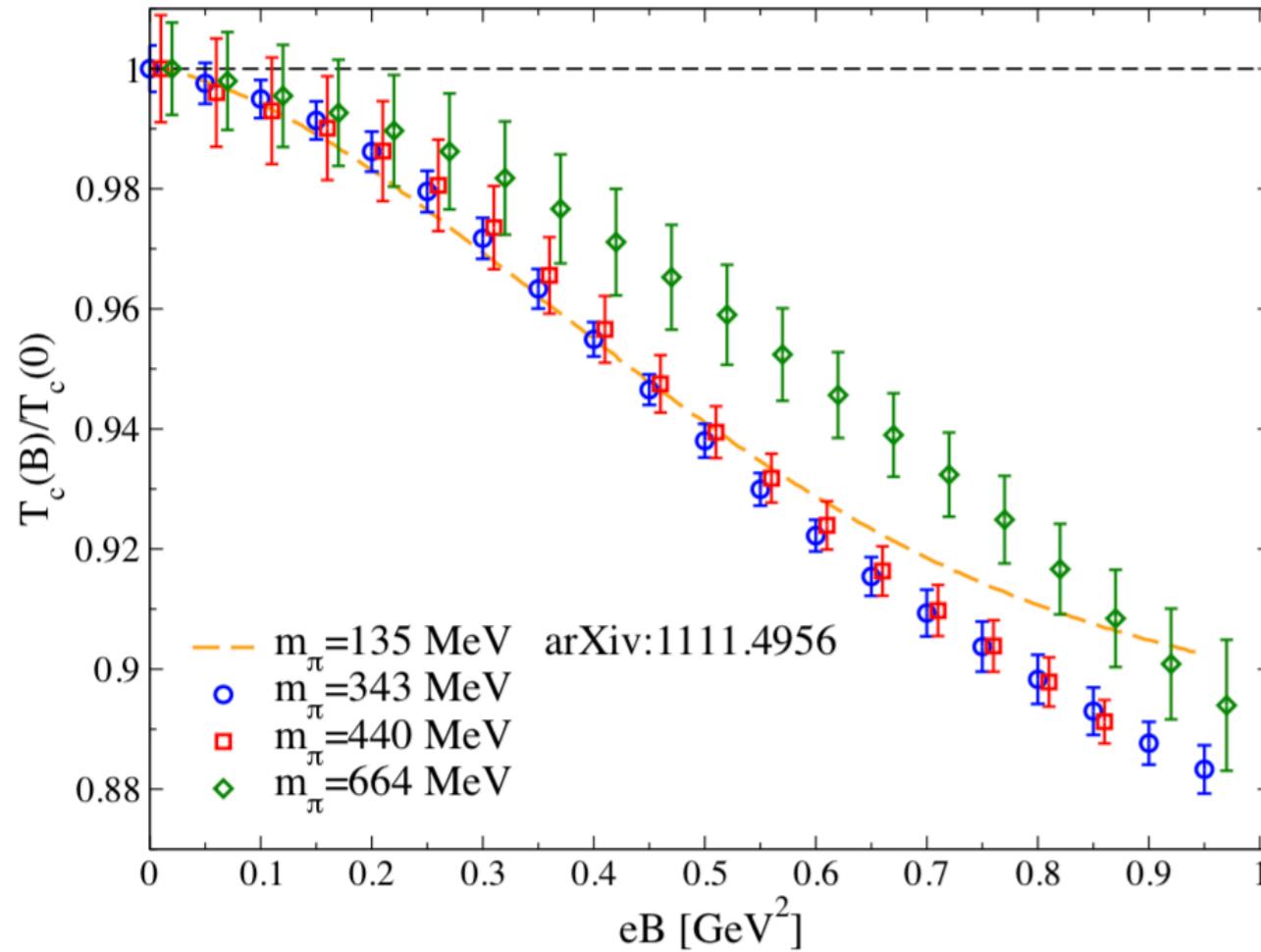


A. Tomiya, HTD, X.-D. Wang (汪晓丹), Y. Zhang (张瑜) et al., Lattice 2018, arXiv: 1904.01276

sea quarks play an important role in the inverse magnetic catalyses

Similar findings from D'Elia et al., 1808.07008, Bruckmann, Endrodi, Kovcas, JHEP 1304(2013)112

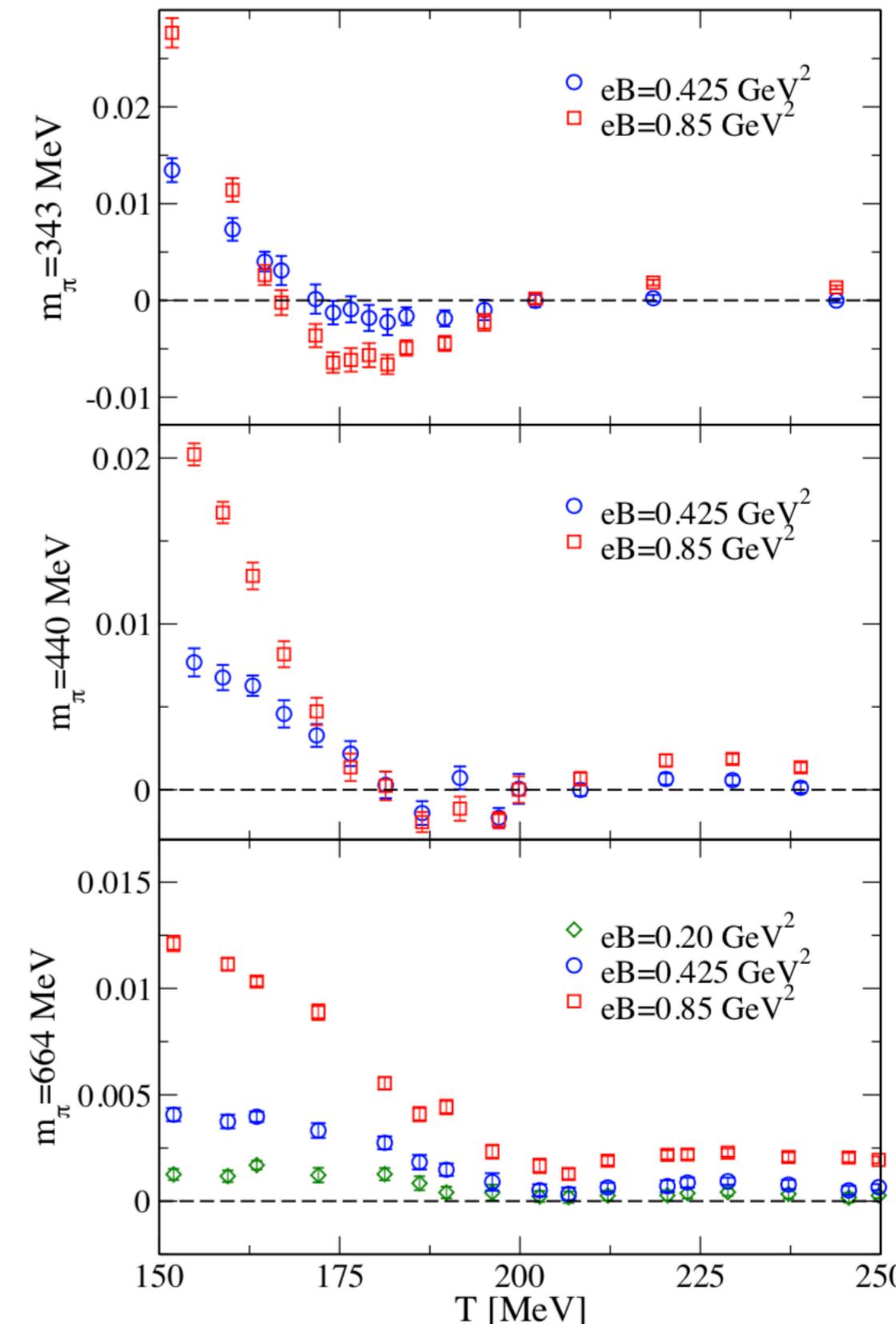
Cases with different pion masses



D'Elia et al., 1808.07008

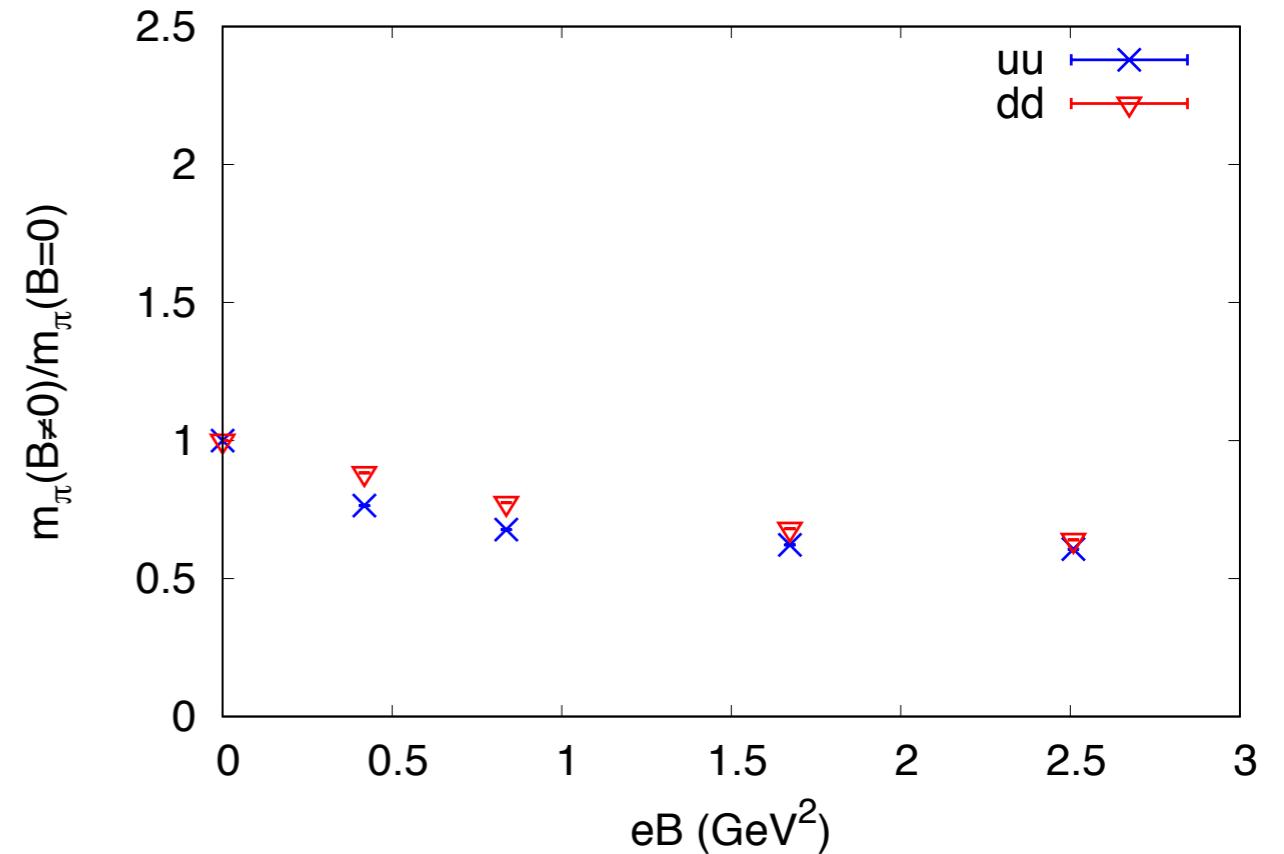
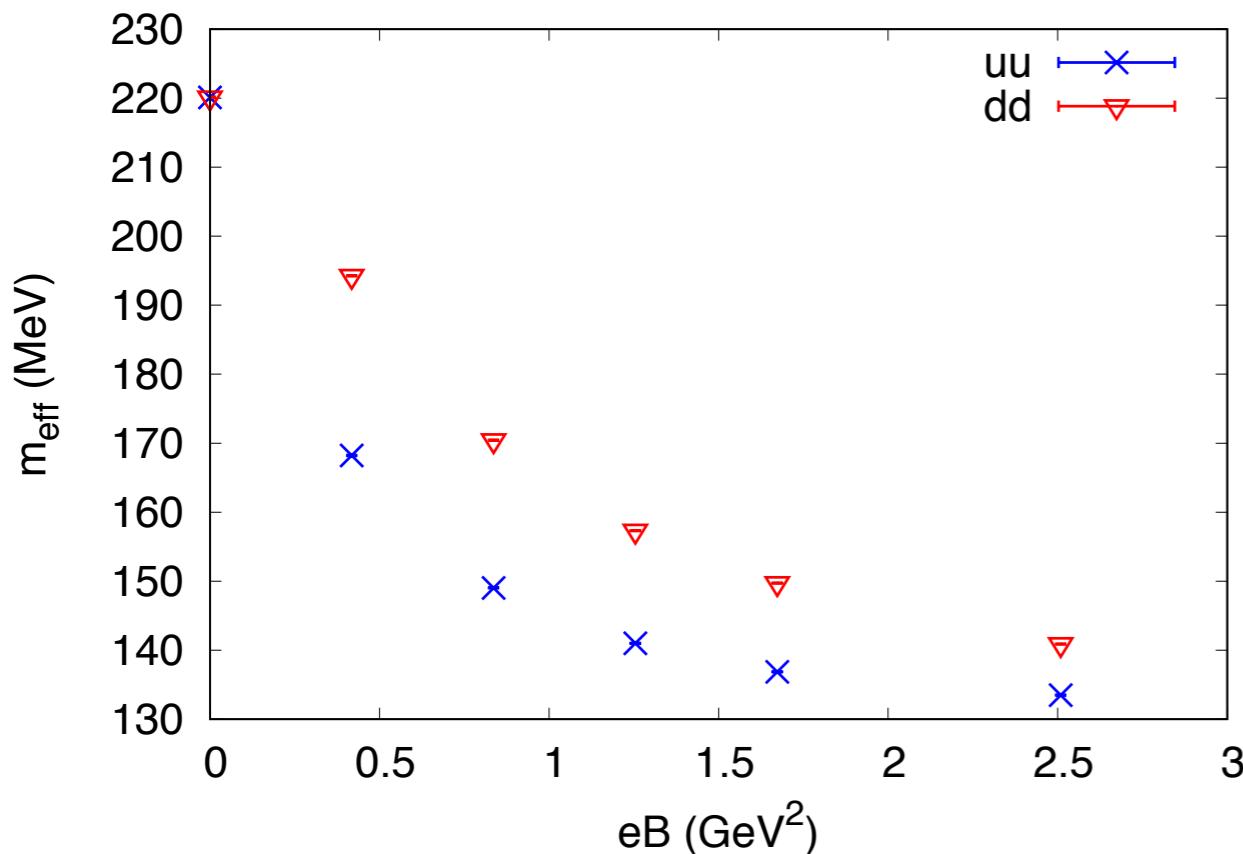
$T_c(B)$ decreases with B
along with magnetic catalyses

$$\langle \bar{\psi}\psi \rangle(B \neq 0) - \langle \bar{\psi}\psi \rangle(B = 0)$$



Change of lightest pion in external magnetic field at T=0

$32^3 \times 96$, $T=0$, $m_\pi \approx 220$ MeV, $N_f=2+1$ with HISQ/tree action



HTD, A. Tomiya, S. Mukherjee, Xiao-Dan Wang (汪晓丹) et al., work in progress

Natural pions become lighter, similar to results from
quenched QCD

quenched QCD: Bali et al., PRD 18'

Conclusions

- chiral crossover temperature is determined with better precision, i.e. $T_{pc} = 156.5(1.5)$ MeV
- The chiral T_c of $N_f=2+1$ QCD is 132^{+3}_{-6} MeV, and the $O(N)$ universality class of the chiral phase transition is favored
- Decreasing of T_c with B may be more relevant with the reduction of neutral pion mass

谢谢！

Thanks for your attention!

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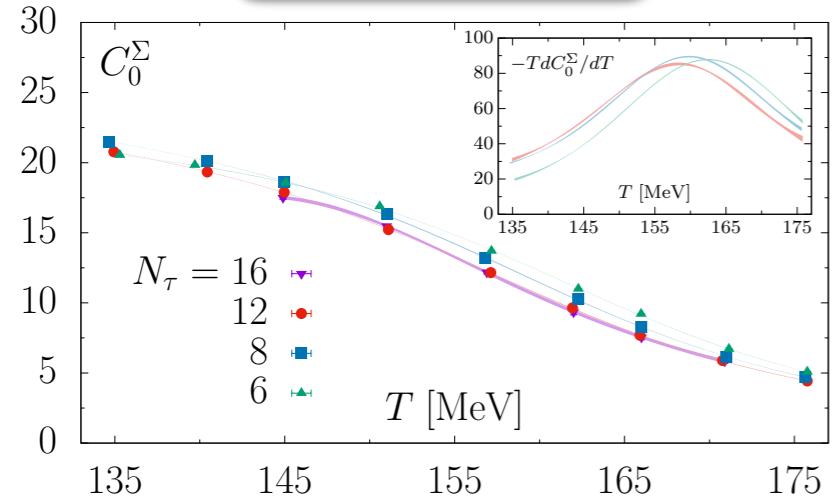
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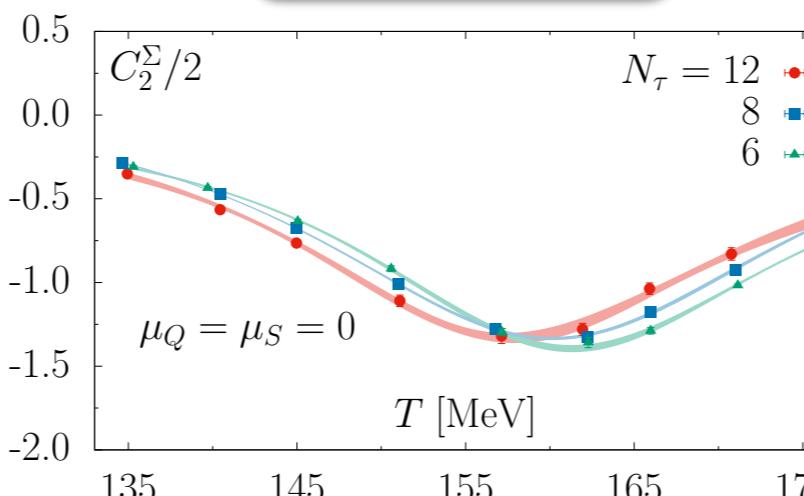
- Heng-Tong Ding (Chair, CCNU)
- Ying Chen (IHEP, CAS)
- Xu Feng (Peking U.)
- Ziwen Fu (Sichuan U.)
- Ming Gong (IHEP, CAS)
- Longcheng Gui (Hunan Normal U.)
- Olaf Kaczmarek (Bielefeld U./CCNU)
- Ning Li (Xi'an Technological U.)
- David Lin (NCTU, Hsinchu)
- Chuan Liu (Peking U.)
- Liuming Liu (IMP, CAS)
- Yubin Liu (Nankai U.)
- Zhaofeng Liu (IHEP, CAS)
- Jian-Ping Ma (ITP, CAS)
- Swagato Mukherjee (BNL/Tsinghua U.)
- Yibo Yang (ITP, CAS)
- Liangkai Wu (Jiangsu U.)
- Jianbo Zhang (Zhejiang U.)



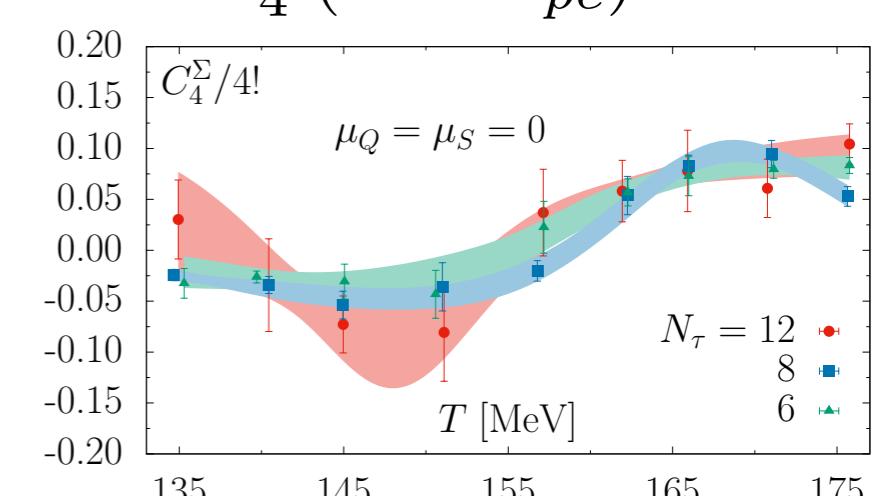
$$\partial_T^2 C_0^\Sigma(T) = 0$$



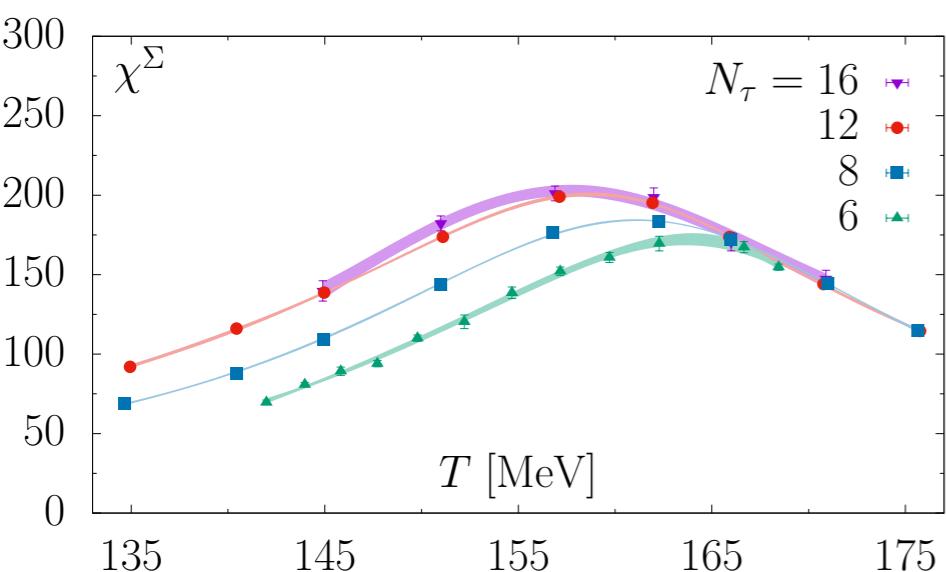
$$\partial_T C_2^\Sigma(T) = 0$$



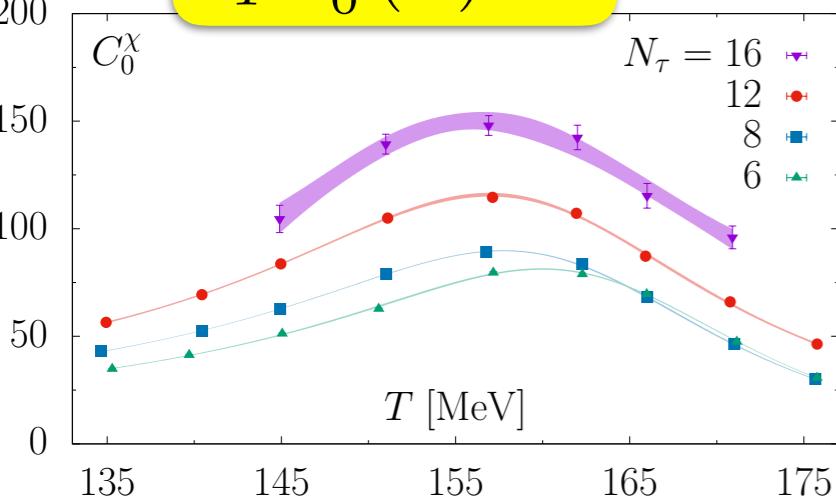
$$C_4^\Sigma(T = T_{pc}) = 0$$



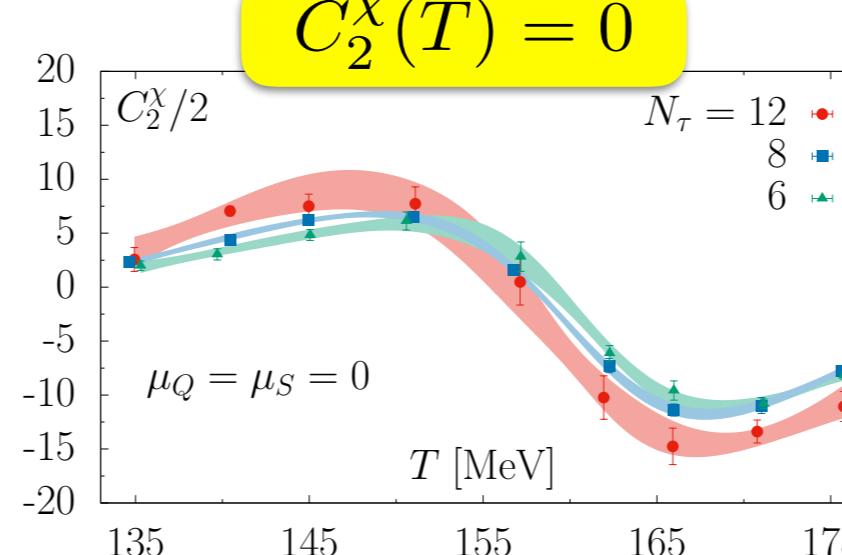
$$\partial_T \chi^\Sigma(T) = 0$$



$$\partial_T C_0^\chi(T) = 0$$



$$C_2^\chi(T) = 0$$



$$\partial_T C_4^\chi(T = T_{pc}) = 0$$

