NuclearScience Computing Center at CCNU

Chiral crossover and phase transition in (2+1)-flavor QCD

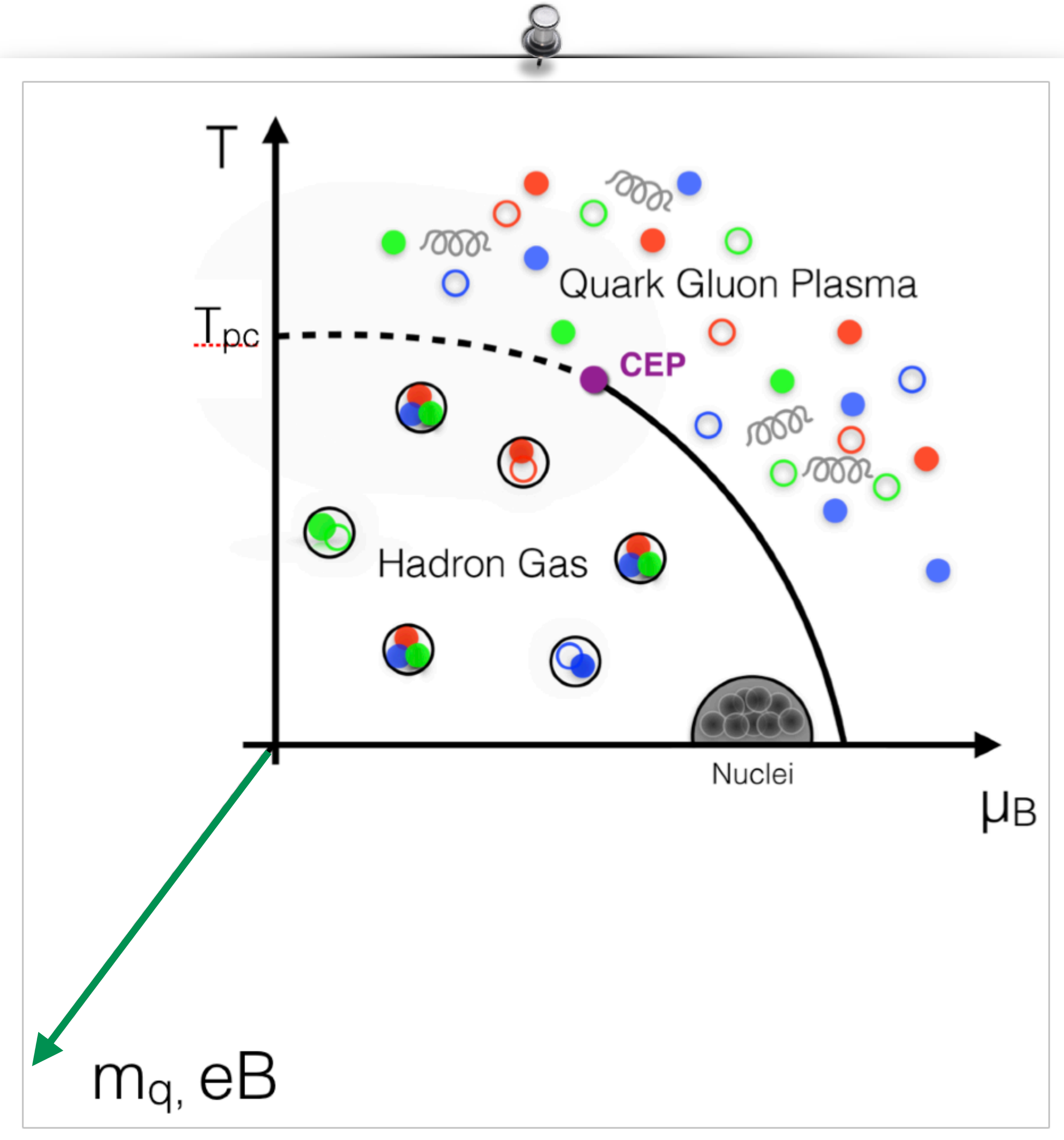
Heng-Tong Ding (丁亨通)

Central China Normal University

The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

8-12 April, 2019@Tsinghua University

Outline: QCD phase structure



HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

- Chiral crossover at zero and small μ_B

  A. Bazavov, HTD, P. Hegde et al. [HotQCD], arXiv:1812.08235

- Chiral phase transition temperature

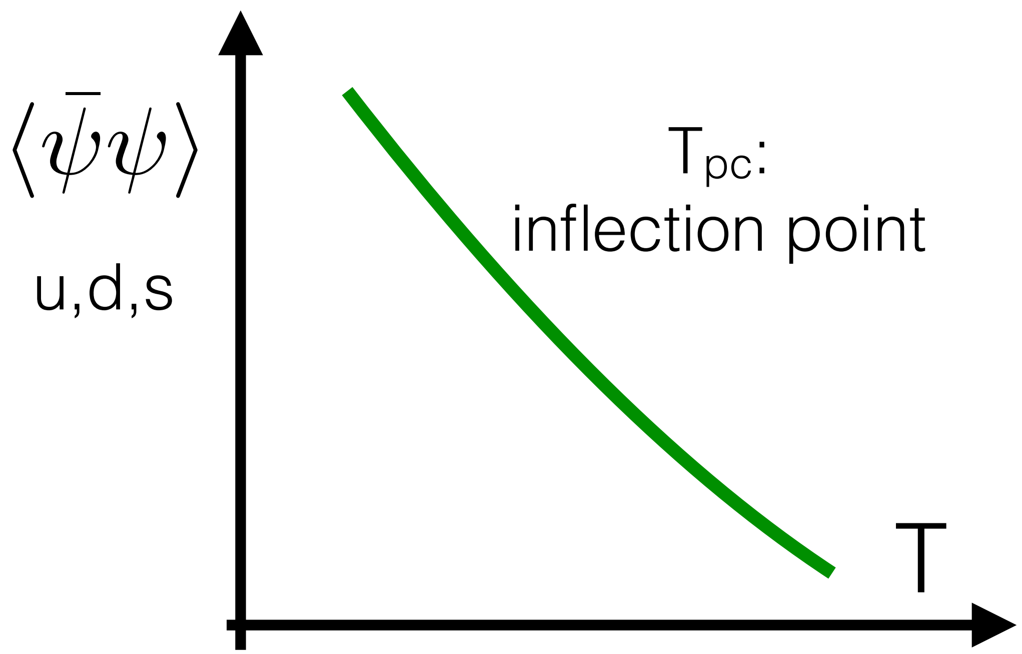
  HTD, P. Hegde, O. Kaczmarek et al. [HotQCD], arXiv:1903.04801

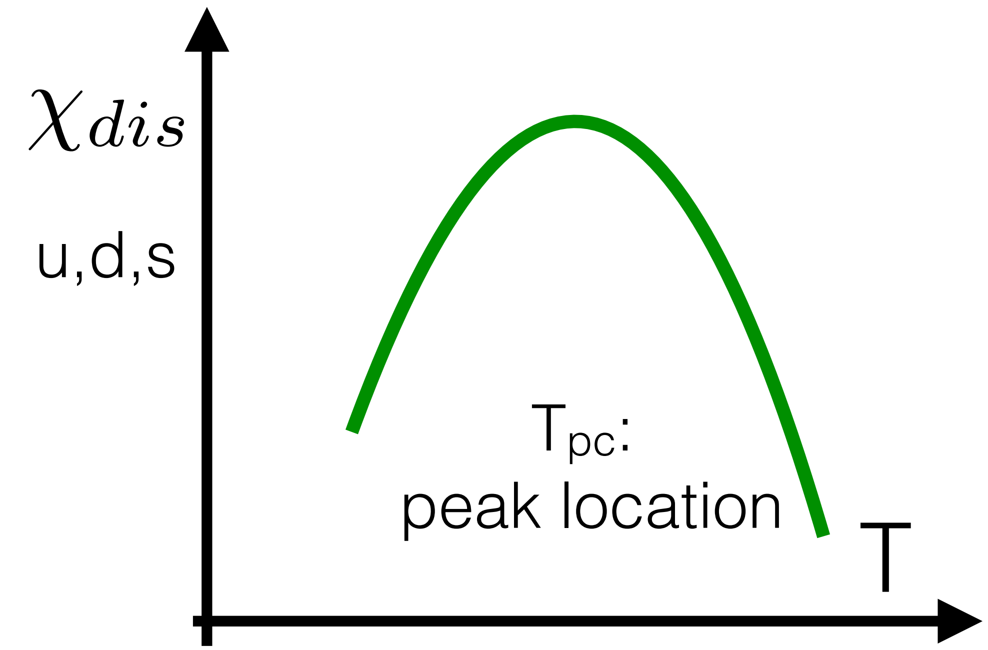
- QCD transition in external B

  Xiao-Dan Wang (汪晓丹) et al., work in progress & arXiv: 1904.01276

Crossover transition temperature T_{pc} in the real world

- Crossover nature of the transition





- Chiral phase transition: most likely 2nd order, 3d O(4)

Ejiri et al., PRD 80(2009)094505,
HotQCD, arXiv:1903.04801
...

- A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HTD, P. Hegde, O. Kaczmarek et al. [HotQCD], arXiv:1903.04801

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^{\Sigma}(T,\mu_B) \sim m^{1/\delta-1} f_\chi$

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T,\mu_B) \sim m^{1/\delta - 1} f_\chi$

Taylor expansions:

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n} \qquad \chi(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\chi}(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T,\mu_B) \sim m^{1/\delta-1} f_\chi$

Taylor expansions:

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad \chi(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n}$$

$\partial_T \chi^\Sigma(T)$
$\partial_T C_0^\chi(T)$
$C_2^\chi(T)$

$$\sim \quad m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$\partial_T^2 C_0^\Sigma(T)$
$\partial_T C_2^\Sigma(T)$

$$\sim \quad m^{1/\delta-2/\beta\delta} f''_G(z)$$

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T,\mu_B) \sim m^{1/\delta} f_G$

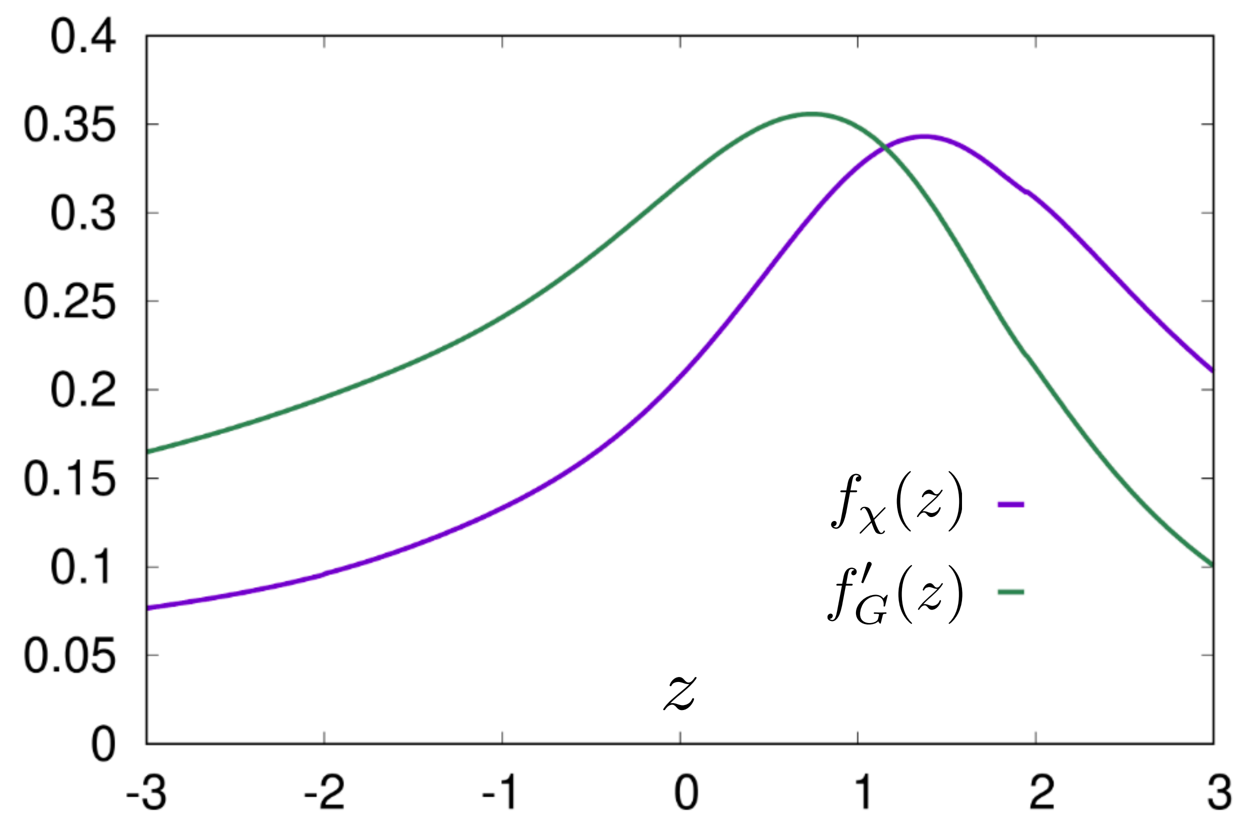
chiral susceptibility: $\chi^{\Sigma}(T,\mu_B) \sim m^{1/\delta-1} f_\chi$

Taylor expansions:

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad \chi(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n}$$

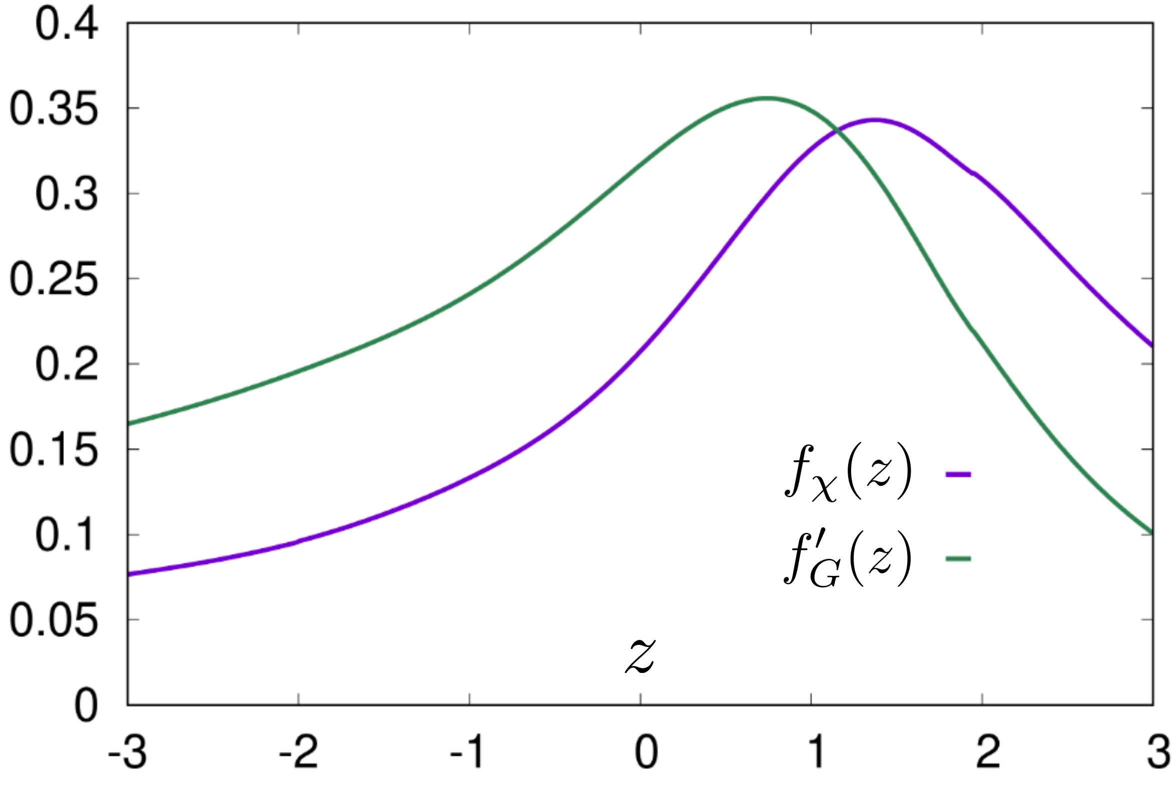
$\partial_T \chi^{\Sigma}(T)$, $\partial_T C_0^{\chi}(T)$, $C_2^{\chi}(T)$ $\sim \; m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$

$\partial_T^2 C_0^{\Sigma}(T)$, $\partial_T C_2^{\Sigma}(T)$ $\sim \; m^{1/\delta-2/\beta\delta} f''_G(z)$



$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$

Well-defined notation of chiral crossover transition temperature



$z \sim \left((T - T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$

$\partial_T \chi^\Sigma(T)$, $\partial_T C_0^\chi(T)$, $C_2^\chi(T)$ $\sim \quad m^{1/\delta - 1 - 1/\beta\delta} f'_\chi(z)$

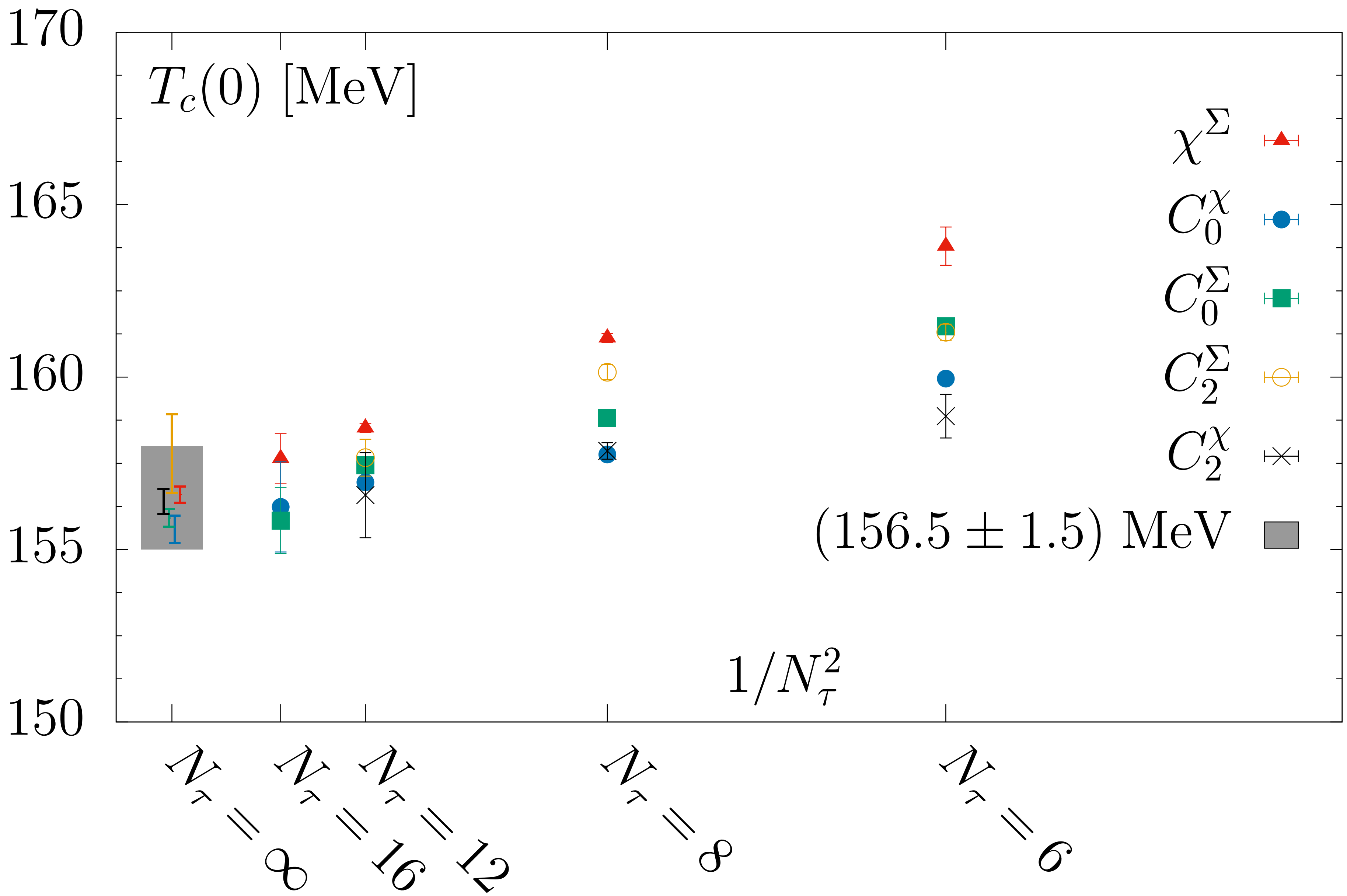
$\partial_T^2 C_0^\Sigma(T)$, $\partial_T C_2^\Sigma(T)$ $\sim \quad m^{1/\delta - 2/\beta\delta} f''_G(z)$

- 5 conditions to extract Tc: maxima of f_χ and f'_G

$\partial_T \chi^\Sigma(T) = 0$ $\quad \partial_T C_0^\chi(T) = 0$ $\quad C_2^\chi(T) = 0$ $\quad \partial_T^2 C_0^\Sigma(T) = 0$ $\quad \partial_T C_2^\Sigma(T) = 0$

- m=0: all these susceptibilities diverge at a unique T
- m=/=0: non-unique temperatures, crossover

QCD transition with m_π =140 MeV at μ_B=0

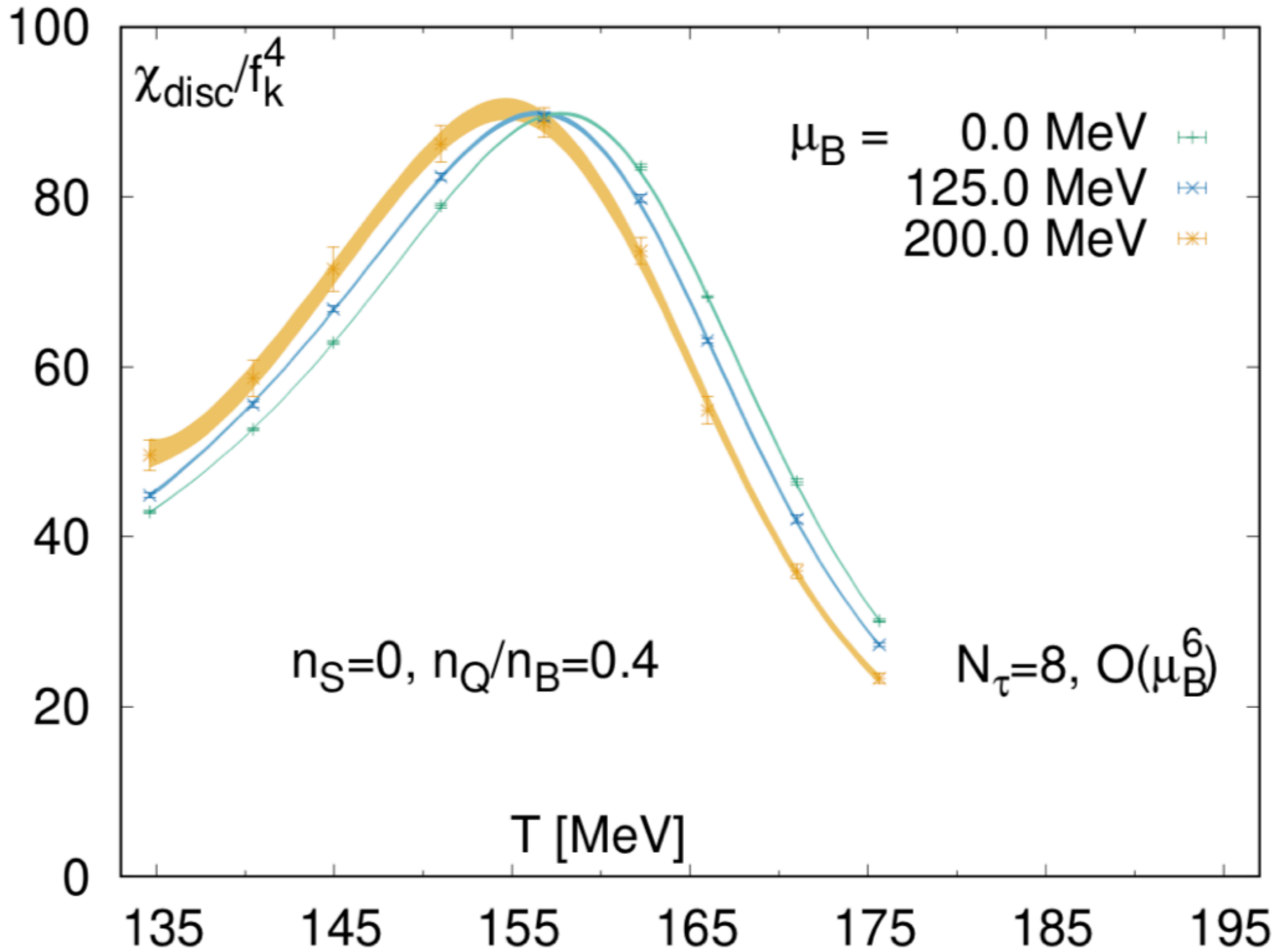


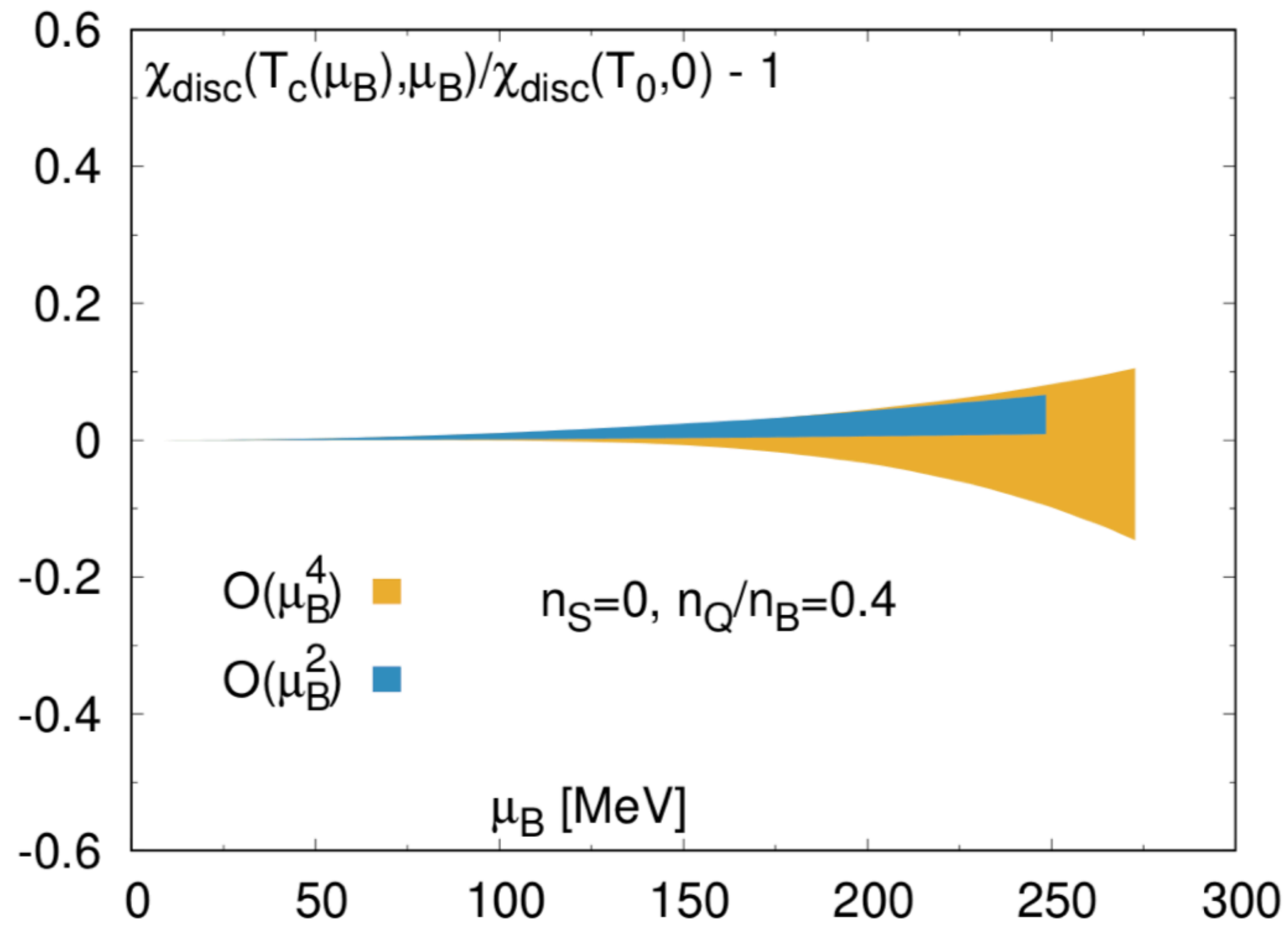
Higher precision in the continuum limit:

T_{pc} =156.5(1.5)MeV

HotQCD, arXiv:1812.08235

Order Parameter Susceptibility at μ_B=/=0

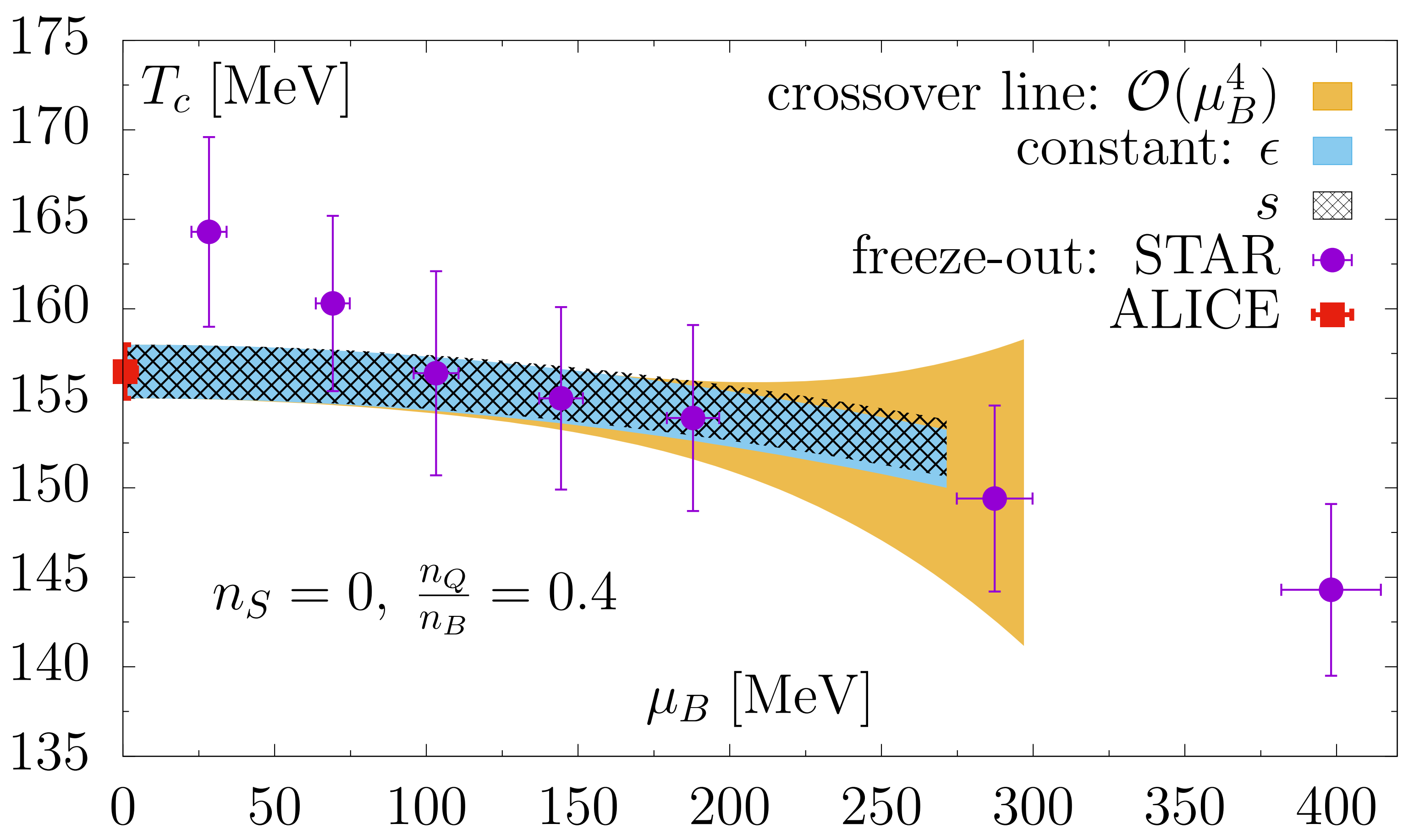




No indication of a stronger phase transition at larger μ_B

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right)$$



A. Bazaovo, HTD et al. [HotQCD], arXiv:1812.08235

curvature of crossover line

$$\kappa_2 = 0.0123 \pm 0.003$$

$$\kappa_4 = 0.000131 \pm 0.0041$$

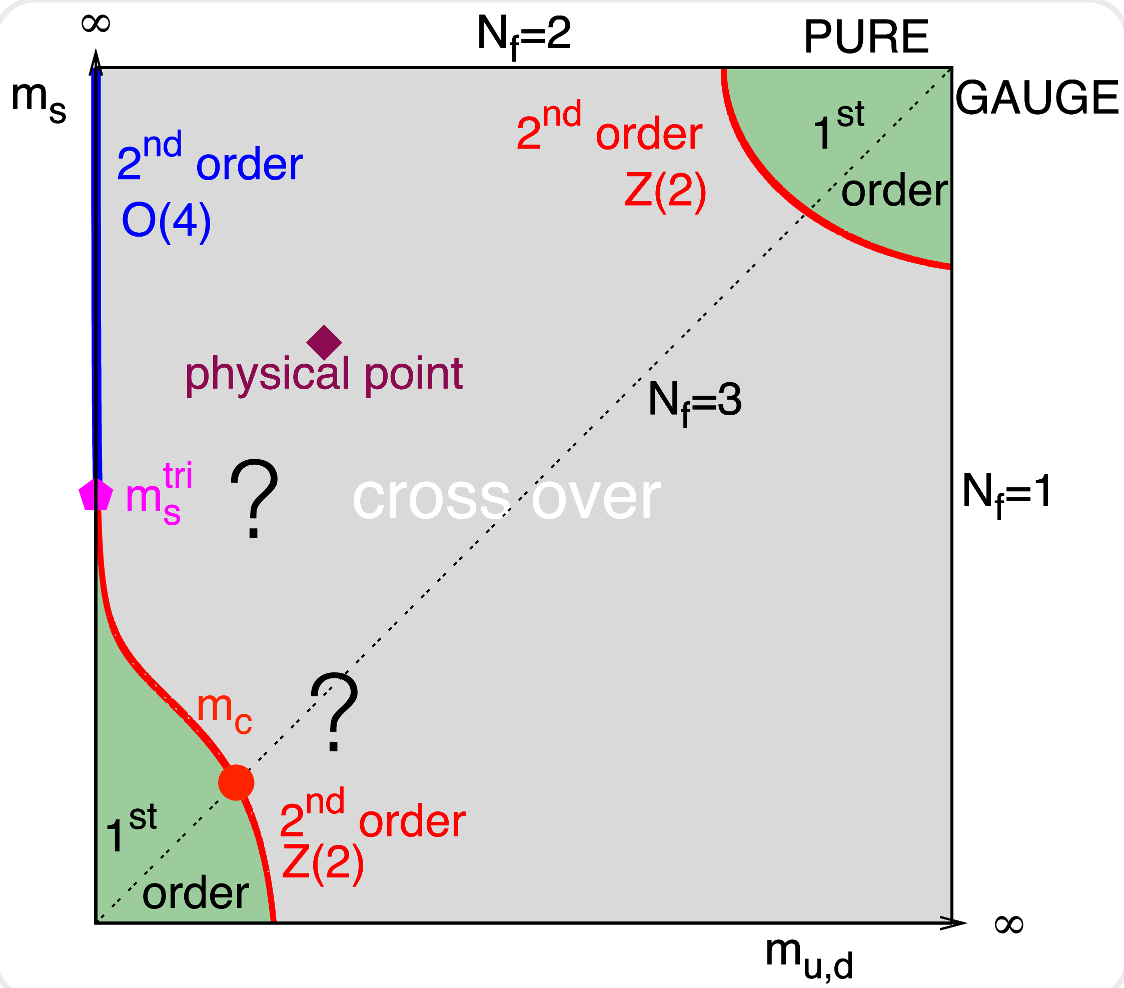
curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

QCD phase diagram in the quark mass plane

Columbia plot:

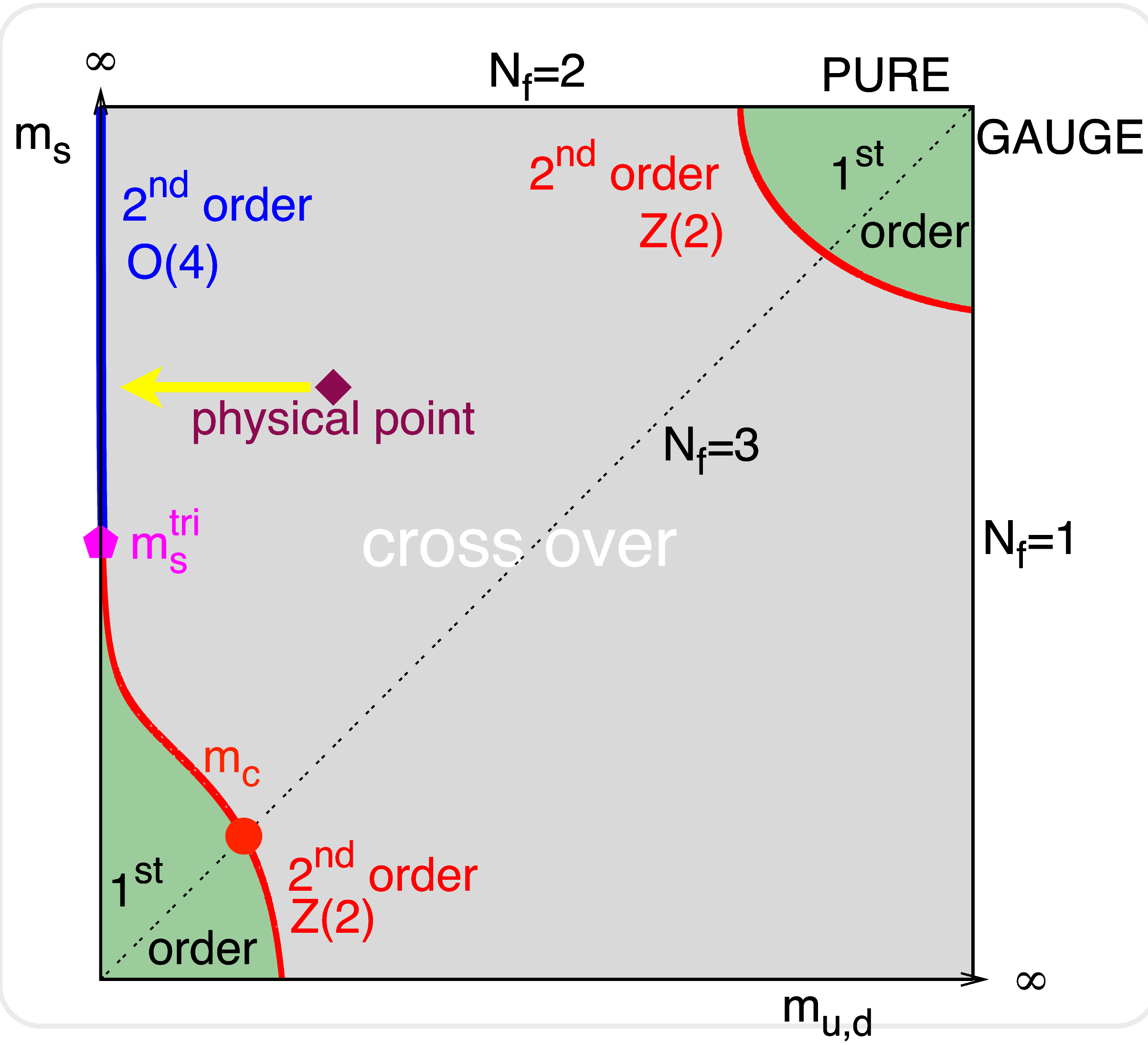


- At physical point: cross over, T_{pc} = 156.5(1.5) MeV HotQCD, arXiv:1812.08235
- N_f=2(+1): $U_A(1)$ remains broken at $T_{\chi SB}$ JLQCD '13,'14,'15, HotQCD '13,'14
- Critical lines of second order transition Pisarski & Wilczek PRD '84
  N_f=2: O(4) universality class Kogut & Sinclair, PRD '06
  N_f=3: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

Towards the chiral limit:

- N_f=2+1 QCD: m_s^{tri} ? m_s^{phy}
- Fundamental scale of QCD: chiral T_c ?

Towards chiral limit of (2+1)-flavor QCD



- HISQ/tree action
- **N_f=2+1:**

$m_u = m_d \rightarrow 0$

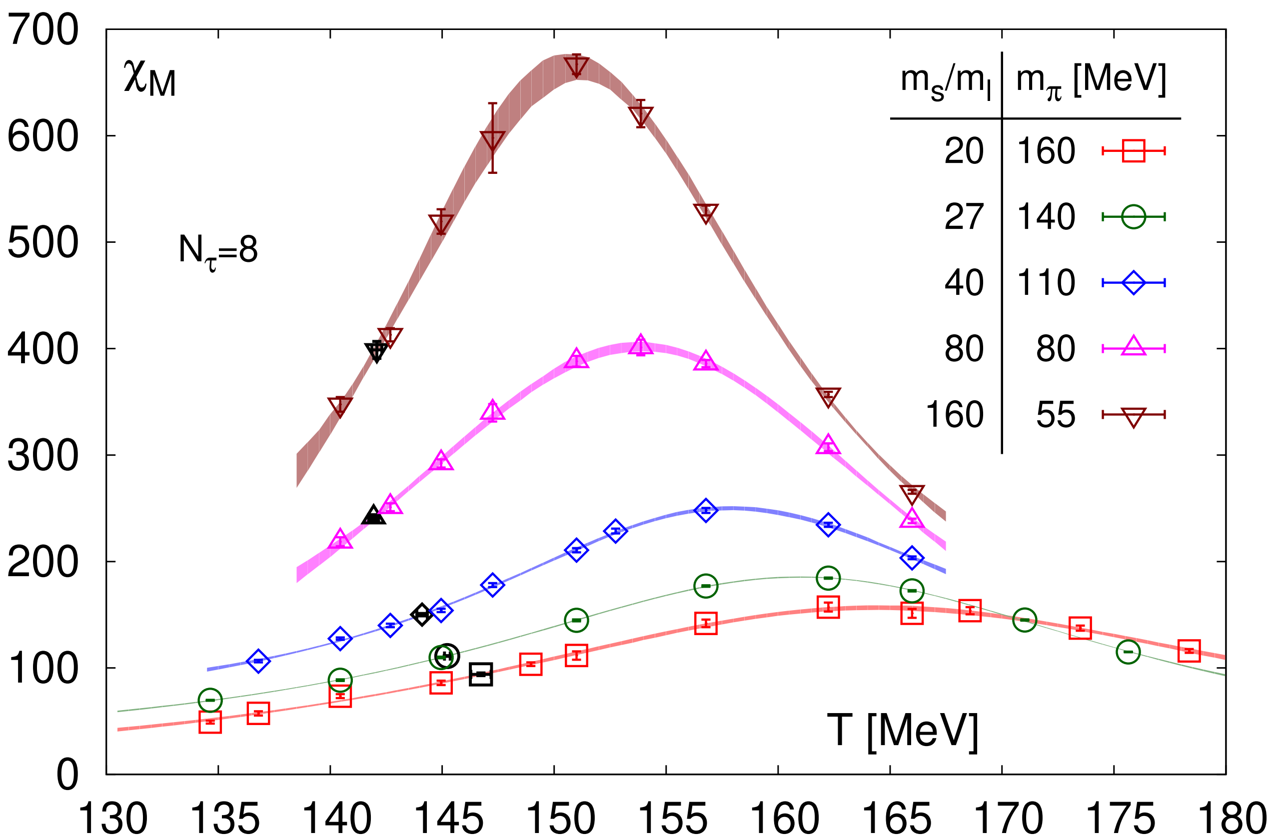
$m_s = m_s^{phy}$

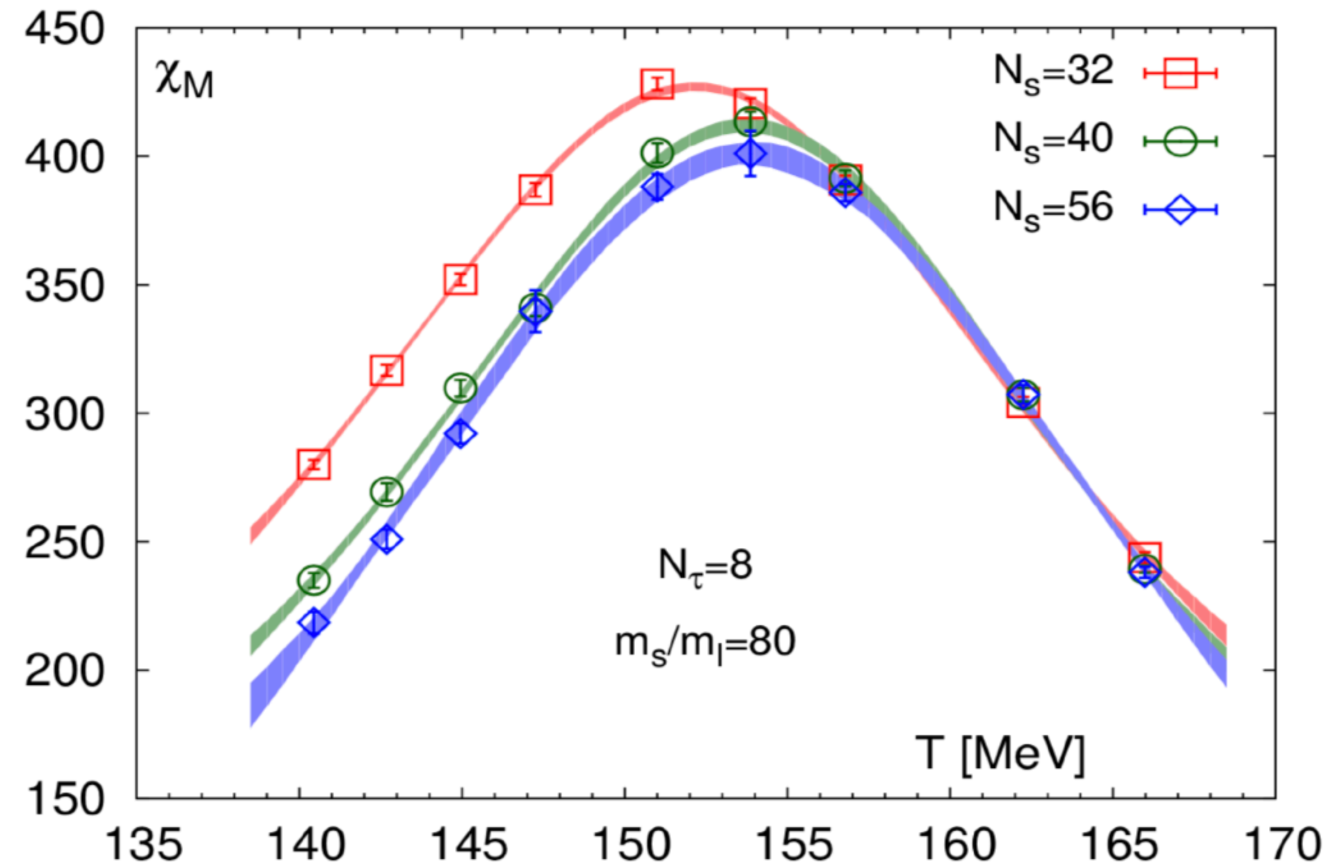
- ☑ Nt=6,8,12
- ☑ $m_s^{phy}/\mathbf{m_l}$ = 20, 27, 40, 60, 80,160

  $m_\pi \approx$160,140,110,90,80,55 MeV
- ☑ $7 \geq N_s/N_t \geq 4 \;\Leftrightarrow\; 5 \gtrsim m_\pi L \gtrsim 3$

This allows us to perform
infinite volume, continuum and then chiral extrapolation!

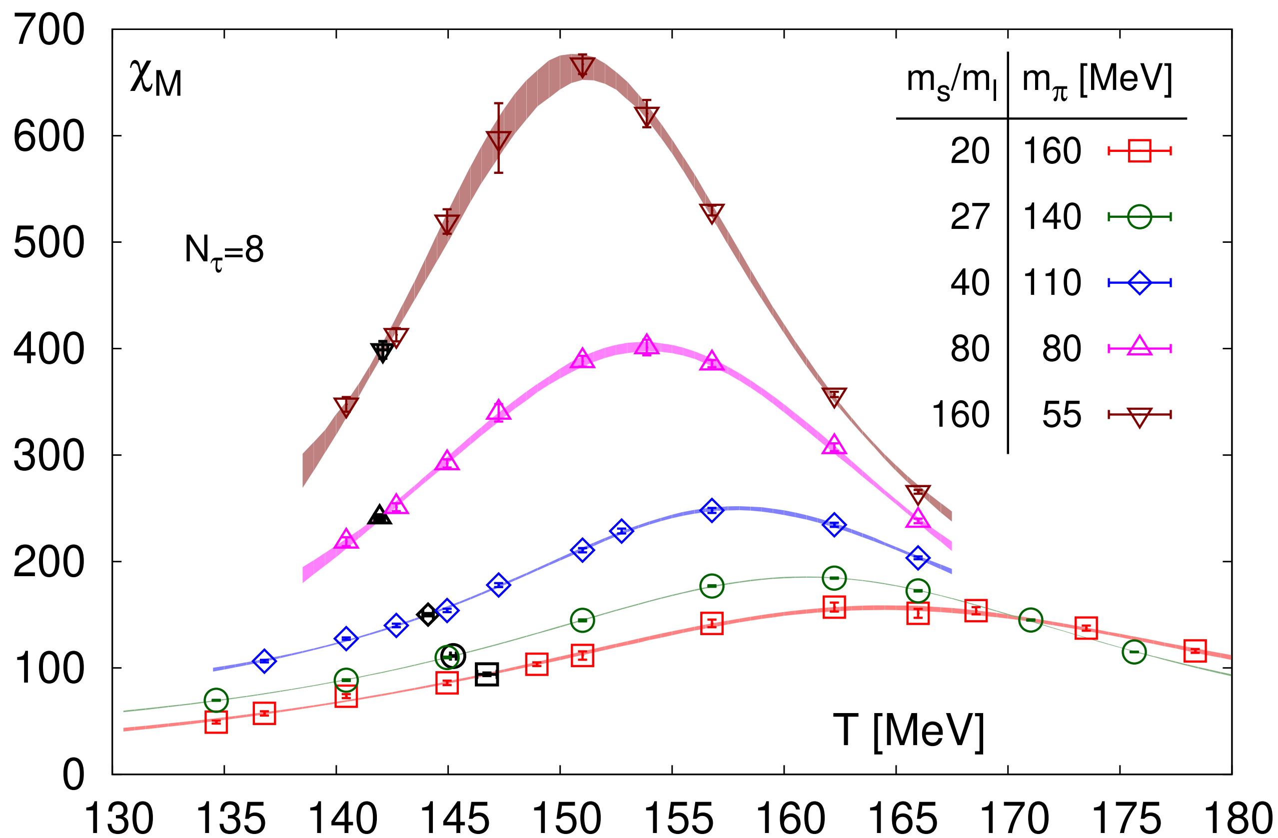
Quark mass and volume dependences of chiral susceptibility

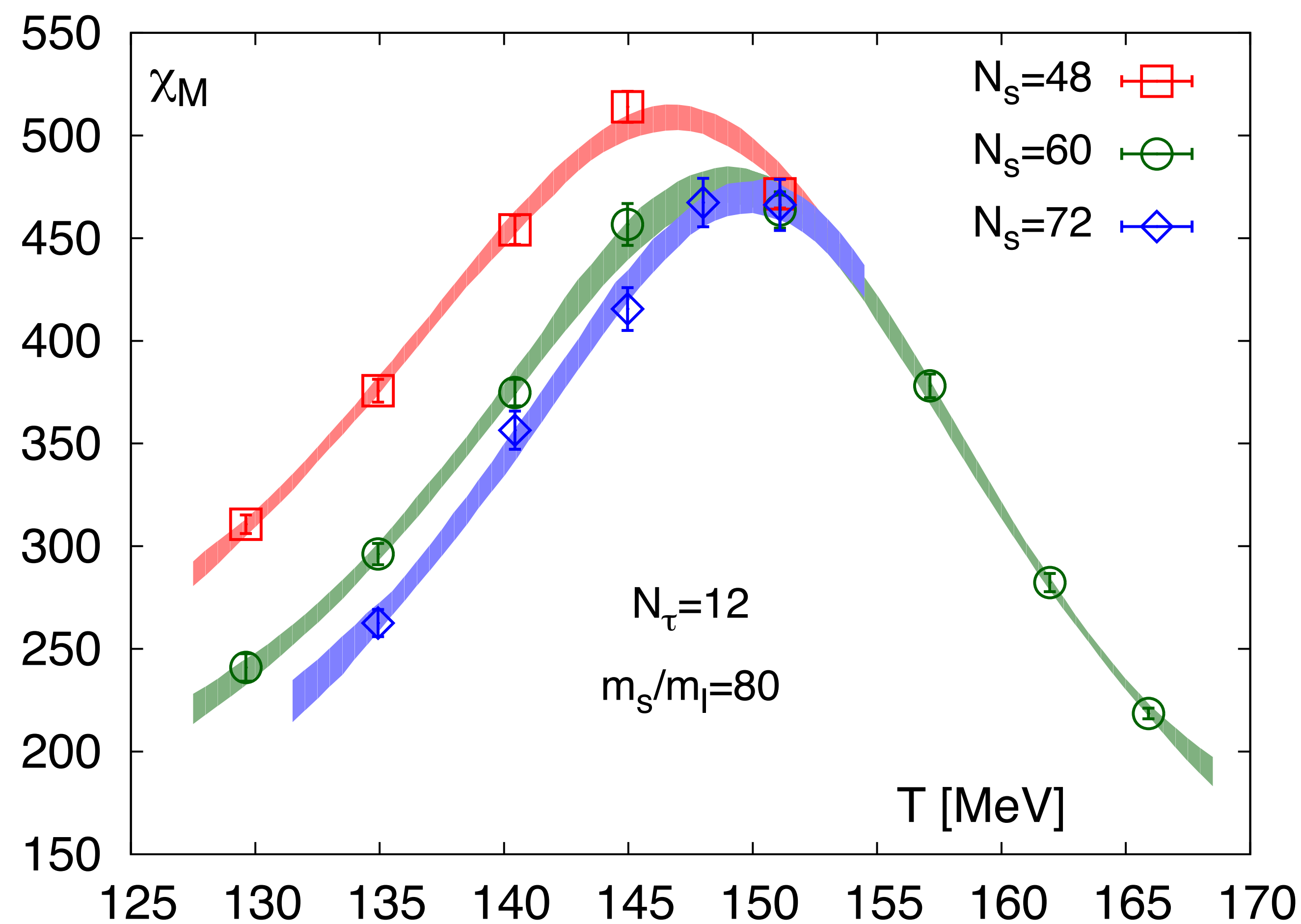




- Susceptibility increases as $m_l^{1/\delta-1}$+const, here $\delta \simeq 4.8$
- Peak height of susceptibility slightly changes with Volume
- Consistent with a continuous phase transition with O(N) universality class in the chiral limit of m_l

Quark mass and volume dependences of chiral susceptibility





- Susceptibility increases as $m_l^{1/\delta-1}$+const, here $\delta \simeq 4.8$
- Peak height of susceptibility slightly changes with Volume
- Consistent with a continuous phase transition with O(N) universality class in the chiral limit of m_l

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m,T) = h^{1+1/\delta} f_s(z) \,, \qquad z = t/h^{1/\beta\delta}$$

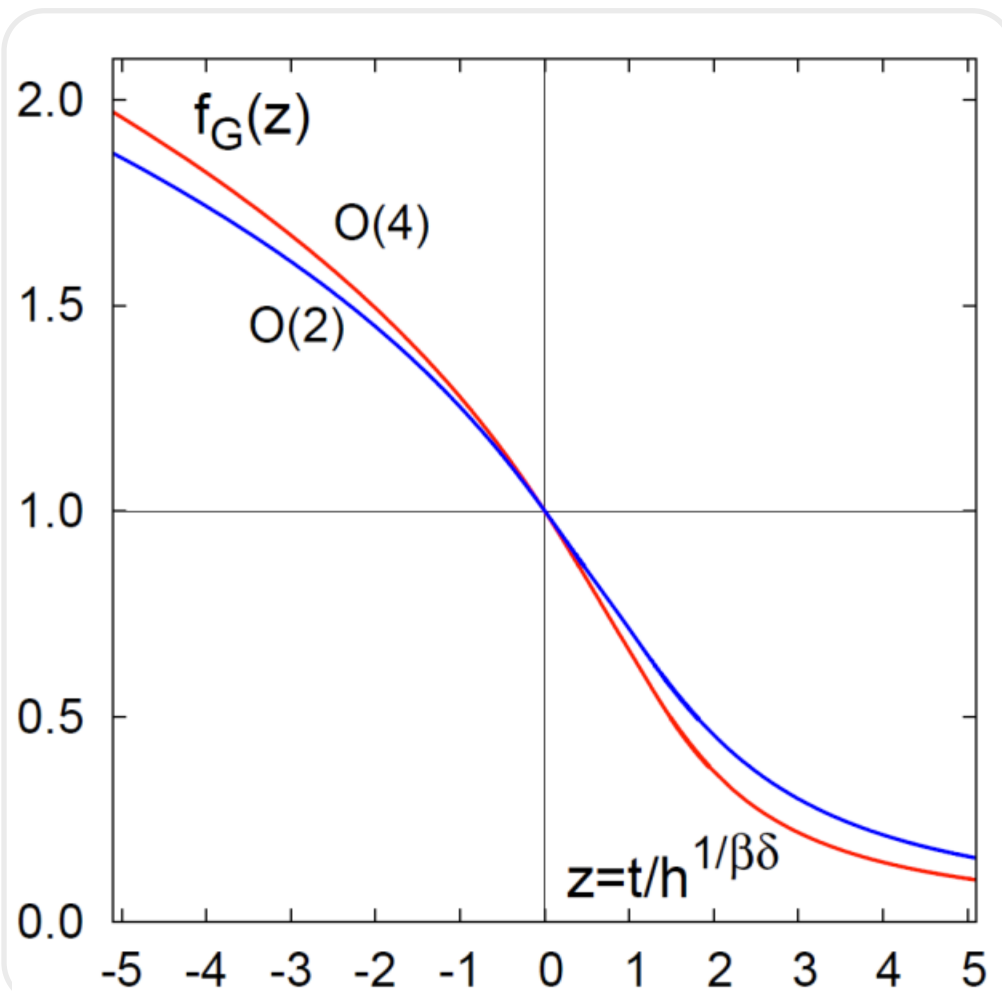
h: external field, t: reduced temperature, β,δ: universal critical exponents

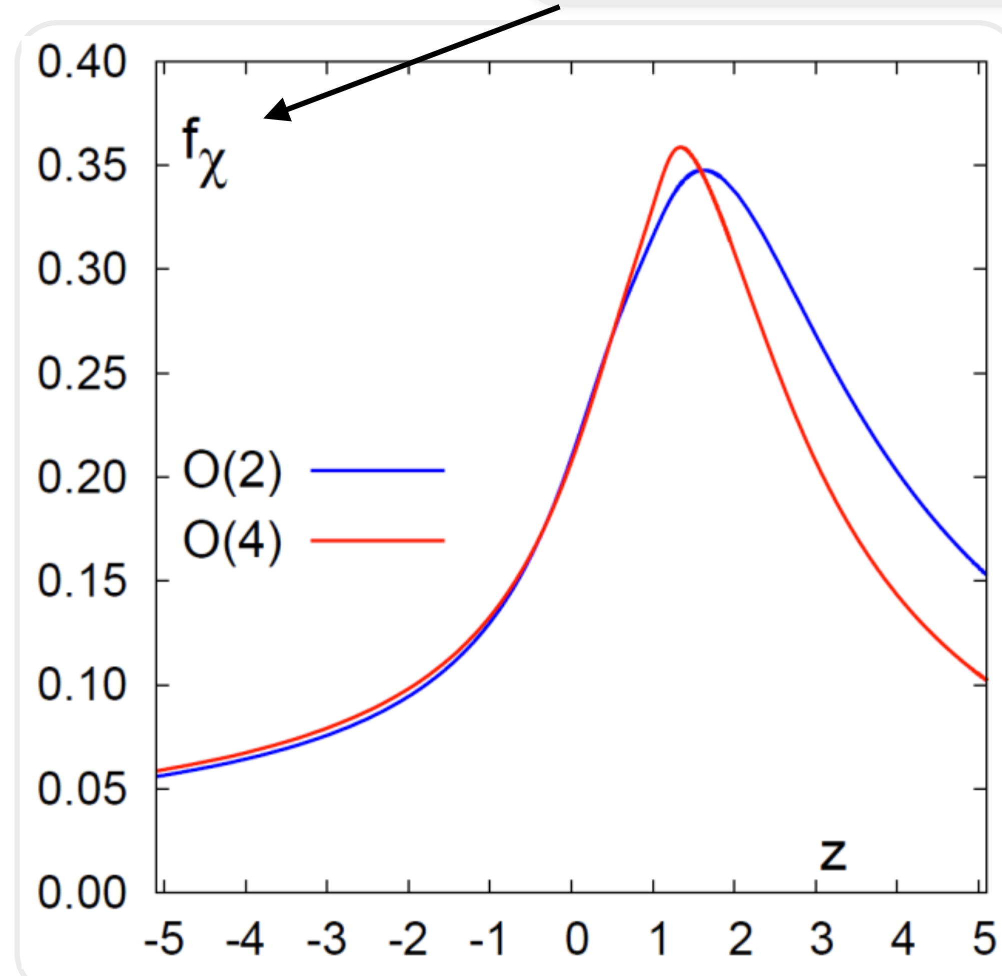
$f_s(z)$: universal scaling function, O(N) etc.

$$h = \frac{1}{h_0}\frac{m_l}{m_s} \qquad t = \frac{1}{t_0}\frac{T-T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z) \qquad \chi_M = \partial M/\partial H = \frac{h^{1/\delta}}{H}\frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$





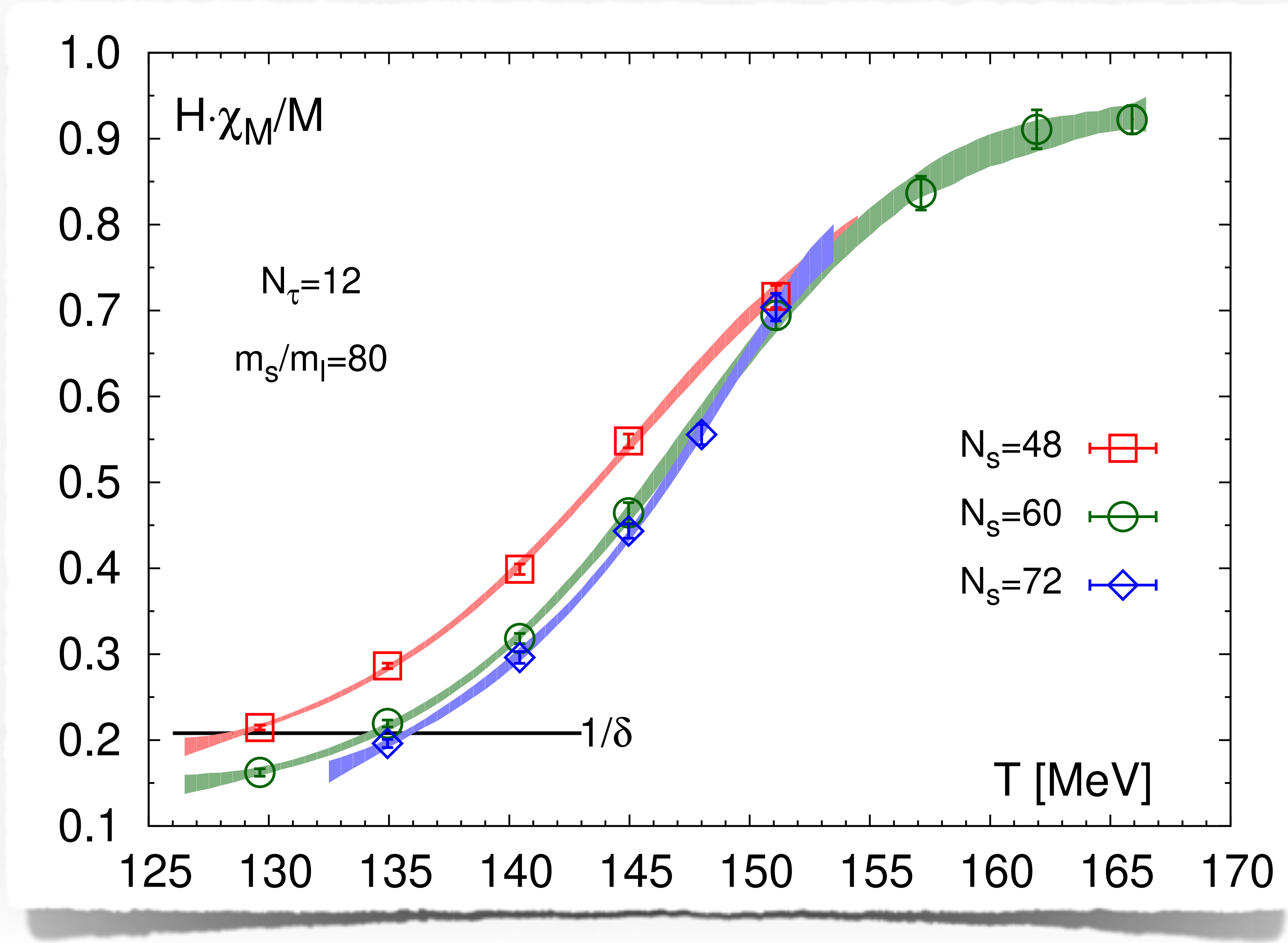
Chiral phase transition temperature T_c^0

$$M = -\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z)$$

$$\chi_M = \partial M/\partial H = \frac{h^{1/\delta}}{H}\frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$

$H\chi_M/M \rightarrow 1/\delta$ @ T_c^0

H: m_l/m_s
M: chiral condensate
χ_M: chiral susceptibility



A novel approach to estimate T_c^0

- Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

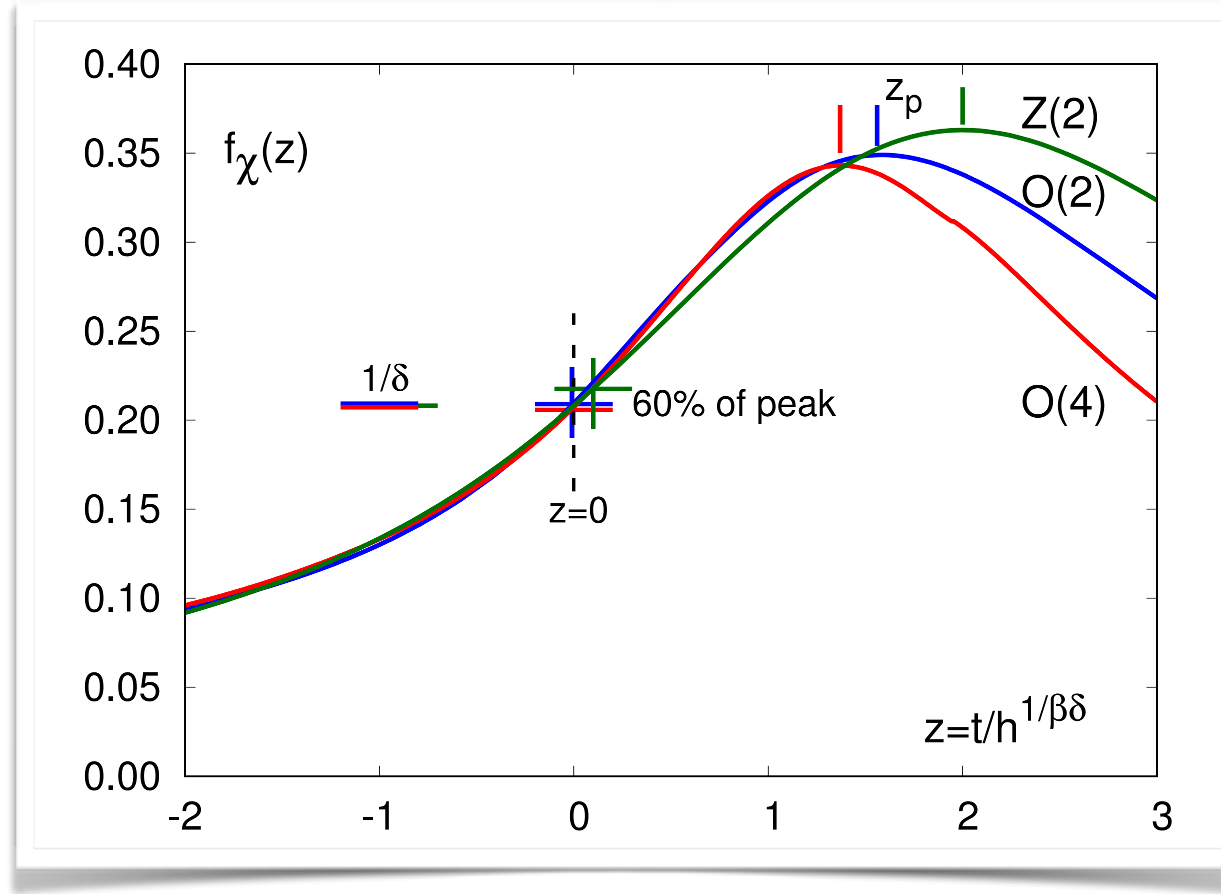
$$z = \frac{1}{t_0}\frac{T-T_c^0}{T_c^0}\left(\frac{H}{h_0}\right)^{-1/\beta\delta} = z_0 \frac{T-T_c^0}{T_c^0} H^{-1/\beta\delta}$$

- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \quad \Longleftrightarrow \quad z(T_\delta)=0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \quad \Longleftrightarrow \quad z(T_{60}) \simeq 0$$

- ☑ small quark mass dependence
- ☑ small variations among universality classes



z_p: peak location of the susceptibility

z_{60}: location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

Things need to be taken care of

- Thermodynamic limit
- Continuum limit
- Chiral limit

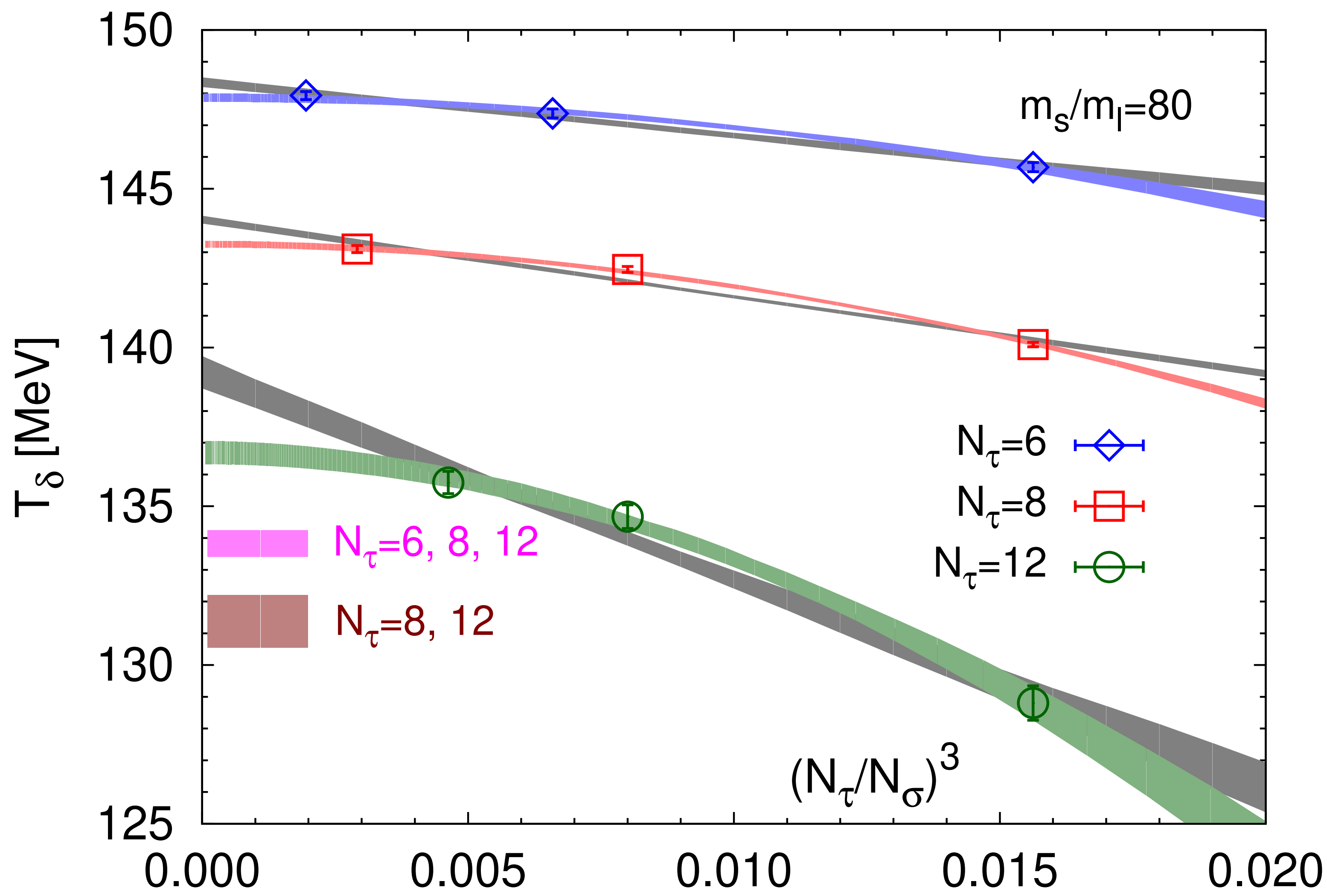
$$T_X(H,L) = T_c^0\left(1+\left(\frac{z_X(z_L)}{z_0}\right)H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$$

Singular Regular

X=60,δ

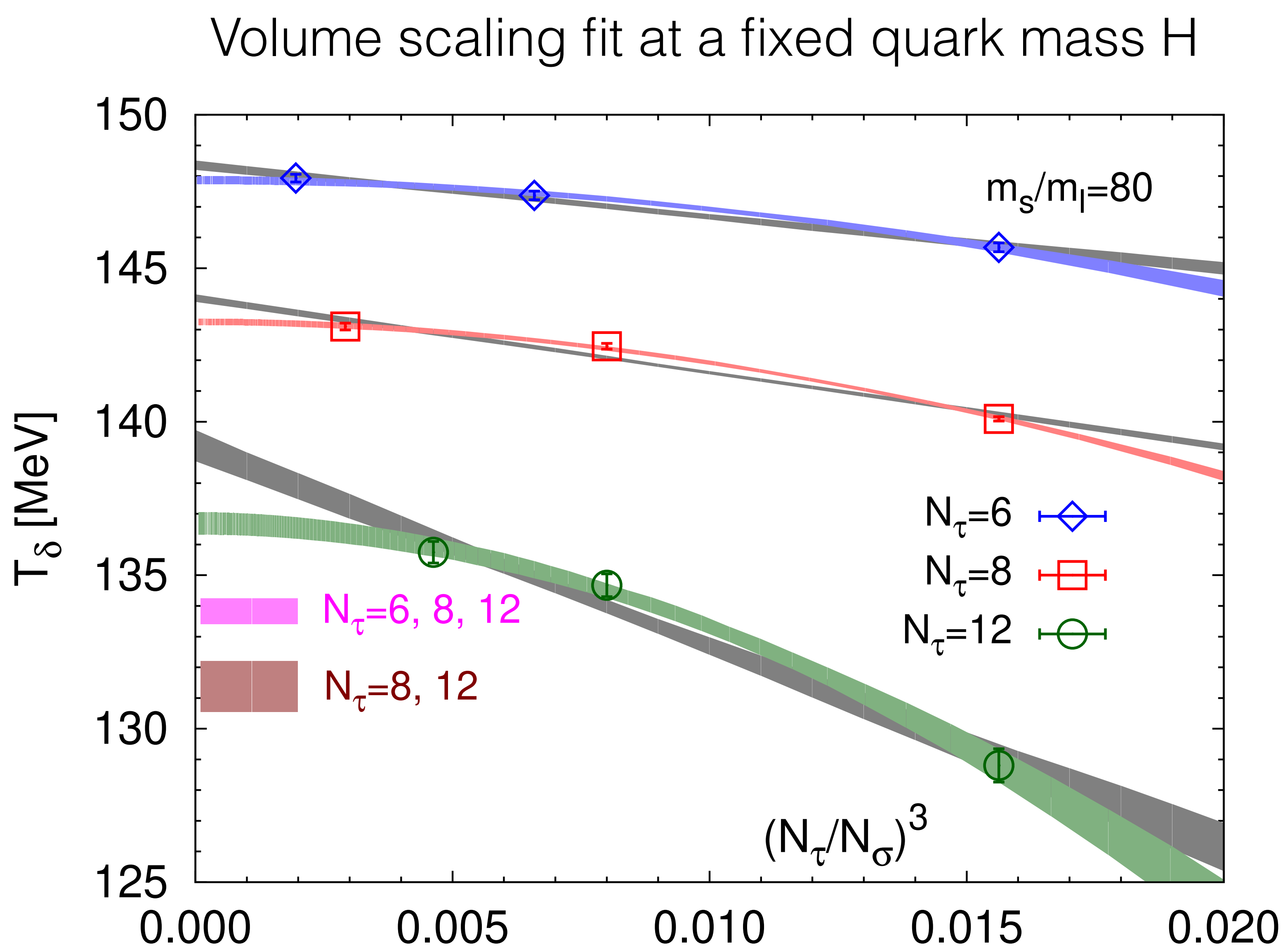
Tδ: Infinite V limit → continuum limit → chiral limit

Volume scaling fit at a fixed quark mass H



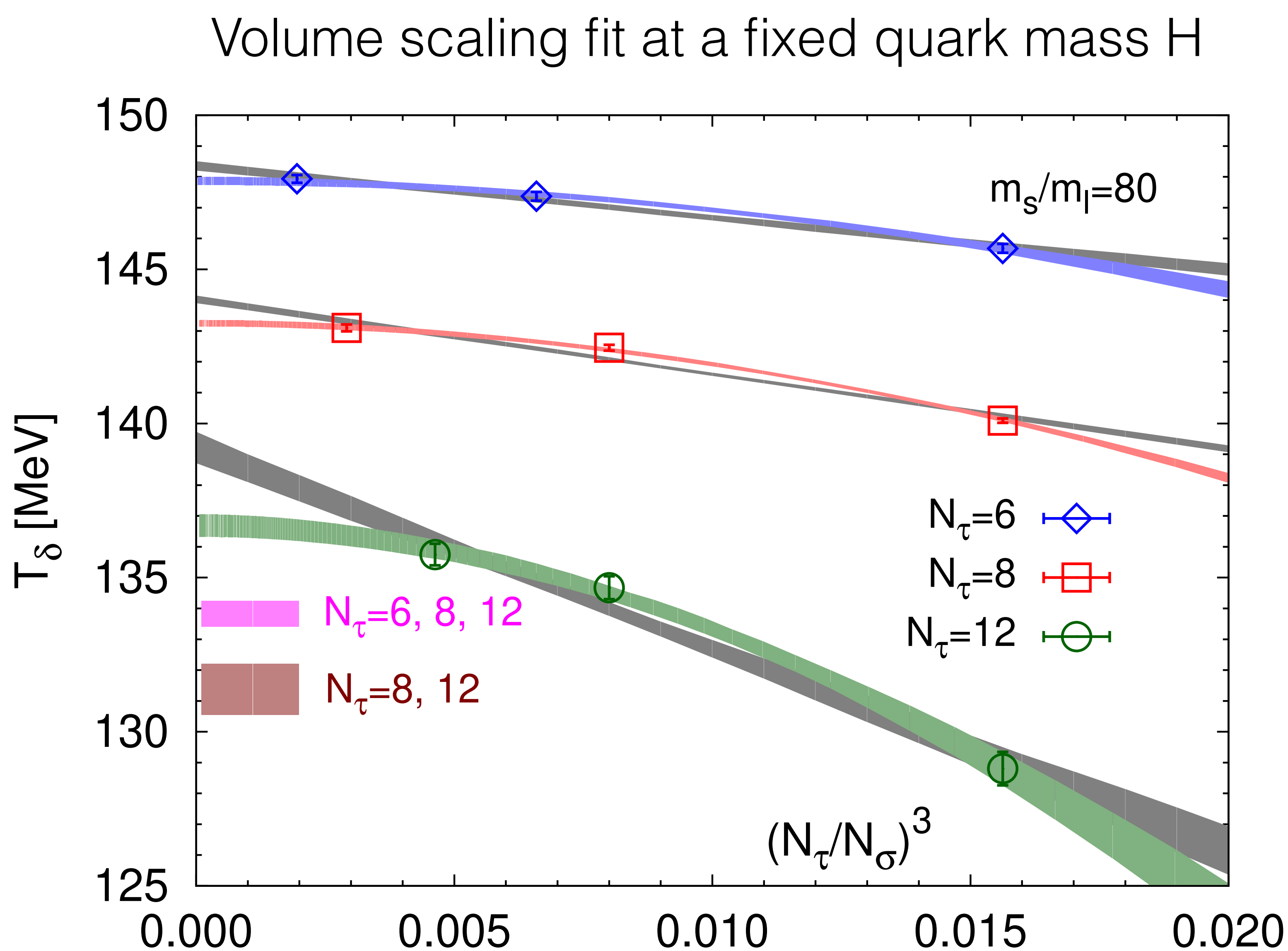
	T_δ(H,V,a) [MeV] with Nt=6,8&12	T_δ(H,V,a) [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)

T_δ: Infinite V limit → continuum limit → chiral limit



	T_δ(H,V,a) [MeV] with Nt=6,8&12	T_δ(H,V,a) [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)

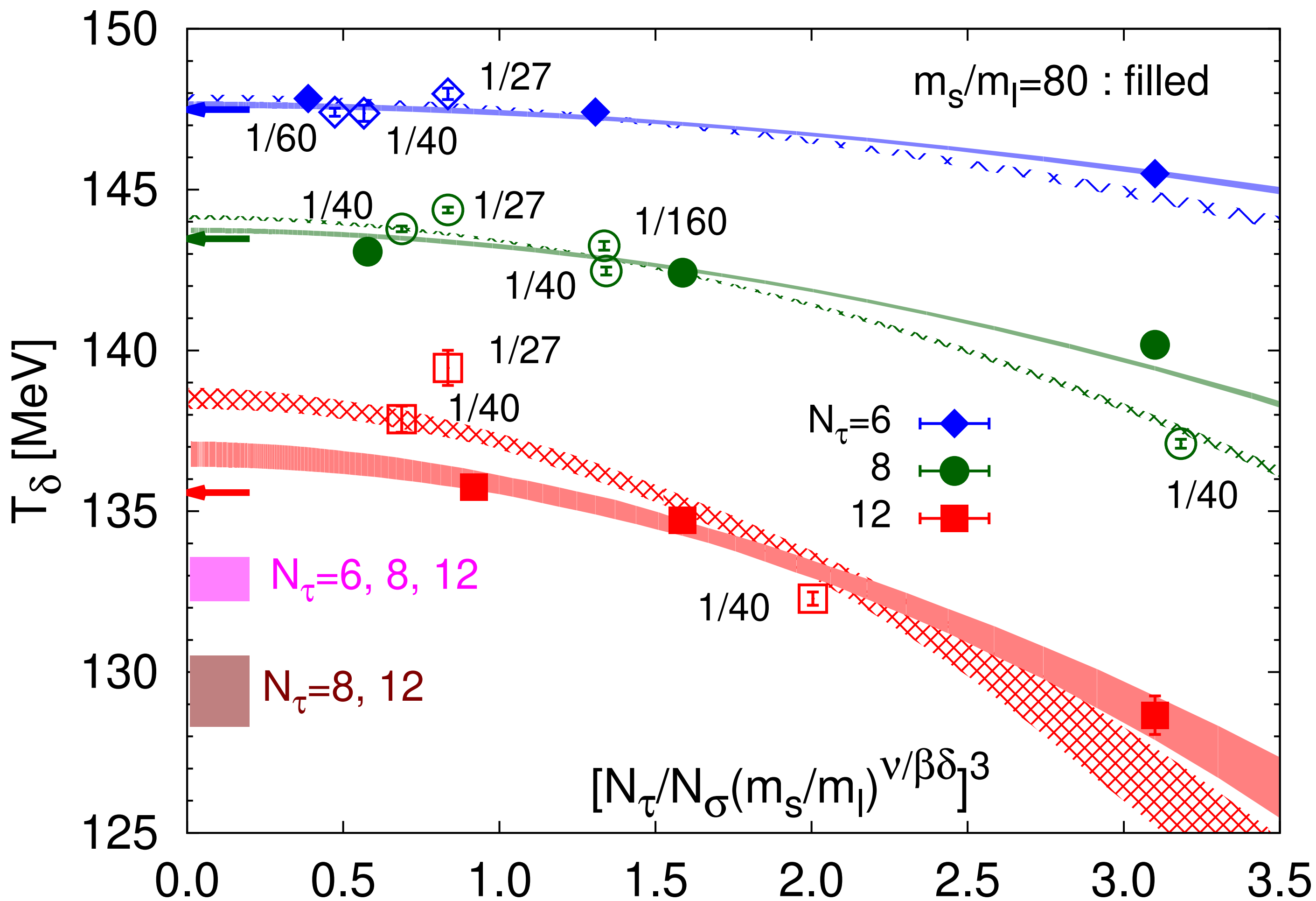
T_δ: Infinite V limit → continuum limit → chiral limit



	$T_\delta(H,V,a)$ [MeV] with Nt=6,8&12	$T_\delta(H,V,a)$ [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

T_δ: Infinite V limit → chiral limit → continuum limit

Joint volume scaling fit with all quark masses

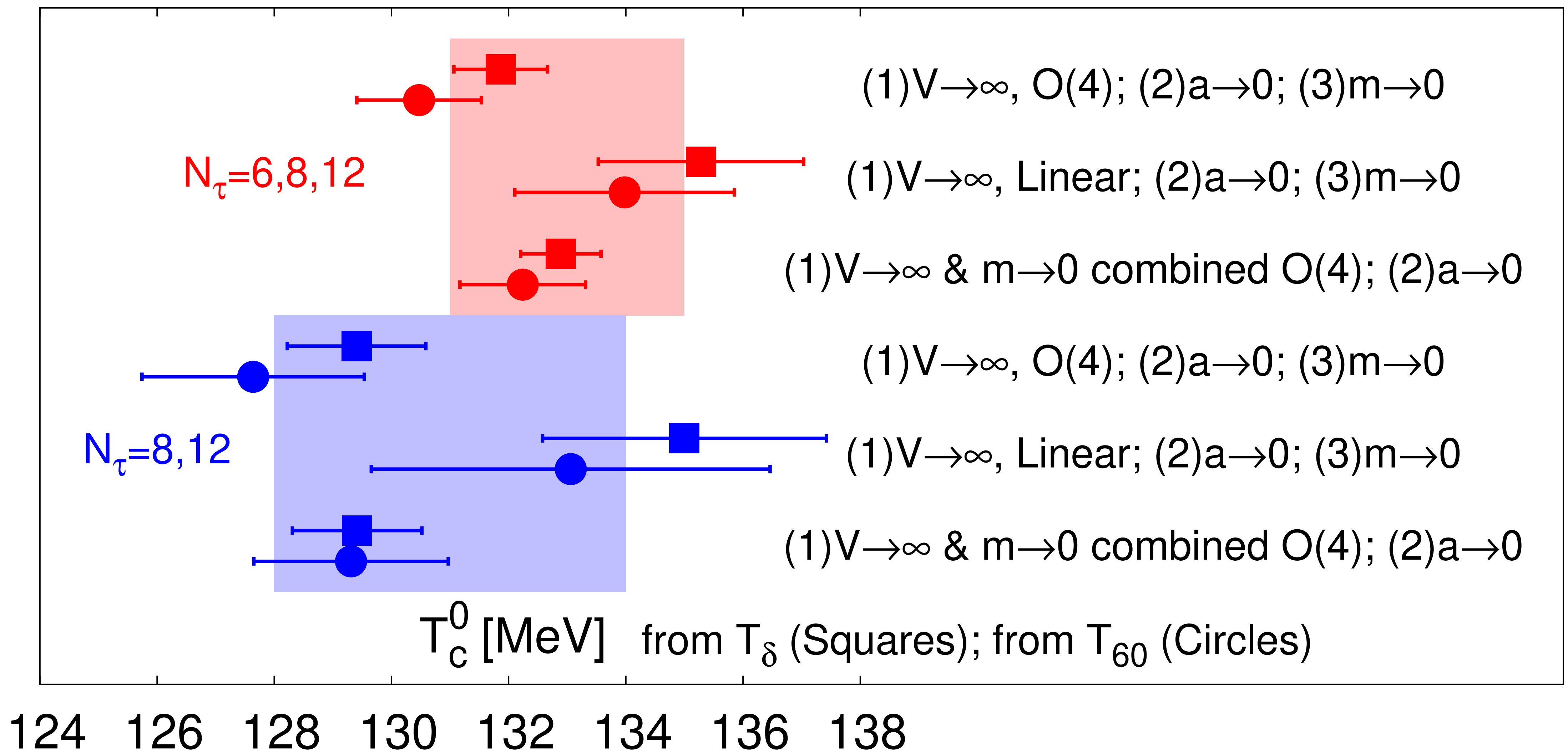


$T_\delta(H,V,a)$ [MeV]

	Nt=6,8&12	Nt=8&12
V→∞, H→0, a→0	132.9(6)	128.6(1.1)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

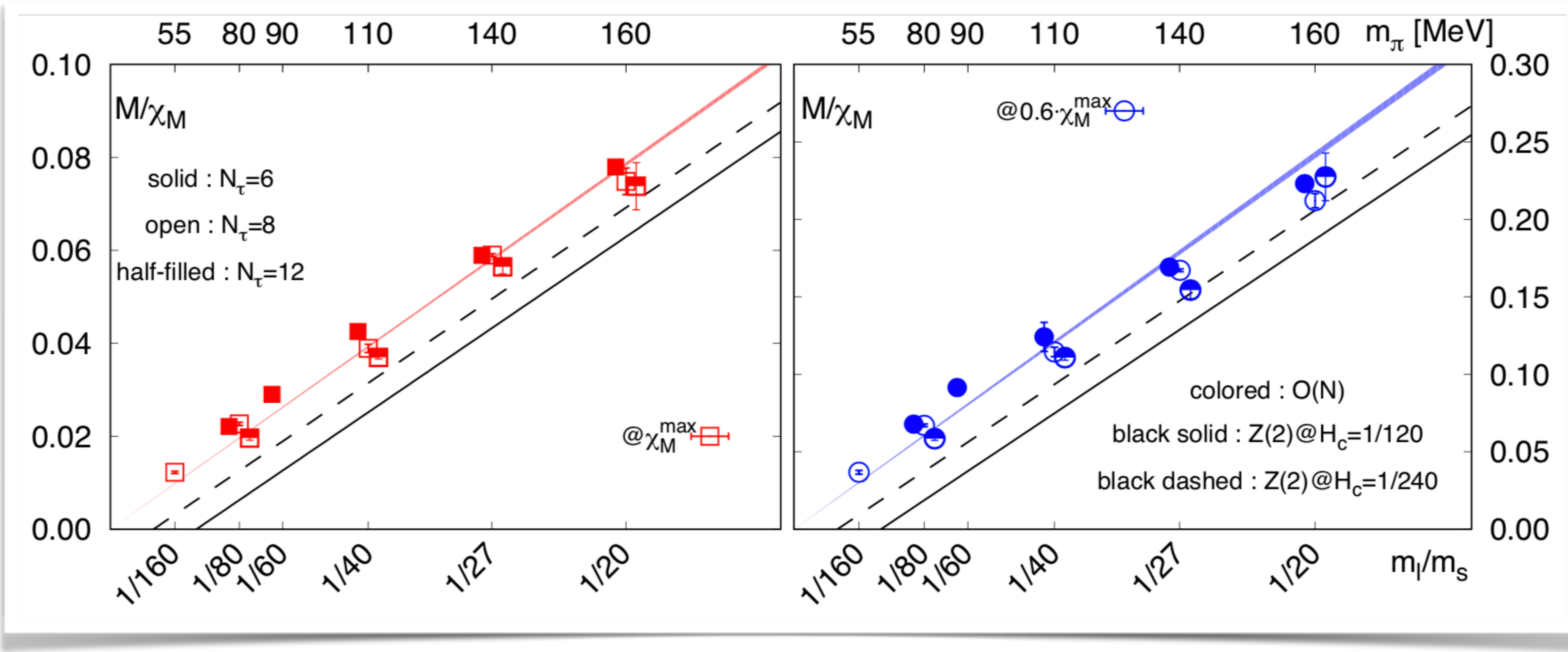
Chiral and continuum limits are Interchangeable

$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$



- T60 and T_δ give consistent results
- About 25 MeV lower than T_{pc} at the physical point!

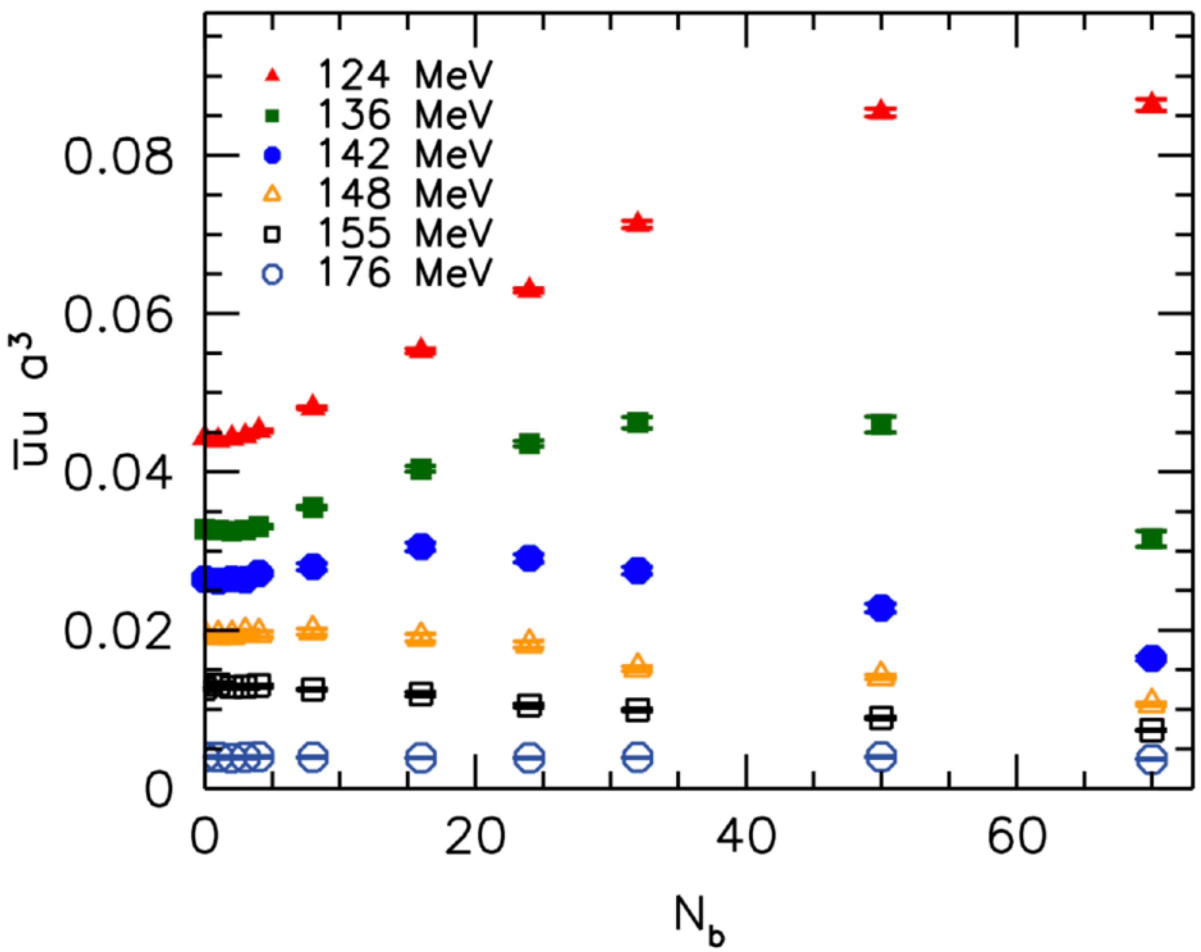
Consistency of QCD chiral phase transition with O(N) universality class

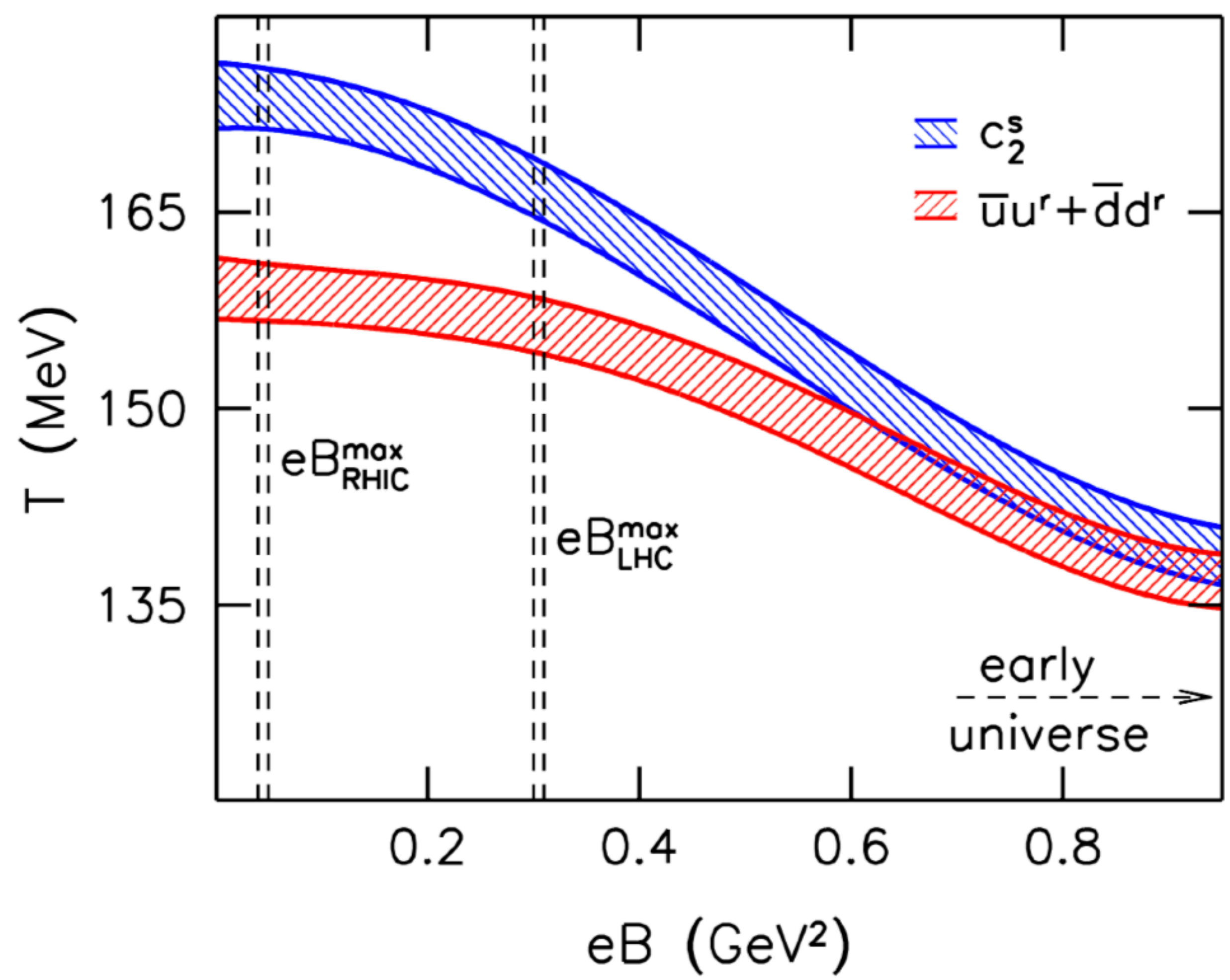


S.-T. Li(李胜泰), Lattice 2018, A. Lahiri, QM 2018

$$M/\chi_M = \frac{m_l - m_l^{critical}}{m_s^{phys}} \frac{f_M}{f_{\chi_M}}$$

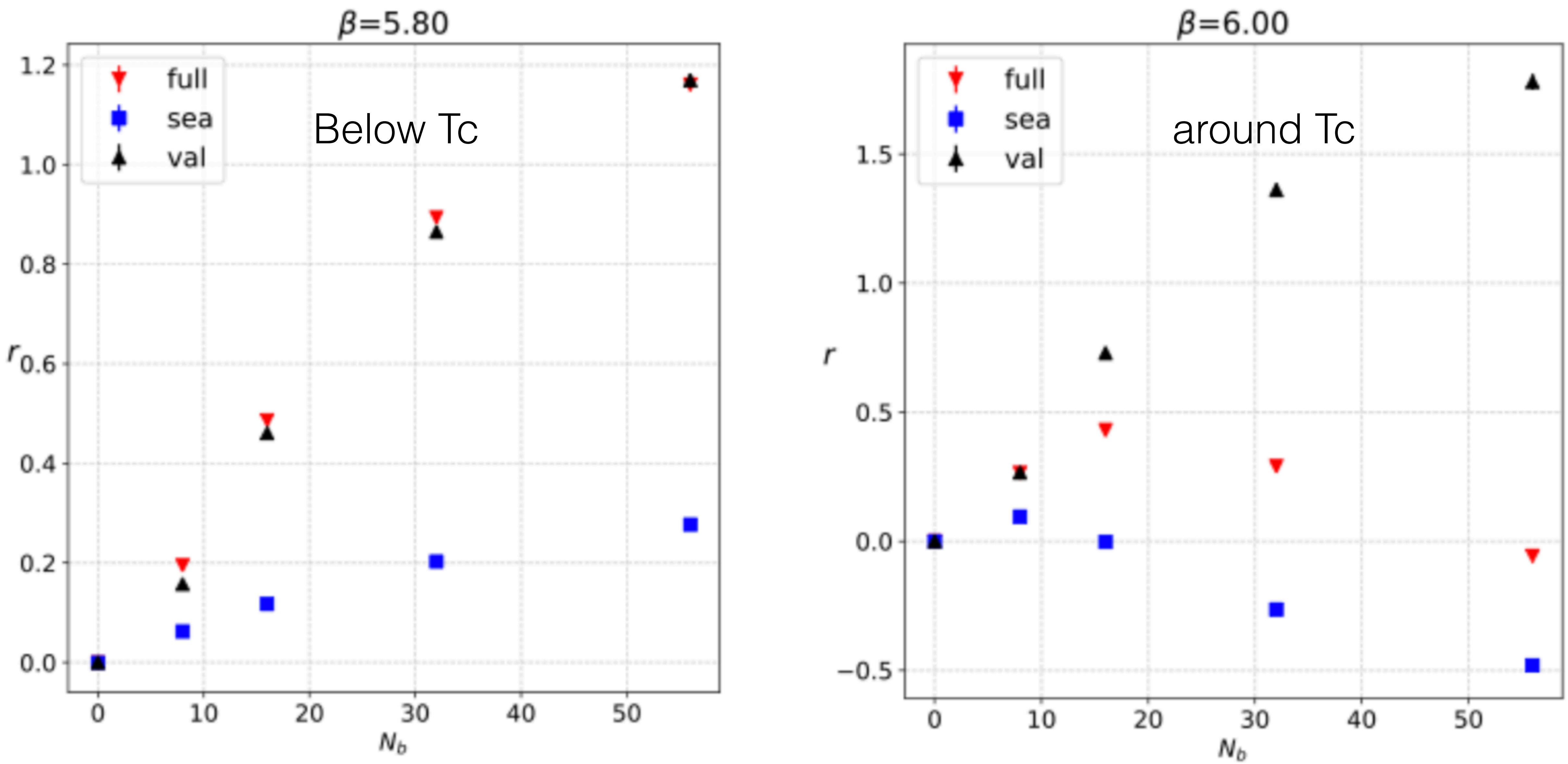
Inverse magnetic catalyses v.s. $T_C(B)$





Bali et al., JHEP02(2012)044

Effects of dynamical quarks

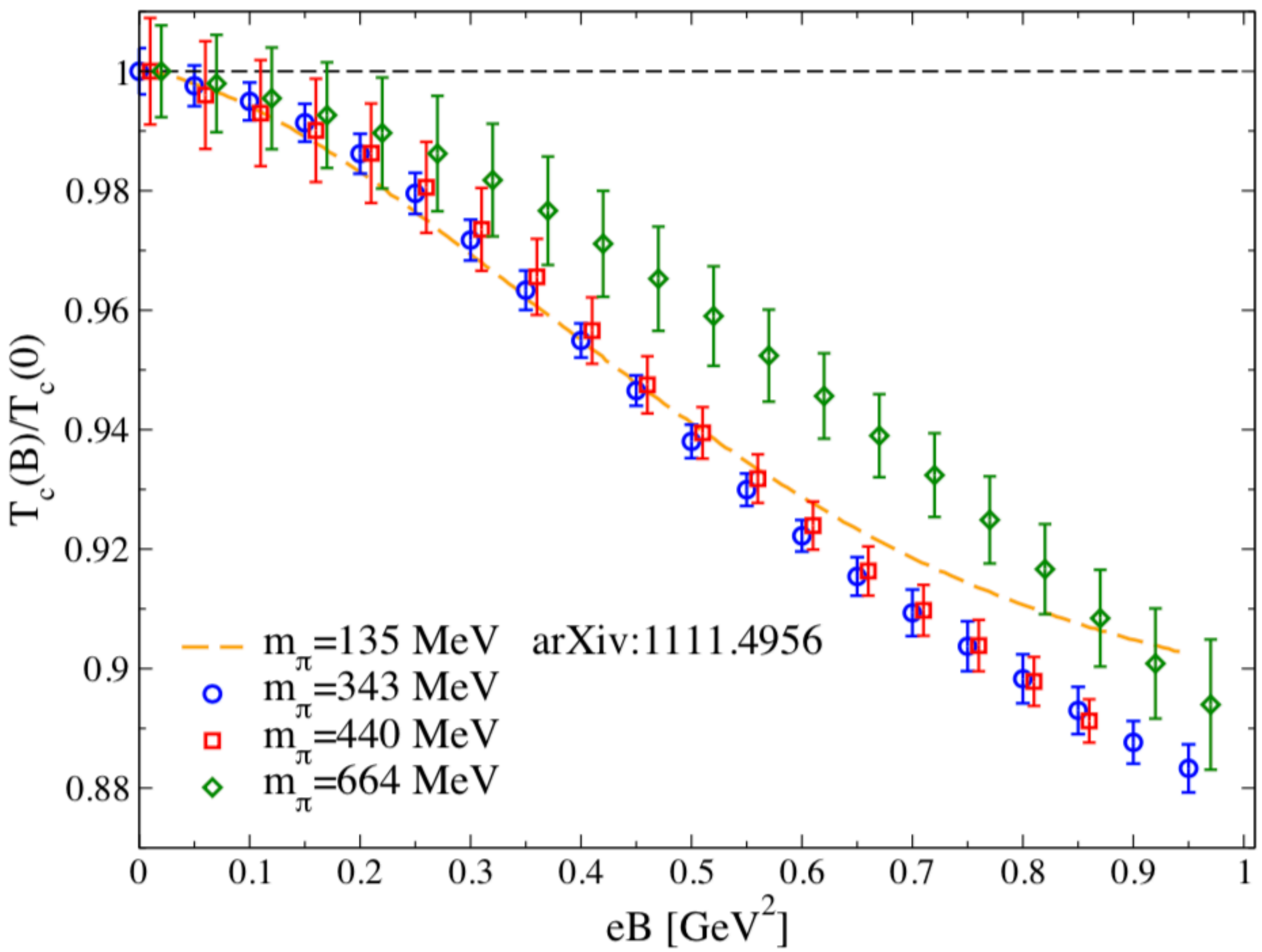


A. Tomiya, HTD, X.-D. Wang (汪晓丹), Y. Zhang (张瑜) et al., Lattice 2018, arXiv: 1904.01276

sea quarks play an important role in the inverse magnetic catalyses

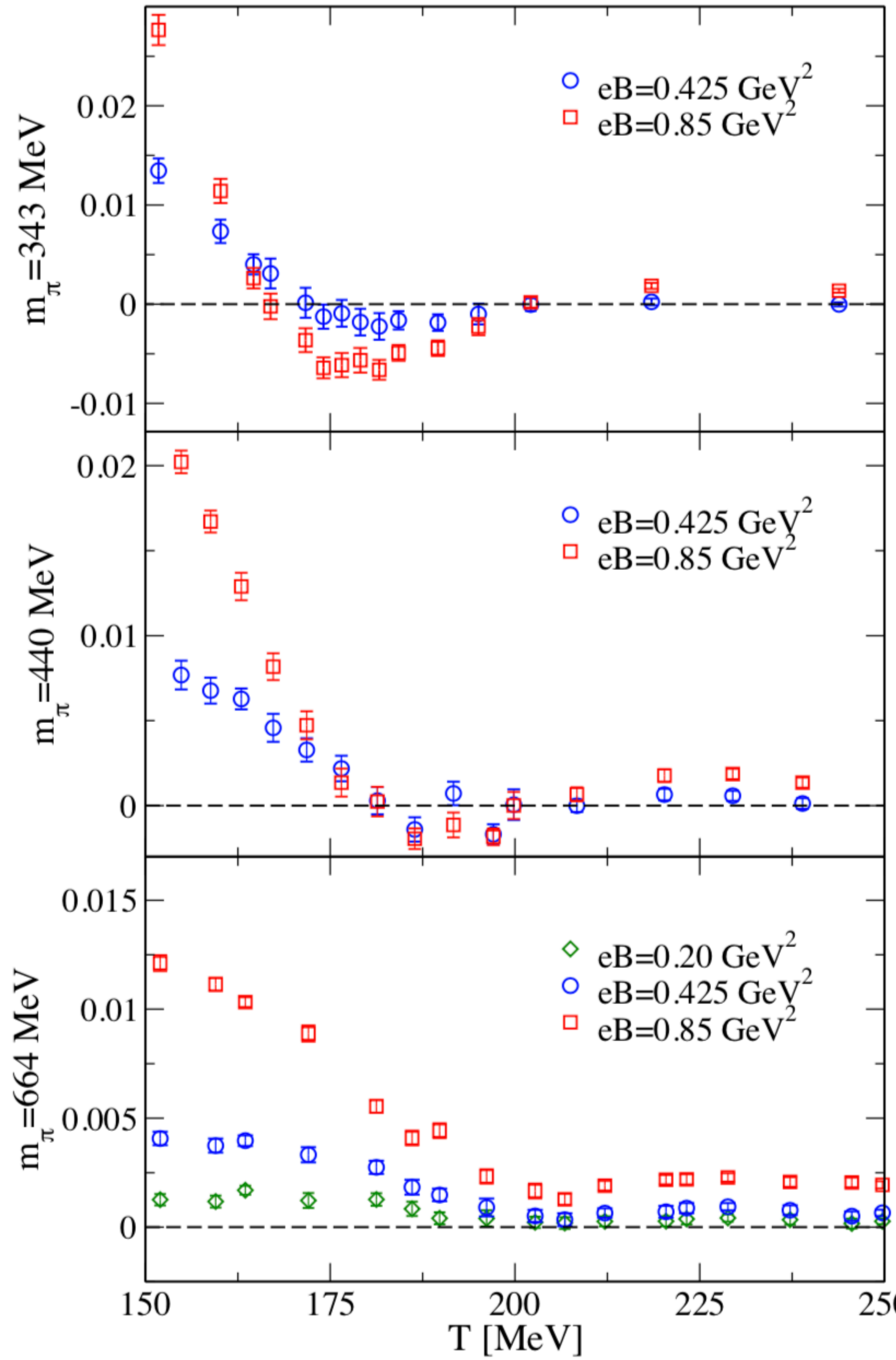
Similar findings from D'Elia et al., 1808.07008, Bruckmann, Endrodi, Kovcas, JHEP 1304(2013)112

Cases with different pion masses



D'Elia et al., 1808.07008

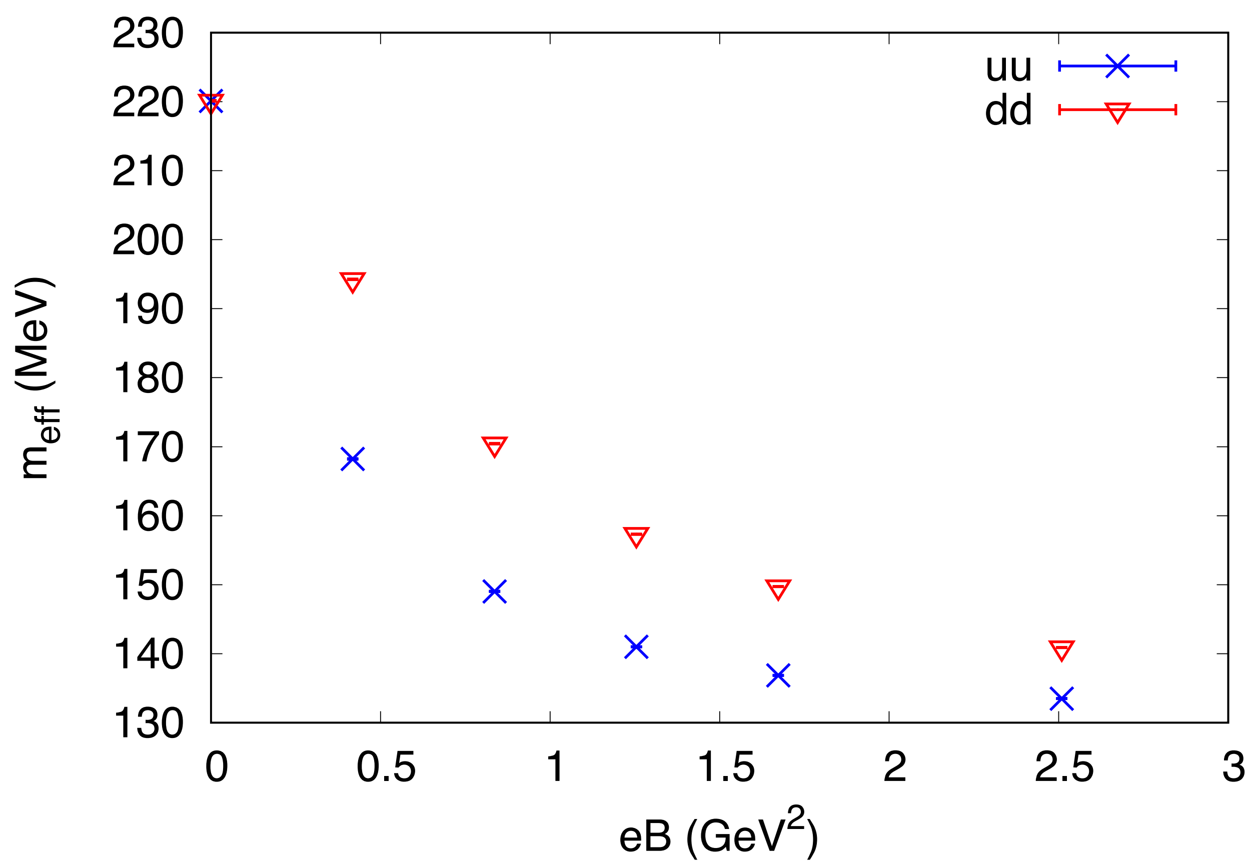
$$\langle\bar{\psi}\psi\rangle(B\neq 0)-\langle\bar{\psi}\psi\rangle(B=0)$$

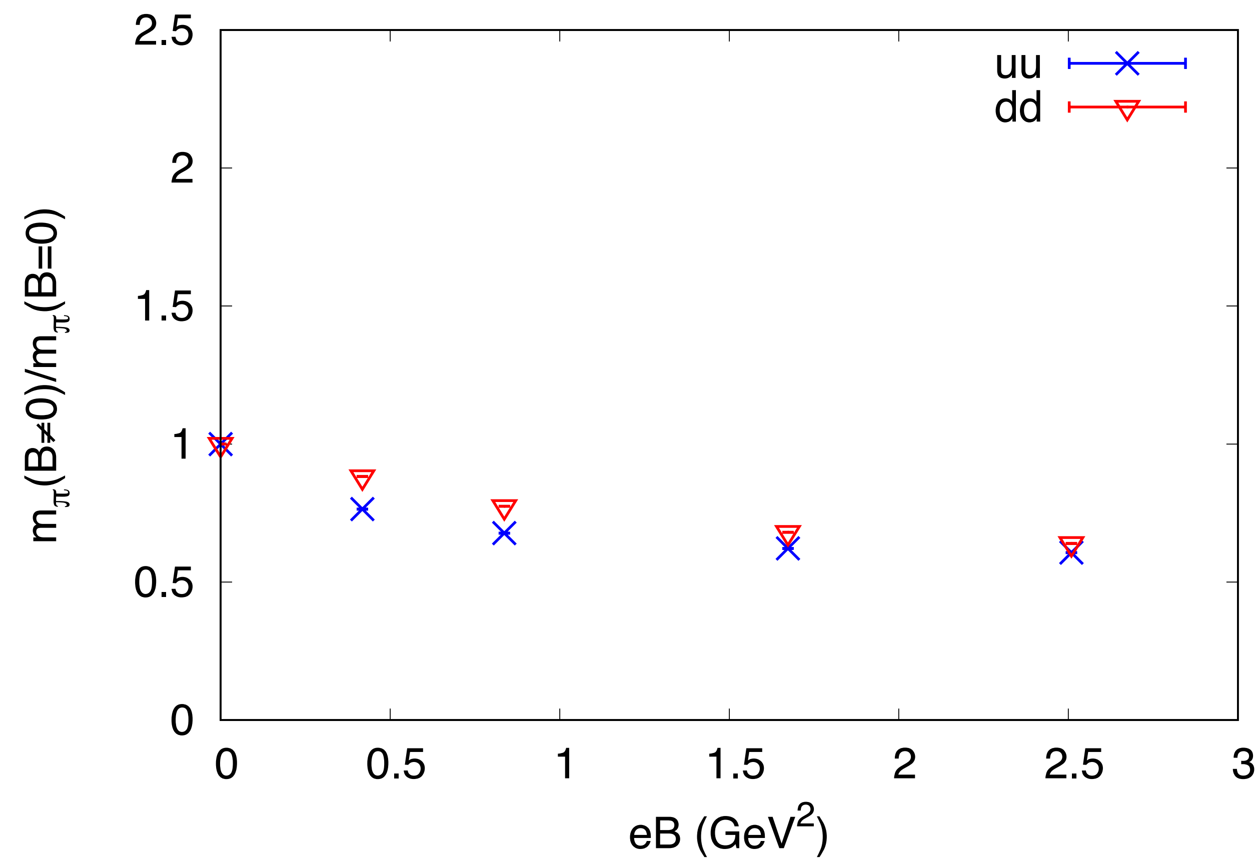


$T_c(B)$ decreases with B
along with magnetic catalyses

Change of lightest pion in external magnetic field at T=0

32^3x96, T=0, $m_\pi \approx$ 220 MeV, N_f=2+1 with HISQ/tree action





HTD, A. Tomiya, S. Mukherjee, Xiao-Dan Wang (汪晓丹) et al., work in progress

Natural pions become lighter, similar to results from quenched QCD

quenched QCD: Bali et al., PRD 18'

Conclusions

☑ chiral crossover temperature is determined with better precision, i.e. T_{pc} = 156.5(1.5) MeV

☑ The chiral Tc of Nf=2+1 QCD is 132^{+3}_{-6} MeV, and the O(N) universality class of the chiral phase transition is favored

☑ Decreasing of T_c with B may be more relevant with the reduction of neutral pion mass

谢谢！

Thanks for your attention!

Lattice 2019

37th series

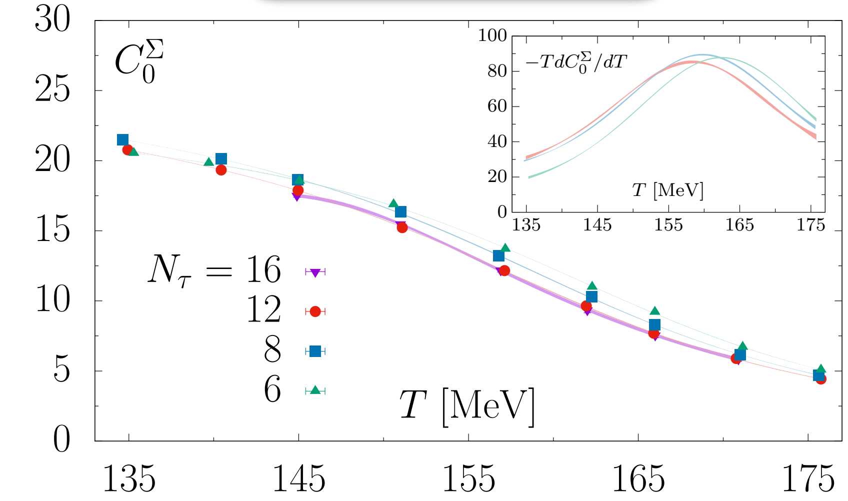
June 16-22, 2019
Wuhan, China

~ 400 participants annually

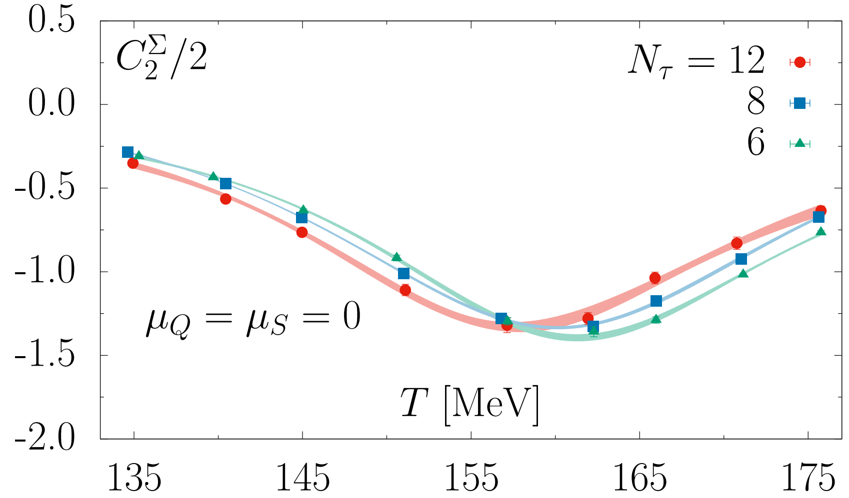
http://lattice2019.ccnu.edu.cn

You are welcome
to join us!

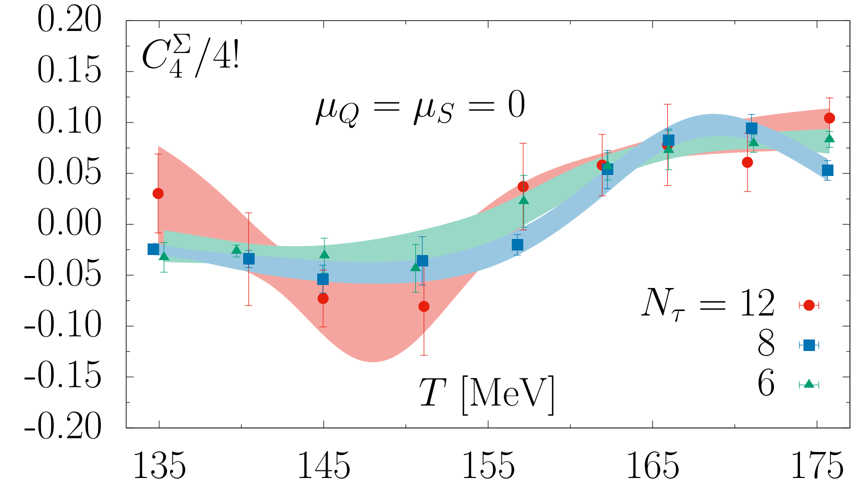
$$\partial_T^2 C_0^\Sigma(T) = 0$$

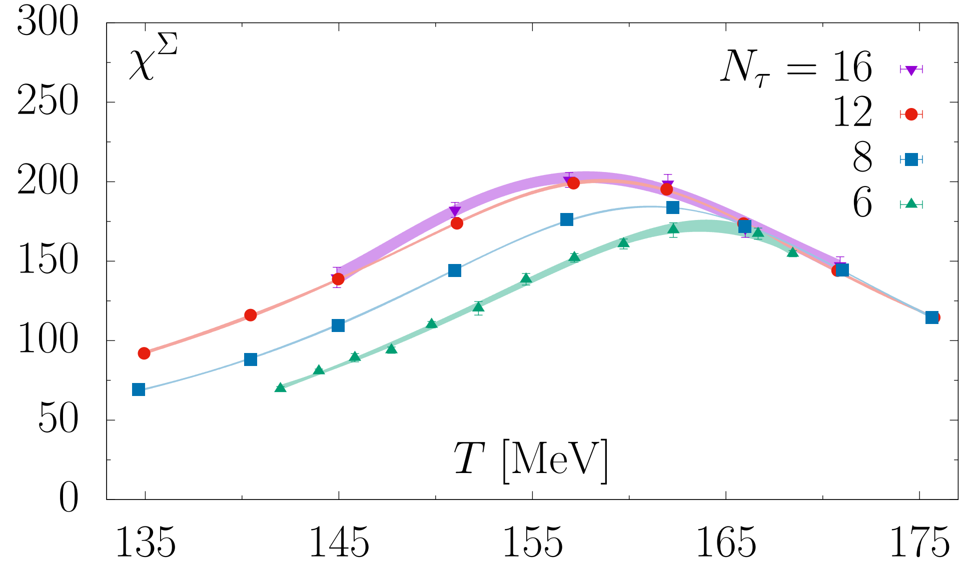


$$\partial_T C_2^\Sigma(T) = 0$$



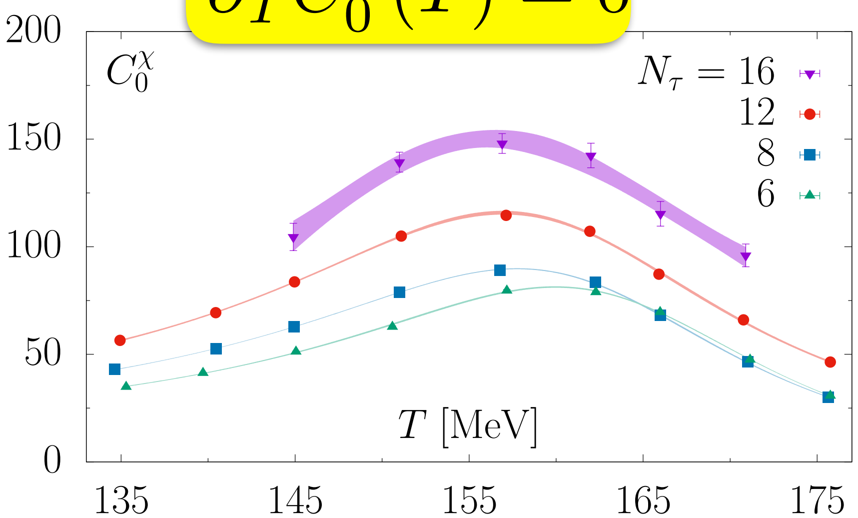
$$C_4^\Sigma(T = T_{pc}) = 0$$



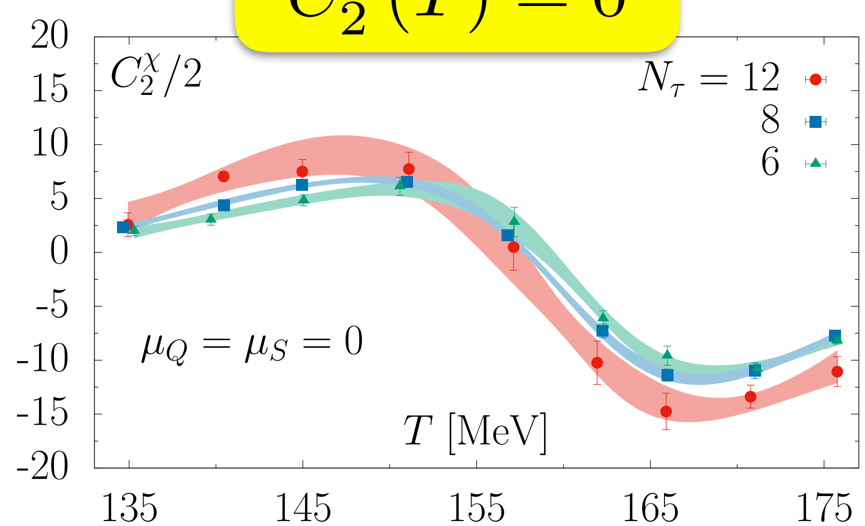


$$\partial_T \chi^\Sigma(T) = 0$$

$$\partial_T C_0^\chi(T) = 0$$



$$C_2^\chi(T) = 0$$



$$\partial_T C_4^\chi(T = T_{pc}) = 0$$