SMEFT from on-shell amplitudes

Jing Shu

ITP-CAS

T. Ma, J. Shu., M-L. Xiao, 1902.06752

Outline

Why I am thinking about this?

EFT has wide applications on various subjects, from high energy physics to QCD, condense matter, even atomic or molecule physics. (kernel of modern QFT)

For high energy physics, there are already marvelous applications to flavor physics, precision EW physics, DM categorization, etc. More recently, precision Higgs measurements from LHC data and future colliders

All above things based on Wilson's EFT approach!

Outline

Why I am thinking about this?

Very recently, there has been several nice very cute results on specific EFT related to phenomenology by using on-shell amplitudes.

Non-renormalization: C. Cheung, C.H. Shen, Phys.Rev.Lett. 115, 7, 071601 (2015)

Helicity selection rule: D. Azatov, R. Contino, C. Machado, F. Riva., Phys. Rev. D. 95, 6, 065014 (2017)

Soft theorem & NGBs
I. Low, Phys.Rev.D 93, 4, 045032 (2016)
I. Low, Z-w Yin, Phys.Rev.Lett. 120, 6, 061601 (2018)
D. Liu, I. Low, Z-w Yin, Phys.Rev.Lett. 121, 26, 261802 (2018)

Pretty sure I may leave out some references, many thanks if you remind me more

What I believe instead is that it is really time to think about general EFTs like SMEFT, using on-shell amplitude methods systematically.

no real special symmetry constrains like (soft-limit, N=4, etc), Wilson coefficient as theory input!

Outline

In contrast to the big landscape I mention previously, here we come to some specific questions:

With revived interests of SMEFT, how to easily define the complete sets of independent operators?

Dim 5: Weinberg operator — S. Weinberg, Phys. Rev. D 22, (1980) 1694

Dim 6: "Warsaw basis" — W. Buchmuller, D. Wyler, Nucl. Phys. B 268, (1986); B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, JHEP 1010, 085 (2010)

Dim 7: L. Lehman, Phys. Rev. D. 90, (2004) 125023; Y. Liao, X.D. Ma, JHEP 1611, 043, (2016)

See also Prof. Liao's talk

Partially Dim 8: L. Lehman, A. Martin, JHEP 1602, 081, (2016); B. Henning, X. Lu, T. Melia, H. Murayama, JHEP 1708, 016, (2017)

Outline

Operators can be redundant because of

- Equation of motion (EOM)
- Integration by parts (IBP).

- How to count independent operators using on-shell amplitude methods (amplitude basis).

In SMEFT, previous studies has been focused on using the conformal symmetries based on Hilbert series

E. Jenkins, A. Manohar, JHEP 0910, 094, (2009)

L. Lehman, A. Martin, JHEP 1602, 081, (2016)

B. Henning, X. Lu, T. Melia, H. Murayama, JHEP 1708, 016, (2017)

B. Henning, X. Lu, T. Melia, H. Murayama, JHEP 1710, 199, (2017)

Using the on-shell methods

However, for real on-shell amplitudes, there is no such issues of redundancies, see also nice discussions in

Y. Shadmi, Y. Weiss, arxiv: 1809.09644

There is a well defined one-to-one amplitude operator correspondence (amplitude basis) that can help us count and write down the independent operates

- On shell condition => E.O.M.
- IBP = momentum conservation.

We simply looking at independent (unfactorizable) amplitudes!!!

Unfactorizable amplitude

For a renormalizable theory:

Very small independent amplitudes, mostly 3-point functions!!!

4-point amplitudes in Yang Mills theory is a product of two 3-point amplitudes with single poles (t or u)

For a non-renormalizable theory:

Without any extra symmetry, only the "covariant" part can be generated recursively (not independent), related through the redundant gauge symmetry

The leading operators, technically the following replacements give the unfactorizable amplitudes (infinite number of them)

$$\text{“}F_{\mu\nu} \to \partial_\mu A_\nu - \partial_\nu A_\mu\text{”} \qquad \text{“}D_\mu \to \partial_\mu\text{”}$$

Any amplitudes not constructed redundantly by gauge symmetry

Structure of amplitudes

$$\mathcal{M}_{\{\alpha\}} = f(\lambda_i, \tilde{\lambda}_i) g(s_{ij}) T_{\{\alpha\}},$$

$T_{\{\alpha\}}$ group factor

$f(\lambda_i, \tilde{\lambda}_i)$ determined by the external legs (trivial for scalars)

notice some times need to insert p to make it Lorentz invariant

$g(s_{ij})$ polynomial (positive power) of Mandelstam variables

adding the derivatives (covariant derivative expansion)

For EFT, Talyer expanding the amplitude around the IR origin as the polynomials

For leading one, we set g=1, primary amplitudes (minimal scalars)

Polynomials has no physical poles, so all unfactorizable!!!

Building blocks of SM

Basic building blocks in operators

$F^{\pm}_{\mu\nu} \equiv \frac{1}{2}(F_{\mu\nu} \pm i\tilde{F}_{\mu\nu})$ $(\tilde{F}_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma})$ SMEFT: massless

$\psi_L,\ \psi^c_L,\ \phi$ transforms as $(1,0),\ (0,1),\ (1/2,0),\ (0,1/2)$ $(0,0)$.

All we have to do is to count the dimension of amplitudes & operators

In the spin-helicity formulism, the dim of "brackets" is one less than the fields from operators

$$d = n + m = n + [f] + [g]$$

Up to dim 6, list all the possible combinations (n_ψ, n_A, h)

h>=0 h is the total helicity The other is just the hermitian conjugate

Why on-shell amplitudes?

Maximally use the Lorentz symmetry, locality

Spinor helicity variables:

$|p> = u_+(p)$ (dotted)

$|p] = u_-(p)$ (undotted)

$$\langle ij\rangle = \epsilon^{\alpha\beta}|i\rangle_\alpha|j\rangle_\beta \quad [ij] = \epsilon^{\dot\alpha\dot\beta}|i]_{\dot\alpha}|j]_{\dot\beta}$$

$$p_{\alpha\dot\alpha} = p_\mu\sigma^\mu_{\alpha\dot\alpha}$$

$$s_{ij} = (p_i + p_j)^2 = 2p_i.p_j = \langle ij\rangle[ij].$$

$$p_{\alpha\dot\alpha} = -|p]_\alpha\langle p|_{\dot\alpha}.$$

$$\det[p] = p_\mu p^\mu,$$

Little group rescaling

$$|p\rangle \to t|p\rangle \quad [p| \to t^{-1}[p|.$$

$p = (p, 0,0, p)$: LG = SO(2)

$$A_n(1^{h_1}, 2^{h_2}, ..., n^{h_n}) \to \prod_i t^{2h_i} A_n(1^{h_1}, 2^{h_2}, ..., n^{h_n})$$

Weight is the helicity

How to write the amplitudes?

Examples:

$< ij > \cdot [ji] = s_{ij} = 0$

$$f(\psi^+\psi^+\phi^2) \sim \lambda_1\lambda_2,$$
$$f(\psi^+\psi^-\phi^2) \sim \lambda_1\tilde{\lambda}_2.$$

$$(\lambda_i\sigma^\mu\tilde{\lambda}_j)p_{k\mu} \equiv [i|p_k|j\rangle = [ik]\langle kj\rangle$$

$$f(\psi^+\psi^+\phi^2) \sim (\lambda_1\epsilon\lambda_2) \equiv [12],$$
$$f(\psi^+\psi^-\phi^2) \sim (\lambda_1\epsilon\sigma^\mu\tilde{\lambda}_2)p_{3\mu} \equiv [1|p_3|2\rangle.$$

Further adding powers of derivatives:

$$[12] \to [1|p_{ij}^2|2] \equiv [1|\sigma^{\mu\nu}|2]p_{i\mu}p_{j\nu},$$
$$[1|p|2\rangle \to [1|p_{ijk}^3|2\rangle \equiv [1|\sigma^{\mu\nu\rho}|2\rangle p_{i\mu}p_{j\nu}p_{k\rho},$$

Schouten identity $[12][34] + [13][42] + [14][23] = 0.$

The rest is not that powerful for on-shell methods (Lorentz)

The primary amplitudes

(n_ψ, n_A, h)	Primary amplitude	m_{min}	n_s	d_{min}
(0,0,0)	$f(\phi^{n_s}) = 1$	0	$n_s \geqslant 3$	3
(0,2,2)	$f(A^+A^+\phi^{n_s}) = [12]^2$	2		5
(0,3,3)	$f(A^+A^+A^+) = [12][23][31]$	3		6
(2,0,1)	$f(\psi^+\psi^+\phi^{n_s}) = [12]$	1		4
(2,0,0)	$f(\psi^+\psi^-\phi^2) = [1\|p_3\|2\rangle$	2	$n_s \geqslant 2$	6
(2,1,2)	$f(A^+\psi^+\psi^+\phi^{n_s}) = [12][13]$	2		5
(4,0,2)	$f(\psi^+\psi^+\psi^+\psi^+) = [12][34]^*$	2		6
(4,0,0)	$f(\psi^+\psi^+\psi^-\psi^-) = [12]\langle 34\rangle$	2		6

$$\frac{3}{2}n_\psi + 2n_A \leq d$$

$$m \geq \frac{1}{2}n_\psi + n_A .$$

This is almost the result

technical details see the paper

$$f^\pm(\psi^+\psi^+\psi^+\psi^+) = ([13][24] \pm [14][23])$$

Some comments

EOM: on-shell condition.

$\Box\phi$, $\not{D}\psi$ or $D_\mu F^{\mu\nu}$ vanishes: massless(on-shell)

That is why in the end it is like Warsaw basis!

All E.O.M. will convert the terms with derivative of a field into something else. In our case, it is zero.

IBP: momentum conservation. Total momentum is zero.

You impose that when writing the amplitudes

d=5 for SMEFT

For SMEFT, consider the quantum numbers & spin-statistics

SM gauge singlet

$f(\psi^+\psi^+\phi^2)$ $f(\psi^-\psi^-\phi^2)$

indices of SU(2)

$$\mathcal{M}(L_\alpha L_\beta H_\gamma H_\delta) = [12](\epsilon_{\alpha\gamma}\epsilon_{\beta\delta} + \epsilon_{\alpha\delta}\epsilon_{\beta\gamma}),$$

The amplitude

$$\mathcal{O}^{(5)} = \frac{1}{\Lambda}(HL)^2 + h.c.$$

d-5 Weinberg operators

d=6 for SMEFT

For d-6, it is almost the Warsaw basis

1. Class $\mathcal{M}(\phi^{n_s})$ ($\mathcal{O} \sim \varphi^6$ and $\varphi^4 D^2$):

Operator	Amplitude Basis
$\mathcal{O}_H$	$\mathcal{M}(H^3_{\alpha\beta\gamma} H^{\dagger 3}_{\dot\alpha\dot\beta\dot\gamma}) = T^+_{\alpha\beta\gamma\dot\alpha\dot\beta\dot\gamma}$
$2\mathcal{O}_{HD} - \mathcal{O}_{H\Box}$	$\mathcal{M}^+(H^2_{\alpha\beta} H^{\dagger 2}_{\dot\alpha\dot\beta}) = s_{12} T^+_{\alpha\beta\dot\alpha\dot\beta}$
$2\mathcal{O}_{HD} + \mathcal{O}_{H\Box}$	$\mathcal{M}^-(H^2_{\alpha\beta} H^{\dagger 2}_{\dot\alpha\dot\beta}) = (s_{13} - s_{23}) T^-_{\alpha\beta\dot\alpha\dot\beta}$

symmetric

anti-symmetric

where $T^+_{\alpha\beta\gamma\dot\alpha\dot\beta\dot\gamma} \equiv \delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta}\delta_{\gamma\dot\gamma} + \delta_{\beta\dot\alpha}\delta_{\alpha\dot\beta}\delta_{\gamma\dot\gamma}$
$\delta_{\gamma\dot\alpha}\delta_{\beta\dot\beta}\delta_{\alpha\dot\gamma} + \delta_{\beta\dot\alpha}\delta_{\gamma\dot\beta}\delta_{\alpha\dot\gamma} + \delta_{\alpha\dot\alpha}\delta_{\gamma\dot\beta}\delta_{\beta\dot\gamma} + \delta_{\gamma\dot\alpha}\delta_{\alpha\dot\beta}\delta_{\beta\dot\gamma}$

$T^\pm_{\alpha\beta\dot\alpha\dot\beta} \equiv \delta_{\alpha\dot\alpha}\delta_{\beta\dot\beta} \pm \delta_{\beta\dot\alpha}\delta_{\alpha\dot\beta}$

d=6 for SMEFT

2. Class $\mathcal{M}(A^+A^+\phi^2)$ and $\mathcal{M}(A^-A^-\phi^2)$ ($\mathcal{O} \sim X^2\varphi^2$):

Warsaw	Amplitude Basis
$\mathcal{O}_{HB} + \mathcal{O}_{H\tilde{B}}$	$\mathcal{M}(B^+B^+H_\alpha H^\dagger_{\dot\alpha}) = [12]^2\delta_{\alpha\dot\alpha}$
$\mathcal{O}_{HB} - \mathcal{O}_{H\tilde{B}}$	$\mathcal{M}(B^-B^-H_\alpha H^\dagger_{\dot\alpha}) = \langle 12\rangle^2\delta_{\alpha\dot\alpha}$
$\mathcal{O}_{HWB} + \mathcal{O}_{H\tilde{W}B}$	$\mathcal{M}(B^+W^{i+}H_\alpha H^\dagger_{\dot\beta}) = [12]^2\tau^i_{\alpha\dot\beta}$
$\mathcal{O}_{HWB} - \mathcal{O}_{H\tilde{W}B}$	$\mathcal{M}(B^-W^{i-}H_\alpha H^\dagger_{\dot\beta}) = \langle 12\rangle^2\tau^i_{\alpha\dot\beta}$
$\mathcal{O}_{HW} + \mathcal{O}_{H\tilde{W}}$	$\mathcal{M}(W^{i+}W^{j+}H_\alpha H^\dagger_{\dot\beta}) = [12]^2T^{ij+}_{\alpha\dot\beta}$
$\mathcal{O}_{HW} - \mathcal{O}_{H\tilde{W}}$	$\mathcal{M}(W^{i-}W^{j-}H_\alpha H^\dagger_{\dot\beta}) = \langle 12\rangle^2T^{ij+}_{\alpha\dot\beta}$
$\mathcal{O}_{HG} + \mathcal{O}_{H\tilde{G}}$	$\mathcal{M}(G^{A+}G^{B+}H_\alpha H^\dagger_{\dot\beta}) = [12]^2T^{AB+}_{\alpha\dot\beta}$
$\mathcal{O}_{HG} - \mathcal{O}_{H\tilde{G}}$	$\mathcal{M}(G^{A-}G^{B-}H_\alpha H^\dagger_{\dot\beta}) = \langle 12\rangle^2T^{AB+}_{\alpha\dot\beta}$

$$T^{ij+}_{\alpha\dot\beta} \equiv \delta^{ij}\delta_{\alpha\dot\beta}$$

$$T^{AB+}_{\alpha\dot\beta} \equiv \delta^{AB}\delta_{\alpha\dot\beta}$$

d=6 for SMEFT

3. Class $\mathcal{M}(A^+A^+A^+)$ and $\mathcal{M}(A^-A^-A^-)$ ($\mathcal{O} \sim X^3$):

Warsaw	Amplitude Basis
$\mathcal{O}_W + \mathcal{O}_{\tilde{W}}$	$\mathcal{M}(W^{i+}W^{j+}W^{k+}) = [12][23][31]\epsilon^{ijk}$
$\mathcal{O}_W - \mathcal{O}_{\tilde{W}}$	$\mathcal{M}(W^{i-}W^{j-}W^{k-}) = \langle 12\rangle\langle 23\rangle\langle 31\rangle\epsilon^{ijk}$
$\mathcal{O}_G + \mathcal{O}_{\tilde{G}}$	$\mathcal{M}(G^{A+}G^{B+}G^{C+}) = [12][23][31]f^{ABC}$
$\mathcal{O}_G - \mathcal{O}_{\tilde{G}}$	$\mathcal{M}(G^{A-}G^{B-}G^{C-}) = \langle 12\rangle\langle 23\rangle\langle 31\rangle f^{ABC}$

4. Class $\mathcal{M}(\psi^+\psi^+\phi^3)$ ($\mathcal{O} \sim \psi^2\varphi^3$) + h.c.:

Warsaw	Amplitude Basis
$\mathcal{O}_{eH}$	$\mathcal{M}(L_\alpha e H_\beta H^{\dagger 2}_{\dot\alpha\dot\beta}) = [12]T^+_{\alpha\beta\dot\alpha\dot\beta}$
$\mathcal{O}_{dH}$	$\mathcal{M}(Q_{a\alpha} d_{\dot a} H_\beta H^{\dagger 2}_{\dot\alpha\dot\beta}) = [12]T^+_{\alpha\beta\dot\alpha\dot\beta}\delta_{a\dot a}$
$\mathcal{O}_{uH}$	$\mathcal{M}(Q_{a\alpha} u_{\dot a} H^2_{\beta\gamma} H^\dagger_{\dot\alpha}) = [12]T^+_{\alpha(\beta\gamma)\dot\alpha}\delta_{a\dot a}$

d=6 for SMEFT

5. Class $\mathcal{M}(\psi^+\psi^-\phi^2)$ $(\mathcal{O} \sim \psi^2\varphi^2 D)$:

Warsaw	Amplitude Basis
$\mathcal{O}_{He}$	$\mathcal{M}(ee^\dagger H_\alpha H^\dagger_{\dot\alpha}) = [1\vert p_3\vert 2\rangle \delta_{\alpha\dot\alpha}$
$\mathcal{O}_{Hu}$	$\mathcal{M}(u_{\dot a} u^\dagger_a H_\alpha H^\dagger_{\dot\alpha}) = [1\vert p_3\vert 2\rangle \delta_{\alpha\dot\alpha}\delta_{a\dot a}$
$\mathcal{O}_{Hd}$	$\mathcal{M}(d_{\dot a} d^\dagger_a H_\alpha H^\dagger_{\dot\alpha}) = [1\vert p_3\vert 2\rangle \delta_{\alpha\dot\alpha}\delta_{a\dot a}$
$\mathcal{O}_{Hud}$ $\mathcal{O}^\dagger_{Hud}$	$\mathcal{M}(d_{\dot a} u^\dagger_a H^2_{\alpha\beta}) = \frac{1}{2}[1\vert p_3 - p_4\vert 2\rangle \epsilon_{\alpha\beta}\delta_{a\dot a}$ $\mathcal{M}(u_{\dot a} d^\dagger_a H^{\dagger 2}_{\dot\alpha\dot\beta}) = \frac{1}{2}[1\vert p_3 - p_4\vert 2\rangle \epsilon_{\dot\alpha\dot\beta}\delta_{a\dot a}$
$\mathcal{O}^{(3)}_{HL} + \frac{3}{4}\mathcal{O}^{(1)}_{HL}$	$\mathcal{M}^+(L_\alpha L^\dagger_{\dot\alpha} H_\beta H^\dagger_{\dot\beta}) = [1\vert p_3\vert 2\rangle T^+_{\alpha\beta\dot\alpha\dot\beta}$
$\mathcal{O}^{(3)}_{HL} - \frac{1}{4}\mathcal{O}^{(1)}_{HL}$	$\mathcal{M}^-(L_\alpha L^\dagger_{\dot\alpha} H_\beta H^\dagger_{\dot\beta}) = [1\vert p_3\vert 2\rangle T^-_{\alpha\beta\dot\alpha\dot\beta}$
$\mathcal{O}^{(3)}_{HQ} + \frac{3}{4}\mathcal{O}^{(1)}_{HQ}$	$\mathcal{M}^+(Q_{a\alpha} Q^\dagger_{\dot a\dot\alpha} H_\beta H^\dagger_{\dot\beta}) = [1\vert p_3\vert 2\rangle T^+_{\alpha\beta\dot\alpha\dot\beta}\delta_{a\dot a}$
$\mathcal{O}^{(3)}_{HQ} - \frac{1}{4}\mathcal{O}^{(1)}_{HQ}$	$\mathcal{M}^-(Q_{a\alpha} Q^\dagger_{\dot a\dot\alpha} H_\beta H^\dagger_{\dot\beta}) = [1\vert p_3\vert 2\rangle T^-_{\alpha\beta\dot\alpha\dot\beta}\delta_{a\dot a}$

d=6 for SMEFT

6. Class $\mathcal{M}(A^+\psi^+\psi^+\phi)$ $(\mathcal{O} \sim \psi^2 X\varphi)$ +h.c.:

Warsaw	Amplitude Basis
$\mathcal{O}_{eB}$	$\mathcal{M}(B^+ e L_\alpha H^\dagger_{\dot\alpha}) = [12][13]\delta_{\alpha\dot\alpha}$
$\mathcal{O}_{dB}$	$\mathcal{M}(B^+ d_{\dot a} Q_{a\alpha} H^\dagger_{\dot\alpha}) = [12][13]\delta_{\alpha\dot\alpha}\delta_{a\dot a}$
$\mathcal{O}_{dG}$	$\mathcal{M}(G^{A+} d_{\dot b} Q_{a\alpha} H^\dagger_{\dot\alpha}) = [12][13]\delta_{\alpha\dot\alpha}\lambda^A_{a\dot b}$
$\mathcal{O}_{eW}$	$\mathcal{M}(W^{i+} e L_\alpha H^\dagger_{\dot\beta}) = [12][13]\tau^i_{\alpha\dot\beta}$
$\mathcal{O}_{dW}$	$\mathcal{M}(W^{i+} d_{\dot a} Q_{a\alpha} H^\dagger_{\dot\beta}) = [12][13]\tau^i_{\alpha\dot\beta}\delta_{a\dot a}$
$\mathcal{O}_{uB}$	$\mathcal{M}(B^+ u_{\dot a} Q_{a\alpha} H_\beta) = [12][13]\epsilon_{\alpha\beta}\delta_{a\dot a}$
$\mathcal{O}_{uW}$	$\mathcal{M}(W^{i+} u_{\dot a} Q_{a\alpha} H_\beta) = [12][13]\tau^{i\beta}_\alpha\delta_{a\dot a}$
$\mathcal{O}_{uG}$	$\mathcal{M}(G^{A+} u_{\dot b} Q_{a\alpha} H_\beta) = [12][13]\epsilon_{\alpha\beta}\lambda^A_{a\dot b}$

There are also many 4 fermion operators

$3 + 8 + 4 + 6 + 9 + 16 + 12 + 26 = 84$ basis

Grassmanian

For more complicated cases like d=8, the p conservation has to be done systematically, not case by case.

Examples:

$\psi^4\phi^n$

$f_1(\psi^+\psi^+\psi^+\psi^+) = ([13][24]+[14][23])\,s'$ 3

Not 2*2 =4 different types because of p conservation

Technics to deal with those issues:

- Reduced semi-simple Young tablet
  B. Henning, T. Melia, arxiv: 1902.06754
- Moment Twistor.

Some only works in the massless case

Forgive me too late to find the references (Nima's book)

Up to d=8

Henning & Melia work out the case of d=6

We actually work out all cases for d=8 following them

21. Type $f(A^+\psi^+\psi^+\psi^-\psi^-\phi^{n-5})$

- $n = 5, k = 0$

$$f(A^+\psi^+\psi^+\psi^-\psi^-) = [12][13]\langle 45\rangle, \quad \# = 1$$

SSYT: $(n, \tilde{n}) = (4, 2)_{N=5}$

1	1	1
2	2	3
3		

But dealing with symmetry factor of same particles very difficult.

Things get even more complicated together with SM quantum numbers

However, if one goes to the amplitude case by case, then it is not an issue of problem from Feymann diagrams

I basically give up the systematic approach here

Outlook

- Just an initial taste of on-shell amplitude power.
- Going to the massive cases.
- To get the results from loops (unitarity cuts), reproduce the results of CDE, etc, anomalous d matrices, etc.
- Can easily applied to positivities. (Appendix C, no dim 6 operators for elastic Wh->Wh)

Backup slice