Non-minimal interactions of Higgs boson

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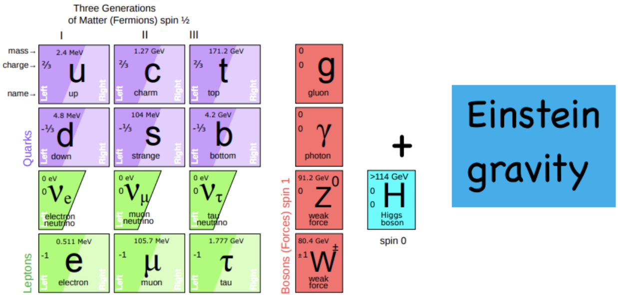
Do we live at an exciting time as particle physicist?

- We found Higgs boson $m_h = 125$ GeV.
- We only found Higgs boson.

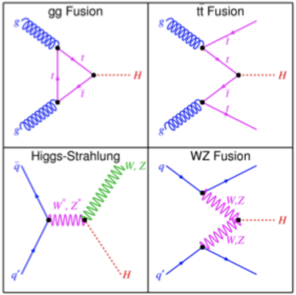
Is there something special about the Higgs boson from a gravitational point of view?

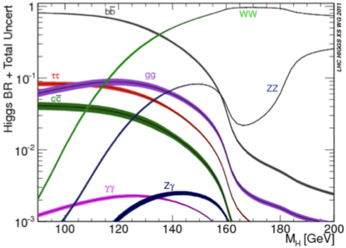
gravity might help us control hierarchy problem!



It is the Higgs boson that we all know

- The Standard Model predicts precisely how the Higgs boson should be produced.
- The Standard Model predicts precisely how the Higgs boson should decay.





Something special about the Higgs boson

Einstein-Hilbert action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2}(R - 2\Lambda) \;+\; \xi R H^\dagger H \right]$$

Non-minimal coupling: required by renormalisation

Is there any bound on its value?

This is a dimension-4 operator: it is a fundamental constant of nature.

The non-decoupling effect

- Consider Standard Model with a non-minimal coupling to R in Jordan frame

$$S_J = \int d^4x \sqrt{-g^J} \left(\frac{1}{2} M^2 + \xi H^\dagger H \right) R^J - \frac{1}{4} \sum_j F_{\mu\nu j} F_j^{\mu\nu}$$
$$+ \left(D_\mu H \right)^\dagger \left(D^\mu H \right) - V(H)$$

Gravity is not canonical

Equation of motion after differentiating with action is

$$\left\{ R^J_{\mu\nu} - \frac{1}{2} g^J_{\mu\nu} R^J + \left(g^J_{\mu\nu} \Delta_\lambda \Delta^\lambda - \Delta_\mu \Delta_\nu \right) \right\} \Omega^2 = -\frac{1}{M_P^2} T_{\mu\nu}$$

Obtain a canonical Einstein equation

Weyl transformation

We go from the Jordan frame to the Einstein frame

$$
\begin{aligned}
g^J_{\mu\nu} &= \Omega^{-2} g_{\mu\nu} \\
g^{\mu\nu J} &= \Omega^2 g^{\mu\nu} \\
\sqrt{-g^J} &= \Omega^{-4}\sqrt{-g} \\
R^J &= \Omega^2 \left(R - 6\Delta^\lambda \Delta_\lambda \log\Omega + 6 g^{\mu\nu}\Delta_\mu \log\Omega \Delta_\nu \log\Omega \right)
\end{aligned}
$$

Scale factor

$\Omega^2 = \frac{M^2 + 2\xi H^\dagger H}{M_P^2}$

Non-decoupling effect in Einstein frame

gravition is now canonical with higgs not being canonical.

Here is the action after Weyl transformation

$$S_E = -\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}M_P^2\tilde{R} - \frac{3\xi^2}{M_P^2\Omega^4}\partial^\mu\left(H^\dagger H\right)\partial_\mu\left(H^\dagger H\right)\right.$$
$$\left. -\frac{1}{\Omega^2}\left(D^\mu H\right)^\dagger\left(D_\mu H\right) + \frac{\mathcal{L}_{SM}}{\Omega^4}\right.$$

.

One notices that the Higgs boson kinetic term is not canonically normalized. We need to diagonalize this term.

- use the unitary gauge

$$H = \frac{1}{\sqrt{2}}(0, h+v)^\top$$

- The Planck mass is defined by

$$\left(M^2 + \xi v^2\right) = M_P^2$$

Additional transformation?

Higgs doublet must be transformed too

Ways Forward: canonical normalization

- To diagonalize the Higgs boson kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2 v^2}{M_P^2 \Omega^4}}$$

- To leading order in Ω^{-1}

$$h = \frac{1}{\sqrt{1+\beta}}\chi \quad \beta = 6\xi^2 v^2 / M_P^2$$

- The couplings of the Higgs boson to particles of the SM are rescaled!

$$y h \overline{\psi}\psi \to \frac{y}{\sqrt{1+\beta}}\chi\overline{\psi}\psi$$

Jordan Frame Detection

Schematic form of the Jordan frame equation

The decoupling can also be seen in the Jordan frame

$$\mathcal{L}^{(2)} = -\tfrac{M^2+\xi v^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu \Box h^\nu_\nu\right) + \tfrac{1}{2}\left(\partial_\mu h\right)^2 + \xi v\left(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu}\right)h$$

same renormalization factor!

$$h = \frac{1}{\sqrt{1+\beta}}\chi$$
$$h_{\mu\nu} = \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\overline{g}_{\mu\nu}\chi$$

Bound on the non-minimal coupling from the LHC

- The LHC experiments produce fits to the data assuming that all Higgs boson couplings are modified by a single parameter

$$\kappa = 1/\sqrt{1+\beta}$$

- In the narrow width approximation, one finds

$$\begin{aligned}\sigma(ii \to h \to ff) &= \sigma(ii \to h) \cdot \mathrm{BR}(h \to ff) \\ &= \kappa^2 \sigma_{\mathrm{SM}}(ii \to h) \cdot \mathrm{BR}_{\mathrm{SM}}(h \to ff)\end{aligned}$$

Bound on the non-minimal coupling from the LHC

- The signal strength of a single Higgs boson in a specific analysis

$$\mu = \sum_i c_i \omega_i$$

- For i-th channel, the signal strength can be calculated

$$c_i = \frac{[\sigma \times BR]_i}{[\sigma_{SM} \times BR_{SM}]_i}, \quad \omega_i = \frac{\epsilon_i \left[\sigma_{SM} \times BR_{SM}\right]_i}{\sum_j \epsilon_j \left[\sigma_{SM} \times BR_{SM}\right]_j}$$

Actual bound from numerical analysis

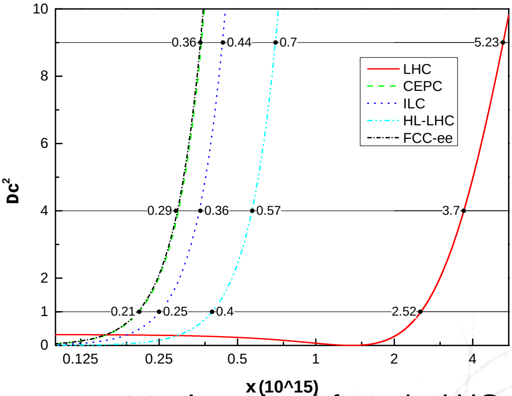


Figure: Bounds on non-minimal coupling ξ from the LHC (run1+run2), HL-LHC(14 TeV, 3 fb^{-1}), ILC(250GeV, 2 fb^{-1}), FCC-ee (240 GeV, 5 fb^{-1}) and CEPC(240 GeV, 5 fb^{-1}).

Unitary constraint on cut-off

Does the theory become unnatural at the scale $\Lambda = M_P/\xi$

In terms of power counting and unitary constraint, one gets the cut-off $\Lambda = M/\xi$

- For larger ξ satisfying the bound, we will have a much lower cut-off i.e. $\Lambda = 10^3$ GeV

Does our bound make sense?

Do we need new physics above the bound like supersymmetry or strong coupling?

Self-healing of unitarity

There is no need to worry about cut-off problem!

- Perturbative unitarity of the S-matrix implies at one-loop.

$$|T^{\text{tree}}|^2 = \text{Im}\left(T^{1-\text{loop}}\right)$$

- This is optical theorem and is a test of unitarity
- Look at the gravitational scattering between the Higgs boson and pseudo-Goldstone particles.

If so

New physics is not needed above M/ξ. The actual cut-off can be arbitrary as Plank scale.

- at tree-levels

$$A_{\text{tree}} = \frac{8\pi G_N(\bar{\phi})}{s}\left[s^2\left(6\xi_1\xi_2 + \xi_1 + \xi_2\right) + ut\right]$$

- At one-loop

$$A_{1-\text{loop}} = -\frac{G_N^2(\phi)}{15}\left[s^2 F\left(\xi_A, \xi_B, \xi\right) - ut\right]\log(-s)$$

- we thus verify the cutting relation implied by unitarity

$$|a_{0,\text{ tree },\max}|^2 = \text{Im}\left(a_{0,1-\text{loop},\max}\right)$$

Implication on naturalness

- Since cut-off can be very high, Higgs naturalness problems re-emerges!
- non-minimal coupling ξ provides a possible method to solve hierarchy problem.
- at spontaneously broken phase.

$$v^2 = \frac{-m_h^2 M_P^2 - 4V_0\xi}{M_P^2\lambda_h + m_h^2\xi}$$

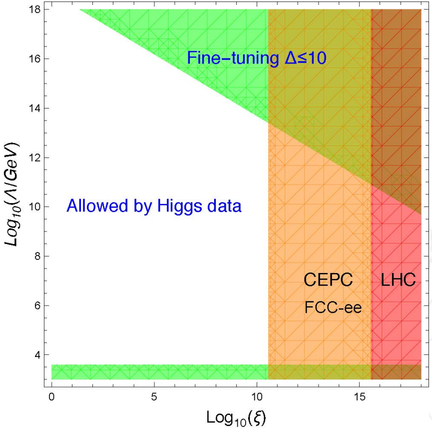
we have the one-loop correction to vev

$\delta v^2 \sim \left(1 - \frac{\Lambda^2}{M_P^2}\xi\right)\Lambda^2$

Fine-tuning measure

Here we define a dimensionless fine-tuning measure

$$\Delta = \delta v^2 / v^2$$



Conclusions

- Large non-minimal coupling of higgs boson to gravity can be bounded by LHC data.
- Large non-minimal coupling of higgs boson to gravity does not suffer from unitarity constraints.
- Large non-minimal coupling of higgs boson to gravity can soften the fine-tuning problem of itself.