Shaanxi University of Technology

## Probing CP－violating Yukawa Interactions of Top－Quark

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arXiv：1602．00684；JHEP，1606，048（2016）．In collaboration with K．Hagiwara and H．Yokoya．

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## Motivations

1. Top-quark is a natural probe of new physics.

- heaviest mass in SM
- short decay width

2. Spin correlations between top-quark and anti-top-quark in pair production are sensitive to physics beyond the standard model.
3. How to avoid the momenta reconstruction for top-quarks in measuring of the spin correlation.
4. Spin correlations have been observed by both ATLAS and CMS by measuring the observable which is defined as the azimuthal angle difference between the two charged leptons in the laboratory frame.

## Motivations

PRL, 108, 212001(2012).


- The hypothesis of zero spin correlation was excluded at $5.1 \sigma$


## Motivations

PRL, 108, 212001(2012).


- The hypothesis of zero spin correlation was excluded at $5.1 \sigma$
- What are the observables for a complete measurement of the possible spin correlations in the top-quark pair production?


## The Question



## The Question



## The Question

For single top-quark decay,

$$
\begin{equation*}
\mathcal{M}_{D}\left(s_{1}\right)=-g_{W}^{2} D_{W_{1}}^{-1}\left\{\overline{u_{b}} \gamma^{\mu} P_{L} u_{t}\left(s_{1}\right)\right\}\left\{\overline{u_{\nu_{\ell}}} \gamma_{\mu} P_{L} v_{\bar{\ell}}\right\} . \tag{1}
\end{equation*}
$$

Since the amplitude is a scalar, so,

$$
\begin{equation*}
\left\{\overline{u_{\nu_{\ell}}} \gamma_{\mu} P_{L} \bar{\nu}_{\bar{\ell}}\right\}=\left[\overline{u_{\nu_{\ell}}} \gamma_{\mu} P_{L} \bar{\nu}_{\bar{\ell}}\right]^{T}=\left(v_{\bar{\ell}}^{*}\right)^{\dagger} P_{L} \gamma_{\mu}^{T} \gamma^{0} u_{\nu_{\ell}}^{*} . \tag{2}
\end{equation*}
$$

Inserting the relation $C \gamma_{\mu}^{T} C^{-1}=-\gamma_{\mu}$, where $C=-i \gamma^{0} \gamma^{2}$, then,

$$
\begin{equation*}
\left\{\overline{u_{\nu_{\ell}}} \gamma_{\mu} P_{L} \nu_{\bar{\ell}}\right\}=-\left(v_{\bar{\ell}}^{*}\right)^{\dagger} P_{L} C^{-1} \gamma_{\mu} C \gamma^{0} u_{\nu_{\ell}}^{*}=-\left[\left(C \gamma^{0} v_{\bar{\ell}}^{*}\right)^{\dagger} \gamma^{0}\right] \gamma_{\mu} P_{R}\left(C \gamma^{0} u_{\nu_{\ell}}^{*}\right) . \tag{3}
\end{equation*}
$$

In terms of the wave functions of anti-particle $\psi^{c}=C \gamma^{0} \psi^{\dagger}$, it can be written as

$$
\begin{equation*}
\left\{\overline{u_{\nu_{\ell}}} \gamma_{\mu} P_{L} v_{\bar{\ell}}\right\}=-\overline{u_{\bar{\ell}}} \gamma_{\mu} P_{R} v_{\nu_{\ell}} . \tag{4}
\end{equation*}
$$

Applying the Fierz identity

$$
\begin{equation*}
\left(\bar{\chi}_{1} \gamma^{\mu} P_{L} \psi_{1}\right)\left(\bar{\psi}_{2} \gamma^{\mu} P_{R} \chi_{2}\right)=2\left(\bar{\psi}_{2} P_{L} \psi_{1}\right)\left(\bar{\chi}_{1} P_{R} \chi_{2}\right) \tag{5}
\end{equation*}
$$

we have

$$
\begin{equation*}
\mathcal{M}_{D}\left(s_{1}\right)=-2 g_{W}^{2} D_{W_{1}}^{-1}\left\{\overline{u_{\bar{\ell}}} P_{L} u_{t}\left(s_{1}\right)\right\}\left\{\overline{u_{b}} P_{R} v_{\nu_{\ell}}\right\}, \tag{6}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\mathcal{M}_{D}\left(s_{1}\right) \propto d_{s_{1},-1 / 2}^{1 / 2}=\sqrt{\frac{1-2 s_{1} \cos \theta}{2}}, \quad\left|\mathcal{M}_{D}( \pm)\right|^{2} \propto 1 \mp \cos \theta \tag{7}
\end{equation*}
$$

## The Question



## The Question



## The Question


W. Bernreuther, D. Heisler, Z.-G. Si, JHEP,12, 026(2015).

## Spin density matrix

No matter what is the dynamics in the top-quark pair production, by employing the narrow width approximation, the helicity amplitude can be written as a product of the production term and the decay term,

$$
\begin{equation*}
\mathcal{M}=\sum_{s_{1}, s_{2}= \pm 1 / 2} \mathcal{M}_{P}\left(s_{1}, s_{2}\right) \mathcal{M}_{D}\left(s_{1}, s_{2}\right) . \tag{1}
\end{equation*}
$$

Correspondingly, the amplitude squared can also be factorized into production and decay density matrices,

$$
\begin{equation*}
\overline{|\mathcal{M}|^{2}}=\sum_{s_{1}, s_{2} ; s_{1}^{\prime}, s_{2}^{\prime}} \mathcal{P}_{s_{1}^{\prime}, s_{2}^{\prime}}^{s_{1}, s_{2}} \mathcal{D}_{s_{1}^{\prime}, s_{2}^{\prime}}^{s_{1}, s_{2}} . \tag{2}
\end{equation*}
$$

However, in a chosen reference frame $R$, the top-quark pair and the lepton pair can be treated as two single systems whose spin can be either 0 or 1 . The helicity of these two systems along the direction $\vec{Q}$ are equal, and can have values $\lambda_{f}=\mathrm{s}, 0, \pm 1$. After spin projection, the amplitude squared $\overline{|\mathcal{M}|^{2}}$ can be decomposed as,

$$
\begin{equation*}
\overline{|\mathcal{M}|^{2}}=\sum_{\lambda_{f}, \lambda_{f}^{\prime}=s, 0, \pm 1} \widetilde{\mathcal{P}}_{\lambda_{f}^{\lambda_{f}^{\prime}}}^{\lambda_{f}} \widetilde{\mathcal{D}}_{\lambda_{f}^{\prime}}^{\lambda_{f}} \tag{3}
\end{equation*}
$$

K. Ma, PRD (R), 96, 071501 (2017)

## Spin Projection

The decay helicity amplitude $\mathcal{M}_{D}\left(s_{1}, s_{2}\right)$ can be written at leading order as a product of helicity amplitudes $\mathcal{M}_{D_{1}}\left(s_{1}\right)$ and $\mathcal{M}_{D_{2}}\left(s_{s}\right)$ for top-quark and anti-top-quark decays, respectively, which can be obtained directly according to the Feynman rules of the SM,

$$
\begin{align*}
& \mathcal{M}_{D_{1}}\left(s_{1}\right)=\frac{g_{W}^{2}}{2 D_{W_{1}}}\left\{\overline{u_{\nu}} \gamma_{\mu} P_{L} v_{\bar{\ell}}\right\}\left\{\overline{u_{b}} \gamma^{\mu} P_{L} u_{t}\left(s_{1}\right)\right\},  \tag{4}\\
& \mathcal{M}_{D_{2}}\left(s_{2}\right)=\frac{g_{W}^{2}}{2 D_{W_{2}}}\left\{\overline{u_{\ell}} \gamma_{\nu} P_{L} v_{\bar{\nu}_{\ell}}\right\}\left\{\overline{v_{t}\left(s_{2}\right)} \gamma^{\nu} P_{L} v_{\bar{b}}\right\} . \tag{5}
\end{align*}
$$

Spin correlations between the anti-lepton (lepton) and the top-quark (anti-top-quark) can be clearly observed by applying Fierz transformations after replacing wave functions of the anti-lepton (lepton) and the top-quark (anti-top-quark) by the ones of their anti-particles,

$$
\begin{align*}
& \mathcal{M}_{D}\left(s_{1}\right)=-g_{W}^{2} D_{W_{1}}^{-1}\left\{\overline{u_{\bar{\ell}}} P_{L} u_{t}\left(s_{1}\right)\right\}\left\{\overline{u_{b}} P_{R} v_{\nu_{\ell}}\right\},  \tag{6}\\
& \mathcal{M}_{D}\left(s_{2}\right)=-g_{W}^{2} D_{W_{2}}^{-1}\left\{\overline{\bar{v}_{t}}\left(s_{2}\right) P_{R} v_{\ell}\right\}\left\{\overline{\bar{u}_{\bar{\nu}}} P_{L} v_{\bar{b}}\right\} . \tag{7}
\end{align*}
$$

## Spin Projection

Factorization of the lepton pair system from top-quark pair system can be realized by simply applying the Fierz transformation one more step. Then we find,

$$
\begin{equation*}
\mathcal{M}\left(s_{1}, s_{2}\right)=\mathcal{H}^{\mu}\left(s_{1}, s_{2}\right) \mathcal{K}_{\mu} \mathcal{X} \overline{\mathcal{X}} \tag{8}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{H}^{\mu}\left(s_{1}, s_{2}\right) & =\bar{v}_{t}\left(s_{2}\right) \gamma^{\mu} P_{L} u_{t}\left(s_{1}\right),  \tag{9}\\
\mathcal{K}_{\mu} & =-\frac{1}{2} \overline{\bar{u}_{\bar{\ell}}} \gamma_{\mu} P_{R} v_{\ell}=\frac{1}{2} \overline{u_{\ell}} \gamma_{\mu} P_{L} v_{\bar{\ell}},  \tag{10}\\
\mathcal{X} & =g_{W}^{2} D_{W_{2}}^{-1}\left[\overline{u_{b}} P_{R} v_{\nu_{\ell}}\right],  \tag{11}\\
\overline{\mathcal{X}} & =g_{W}^{2} D_{W_{1}}^{-1}\left[\overline{u_{\bar{\nu}}} P_{L} v_{\bar{b}}\right] . \tag{12}
\end{align*}
$$

Spin projection along chosen direction $\vec{Q}$ in a reference frame $R$ can be easily obtained by inserting a complete projection relation,

$$
\begin{equation*}
g_{\mu \nu}=\sum_{\lambda_{f}=\mathrm{s}, 0, \pm} \eta_{\lambda_{f}} \epsilon_{\mu}^{*}\left(\vec{Q}, \lambda_{f}\right) \epsilon_{\nu}\left(\vec{Q}, \lambda_{f}\right) \tag{13}
\end{equation*}
$$

## Spin Projection

Then we find,

$$
\begin{equation*}
\mathcal{M}\left(s_{1}, s_{2}\right)=\sum_{\lambda_{f}=s, 0, \pm} \widetilde{\mathcal{H}}_{\lambda_{f}}\left(s_{1}, s_{2}\right) \widetilde{\mathcal{K}}_{\lambda_{f}} \tag{14}
\end{equation*}
$$

where the scalar functions $\widetilde{\mathcal{H}}_{\lambda_{f}}\left(s_{1}, s_{2}\right)$ and $\widetilde{\mathcal{K}}_{\lambda_{f}}$ are defined as

$$
\begin{align*}
\widetilde{\mathcal{H}}_{\lambda_{f}}\left(s_{1}, s_{2}\right) & =\sqrt{E_{1} E_{2}} \mathcal{X} \overline{\mathcal{X}} \mathcal{H}^{\mu}\left(s_{1}, s_{2}\right) \epsilon_{\mu}^{*}\left(\vec{Q}, \lambda_{f}\right),  \tag{15}\\
\widetilde{\mathcal{K}}_{\lambda_{f}} & =\frac{1}{\sqrt{E_{1} E_{2}}} \eta_{\lambda_{f}} \mathcal{K}^{\nu} \epsilon_{\nu}\left(\vec{Q}, \lambda_{f}\right), \tag{16}
\end{align*}
$$

with $E_{1}$ and $E_{2}$ are energy of the anti-lepton and lepton in $R$, respectively. The spin-projected decay density matrix is simply given as

$$
\begin{equation*}
\widetilde{\mathcal{D}}_{\lambda_{f}^{\prime}}^{\lambda_{f}^{\prime}}=\widetilde{\mathcal{K}}_{\lambda_{f}} \widetilde{\mathcal{K}}_{\lambda_{f}^{\prime}}^{\dagger} . \tag{17}
\end{equation*}
$$

And the spin-projected production density matrix can be obtained by summing up the helicity $s_{1}$ and $s_{2}$,

$$
\begin{equation*}
\widetilde{\mathcal{P}}_{\lambda_{f}^{\prime}}^{\lambda_{f}}=\sum_{s_{1}, s_{2} ; s_{1}^{\prime}, s_{2}^{\prime}} \mathcal{P}_{s_{1}, s_{2}^{\prime}}^{s_{1}, s_{2}} \widetilde{\mathcal{H}}_{\lambda_{f}}\left(s_{1}, s_{2}\right) \widetilde{\mathcal{H}}_{\lambda_{f}^{\prime}}^{\dagger}\left(s_{1}^{\prime}, s_{2}^{\prime}\right) . \tag{18}
\end{equation*}
$$

K. Ma, $\operatorname{PRD}(\mathbf{R}), 96,071501$ (2017)

## Spin Projection

Explicit expressions of $\widetilde{\mathcal{D}}_{\lambda_{f}^{\prime}}^{\lambda_{f}}$ can be obtained in a straightforward way once $R$ and $\vec{Q}$ are given. Without loss of generality, we can set $\vec{Q}=$ $(0,0,1)$, i.e., it is the unite vector along the $z$ direction in $R$, then we find

$$
\widetilde{\mathcal{K}}_{\lambda_{f}}=\left\{\begin{array}{ll}
-\sqrt{2} c_{1} s_{2} e^{i \phi_{+}} & \lambda_{f}=+1  \tag{19}\\
\sqrt{2} s_{1} c_{2} e^{-i \phi_{+}} & \lambda_{f}=-1 \\
c_{1} c_{2} e^{i \phi_{-}}-s_{1} s_{2} e^{-i \phi_{-}} & \lambda_{f}=0 \\
c_{1} c_{2} e^{i \phi_{-}}+s_{1} s_{2} e^{-i \phi_{-}} & \lambda_{f}=\mathrm{s}
\end{array},\right.
$$

where $\theta_{1}\left(\phi_{1}\right)$ and $\theta_{2}\left(\phi_{2}\right)$ are the polar (azimuthal) angles of the antilepton and lepton in the reference frame $R$, respectively; and $c_{i}=$ $\cos \left(\theta_{i} / 2\right), s_{i}=\sin \left(\theta_{i} / 2\right)$, and the phases $\phi_{ \pm}=\left(\phi_{1} \pm \phi_{2}\right) / 2$.
Spin correlations between the lepton and top-quark systems are encoded in the following density matrix,

$$
\begin{equation*}
\rho_{\lambda_{f}^{\prime}}^{\lambda_{f}}=\widetilde{\mathcal{P}}_{\lambda_{f}^{\prime}}^{\lambda_{f}^{\prime}} \widetilde{\mathcal{D}}_{\lambda_{f}^{\prime}}^{\lambda_{f}}, \tag{20}
\end{equation*}
$$

## Spin Projections

The diagonal elements are

$$
\begin{aligned}
\rho_{+}^{+} & =\frac{1}{2} \widetilde{\mathcal{P}}_{+}^{+}\left(1+\cos \theta_{1}\right)\left(1-\cos \theta_{2}\right), \\
\rho_{-}^{-} & =\frac{1}{2} \widetilde{\mathcal{P}}_{-}^{-}\left(1-\cos \theta_{1}\right)\left(1+\cos \theta_{2}\right), \\
\rho_{0}^{0} & =\frac{1}{2} \widetilde{\mathcal{P}}_{0}^{0}\left(1+\cos \theta_{1} \cos \theta_{2}-\sin \theta_{1} \sin \theta_{2} \cos \left(2 \phi_{-}\right)\right), \\
\rho_{s}^{s} & =\frac{1}{2} \widetilde{\mathcal{P}}_{s}^{s}\left(1+\cos \theta_{1} \cos \theta_{2}+\sin \theta_{1} \sin \theta_{2} \cos \left(2 \phi_{-}\right)\right), \\
\frac{d^{2} \sigma}{d \cos \theta_{1}^{*} d \cos \theta_{2}^{*}} & \propto 1+B_{1} \cos \theta_{1}^{*}+B_{2} \cos \theta_{2}^{*}-C \cos \theta_{1}^{*} \cos \theta_{2}^{*}
\end{aligned}
$$

W. Bernreuther, D. Heisler, Z.-G. Si, JHEP,12, 026(2015).

## Spin Projections

## The off-diagonal elements are

$$
\begin{aligned}
\operatorname{Re}\left[\rho_{-}^{+}\right] & =-\frac{1}{2}\left|\widetilde{\mathcal{P}}_{-}^{+}\right| \sin \theta_{1} \sin \theta_{2} \cos \left(2 \phi_{+}+\delta_{-}^{+}\right), \\
\operatorname{Re}\left[\rho_{0}^{+}\right] & =\frac{1}{2 \sqrt{2}}\left|\widetilde{\mathcal{P}}_{0}^{+}\right|\left\{\left(1-\cos \theta_{2}\right) \sin \theta_{1} \cos \left(\phi_{1}+\delta_{0}^{+}\right)-\left(1+\cos \theta_{1}\right) \sin \theta_{2} \cos \left(\phi_{2}+\delta_{0}^{+}\right)\right\} \\
\operatorname{Re}\left[\rho_{s}^{+}\right] & =\frac{-1}{2 \sqrt{2}}\left|\widetilde{\mathcal{P}}_{\mathrm{s}}^{+}\right|\left\{\left(1-\cos \theta_{2}\right) \sin \theta_{1} \cos \left(\phi_{1}+\delta_{\mathrm{s}}^{+}\right)+\left(1+\cos \theta_{1}\right) \sin \theta_{2} \cos \left(\phi_{2}+\delta_{\mathrm{s}}^{+}\right)\right\} \\
\operatorname{Re}\left[\rho_{0}^{-}\right] & =\frac{1}{2 \sqrt{2}}\left|\widetilde{\mathcal{P}}_{0}^{-}\right|\left\{\left(1+\cos \theta_{2}\right) \sin \theta_{1} \cos \left(\phi_{1}-\delta_{0}^{-}\right)-\left(1-\cos \theta_{1}\right) \sin \theta_{2} \cos \left(\phi_{2}-\delta_{0}^{-}\right)\right\} \\
\operatorname{Re}\left[\rho_{\mathrm{s}}^{-}\right] & =\frac{1}{2 \sqrt{2}}\left|\widetilde{\mathcal{P}}_{\mathrm{s}}^{-}\right|\left\{\left(1+\cos \theta_{2}\right) \sin \theta_{1} \cos \left(\phi_{1}-\delta_{\mathrm{s}}^{-}\right)+\left(1-\cos \theta_{1}\right) \sin \theta_{2} \cos \left(\phi_{2}-\delta_{\mathrm{s}}^{-}\right)\right\} \\
\operatorname{Re}\left[\rho_{\mathrm{s}}^{0}\right] & =\frac{1}{2}\left|\widetilde{\mathcal{P}}_{\mathrm{s}}^{0}\right|\left\{\left(\cos \theta_{1}+\cos \theta_{2}\right) \cos \left(\delta_{\mathrm{s}}^{0}\right)-\sin \theta_{1} \sin \theta_{2} \sin \left(2 \phi_{-}\right) \sin \left(\delta_{\mathrm{s}}^{0}\right)\right\},
\end{aligned}
$$

## Asymmetry Observables

Apart from the spin-independent total cross section,

$$
\begin{equation*}
\sigma=\frac{1}{2}\left(\overline{\widetilde{\mathcal{P}}_{\mathrm{s}}^{s}}+\overline{\widetilde{\mathcal{P}}_{0}^{0}}+\overline{\widetilde{\mathcal{P}}_{+}^{+}}+\overline{\widetilde{\mathcal{P}}_{-}^{-}}\right), \tag{21}
\end{equation*}
$$

where the over line "-" means summing up PDFs of the initial state and integrating over phase space apart from the following region,

$$
\begin{equation*}
d \Phi_{2}=\frac{1}{16 \pi^{2}} \int_{-1}^{1} d \cos \theta_{1} \int_{0}^{2 \pi} d \phi_{1} \int_{-1}^{1} d \cos \theta_{2} \int_{0}^{2 \pi} d \phi_{2} \tag{22}
\end{equation*}
$$

Our approach can provide 15 independent observables, and can be classified into two kinds of asymmetries:

$$
\begin{equation*}
\mathcal{A}[f(\varsigma)]=\frac{\sigma\left[g_{+}(\varsigma)>0\right]-\sigma\left[g_{+}(\varsigma)<0\right]}{\sigma\left[g_{+}(\varsigma)>0\right]+\sigma\left[g_{+}(\varsigma)<0\right]}, \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{C}[f(\varsigma)]=\frac{\sigma\left[g_{-}(\varsigma)>0\right]-\sigma\left[g_{-}(\varsigma)<0\right]}{\sigma\left[g_{-}(\varsigma)>0\right]+\sigma\left[g_{-}(\varsigma)<0\right]}, \tag{24}
\end{equation*}
$$

where $g_{ \pm}(\varsigma)=f(\varsigma) \pm f(-\varsigma)$ with $f(\varsigma)$ a function of angular variables. K. Ma, PRD (R), 96, 071501 (2017)

## Even Asymmetry Observables

The simplest observables are $\mathcal{A}_{\theta_{i}}=\mathcal{A}\left[\cos \theta_{i}\right]$ for the single side distributions, which measure the polarizations of top-quarks inclusively,

$$
\begin{equation*}
\mathcal{A}_{\theta_{i}}=\frac{1}{4 \sigma}\left\{Q_{i}\left(\overline{\mathcal{P}_{-}^{-}}-\overline{\mathcal{P}_{+}^{+}}\right)+2 \overline{\mathcal{P}_{\mathrm{s}}^{0} \cos \delta_{\mathrm{s}}^{0}}\right\}, \tag{25}
\end{equation*}
$$

where $Q_{i=1,2}$ are electric charges of the anti-lepton and lepton, respectively, in unite of $|e|$. There are also single side even asymmetries for the azimuthal angles, $\mathcal{A}_{\phi_{i}}=\mathcal{A}\left[\cos \phi_{i}\right]$ having following explicit expressions,

$$
\begin{equation*}
\mathcal{A}_{\phi_{i}}=\frac{-1}{2 \sqrt{2} \sigma} \sum_{\lambda= \pm 1}\left\{\lambda \overline{\mathcal{P}_{\mathrm{s}}^{\lambda} \cos \delta_{\mathrm{s}}^{\lambda}}+Q_{i} \overline{\mathcal{P}_{0}^{\lambda} \cos \delta_{0}^{\lambda}}\right\} . \tag{26}
\end{equation*}
$$

$\mathcal{A}_{\theta_{1} \theta_{2}}=\mathcal{A}\left[\cos \theta_{1} \cos \theta_{2}\right]$ is the simplest double side even asymmetry, and measures helicity correlation,

$$
\begin{equation*}
\mathcal{A}_{\theta_{1} \theta_{2}}=\frac{1}{8 \sigma}\left\{\overline{\mathcal{P}_{s}^{s}}+\overline{\mathcal{P}_{0}^{0}}-\overline{\mathcal{P}_{+}^{+}}-\overline{\mathcal{P}_{-}^{-}}\right\} . \tag{27}
\end{equation*}
$$

## Even Asymmetry Observables

Furthermore, linear combinations of the azimuthal angles can also be used to define double side asymmetries as $\mathcal{A}_{\phi_{ \pm}}=\mathcal{A}\left[\cos \left(2 \phi_{ \pm}\right)\right]$. And the explicit expressions are,

$$
\begin{align*}
& \mathcal{A}_{\phi_{+}}=\frac{\pi}{4 \sigma} \overline{\mathcal{P}_{-}^{+} \cos \delta_{-}^{+}}  \tag{28}\\
& \mathcal{A}_{\phi_{-}}=\frac{\pi}{8 \sigma}\left\{\overline{\mathcal{P}_{0}^{0}}-\overline{\mathcal{P}_{\mathrm{s}}^{\mathrm{s}}}\right\} . \tag{29}
\end{align*}
$$

There are also double side even asymmetries involving polar and azimuthal angles of oppositely charged leptons, $\mathcal{A}_{\theta_{i} \phi_{i^{\prime}}}=\mathcal{A}\left[\cos \left(\theta_{i}\right) \cos \left(\phi_{i^{\prime}}\right)\right]$ with $i \neq i^{\prime}$,

$$
\begin{equation*}
\mathcal{A}_{\theta_{i} \phi_{i^{\prime}}}=\frac{1}{4 \sqrt{2} \sigma} \sum_{\lambda= \pm 1}\left\{Q_{i} \overline{\mathcal{P}_{\mathrm{s}}^{\lambda} \cos \delta_{\mathrm{s}}^{\lambda}}-\lambda \overline{\mathcal{P}_{0}^{\lambda} \cos \delta_{0}^{\lambda}}\right\} . \tag{30}
\end{equation*}
$$

## Odd Asymmetry Observables

The simplest single side odd asymmetries are $\mathcal{C}_{\phi_{i}}=\mathcal{C}\left[\sin \phi_{i}\right]$,

$$
\begin{equation*}
\mathcal{C}_{\phi_{i}}=\frac{1}{2 \sqrt{2} \sigma} \sum_{\lambda= \pm 1}\left\{\overline{\mathcal{P}_{\mathrm{s}}^{\lambda} \sin \delta_{\mathrm{s}}^{\lambda}}+Q_{i} \lambda \overline{\mathcal{P}_{0}^{\lambda} \sin \delta_{0}^{\lambda}}\right\} . \tag{31}
\end{equation*}
$$

In contrast, there are no single side odd asymmetry for polar angles. On the other hand, linearly combinations of the azimuthal angles can give two double side asymmetries $\mathcal{C}_{\phi_{+}}=\mathcal{C}\left[\sin \left(2 \phi_{+}\right)\right]$and $\mathcal{C}_{\phi_{-}}=\mathcal{C}\left[\sin \left(2 \phi_{-}\right)\right]$as follows,

$$
\begin{align*}
\mathcal{C}_{\phi_{+}} & =-\frac{\pi}{8 \sigma} \overline{\mathcal{P}_{-}^{+} \sin \delta_{-}^{+}}  \tag{32}\\
\mathcal{C}_{\phi_{-}} & =\frac{\pi}{8 \sigma} \overline{\mathcal{P}_{\mathrm{s}}^{0} \sin \delta_{\mathrm{s}}^{0}} \tag{33}
\end{align*}
$$

In addition, there are two more double side odd asymmetries involving azimuthal and polar angles of oppositely charged particles, $\mathcal{C}_{\theta_{i} \phi_{i^{\prime}}}=$ $\mathcal{C}\left[\cos \left(\theta_{i}\right) \sin \left(\phi_{i^{\prime}}\right)\right]$ with $i \neq i^{\prime}$,

$$
\begin{equation*}
\mathcal{C}_{\theta_{i} \phi_{i^{\prime}}}=\frac{1}{4 \sqrt{2} \sigma} \sum_{\lambda= \pm 1}\left\{\overline{\mathcal{P}_{0}^{\lambda} \sin \delta_{0}^{\lambda}}-Q_{i} \lambda \overline{\mathcal{P}_{\mathrm{s}}^{\lambda} \sin \delta_{\mathrm{s}}^{\lambda}}\right\} . \tag{34}
\end{equation*}
$$

## Correlations Among Density Matrix Elements

However, the 15 matrix elements ( 16 before normalization) may not be independent in general. If the top-quark pair are generated in pure quantum state, the production density matrix can be obtained by using transition amplitude, $\widetilde{\mathcal{P}}_{\lambda_{t}^{\prime}}^{\lambda_{t}}=\widetilde{\mathcal{M}}_{P}\left(\lambda_{t}\right) \widetilde{\mathcal{M}}_{P}^{\dagger}\left(\lambda_{t}^{\prime}\right)$. Then there are only 4 independent magnitudes $\left|\mathcal{M}_{P}\left(\lambda_{t}\right)\right|$, and following relations have to be hold exactly,

$$
\begin{equation*}
\left|\widetilde{\mathcal{P}}_{\lambda_{t}^{\prime}}^{\lambda_{t}}\right|=\sqrt{\widetilde{\mathcal{P}}_{\lambda_{t}}^{\lambda_{t}} \widetilde{\mathcal{P}}_{\lambda_{t}^{\prime}}^{\lambda_{t}^{\prime}}} . \tag{35}
\end{equation*}
$$

Note that this number reduce to 3 after normalization. Furthermore, the number of independent phases is reduced to 3 (after an overall phase is removed), and following relations are also exact,

$$
\begin{equation*}
\delta_{\lambda_{t}^{\prime}}^{\lambda_{t}}=\delta\left(\lambda_{t}\right)-\delta\left(\lambda_{t}^{\prime}\right)=\delta_{\lambda_{t}^{\prime \prime}}^{\lambda_{t}}-\delta_{\lambda_{t}^{\prime}}^{\lambda_{1}^{\prime \prime}}, \tag{36}
\end{equation*}
$$

where $\delta\left(\lambda_{t}\right)$ are phases of the production helicity amplitudes $\widetilde{\mathcal{M}}_{P}\left(\lambda_{t}\right)$. At hadron collider, the colliding protons are mixed states, and the mixture are described by the PDFs. As a result the top-quark pair are not in pure state, and hence the relations (35) and (36) can be violated. However, if only one kind of the subprocess dominates the transition under considering, then the relations should be hold approximately.

## CP observables: $e^{-} e^{+} \rightarrow h t \bar{t} @ 500 \mathrm{GeV}$

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## Asymmetries: $e^{-} e^{+} \rightarrow h t \bar{t} @ 500 \mathrm{GeV}$

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## Why there is an enhancement?

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The observables $\mathcal{C}_{\theta_{i} \phi_{i^{\prime}}}$, are due to interferences between the transverse and longitudinal as well as scalar spin-projected components,

$$
\begin{equation*}
\mathcal{C}_{\theta_{i} \phi_{i^{\prime}}}=\frac{1}{4 \sqrt{2} \sigma} \sum_{\lambda= \pm 1}\left\{\overline{\mathcal{P}_{0}^{\lambda} \sin \delta_{0}^{\lambda}}-Q_{i} \lambda \overline{\mathcal{P}_{\mathrm{s}}^{\lambda} \sin \delta_{\mathrm{s}}^{\lambda}}\right\} . \tag{37}
\end{equation*}
$$

i.e., proportional to $\left|\mathcal{P}_{0}^{ \pm}\right|=\left|\mathcal{M}_{P}^{ \pm} \mathcal{M}_{P}^{0}\right|$ and $\left|\mathcal{P}_{s}^{ \pm}\right|=\left|\mathcal{M}_{P}^{ \pm} \mathcal{M}_{P}^{s}\right|$.

## Why there is an enhancement?

On the other hand, the spin-projected production helicity amplitudes can be generally written as

$$
\begin{equation*}
\mathcal{M}_{P}\left(\lambda, \lambda_{f}\right)=\sum_{\lambda_{f}} \mathcal{G}^{\mu \nu}\left(\lambda_{f}\right) L_{\mu}(\lambda) \epsilon_{\nu}\left(\lambda_{f}\right), \tag{37}
\end{equation*}
$$

where $L_{\mu}(\lambda)$ is current of the initial electron pair, and $\lambda$ stands for its polarization, $\epsilon_{\nu}\left(\lambda_{f}\right)$ is the spin-projection factor, and $\mathcal{G}\left(\lambda_{f}\right)^{\mu \nu}$ accounts for Lorentz structures determined by the production dynamics. In case of that $\mathcal{G}_{\mu \nu} \propto g_{\mu \nu}$, then the production helicity amplitudes with $\lambda_{f}=s$ in these two reference frames are given as,

$$
\mathcal{M}_{P}\left(\lambda_{f}=s\right) \propto \begin{cases}\gamma_{\psi} \beta_{\psi} & \text { in } R_{\psi}  \tag{38}\\ \gamma_{h} \beta_{h} & \text { in } R_{h}\end{cases}
$$

where $\gamma_{\psi}\left(\gamma_{h}\right)$ and $\beta_{\psi}\left(\beta_{h}\right)$ are the $\gamma$-factors and velocities of the top-quark pair system (Higgs boson) in the laboratory frame, respectively. Their ratio can be easily calculated,

$$
\begin{equation*}
\frac{\gamma_{h} \beta_{h}}{\gamma_{\psi} \beta_{\psi}}=\frac{m_{\psi}}{m_{h}} \gtrsim 2.77 \tag{39}
\end{equation*}
$$

where $m_{\psi}$ is the invariant mass of the top-quark pair, and the approximation stands for $m_{\psi}<2 m_{t}$ due to bound state effect near the threshold. Similarly, there is also an enhancement in the longitudinal component $\left(\lambda_{f}=0\right)$, even through the scale factor is relatively small (the ratio is $\gamma_{h} / \gamma_{\psi} \approx 1.2$ at the threshold).

## Furthermore enhancements

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K. Ma, arXiv:1809.07127.

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## Possible Anomaly

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Parton level $\Delta \phi\left(l^{+}, \Gamma\right) / \pi[\mathrm{rad} / \pi]$


Parton level $\Delta \phi\left(I^{+}, \Gamma\right) / \pi[\mathrm{rad} / \pi]$

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Parton level $\Delta \phi\left(\mathrm{l}^{+}, \Gamma\right) / \pi[\mathrm{rad} / \pi]$


Parton level $\Delta \phi\left(\mathrm{I}^{+}, \Gamma\right) / \pi[\mathrm{rad} / \pi]$

- For the inclusive result, $3.2 \sigma$ deviation, when including theoretical uncertainties;
- LO decay of the top quarks are not relevant and do not explain the observed deviation;

ATLAS Collaboration, arXiv:1903.07570.

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- SM predictions?

Anomalous interactions in decay of top-quarks?
New physics ...?

