Gluon-pair-Creation Production Model of Strong Interaction Vertices

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Bing-Dong Wan and Cong-Feng Qiao, arXiv:1904.02067

Contents:

- Glueballs and Glueball Studies
- Construction of the 0^{++} model
- Glueballs production via the 0^{++} model
- Summary

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- **Glueballs and Glueball Studies**
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- **Summary**

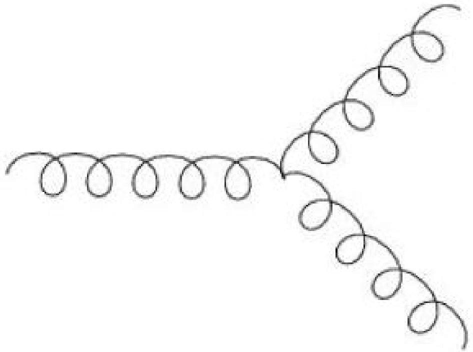
Glueball Studies

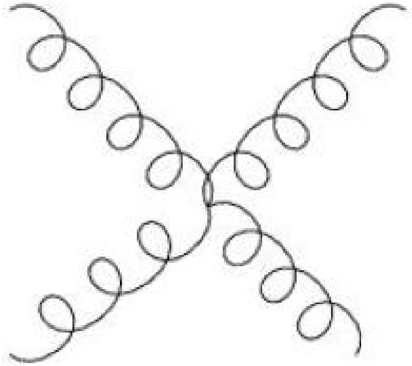
The QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\gamma^\mu D_\mu - m_q) \psi_q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

There exist gluon self-interactions





Glueball Studies

- **Color structure**
  - **Quark= fundamental representation 3**
  - **Gluon= Adjoint representation 8**
  - **Observable particles=color singlet 1**
    - **Mesons** $3 \otimes 3 = 1 \oplus 8$
    - **Baryons** $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
    - **Glueballs** $\begin{cases} 8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27 \\ 8 \otimes \cdots \otimes 8 = 1 \oplus 8 \oplus \cdots \end{cases}$

Glueball Studies

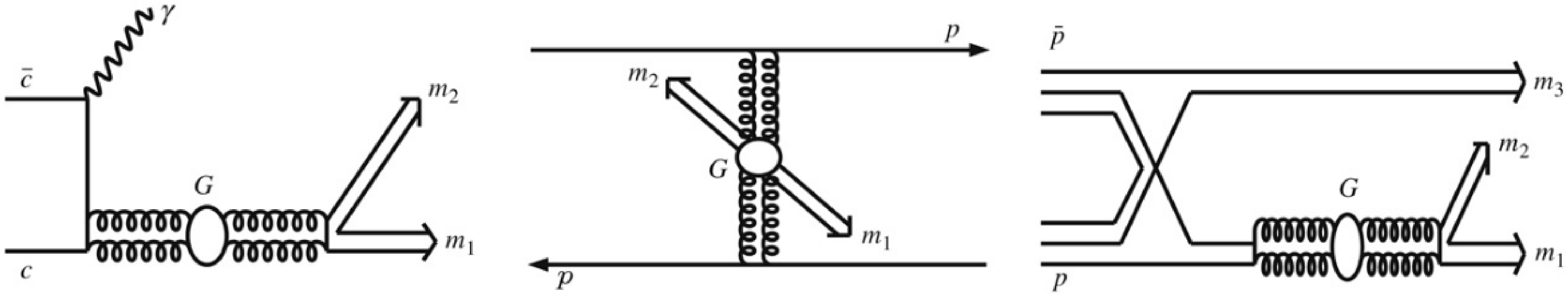
- Glueballs are allowed by QCD
- No definite observation in experiment up to now

The main difficulties in observing glueballs lie in

- lack of the knowledge about their production & decay properties
- mixing with quark states adds difficulty to isolate them.

Glueball Studies

- ➢Gluon-rich processes (Taking gg as an example)



Feynman diagrams describing glue-rich production mechanisms in favor of glueball formation: radiative J/ψ decays, Pomeron–Pomeron collisions in pp central production, and proton–antiproton annihilation. Picture taken from E. Klempt, A. Zaitsev, Phys. Rept. 454 (2007) 1. arXiv:0708.4016 [hep-ph]

Glueball Studies

- $f_0(1500)$ chosen as candidate of scalar glueball $J^{PC} = 0^{++}$
  - Found in J/ψ radiative decay
  - Absence of the $\gamma\gamma \to K\bar{K}$ or $\pi^+\pi^-$
  - Mass fitting the lattice calculation

V. Crede and C.A. Meyer, Prog. Part. Nucl. Phys. 63(2009) 74-116, and refs. Therein.
M. Acciarri et al., Phys. Lett. B 501, 173 (2001)
R. Barate et al., Phys. Lett. B 472, 189 (2000)
A. Abele et al., Phys. Lett. B 385, 425 (1996)
A. Abele et al., Phys. Rev. D 57, 3860 (1998)
D. Barberis et al., Phys. Lett. B 462, 462 (1999)
C. Amsler and F. E. Close, Phys. Lett. B 353, 385 (1995)
F. E. Close and A. Kirk, Phys. Lett. B 483, 345 (2000)

Glueball Studies

- η(1405) chosen as candidate of pseudoscalar glueball $J^{PC} = 0^{-+}$
  - Observed η(1405) → ηππ in J/ψ(1S) decay and $p\bar{p}$ annihilation
  - Suppressed in $\gamma\gamma$ collision
  - Mass fitting the fluxtube model very well

V. Crede and C.A. Meyer, Prog. Part. Nucl. Phys. 63(2009) 74-116, and refs. Therein
M. Ablikim et al., Phys. Rev. Lett. 107, 182001 (2011)
C. Amsler et al., Phys. Lett. B 358, 289 (1995)
M. Acciarri et al., Phys. Lett. B 501, 1 (2001)
L. Faddeev et al., Phys. Rev. D 70, 114033 (2004)
H.-Y. Cheng, H.-N. Li, and K.-F. Liu, Phys. Rev. D 79,014024(2009)

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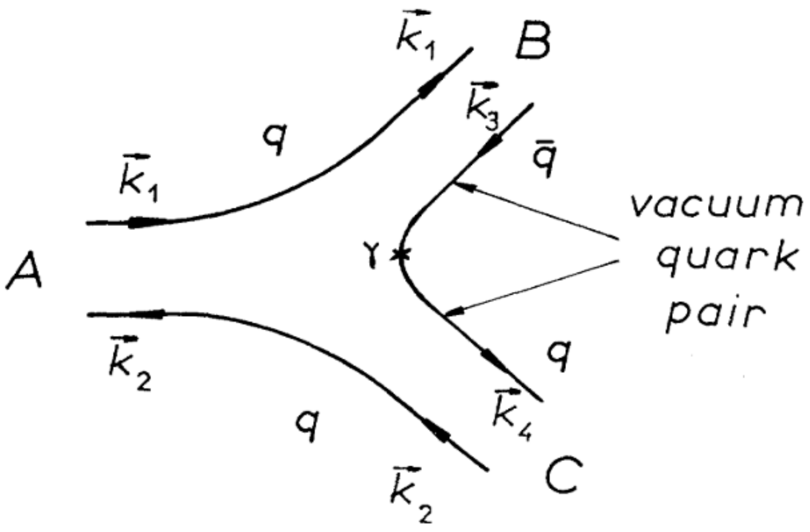
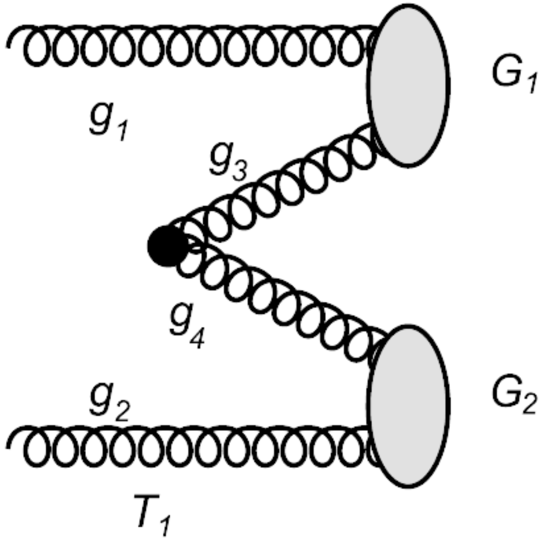
Construction of the 0^{++} model

- $^3P_0$ model vs. 0^{++} model



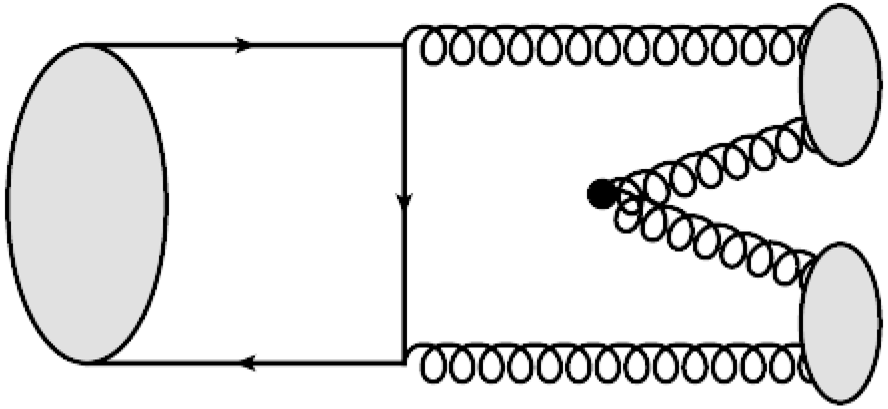
$^3P_0$ model



0^{++} model

A. Le Yaouanc et al. , Phys. Rev. D 8, 2223 (1973)

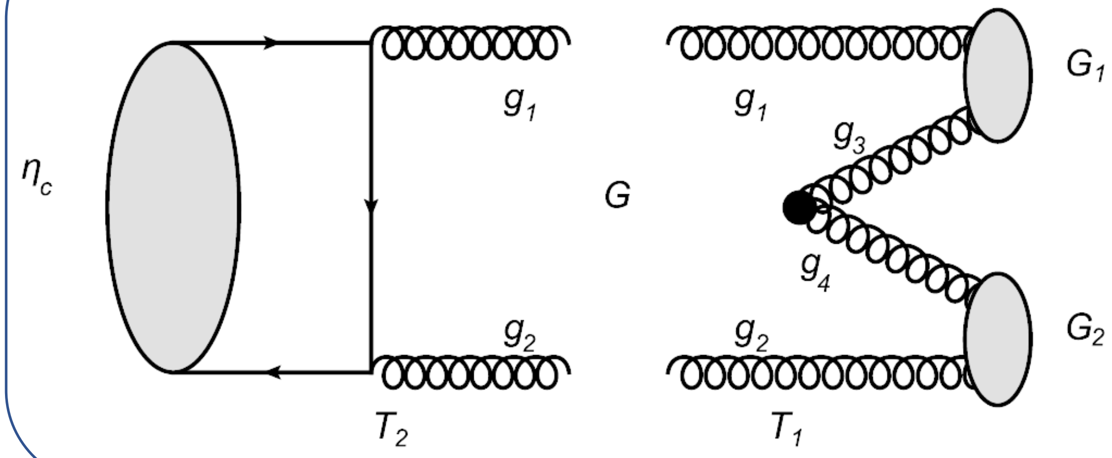
Construction of the 0^{++} model



The transition amplitude writes:

$$\langle G_1G_2|T|\eta_c\rangle = g_s^2\gamma_g\langle G_1G_2|\bar{q}_i t^a_{ij}\gamma_\mu q_j A^\mu_a \bar{q}_m \times t^b_{mn}\gamma_\nu q_n A^\nu_b \delta_{cd}\eta_{\rho\sigma}A^\rho_c A^\sigma_d|\eta_c\rangle$$

- $\delta_{cd}\eta_{\rho\sigma}A^\rho_c A^\sigma_d \rightarrow$ gluon-pair-vacuum
- $\gamma_g \rightarrow$ gluon-pair creation strength



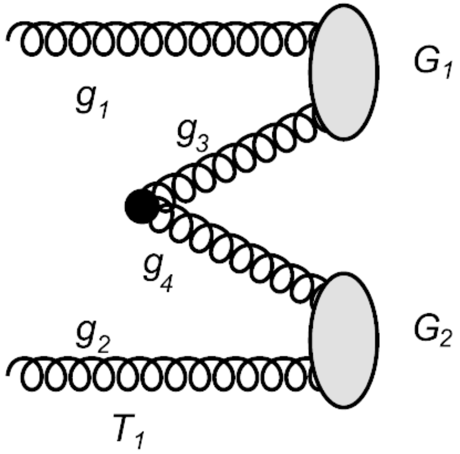
Inserting the completeness relation:

$$\langle G_1G_2|T|\eta_c\rangle = \frac{1}{2E_G}\sum_G \gamma_g\langle G_1G_2|\delta_{cd}\eta_{\rho\sigma}A^\rho_c A^\sigma_d|G\rangle \times g_s^2\langle G|\bar{q}_i t^a_{ij}\gamma_\mu q_j A^\mu_a \bar{q}_m t^b_{mn}\gamma_\nu q_n A^\nu_b|\eta_c\rangle$$

$$\equiv \frac{1}{2E_G}\sum_G \gamma_g\langle G_1G_2|T_1|G\rangle g_s^2\langle G|T_2|\eta_b\rangle$$

Construction of the 0^{++} model

- Gluon-pair-vacuum



$$\hat{T}_1 = I_1 \otimes I_2 \otimes \hat{T}_{vac}$$

$I_i \rightarrow$ Identity matrices for g_i

$\hat{T}_{vac} \rightarrow$ gluon-pair-vacuum

$$J^{PC}_{vac} = 0^{++} \quad \Rightarrow \begin{cases} L = 0 \\ S = 0 \end{cases}$$

$$\Rightarrow \quad \langle LM_L; SM_S | J_{vac} M_{vac} \rangle = 1$$

$$\hat{T}_{vac} = \gamma_g \int \mathrm{d}^3\mathbf{k}_3 \, \mathrm{d}^3\mathbf{k}_4 \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \mathcal{Y}_{00}\left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2}\right) \chi^{34}_{0,0} \, \delta_{cd} a^\dagger_{3c}(\mathbf{k}_3) \, a^\dagger_{4d}(\mathbf{k}_4)$$

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Construction of the 0^{++} model

➢ Expression of $|G\rangle$

$$|G\rangle = \sqrt{2E_G}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta^3\left(\mathbf{K}_\mathrm{G}-\mathbf{k}_1-\mathbf{k}_2\right)$$

$$\times\sum_{M_{L_G},M_{S_G}}\langle L_G M_{L_G} S_G M_{S_G}|J_G M_{J_G}\rangle\,\psi_{n_G L_G M_{L_G}}\left(\mathbf{k}_1,\mathbf{k}_2\right)\chi^{12}_{S_G M_{S_G}}\delta_{ab}\left|g_1^a g_2^b\right\rangle$$

- The normalization conditions are

$$\langle G(\mathbf{K}_G)|G(\mathbf{K}'_G)\rangle = 2E_G\delta^3(\mathbf{K}_G-\mathbf{K}'_G)$$

$$\langle g_i^a(\mathbf{k}_i)|g_j^b(\mathbf{k}_j)\rangle = \delta_{ij}\delta^{ab}\delta^3(\mathbf{k}_i-\mathbf{k}_j)$$

$$\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\delta^3(\mathbf{K}_\mathrm{G}-\mathbf{k}_1-\mathbf{k}_2)\psi_\mathrm{G}(\mathbf{k}_1,\mathbf{k}_2)\psi_{\mathrm{G}'}(\mathbf{k}_1,\mathbf{k}_2)=\delta_{\mathrm{G}'\mathrm{G}}$$

Construction of the 0^{++} model

- The transition amplitude

$$\langle G_1 G_2 | T_1 | G\rangle = \sqrt{8E_G E_{G_1} E_{G_2}}\;\gamma_g \sum_{M_{L_G}, M_{S_G} M_{L_{G_1}}, M_{S_{G_1}} M_{L_{G_2}}, M_{S_{G_2}}}$$
$$\times \langle L_G M_{L_G} S_G M_{S_G} | J_G M_{J_G}\rangle \langle L_{G_1} M_{L_{G_1}} S_{G_1} M_{S_{G_1}} | J_{G_1} M_{J_{G_1}}\rangle \langle L_{G_2} M_{L_{G_2}} S_{G_2} M_{S_{G_2}} | J_{G_2} M_{J_{G_2}}\rangle$$
$$\times \langle \chi^{13}_{S_{G_1} M_{S_{G_1}}} \chi^{24}_{S_{G_2} M_{S_{G_2}}} | \chi^{12}_{S_G M_{S_G}} \chi^{34}_{00}\rangle I_{M_{L_G}, M_{L_{G_1}}, M_{L_{G_2}}}(\mathbf{K}) (\delta_{ab}\delta_{cd}\delta_{ac}\delta_{bd})_{color-octet}\,.$$

$$I_{M_{L_G}, M_{L_{G_1}}, M_{L_{G_2}}}(\mathbf{K}) = \int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \mathrm{d}^3\mathbf{k}_3 \mathrm{d}^3\mathbf{k}_4\, \delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta^3(\mathbf{k}_3+\mathbf{k}_4)\delta^3(\mathbf{K}_{G_1}-\mathbf{k}_1-\mathbf{k}_3)\delta^3(\mathbf{K}_{G_2}-\mathbf{k}_2-\mathbf{k}_4)$$
$$\times\; \psi^*_{n_{G_1} L_{G_1} M_{L_{G_1}}}(\mathbf{k}_1,\mathbf{k}_3)\psi^*_{n_{G_2} L_{G_2} M_{L_{G_2}}}(\mathbf{k}_2,\mathbf{k}_4)\psi_{n_G L_G M_{L_G}}(\mathbf{k}_1,\mathbf{k}_2)\mathcal{Y}_{00}\left(\frac{\mathbf{k}_3-\mathbf{k}_4}{2}\right).$$

Construction of the 0^{++} model

- Spatial wave function expressed as harmonic oscillator

$$\psi_{nLM}(\mathbf{k}) = \mathcal{N}_{nL} \exp\left(-\frac{R^2\mathbf{k}^2}{2}\right) \mathcal{Y}_{LM}(\mathbf{k})\, \mathcal{P}(\mathbf{k}^2)$$

Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D 79, 074020 (2009).

- Spin coupling expressed as winger's 9j symbol

$$\langle \chi^{13}_{S_{G_1}M_{S_{G_1}}} \chi^{24}_{S_{G_2}M_{S_{G_2}}} | \chi^{12}_{S_G M_{S_G}} \chi^{34}_{00} \rangle = (-1)^{S_{G_2}+1} \Big[(2S_{G_1}+1)(2S_{G_2}+1)(2S_G+1) \Big]^{1/2}$$

$$\times \sum_{S,M_s} \langle S_{G_1} M_{S_{G_1}}; S_{G_2} M_{S_{G_2}} | S M_s \rangle \langle S M_s | S_G M_{S_G}; 00 \rangle \begin{Bmatrix} s_1 & s_3 & S_{G_1} \\ s_2 & s_4 & S_{G_2} \\ S_G & 0 & S \end{Bmatrix}$$

A. Le Yaouanc, L. Oliver, O. Pene and J. Raynal, Hadron Transitions in the Quark Model, Gordon and Breach Science Publishers, New York, (1987).

Construction of the 0^{++} model

Helicity amplitude can be extracted the from

$$\langle G_2|T_1|G\rangle = \delta^3(\mathbf{K}_{G_1} + \mathbf{K}_{G_2} - \mathbf{K}_G) \times \mathcal{M}_1^{M_{J_G} M_{J_{G_1}} M_{J_{G_2}}}$$

T_2 can be calculated by perturbative QCD,

$$|\mathcal{M}_2|^2 = \frac{4g_s^4 |R(0)_{\eta_c}|^2}{3\pi m_c}$$

$$\mathcal{M}^{JL} = \frac{\mathcal{M}_1^{JL}\mathcal{M}_2}{2E_G}$$

$$\mathcal{M}_1^{JL} = \frac{\sqrt{2L+1}}{2J_G+1} \sum_{M_{G_1}, M_{G_2}} \langle L0JM_{J_G}|J_G M_{J_G}\rangle \times \langle J_{G_1} M_{J_{G_1}} J_{G_2} M_{J_{G_2}} | J M_{J_G}\rangle \mathcal{M}_1^{M_{J_G} M_{J_{G_1}} M_{J_{G_2}}}$$

M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959)

The decay width

$$\Gamma = \pi^2 \frac{|\mathbf{K}|}{M_{\eta_c}^2} \sum_{JL} \left|\mathcal{M}^{JL}\right|^2$$

Contents:

Glueballs production via the 0^{++} model

- Quantum numbers

The color contraction → 8

$$J_G^{PC} = J_{\eta_c}^{PC} = 0^{-+}$$

	J^{PC}	L	M	S	M_S
η_c	0^{-+}	1	M_0	1	$-M_0$
$f_0(1500)$	0^{++}	0	0	0	0
$\eta(1405)$	0^{-+}	1	M_2	1	$-M_2$

$$\begin{aligned}\langle G_1 G_2 | T_1 | G\rangle &= \sum_{M_G, M_{G_2}} 8\gamma_g \sqrt{8E_G E_{G_1} E_{G_2}} \\ &\times \langle 1 M_0; 1 - M_0 | 00\rangle \\ &\times \langle 1 M_2; 1 - M_2 | 00\rangle \\ &\times \langle \chi_{00}^{13} \chi_{1-M_2}^{24} | \chi_{1-M_0}^{12} \chi_{00}^{34}\rangle I_{M_0,0,M_2}(\mathbf{K}) \,.\end{aligned}$$

Glueballs production via the 0^{++} model

- Angular momentum coupling

  - Wigner's 3j symbol:

$$\left\{ \begin{matrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{matrix} \right\} = \frac{(-1)^{j_1-j_2-m}}{\sqrt{2j+1}} \langle j_1 j_2 m_1 m_2 | j_1 j_2 j, -m \rangle$$

  - Wigner's 9j symbol:

$$\left\{ \begin{matrix} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \\ j_{13} & j_{24} & j \end{matrix} \right\} = \sum_m \left\{ \begin{matrix} j_1 & j_2 & j_{12} \\ m_1 & m_2 & m_{12} \end{matrix} \right\} \left\{ \begin{matrix} j_3 & j_4 & j_{34} \\ m_3 & m_4 & m_{34} \end{matrix} \right\} \left\{ \begin{matrix} j_{13} & j_{24} & j \\ m_{13} & m_{24} & m \end{matrix} \right\}$$

$$\times \left\{ \begin{matrix} j_1 & j_3 & j_{13} \\ m_1 & m_3 & m_{13} \end{matrix} \right\} \left\{ \begin{matrix} j_2 & j_4 & j_{24} \\ m_2 & m_4 & m_{24} \end{matrix} \right\} \left\{ \begin{matrix} j_{12} & j_{34} & j \\ m_{12} & m_{34} & m \end{matrix} \right\}$$

Glueballs production via the 0^{++} model

- Momentum-space distribution integral

$$\langle G_1 G_2 | T_1 | G \rangle = -\frac{\gamma_g}{27}\sqrt{8E_G E_{G_1} E_{G_2}}\Big(I_{1,0,1}(\mathbf{K}) + I_{0,0,0}(\mathbf{K}) + I_{-1,0,-1}(\mathbf{K}) \Big)$$

$$I_{1,0,1} = I_{-1,0,-1} = 0$$

$$I_{0,0,0} = -\delta^3(\mathbf{K}_G - \mathbf{K}_{G_1} - \mathbf{K}_{G_2}) \frac{R_1^{3/2} R_2^{5/2} R_0^{5/2}}{6\sqrt{2}\pi^{5/4}(R_0^2 + R_1^2 + R_2^2)^{9/2}} \exp\left(-\frac{\mathbf{K}^2 R_0^2 (R_1^2 + R_2^2)}{8(R_0^2 + R_1^2 + R_2^2)} \right)$$

$$\times\left\{ R_0^2\left(R_1^2+R_2^2\right)\left[\mathbf{K}^4\left(R_1^2+R_2^2\right)^2 - 96\right] + 12R_0^4\left[\mathbf{K}^2\left(R_1^2+R_2^2\right) - 4\right] - 12\left(R_1^2+R_2^2\right)^2\left[\mathbf{K}^2\left(R_1^2+R_2^2\right) + 4\right] \right\}$$

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Glueballs production via the 0^{++} model

- R : the most probable radius
  - In simple harmonic oscillator :

$$E_{in} = (2n + L + 3/2)\hbar\omega, \qquad \alpha = \sqrt{\mu\omega/\hbar}, \qquad R = 1/\alpha.$$

R and other parameters

	E (GeV)	E_{in}(GeV)	ω (GeV)	α (GeV)	R (GeV)$^{-1}$
G	2.98	2.98	0.66	0.45	2.24
G_1	1.53	1.50	0.43	0.36	2.79
G_2	1.45	1.41	0.31	0.31	3.26

$$I_{0,0,0} = -0.41\delta^3(\mathbf{K}_G - \mathbf{K}_{G_1} - \mathbf{K}_{G_2}),$$

$$\mathcal{M}_1^{JL} = \mathcal{M}_1^{00} = 0.11\gamma_g.$$

Glueballs production via the 0^{++} model

- Estimating the strength of Gluon-pair-vacuum production

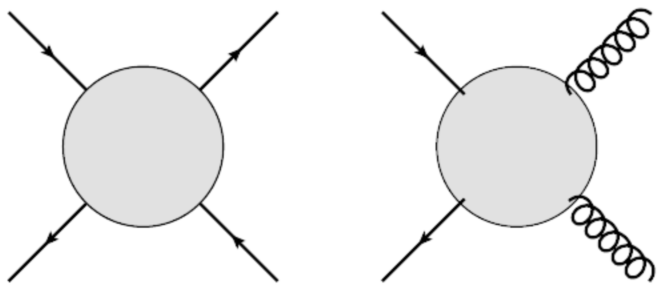
γ_g estimated by comparing to the $^3P_0$ model.

$\gamma = g/2m \rightarrow$ quark-pair creation strength.

Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D 79, 074020 (2009),
J. Segovia, D.R. Entem, and F. Fernandez Grupo, Phys. Lett. B 715, 322 (2012).

$$\gamma_g^2/g^2 \approx \frac{\sigma(q\bar{q} \rightarrow gg)}{\sigma(q\bar{q} \rightarrow q\bar{q})} \approx 0.0288$$

$$\gamma_g^2 \approx 0.0288 \times \gamma^2 \times (2m_u)^2 = 0.239^{+0.146}_{-0.079}\ \mathrm{GeV}^2$$



$$|\bar{M}(q\bar{q} \rightarrow q\bar{q})|^2 = \frac{4}{9}\left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} - \frac{2u^2}{3st}\right)$$

$$|\bar{M}(q\bar{q} \rightarrow gg)|^2 = \frac{32}{27}\left(-\frac{9\,(t^2+u^2)}{4s^2} + \frac{t}{u} + \frac{u}{t}\right)$$

Glueballs production via the 0^{++} model

- The decay width

$$\Gamma = \pi^2 \frac{|\mathbf{K}|}{M_{\eta_c}^2} \sum_{JL} \left| \mathcal{M}^{JL} \right|^2 = \pi^2 \frac{|\mathbf{K}|}{4M_{\eta_c}^4} |\mathcal{M}_1^{00}|^2 |\mathcal{M}_2|^2$$

$$= \frac{8\pi^2 g_s^4 |R(0)_{\eta_c}|^2 \gamma_g^2 |\mathbf{K}| E_G E_{G_1} E_{G_2} I^2}{3^7 \times \pi m_c M_{\eta_c}^4} = 50.73^{+35.47}_{-18.37}\ \mathrm{keV}$$

- The branching ratio

$$Br_{\eta_c \to f_0(1500)\eta(1405)} = \frac{\Gamma_{\eta_c \to f_0(1500)\eta(1405)}}{\Gamma_{total}} = 1.80^{+0.98}_{-0.81} \times 10^{-3}$$

Contents:

Summary

- We introduce 0^{++} model as mimic phenomenologically the gluon-pair-vacuum interaction vertices.
- We obtain the decay width of $\eta_c \to f_0(1500)\eta(1405)$ about 50.73 KeV and the branching ratio about 1.8×10^{-3}. These processes deserve to be investigated in BESIII, BelleII, LHCb and other experiments.
- 0^{++} model is quite premature and refinement of the 0^{++} model still needs a lot of tedious works.

Thanks!