The Effective Quintessence Potential from Landscape

Xun Xue

East China Normal University

Collaborator: Hanyu Zhai and Jiayin Shen

April 22, 14th Workshop on TeV Physics, NNU

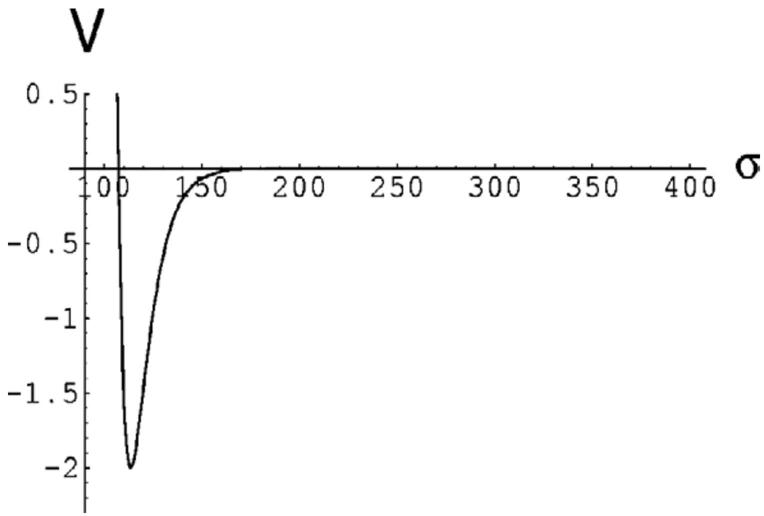
Outline

- 1) The String Swampland and String Landscape
- 2) The origin and sign of Large Scale Lorentz Violation
- 3) The Cosmology with large scale Lorentz Violation in swampland and string landscape
- 4) Summary

- The possible string vacua compactification choice can be of order 10^500. Among them, inequivalent ones constitute the string landscape.
- it is likely that any consistent looking lower dimensional effective field theory (EFT) coupled to gravity can arise in some way from a string theory compactification
- the set of all EFT which do not admit a string theory UV completion as the swampland.
- The generic AdS vacuum is SUSY preserving.
- To account the inflation and accelerating expansion one needs to lift the AdS to dS vacuum.

The uplifting the AdS type of vacua to dS ones comes from $\bar{D}_3$ branes tension in a sufficiently warped background, in the presence of quantum corrections, by carefully adding $\bar{D}_3$ branes into the compactification. –KKLT construction Kachru, PRD 03

Shortage: no-go theorems, restrictions on ingredients used in string theory, typically specific combinations of fluxes, D-branes, orientifolds



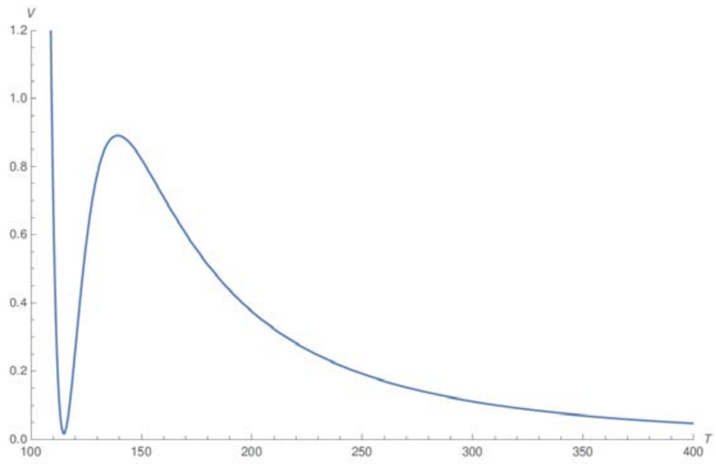


Figure 7. The scalar potential for a metastable dS.

The second criterion of swampland conjecture excludes the EFT with a meta-stable dS vacuum as theory with UV completion. Metastable dS belongs to the swampland.

Quintessence model can satisfy the second criterion.

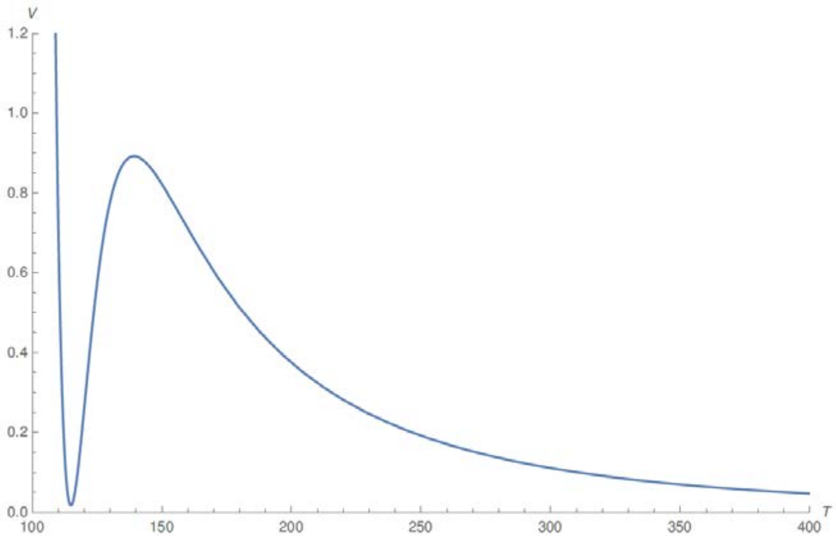


Figure 7. The scalar potential for a metastable dS.

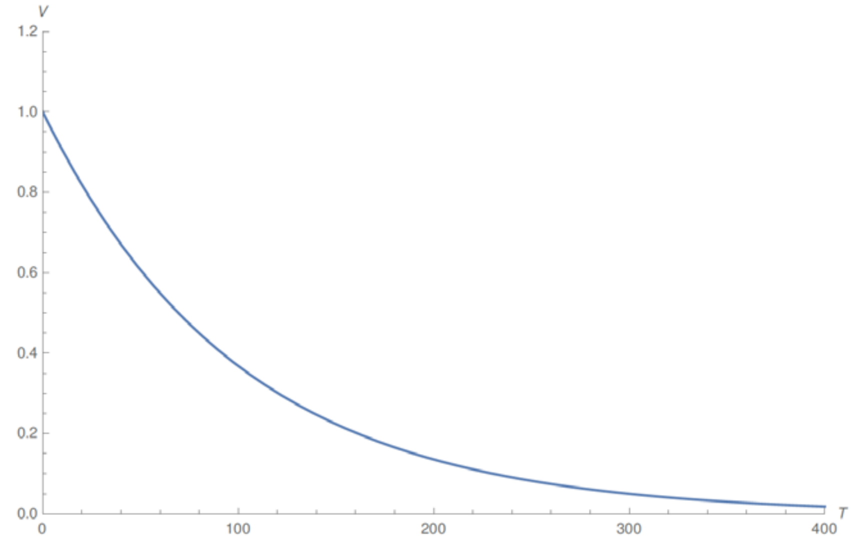


Figure 8. The scalar potential for an unstable dS.

- dS Swampland Conjecture: Obied, Ooguri, Spodyneiko, Vafa ‘18
- In the dS regime, there exists always a scalar field direction where the potential is sliding with a slope bigger than the gravitational strength
- This excludes all dS stationary solutions (dS local minima, maxima(refined), saddle points), implies any dS state should run away fast enough.
- The dark energy of our Universe can not be a C.C, but is the potential energy of a sliding scalar field Q=quintessence
- Refined dS swampland conjecture: Garg, Krishnan ’18 Ooguri, Palti, Shiu, Vafa ‘18
- All unstable dS stationary points (Higgs & pion maxima) are compatible with the refined dS SC

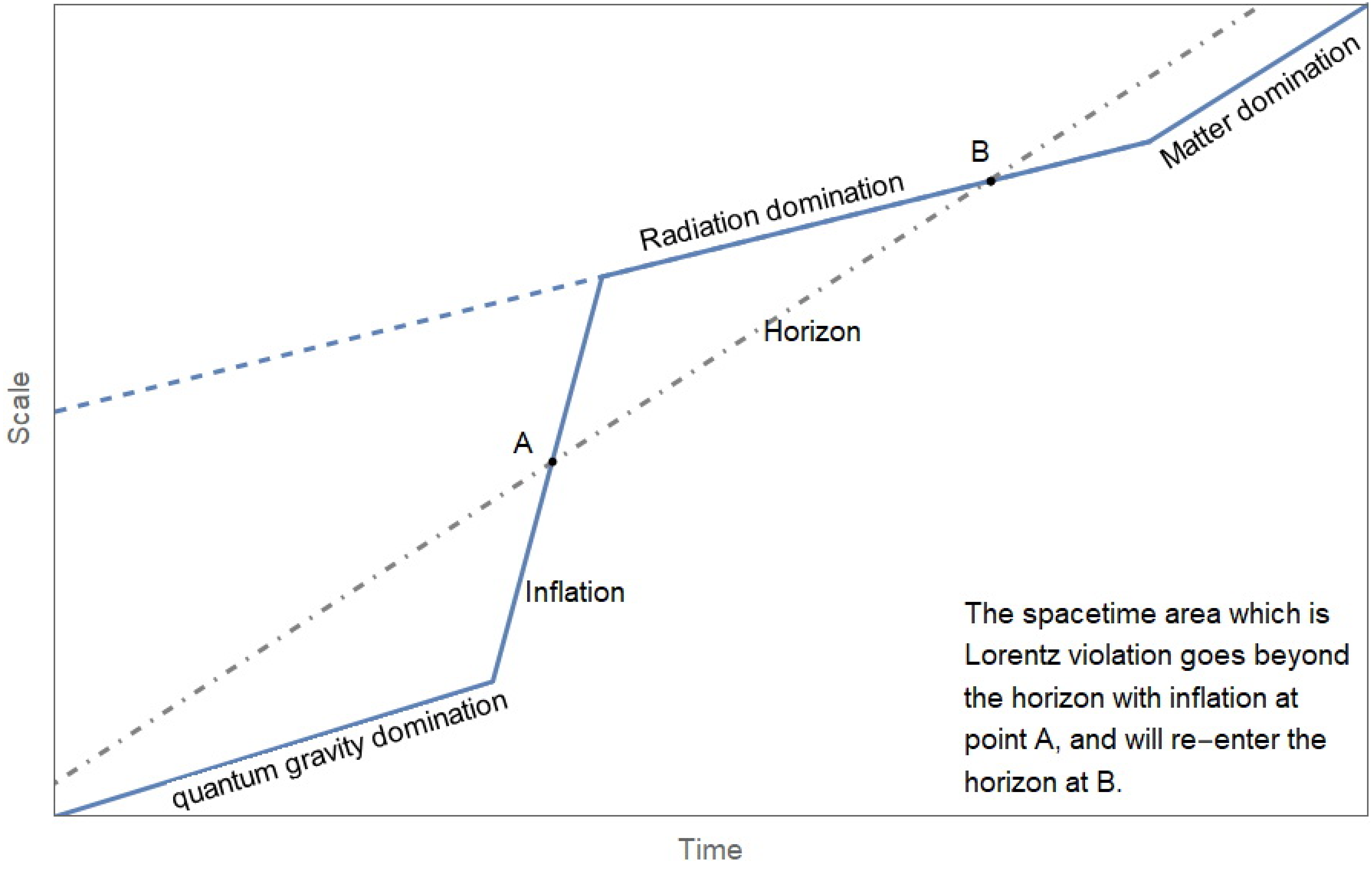
The ΛCDM model

- Einstein equation with cosmological constant

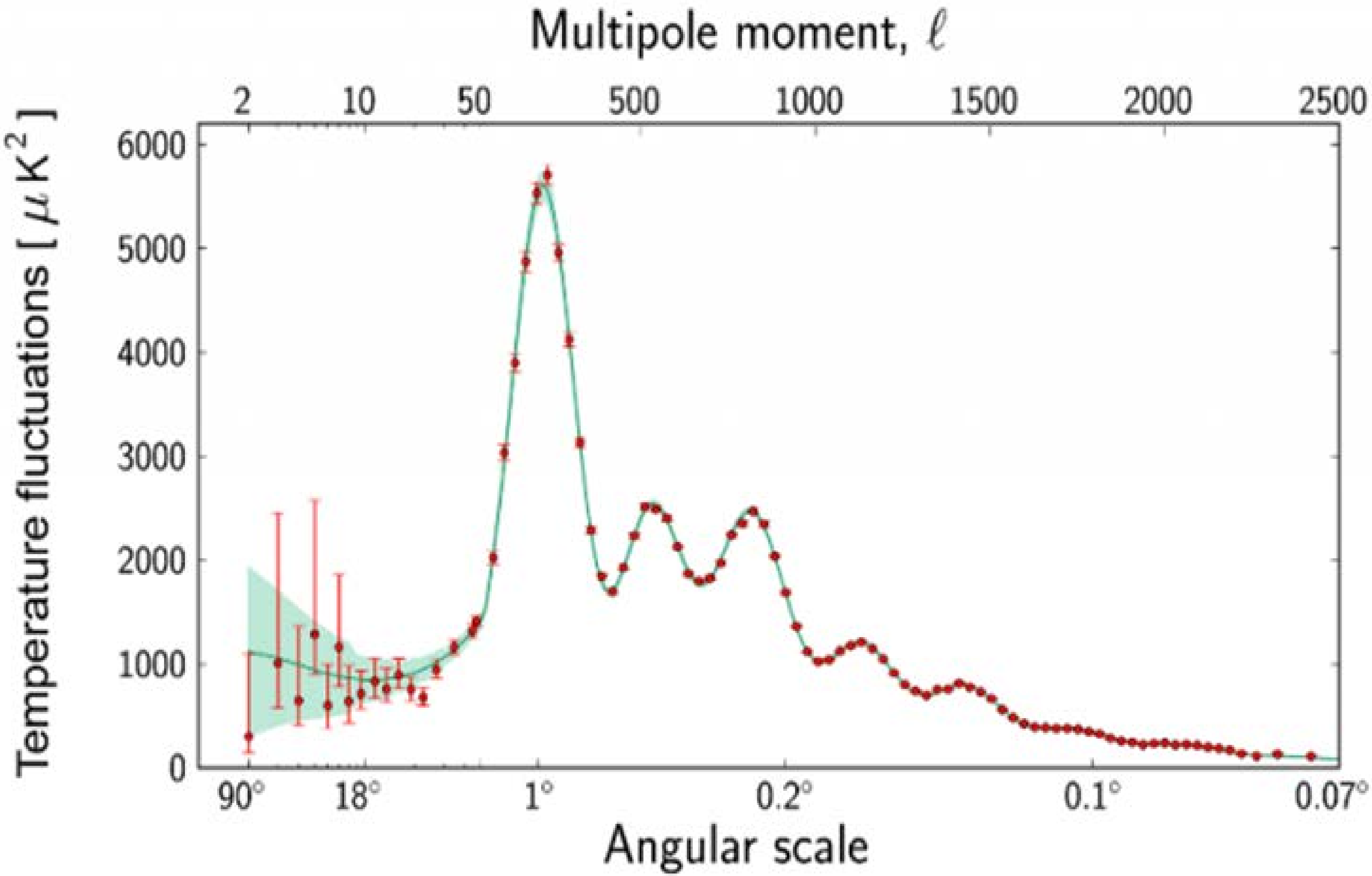
$$S_E = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R-2\Lambda\right)$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\left(T_M\right)_{\mu\nu}$$

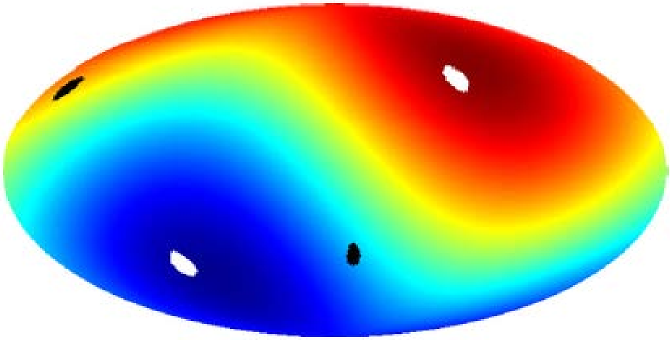
- Difficulty: There is huge dismatch between theoretical prediction and observation of Λ, ranging from 54 to 112 order of 10

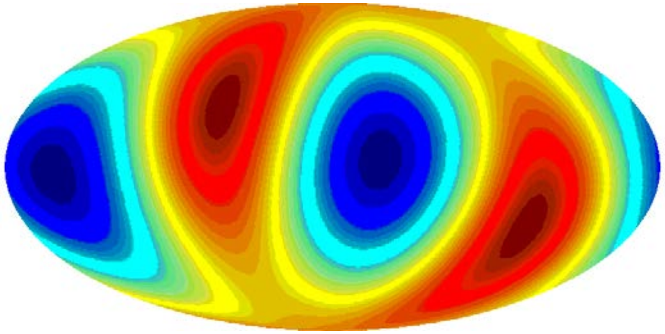

Scale
Time
Matter domination
B
Radiation domination
Horizon
A
Inflation
quantum gravity domination
The spacetime area which is Lorentz violation goes beyond the horizon with inflation at point A, and will re−enter the horizon at B.

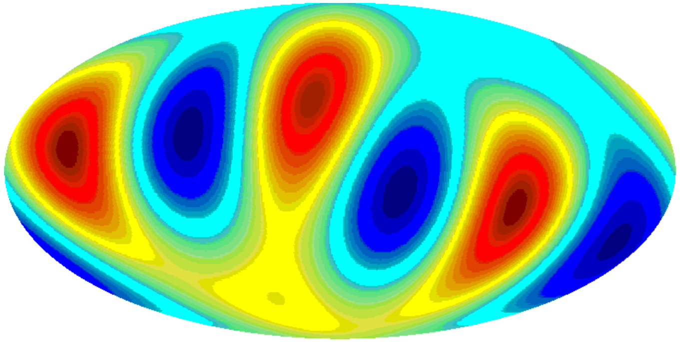
Anistropies of CMB

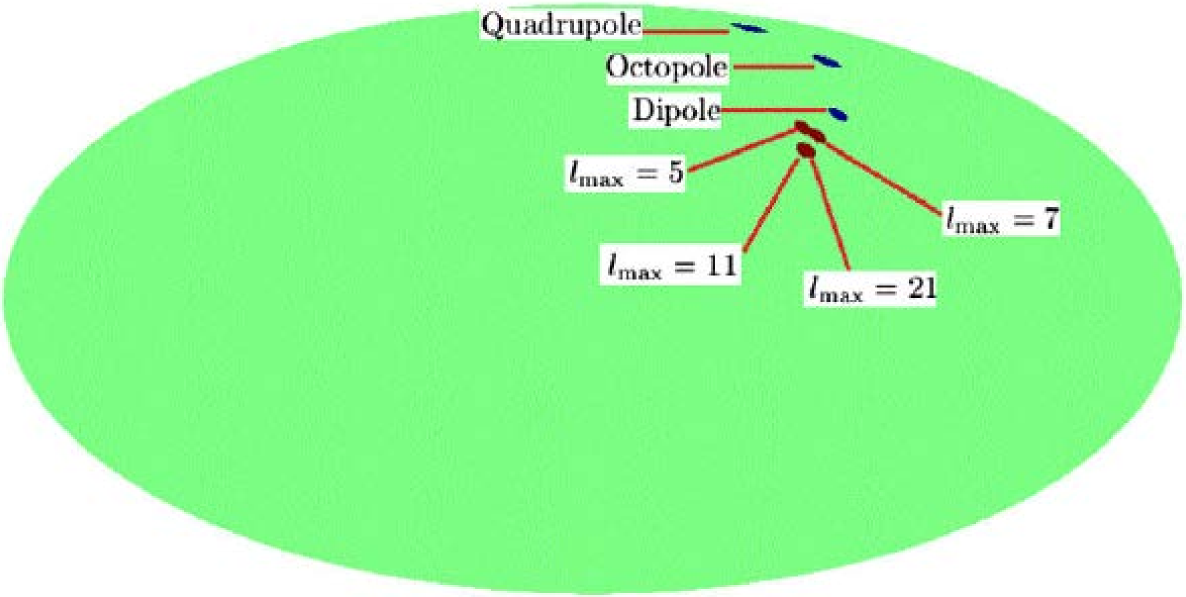


Comparing with preferred directions in CMB dipole, quadrupole and octopole









The Lorentz Violated EFT of Gravity

- The action for a sim(2) gravity

$$S_E = \frac{1}{16\pi G}\int d^4 x h\left(R^{ab}{}_{ab} + \lambda_1{}^{\mu}\left(A^{10}{}_{\mu} - A^{31}{}_{\mu}\right) + \lambda_2{}^{\mu}\left(A^{20}{}_{\mu} + A^{23}{}_{\mu}\right)\right)$$

- The Lagrange-multipliers term can be regarded as an effective angular momentum distribution $C_{M\,eff}$

$$\mathscr{D}_{\nu}\left(h\left(h_a{}^{\nu} h_b{}^{\mu} - h_a{}^{\mu} h_b{}^{\nu}\right)\right) = 16\pi G\left(C_M + C_{M\,eff}\right)_{ab}{}^{\mu}$$

- Lorentz violation leads to non-trivial distribution of contortion
- The non-trivial effective concribution to the energy-momentum distribution by contortion is expected to be responsible for the dark partner of the matter.

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G\left(T_{eff} + T_M\right)_c{}^a$$

- The Bianchi Identities imply the conservation of T_{eff}

arXiv:1802.03502

The Modified Constrain for SO(3)

- For SO(3) $\Lambda_0{}^j(x) = 0$

$$A'^i{}_{0\mu} = \Lambda^i{}_j(x) A^j{}_{0\mu} \Lambda_0{}^0(x) + \Lambda^i{}_j(x) \partial_\mu \Lambda_0{}^j(x)$$

$$= \Lambda^i{}_j(x) A^j{}_{0\mu}$$

- The Modified Constrain for SO(3) can be

$$S_E = \frac{c^4}{16\pi G} \int d^4 x h \left(R - 2\Lambda_0 + \lambda^u \left(\left(A^0{}_{1u} \right)^2 + \left(A^0{}_{2u} \right)^2 + \left(A^0{}_{3u} \right)^2 - f_u^2 \right) \right)$$

- Where f_μ can be regarded as the measurement of Lorentz violation.

Accelerating Expansion of the Universe

- To construct the FRW like solution of the model

$$ds^2 = \mathrm{d}t^2 - a(t)^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right)$$

- The naïve commoving tetrad can be chosen as

$$h^0 = \mathrm{d}t,\ h^1 = \frac{a(t)}{\sqrt{1-kr^2}}\mathrm{d}r,\ h^2 = ra(t)\mathrm{d}\theta,\ h^3 = r\sin\theta a(t)\mathrm{d}\varphi$$

- And

$$h_0 = \frac{\partial}{\partial t},\ h_1 = \frac{\sqrt{1-kr^2}}{a(t)}\frac{\partial}{\partial r},\ h_2 = \frac{1}{ra(t)}\frac{\partial}{\partial\theta},\ h_3 = \frac{1}{r\sin\theta a(t)}\frac{\partial}{\partial\varphi}$$

Accelerating Expansion of the Universe

- The field eqn for the tetrad field by $\dfrac{\delta S}{\delta h^{a}{}_{\mu}}$

$$G^{a}{}_{b} \equiv R^{a}{}_{b} - \frac{1}{2} R \delta^{a}{}_{b} + \Lambda_0 \delta^{a}{}_{b} = \frac{8\pi G}{c^4} T^{a}{}_{b}$$

Cosmic solution of contortion

- The perfect fluid of cosmic media demands

$$G^1{}_1 = G^2{}_2 = G^3{}_3$$

- With decomposition of connections, $A^a{}_{b\mu} = \Gamma^a{}_{b\mu} + K^a{}_{b\mu}$ a simple solution can be chosen as

$$K^0{}_{11} = K^0{}_{22} = K^0{}_{33} = \mathcal{K}(t)$$

- With other contortion components vanish.
- And the relation with $f_\mu(x)$ is

$$\left(f_t, f_r, f_\theta, f_\varphi\right) = \left(a(t)\mathcal{K}(t) + \dot{a}(t)\right) \cdot \left(0, \frac{1}{\sqrt{1-kr^2}}, r, r\sin\theta\right)$$

- The degree of freedom of $f_\mu(x)$ is actually 4, which hide in the choice of frames by Lorentz boost.

- Denoting $\tilde{G}^a_c$ the Einstein tensor of Levi-Civita Connection

$$G^a{}_c = \tilde{G}^a_c + 2\left(\tilde{\nabla}_{[c} K^{ab}{}_{b]} + K^a{}_{e[c} K^{eb}{}_{b]} - \frac{1}{2}\left(\tilde{\nabla}_d K^{db}{}_b + K^d{}_{e[d} K^{eb}{}_{b]}\right)\delta^a{}_c\right) + \Lambda_0 \delta^a{}_c$$

- The gravitation field equation

$$\tilde{R}^a_c - \frac{1}{2}\tilde{R}\delta^a{}_c = 8\pi G\left(T + T_\Lambda\right)^a{}_c, \quad T_\Lambda{}^a{}_c = \frac{1}{8\pi G}\Lambda^a{}_c = \frac{1}{8\pi G}\left(\tilde{G}^a_c - G^a{}_c\right)$$

- The gravitation field equations for the naïve tetrad of RW metric of $k = 0$

$$3\left(\mathscr{K} + \frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{c^4}\left(\rho + \Lambda_0\right)$$

$$\left(\mathscr{K} + \frac{\dot{a}}{a}\right)^2 + 2\left(\dot{\mathscr{K}} + \frac{\dot{a}\mathscr{K}}{a} + \frac{\ddot{a}}{a}\right) = \frac{8\pi G}{c^4}\left(-p + \Lambda_0\right)$$

- And

$$[T_\Lambda]^a{}_c = Diag(\rho_\Lambda, -p_\Lambda, -p_\Lambda, -p_\Lambda)$$

$$\rho_\Lambda = -\frac{c^4}{8\pi G}\left(3\mathscr{K}^2 + 6\mathscr{K}\frac{\dot{a}}{a} - \Lambda_0\right)$$

$$p_\Lambda = \frac{c^4}{8\pi G}\left(\mathscr{K}^2 + 4\mathscr{K}\frac{\dot{a}}{a} + 2\dot{\mathscr{K}} - \Lambda_0\right)$$

- Denote Λ_0 as the bare cosmological constant in our Lorentz violating model from vacuum energy density, Λ as the observed one and take the geometrical unit $\frac{8\pi G}{c^4}=1$ and $x=\frac{\Lambda_0}{\Lambda}$
- the modified Friedmann Equation

$$\left(\mathscr{K}+\frac{\dot{a}}{a}\right)^2=\frac{1}{3}(\rho+\Lambda_0)$$

$$\ddot{a}=-\frac{a}{2}\left(p+\frac{\rho}{3}\right)+\frac{1}{3}\left(a\Lambda_0-3\frac{d}{dt}(a\mathscr{K})\right)$$

- The Friedmann Eqns in ΛCDM

$$\left(\frac{\dot{a}}{a}\right)^2-\frac{\Lambda}{3}=\frac{\rho}{3}$$

$$\ddot{a}=-\frac{a}{2}\left(p+\frac{\rho}{3}\right)+\frac{1}{3}a\Lambda$$

- Accelerating expansion condition:

$$\frac{a}{2}\left(p+\frac{\rho}{3}-\frac{2}{3}\Lambda_0\right)+\frac{d}{dt}\left(a\mathcal{K}\right)<0$$

- the modified Friedmann Equation with the Eq of States for cosmic media p=wρ

$$\dot{H}(t)+\dot{\mathcal{K}}(t)+H(t)\left(H(t)+\mathcal{K}(t)\right)+\frac{3w+1}{2}\left(H(t)+\mathcal{K}(t)\right)^2-\frac{(w+1)}{2}\Lambda_0=0$$

- And w≈0 for matter dominated period
- Define the Effective Cosmological Constant which really responsible to the accelerating expansion

$$\Lambda_{eff}(t)=\Lambda_0-3\left(\mathcal{K}(t)^2+2\mathcal{K}(t)\frac{\dot{a}(t)}{a(t)}\right)$$

- Initial conditions: $$\mathcal{K}(t_0)^2 + 2\mathcal{K}(t_0)\frac{\dot{a}(t_0)}{a(t_0)} = \frac{\Lambda_0}{3} - \frac{\Lambda}{3}$$

$$\mathcal{K}(t_0) = H_0\left(\pm\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0}} - 1\right) \to \Lambda_0 \geq -\left(3H_0 - \Lambda\right) \approx -\frac{2}{5}\Lambda$$

- Three cases of approximation
- Case A: $\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathcal{K}\right) = -\frac{1}{3}a\left(\Lambda - \Lambda_0\right)$
- Or

$$H(t)\mathscr{K}(t) + \dot{\mathscr{K}}(t) = \frac{1}{3}\left(\Lambda_0 - \Lambda\right)$$

- Case B:

$$\dot{\mathscr{K}}(t)+(3w+2)H(t)\mathscr{K}(t)+\frac{3w+1}{2}\mathscr{K}^2(t)=\frac{w+1}{2}(\Lambda_0-\Lambda)$$

- Case C:

$$[T_\Lambda]^a{}_c = Diag(\rho_\Lambda, -p_\Lambda, -p_\Lambda, -p_\Lambda)$$

$$p_\Lambda = w_0\rho_\Lambda$$

$$(3w_0+1)\mathscr{K}^2+(6w_0+4)H\mathscr{K}+2\dot{\mathscr{K}}-(w_0+1)\Lambda_0=0$$

$$\dot{H}+\dot{\mathscr{K}}+H(H+\mathscr{K})+\frac{3w+1}{2}(H+\mathscr{K})^2-\frac{(w+1)}{2}\Lambda_0=0$$

$$\mathscr{K}(t_0)=H_0\left(\pm\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}-1\right),\quad H_0=H(t_0)$$

- Phenomenological, the Λ_{eff} can be regarded as the energy density produced by some auxiliary fields which responsible for the accelerating expansion such as quintessence field etc. e.g.

$$S_q = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\Lambda_{eff} = \frac{\dot{\phi}^2}{2} + V\left(\phi\left(t\right)\right) = -\frac{\dot{\Lambda}_{eff}}{6H} + V\left(\phi\left(t\right)\right)$$

$$\dot{\Lambda}_{eff} = \dot{\phi}\ddot{\phi} + \dot{\phi}V_{,\phi}\left(\phi\left(t\right)\right) = -3H\dot{\phi}^2$$

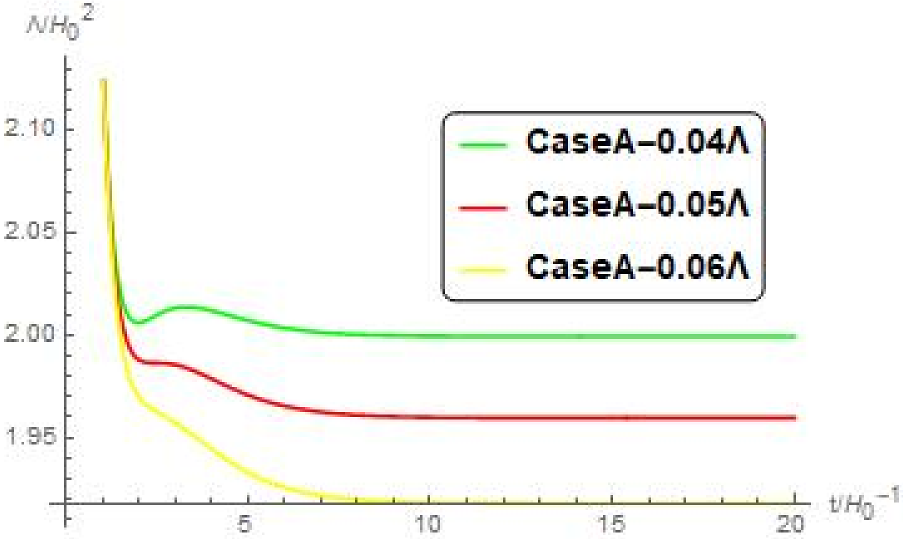
$$V\left(\phi\left(t\right)\right) = \Lambda_{eff} + \frac{\dot{\Lambda}_{eff}}{6H}$$

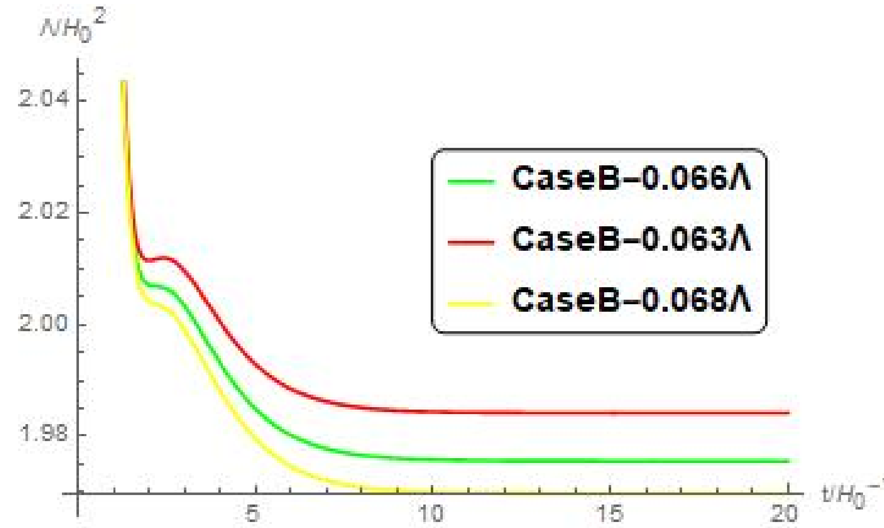
- the cirtical value for Λ_0 which symbolizes the transformation from a monotonically quintessence like $V(\phi(t))$ to the metastable dS potential can be solved for all of the case of approximations. The critical value Λ_{0-crit} centers around $\Lambda_{0-crit} = 0$. It can be conjectured the deviation of Λ_{0-crit} from 0 is caused by the approximations. In a more elaborated model, it should have $\Lambda_{0-crit} = 0$.

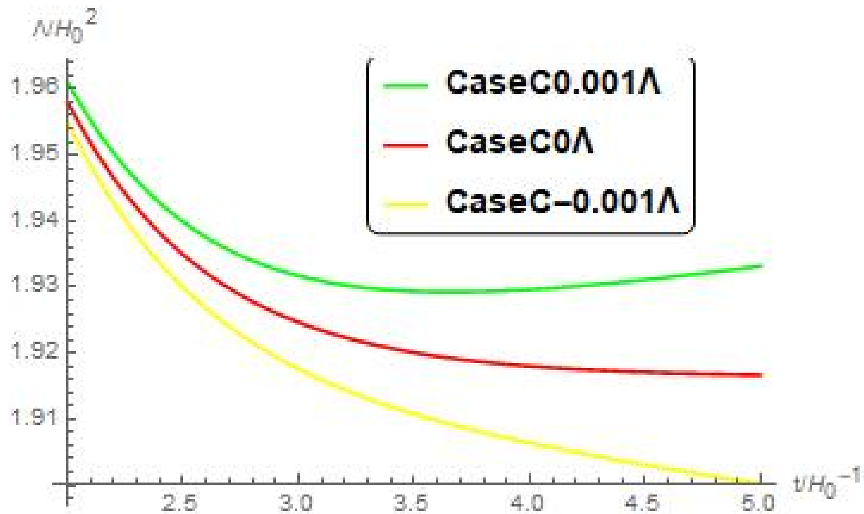
	The initial value $\mathscr{K}(t_0)$	The critical value for Λ_0
CaseA	$\mathscr{K}(t_0)=H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}-1\right)$	-0.05Λ
	$\mathscr{K}(t_0)=-H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}+1\right)$	-0.187Λ
CaseB	$\mathscr{K}(t_0)=H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}-1\right)$	-0.066Λ
	$\mathscr{K}(t_0)=-H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}+1\right)$	-0.2144Λ
CaseC(w_0=-1)	$\mathscr{K}(t_0)=H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}-1\right)$	0.00001
	$\mathscr{K}(t_0)=-H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}+1\right)$	0.00001
CaseC(w_0=-8/9)	$\mathscr{K}(t_0)=H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}-1\right)$	0.119Λ
	$\mathscr{K}(t_0)=-H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}+1\right)$	0.075Λ

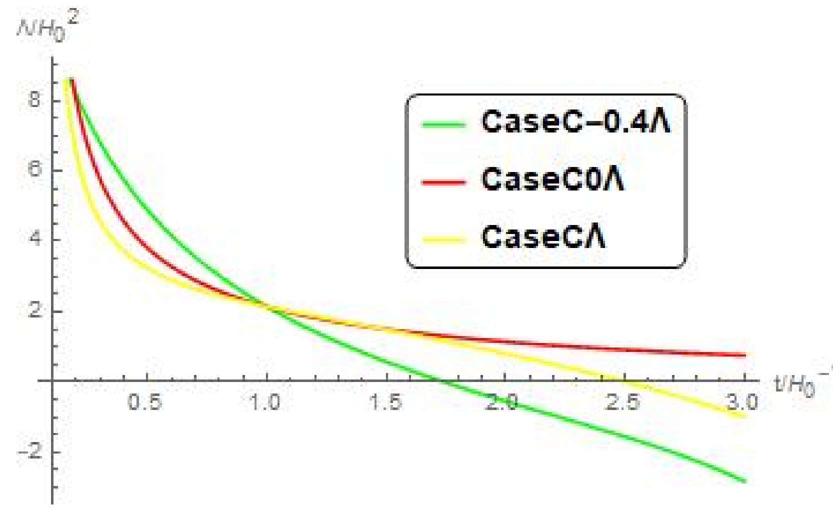
- For Case C, when $w_0 > -8/9$ there doesn't exist a solution of the critical value for Λ_0 which signs the transformation from a monotonically quintessence potential to a metastable dS potential

CaseC(w_0=-7/9)	$\mathscr{K}(t_0) = H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}-1\right)$ $\mathscr{K}(t_0) = -H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}+1\right)$	Monotonic for all Λ_0 0.152Λ
CaseC(w_0=-1/2)	$\mathscr{K}(t_0) = H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}-1\right)$ $\mathscr{K}(t_0) = -H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}+1\right)$	Monotonic for all Λ_0 0.321Λ
CaseC(w_0=-1/3)	$\mathscr{K}(t_0) = H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}-1\right)$ $\mathscr{K}(t_0) = -H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0{}^2}}+1\right)$	Monotonic for all Λ_0 0.397Λ

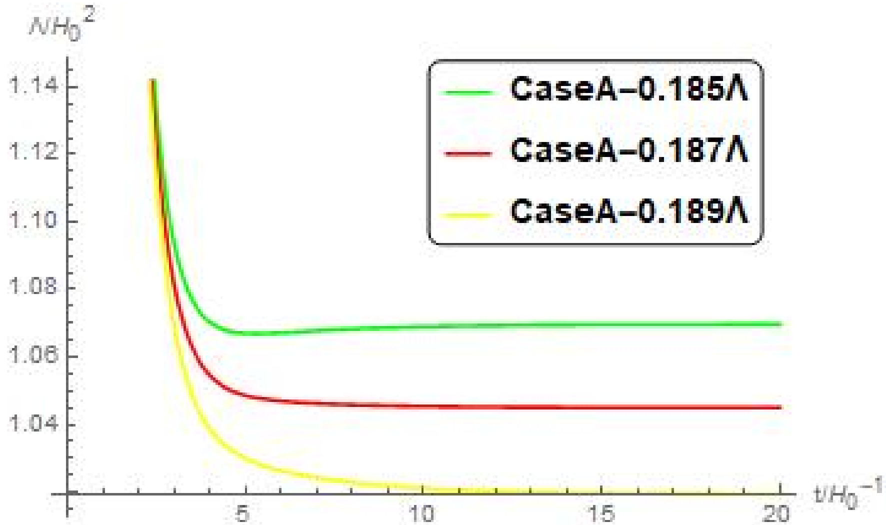


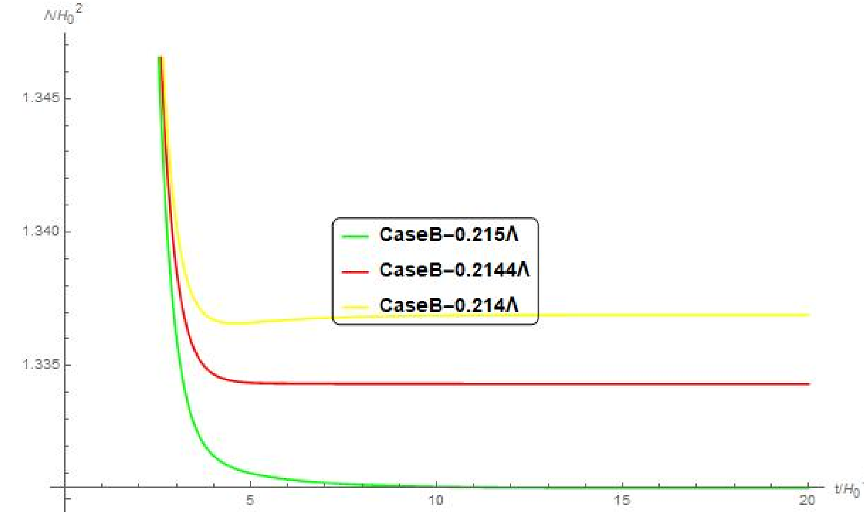


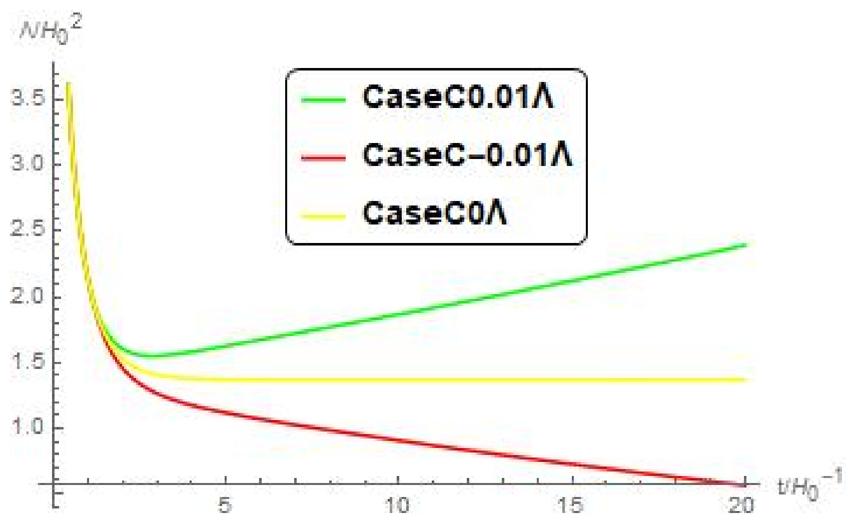


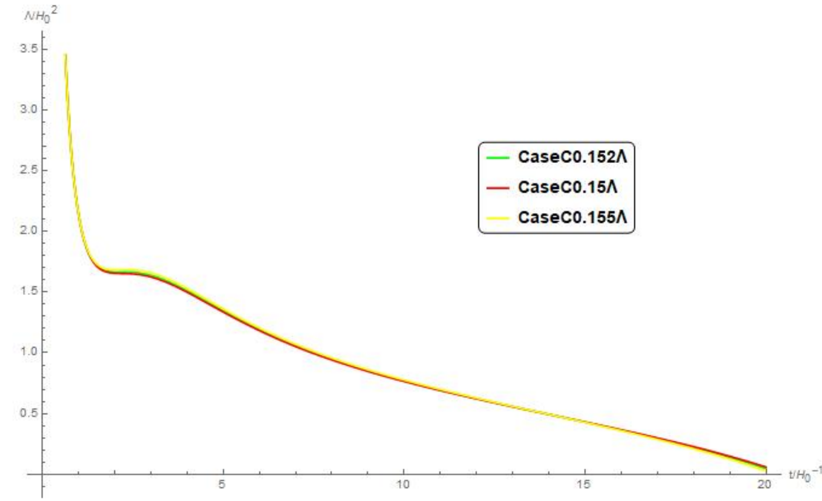


- The transformation from quintessence to dS
- $\mathscr{K}(t_0) = H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\ 2}}}-1\right)$ and $w_0 = -1, -7/9$ for Case C









- The transformation from quintessence to dS
- $\mathscr{K}(t_0) = -H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3{H_0}^2}}+1\right)$ and $w_0 = -1, -7/9$ for Case C

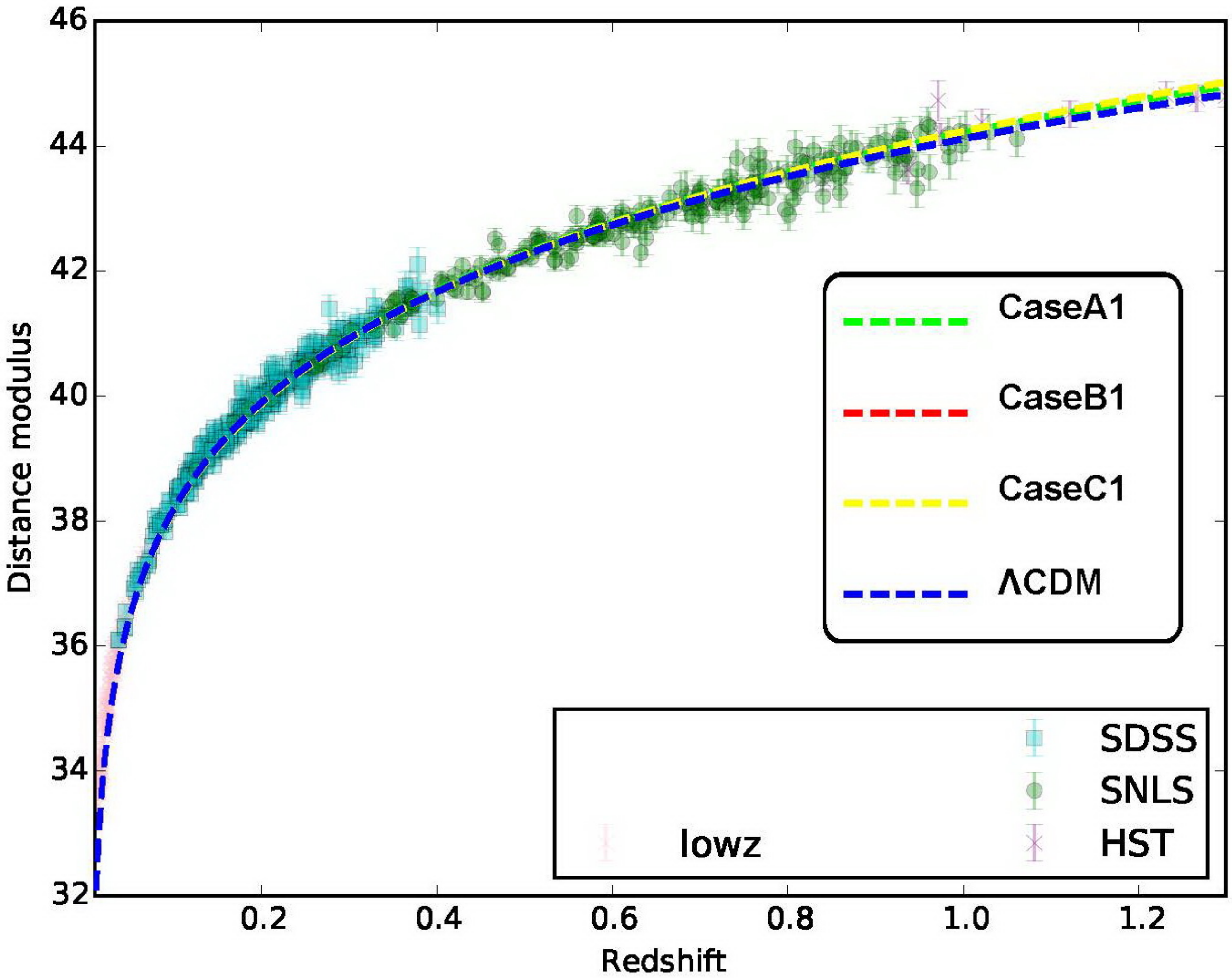
- Actually Case C approximation is not a good one from the comparison between Hubble constant vs t and the luminosity distance vs redshift z.
- The reason may be we use a fixed w_0 in the equation of state of dark partner part. Ignore the case w_0>-8/9 (excluded by observation of luminosity distance with redshift relation),we can make the conclusion:
- Quintessence potential is generated from string landscape AdS vacuum effectively.
- The critical value of cosmological constant separating quintessence from metastable dS is approximately zero

- The formula for the redshift remains unchanged as in the Lorentzian invariant case. $1+z=\frac{a_0}{a}$
- The dependence of luminosity distance d_L with redshift and Hubble constant.

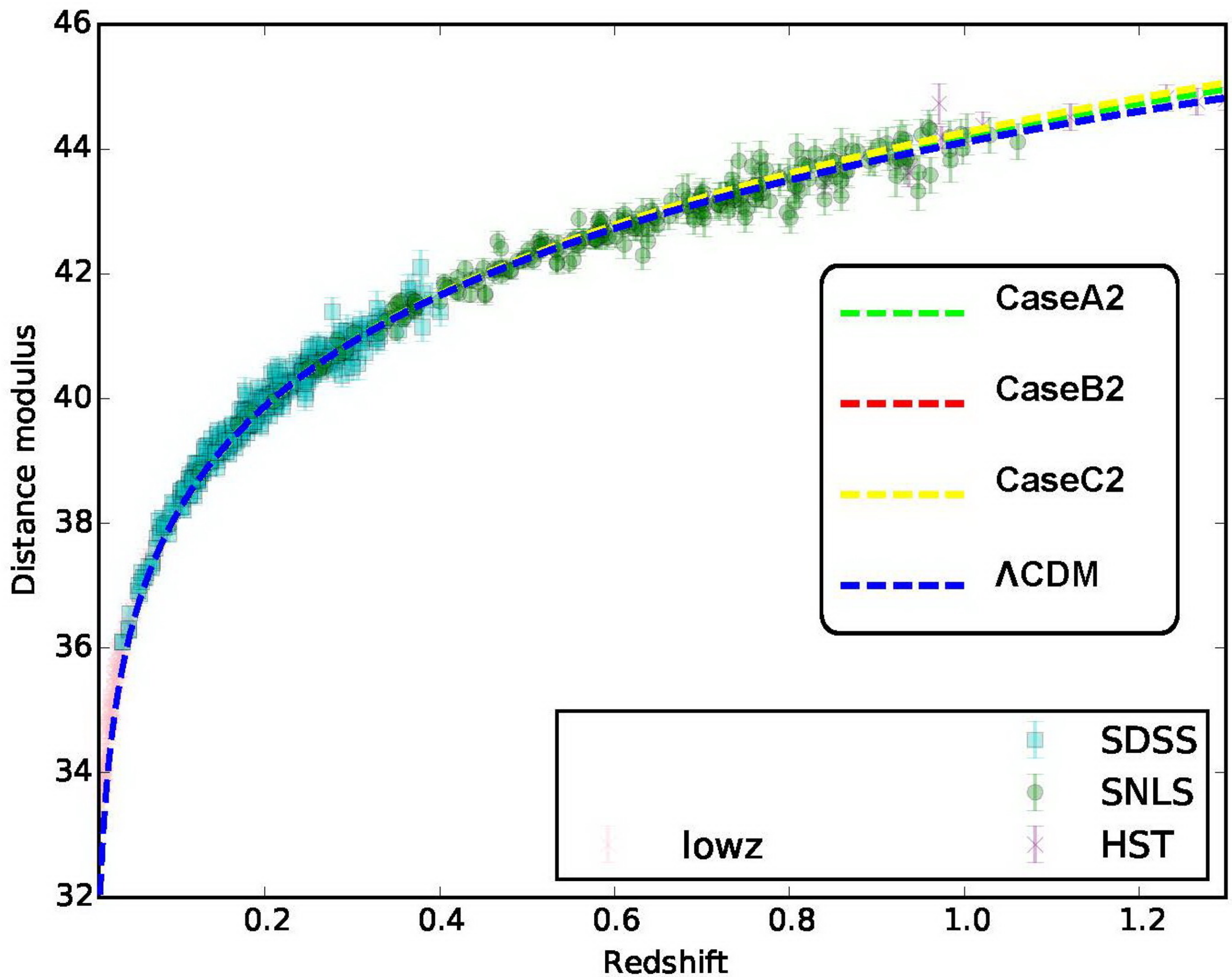
$$H(z)=\left(\frac{\mathrm{d}}{\mathrm{dz}}\frac{d_L}{1+z}\right)^{-1}, \qquad \frac{\mathrm{d}t}{\mathrm{dz}}=-\frac{1}{1+z}\frac{\mathrm{d}}{\mathrm{dz}}\left(\frac{d_L}{1+z}\right)$$

- The distance modulus is defined as

$$\mu=25+5\log_{10}\left(d_L/Mpc\right)$$



- Comparation of distance magnitudes. $\Lambda_0 = -0.4\Lambda$
- $\mathscr{K}(t_0) = H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^2}}-1\right)$ and $w_0 = -0.88$ for Case C



- Comparation of distance magnitudes vs z, $\Lambda_0 = -0.4\Lambda$

- $\mathscr{K}(t_0) = -H_0\left(\sqrt{1-\frac{\Lambda-\Lambda_0}{3H_0^{\,2}}}+1\right)$ and $w_0 = -8/9$ for Case C

Summary

- For string landscape with $\Lambda_0 > -\left(3H_0 - \Lambda\right) \approx -\frac{2}{5}\Lambda$,the effective cosmological constant naturally give a quintessence like potential which satisfies the dS Swampland conjecture
- The uplifting of AdS to a positive effective consmological constant by frozen large scale Lorentz violation mechanism avoids the meta-stable dS swampland puzzle and have a quantum gravity origin
- For string swampland with positive cosmological constant for most reasonably approximation, the effective cosmological constant behaves like a metastable dS potential rather than the quintessence like one when the large scale Lorentz violation is taken into account.
- the scenario prefers approximately zero cosmological constant $\Lambda_{0-crit} = 0$ as a separation of effective quintessence from meta-stable dS

- 第九届海峡两岸粒子物理与宇宙学研讨会6月29-7月2日（28日报到，3日离会）将在乌鲁木齐金谷大酒店举行，会议网址为： https://indico.itp.ac.cn/event/1/
欢迎大家到乌鲁木齐开展学术交流

THANKS!