High-order corrections for Higgs production at CEPC and some selected QCD phenomenology

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Institute of High Energy Physics

Based on a series of paper:

Q. F. Sun, F. Feng, Y.J., W.-L. Sang, PRD (RC) 2017
W.-L. Sang, W. Chen, F. Feng, Y.J., Q. F. Sun, PLB 2017
W. Chen, F. Feng, Y.J., W.-L. Sang, CPC 2019
F. Feng, Y.J., X.-H. Liu, W.-L. Sang, in preparation
F. Feng, Y.J., W.-L. Sang, PRL 2015
F. Feng, Y.J., W.-L. Sang, PRL 2017
F. Feng, Y.J., W.-L. Sang, 1901.08447, submitted to PRL

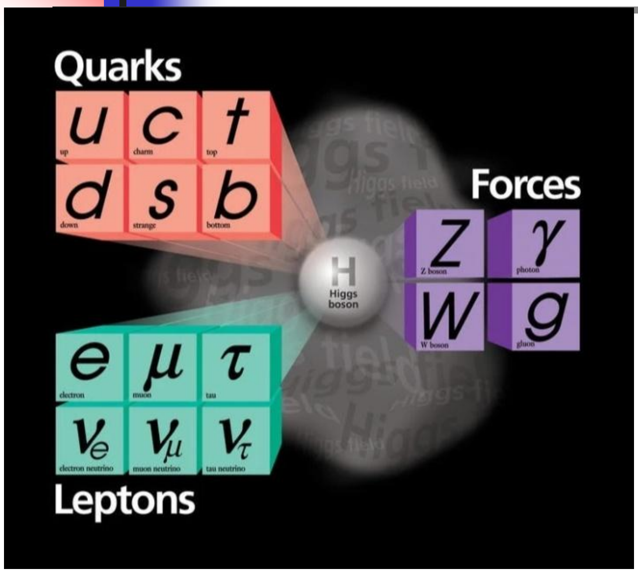
第14届TeV物理工作组学术研讨会，南师大，南京，2019/4/19-4/22

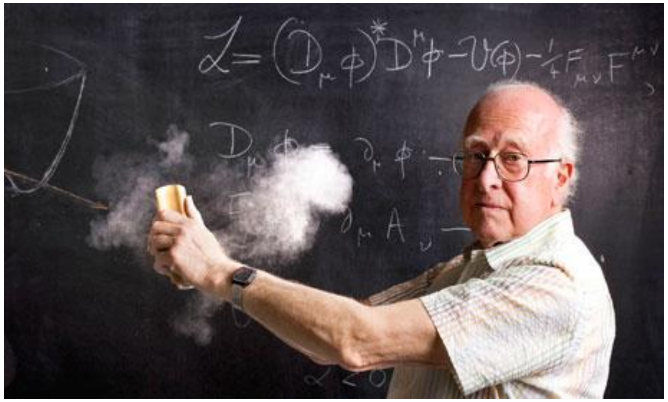
Outline of the talk

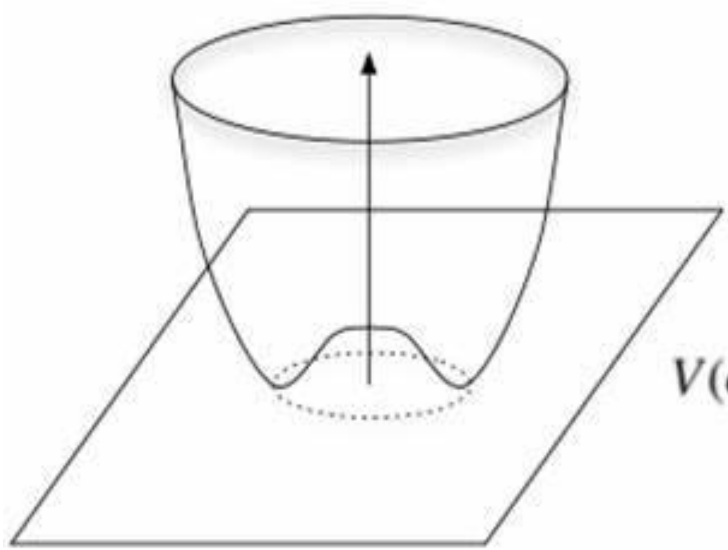
- Overview of Higgs program at CEPC
- NNLO mixed EW-QCD correction to $e^+ e^- \rightarrow H+Z$ at CEPC
- Mixed EW-QCD correction to $e^+ e^- \rightarrow H+Z$ including ISR effect
- Mixed EW-QCD correction to $e^+ e^- \rightarrow H+(Z \rightarrow) \mu^+\mu^-$ (accounting Z width)
- NLO QCD correction to $e^+ e^- \rightarrow H+\gamma$

- NNLO QCD correction to $\gamma\gamma^* \rightarrow \eta_{c,b}$ form factor, $\eta_{c,b}$ total widths in NRQCD
- Summary and Outlook

Higgs boson plays a central role in Standard Model, hence referred to as God particle







$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$

On July 4th 2012, whole world witnessed the historic event: announcement of discovery of long-awaited Higgs boson at CERN

在北京的KITPC也有庆祝活动，韩涛教授专门准备了香槟

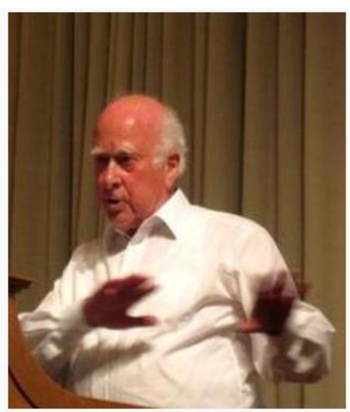
News on the Higgs discovery are all over the world

Nobel physics prize 2013

Photo: A. Mahmoud
François Englert
Prize share: 1/2

Photo: A. Mahmoud
Peter W. Higgs
Prize share: 1/2

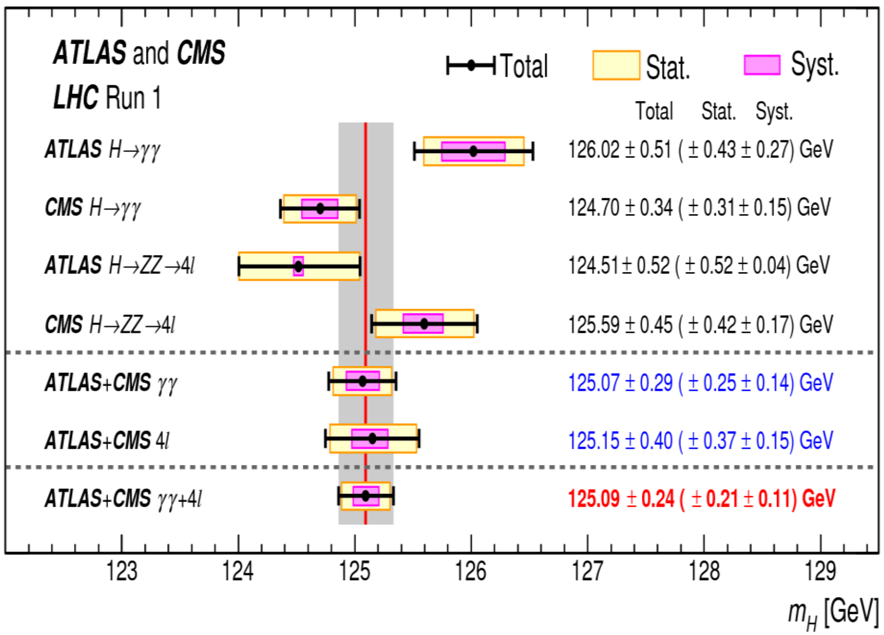
The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs: "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

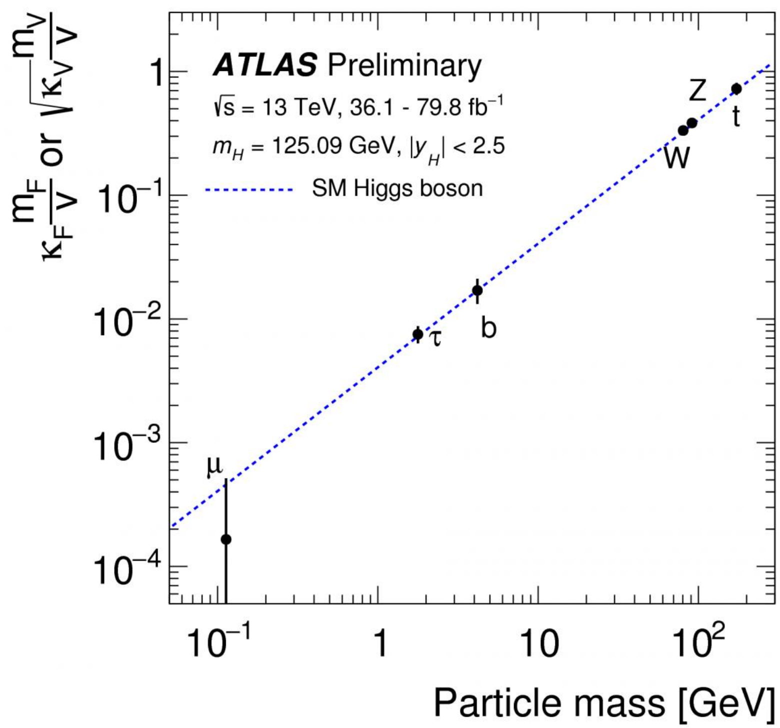


The six authors of a number of 1964 PRL papers, who received the 2010 **J. J. Sakurai Prize** for their work. From left to right: Kibble, Guralnik, Hagen, Englert, Brout, Higgs

P. Anderson is also a pioneer in Higgs mechanism

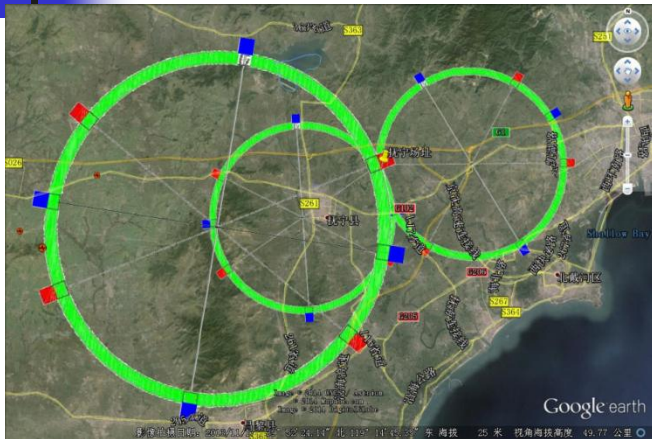
Precise measurements of various Higgs properties, couplings/interactions, are of the top priority in particle physics



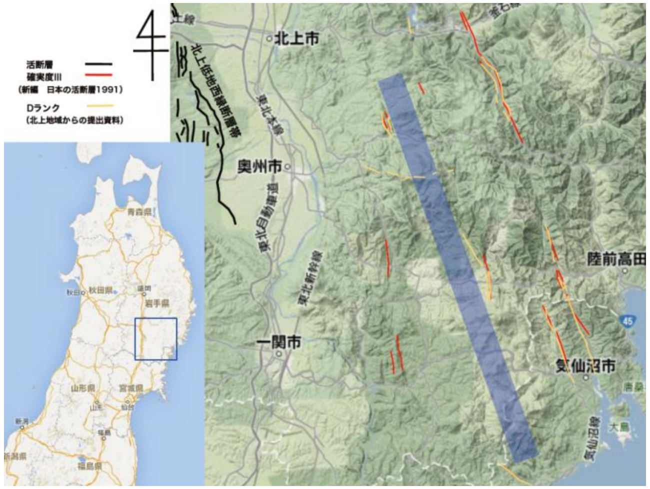


Higgs-charm Yukawa very challenging

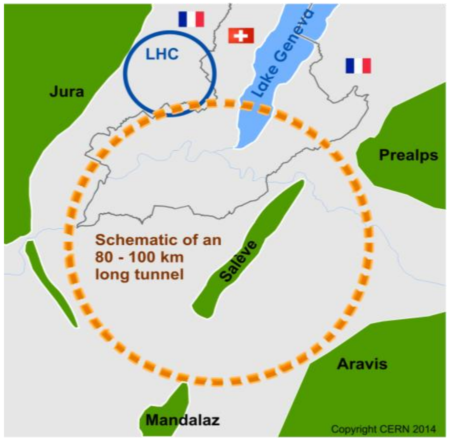
Several Higgs factories under plan



CEPC@90-240 GeV (China)
秦皇岛 or 雄安?



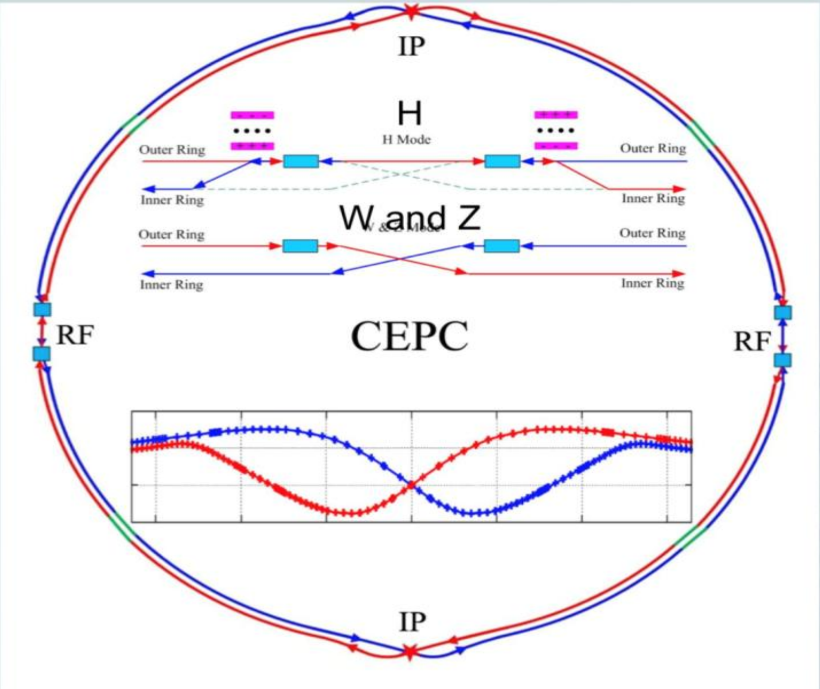
ILC@500,350,250 GeV (Japan)
Kitakami Candidate Site



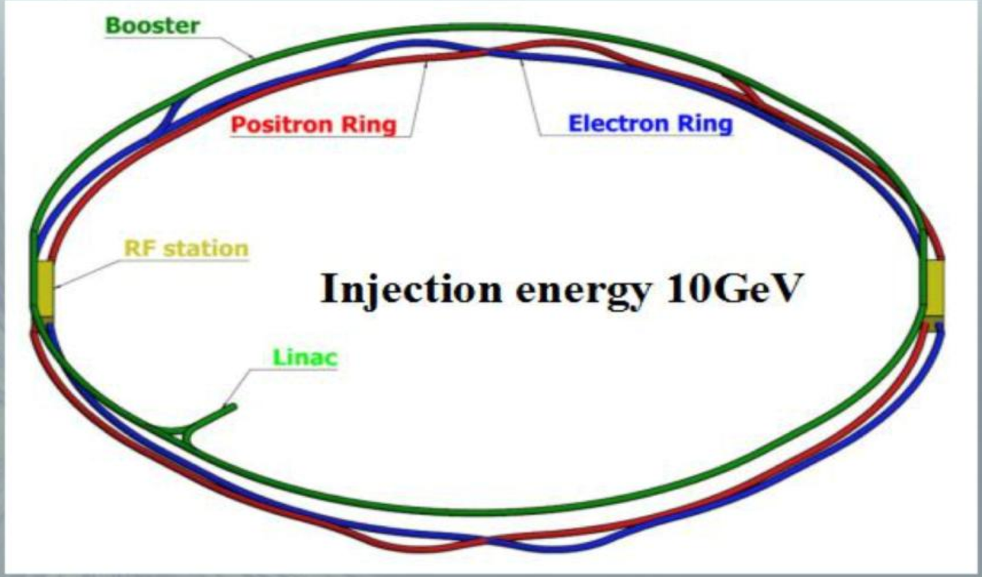
FCC-ee @ 90-400 GeV (Geneva, EU)

CEPC CDR Baseline Layout

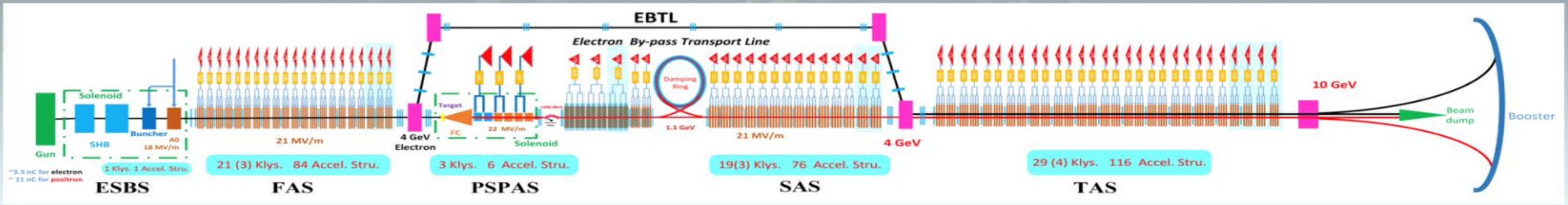
Courtesy to Gang Li



CEPC collider ring (100km)



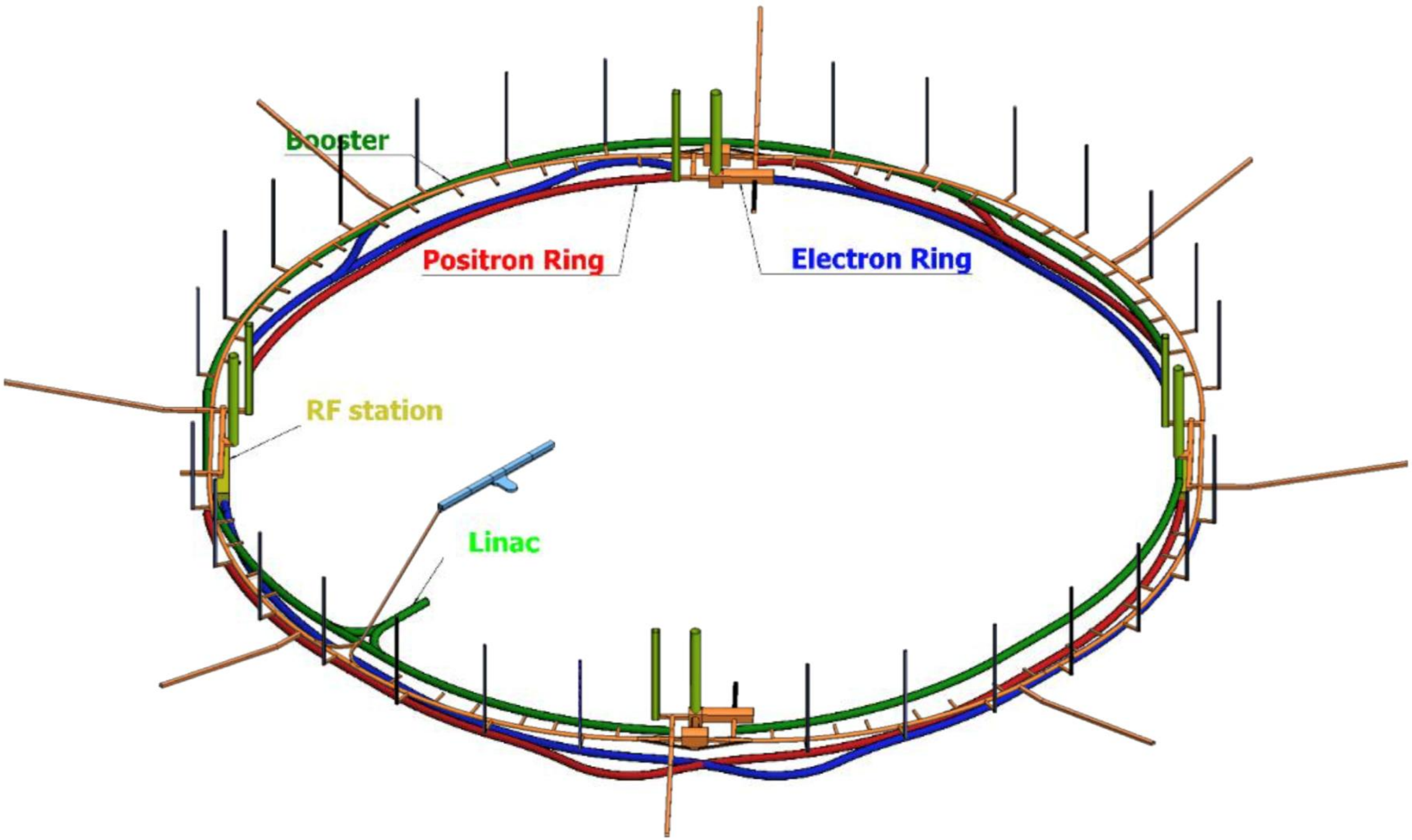
CEPC booster ring (100km)



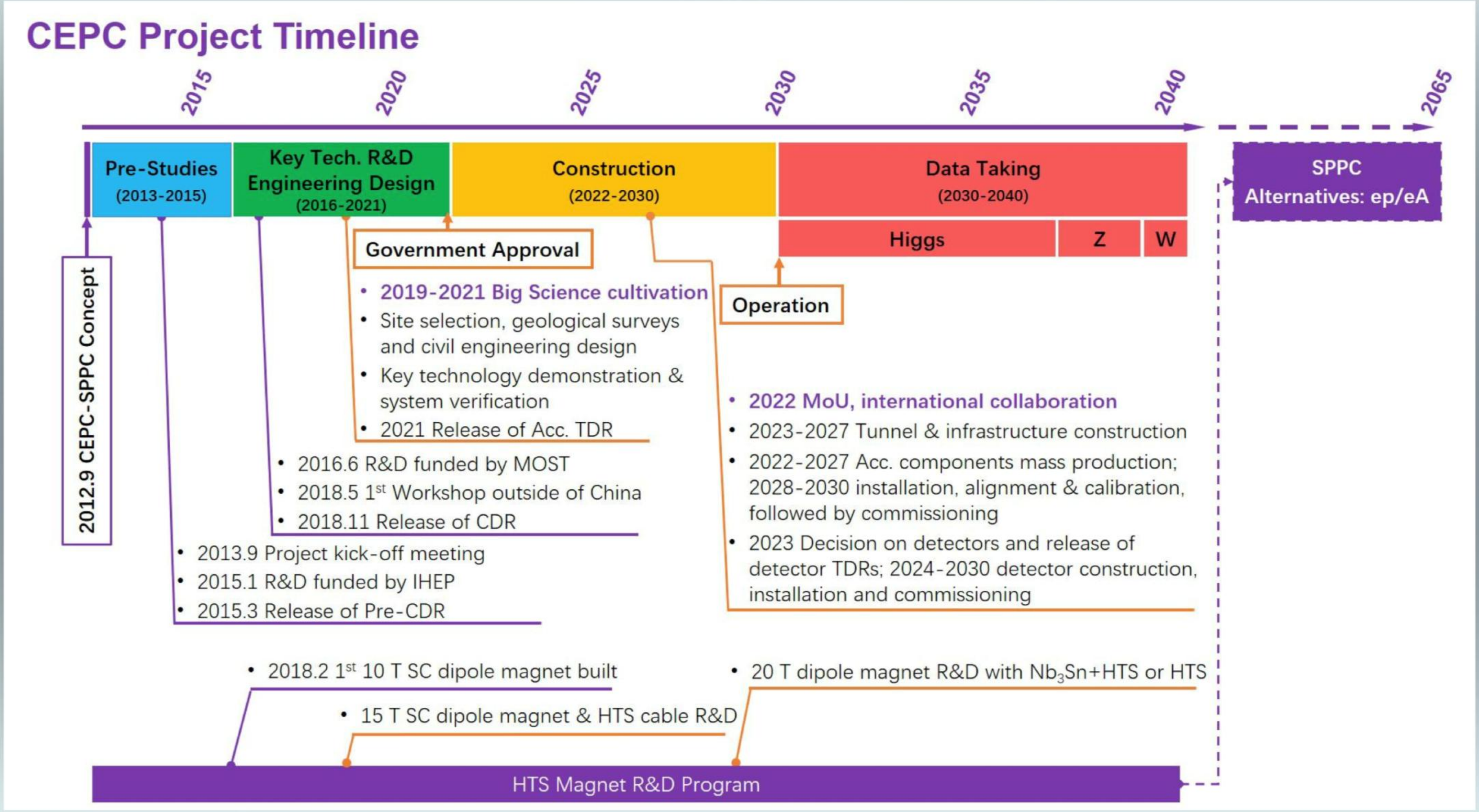
CEPC Linac injector (1.2km, 10GeV)

CEPC represents the future of high energy physics of China

CEPC Civil Engineering Design



CEPC timeline

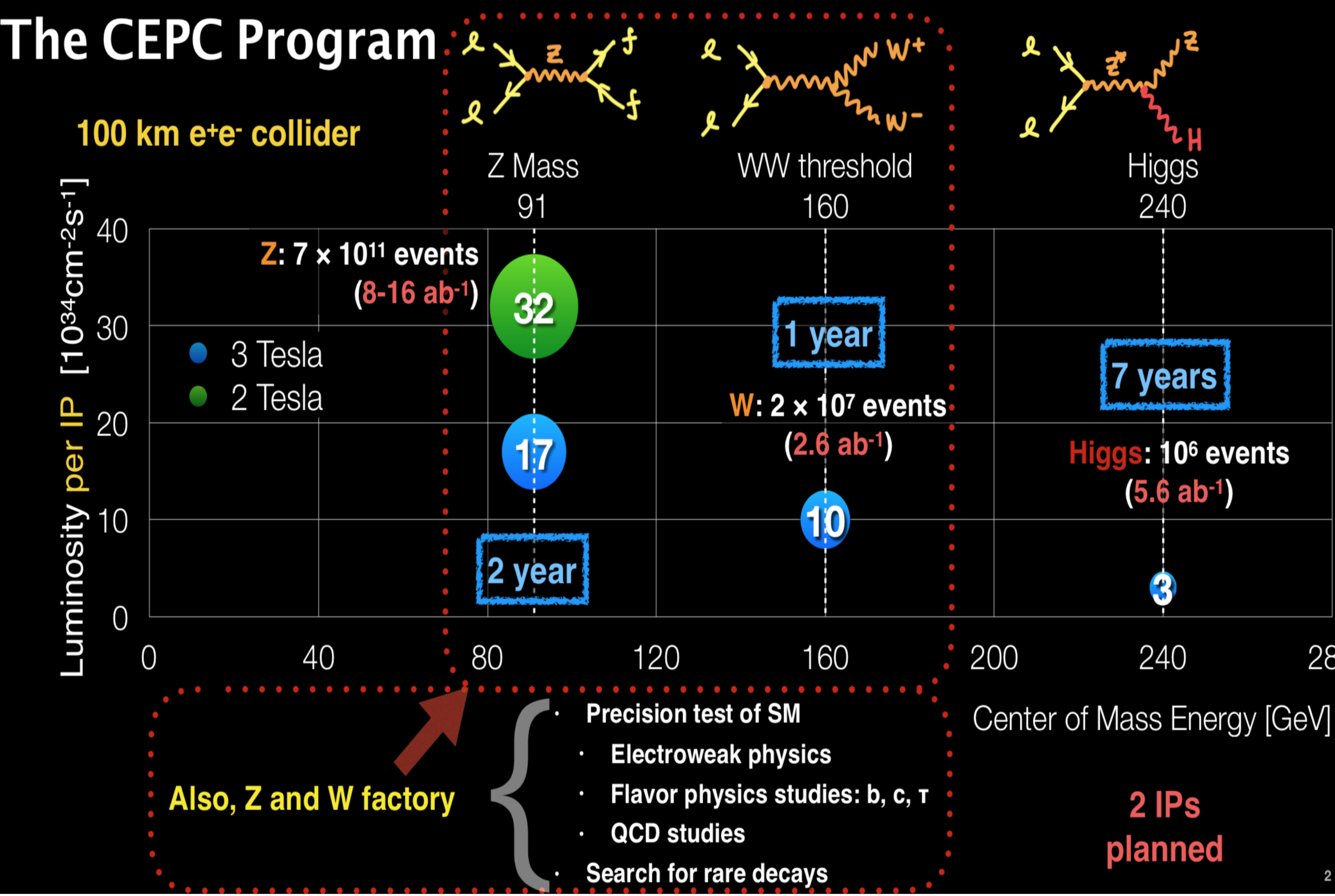


CEPC High Lumi Parameters@Higgs

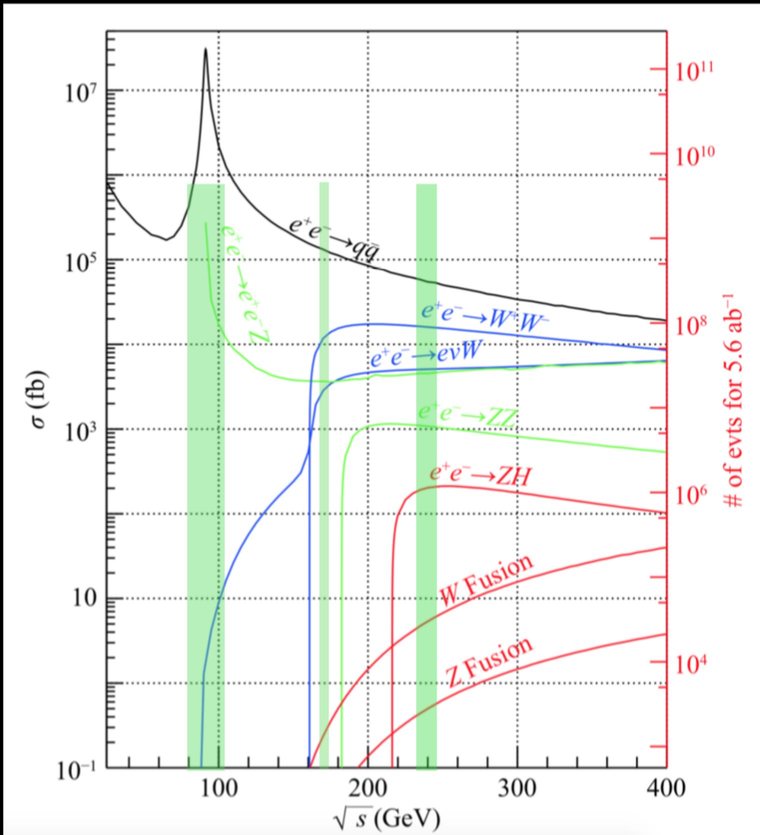
D. Wang

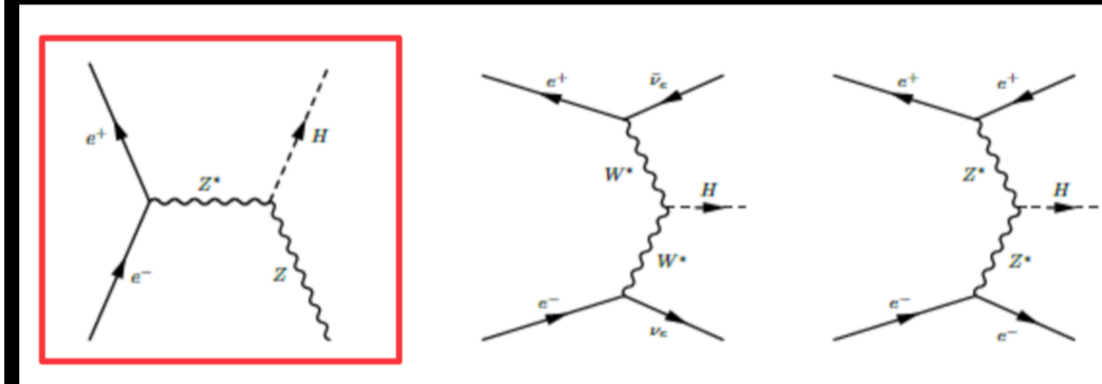
	Higgs	*W*	*Z (3T)*	*Z (2T)*
Number of IPs	2			
Beam energy (GeV)	**120**	**80**	**45.5**	
Circumference (km)	100			
Synchrotron radiation loss/turn (GeV)	1.68	0.33	0.035	
Crossing angle at IP (mrad)	16.5×2			
Piwinski angle	3.78	8.5	27.7	
Number of particles/bunch N_e (10^{10})	17.0	12.0	8.0	
Bunch number (bunch spacing)	**218 (0.76μs)**	**1568 (0.20μs)**	**12000 (25ns+10%gap)**	
Beam current (mA)	17.8	90.4	461.0	
Synchrotron radiation power /beam (MW)	**30**	**30**	**16.5**	
Bending radius (km)	10.7			
Momentum compact (10^{-5})	0.91			
β function at IP β_x^* / β_y^* (m)	**0.33/0.001**	**0.33/0.001**	**0.2/0.001**	
Emittance $\varepsilon_x/\varepsilon_y$ (nm)	0.89/0.0018	0.395/0.0012	0.13/0.003	0.13/0.00115
Beam size at IP σ_x/σ_y (μm)	17.1/0.042	11.4/0.035	5.1/0.054	5.1/0.034
Beam-beam parameters ξ_x/ξ_y	0.024/0.113	0.012/0.1	0.004/0.053	0.004/0.085
RF voltage V_{RF} (GV)	2.4	0.43	0.082	
RF frequency f_{RF} (MHz) (harmonic)	650 (216816)			
Natural bunch length σ_z (mm)	2.2	2.98	2.42	
Bunch length σ_z (mm)	3.93	5.9	8.5	
HOM power/cavity (2 cell) (kw)	0.58	0.77	**1.94**	
Energy spread (%)	0.19	0.098	0.080	
Energy acceptance requirement (%)	**1.7**	**0.90**	**0.49**	
Energy acceptance by RF (%)	3.0	1.27	1.55	
Photon number due to beamstrahlung	0.104	0.050	0.023	
Beamstruhlung lifetime /quantum lifetime* (min)	30/50	>400		
Lifetime (hour)	**0.22**	**1.2**	**3.2**	**2.0**
F (hour glass)	0.85	0.92	0.98	
Luminosity/IP *L* ($10^{34}cm^{-2}s^{-1}$)	**5.2**	**14.5**	**23.6**	**37.7**

*include beam-beam simulation and real lattice


The CEPC Program
100 km e+e- collider
Z Mass
91
WW threshold
160
Higgs
240
Luminosity per IP [10^34cm^-2s^-1]
Z: 7 × 10^11 events
(8-16 ab^-1)
32
3 Tesla
2 Tesla
17
2 year
1 year
W: 2 × 10^7 events
(2.6 ab^-1)
10
7 years
Higgs: 10^6 events
(5.6 ab^-1)
3
Center of Mass Energy [GeV]
Also, Z and W factory
Precision test of SM
Electroweak physics
Flavor physics studies: b, c, τ
QCD studies
Search for rare decays
2 IPs planned
2

CEPC: 1M Higgs, 10^{11}~10^{12} Z bosons and ~10^{8} W Pairs





Process	Cross section	Events in 5.6 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	196.2	1.10×10^6
$e^+e^- \to \nu_e\bar{\nu}_e H$	6.19	3.47×10^4
$e^+e^- \to e^+e^- H$	0.28	1.57×10^3
Total	203.7	1.14×10^6

- Direct: Higgs mass, σ(ZH), Branching ratios, Diff. distributions
- Derived: Higgs width, couplings, quantum numbers, …
- EW Precision, tau physics, Flavor Physics, …

Electroweak observables at CEPC

Electroweak program, in addition to Higgs studies, is essential to constraint new physics

Expect to collect ~7×10[11] Z boson for electroweak precision physics

Observable	LEP precision	CEPC precision	CEPC runs	CEPC $\int \mathcal{L}dt$
m_Z	2.1 MeV	0.5 MeV	Z pole	8 ab^{-1}
Γ_Z	2.3 MeV	0.5 MeV	Z pole	8 ab^{-1}
$A_{FB}^{0,b}$	0.0016	0.0001	Z pole	8 ab^{-1}
$A_{FB}^{0,\mu}$	0.0013	0.00005	Z pole	8 ab^{-1}
$A_{FB}^{0,e}$	0.0025	0.00008	Z pole	8 ab^{-1}
$\sin^2\theta_W^{\text{eff}}$	0.00016	0.00001	Z pole	8 ab^{-1}
R_b^0	0.00066	0.00004	Z pole	8 ab^{-1}
R_μ^0	0.025	0.002	Z pole	8 ab^{-1}
m_W	33 MeV	1 MeV	WW threshold	2.6 ab^{-1}
m_W	33 MeV	2–3 MeV	ZH run	5.6 ab^{-1}
N_ν	1.7%	0.05%	ZH run	5.6 ab^{-1}

Many variables knowledge will improve by more than one order of magnitude relative to LEP

Results in CDR (2018.11)

All scaled to 240 GeV, 5.6ab^{-1}

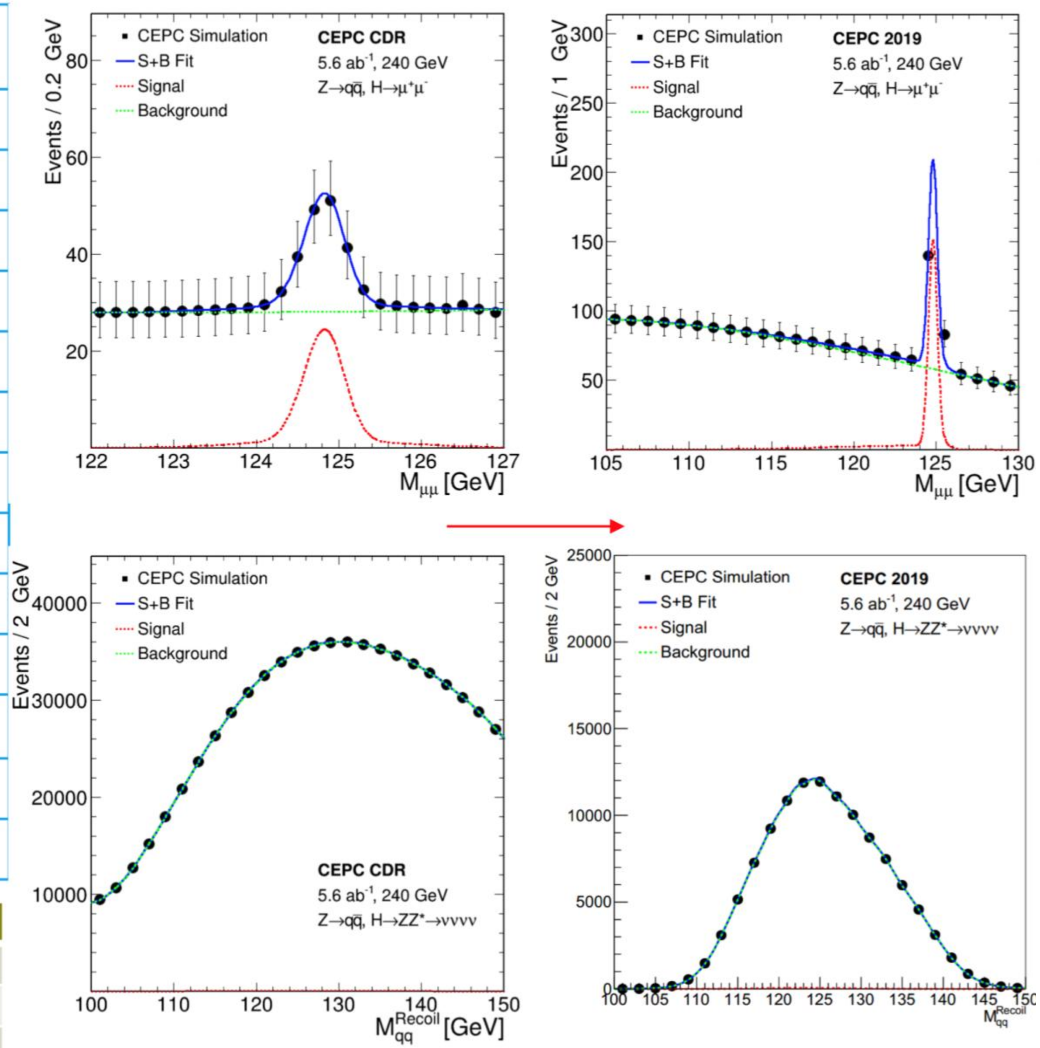
Property	Estimated Precision CEPC-v1		Estimated Precision CEPC-v4	
m_H	5.9 MeV		5.9 MeV	
Γ_H	2.7%		2.8%	
$\sigma(ZH)$	0.5%		0.5%	
$\sigma(\nu\bar{\nu}H)$	3.0%		3.2%	
Decay mode	$\sigma \times$ BR	BR	$\sigma \times$ BR	BR
$H \to b\bar{b}$	0.26%	0.56%	0.27%	0.56%
$H \to c\bar{c}$	3.1%	3.1%	3.3%	3.3%
$H \to gg$	1.2%	1.3%	1.3%	1.4%
$H \to WW^*$	0.9%	1.1%	1.0%	1.1%
$H \to ZZ^*$	4.9%	5.0%	5.1%	5.1%
$H \to \gamma\gamma$	6.2%	6.2%	6.8%	6.9%
$H \to Z\gamma$	13%	13%	16%	16%
$H \to \tau^+\tau^-$	0.8%	0.9%	0.8%	1.0%
$H \to \mu^+\mu^-$	16%	16%	17%	17%
BR_{inv}^{BSM}	–	< 0.28%	–	< 0.30%

Signal Z	Signal H	Precision	Signal Z	Signal H	Precision	Signal Z	Signal H	Precision
H->qq			H->WW			H→γγ, Zγ		
ee	bb	1.32%	ee	lvlv	9.52%	μμ+ττ	γγ	23.7%
	cc	13.5%		evqq	4.56%	vv		10.5%
	gg	7.22%		μvqq	3.93%	qq		9.84%
μμ	bb	0.99%	μμ	lvlv	7.29%	vv	Zγ(qqγ)	15.7%
	cc	9.54%		evqq	3.90%	vvH(WW fusion)		
	gg	5.01%		μvqq	3.90%	vv	bb	3.00%
qq	bb	0.46%	vv	qqqq	1.90%	H→μμ		
	cc	11.1%		evqq	4.65%	qq	μμ	17.1%
	gg	3.64%		μvqq	4.14%	ee		
vv	bb	0.39%		lvlv	11.5%	μμ		
	cc	3.83%	qq	qqqq	1.75%	vv		
	gg	1.47%	H->ZZ			H→ττ		
H->Invisible			vv	μμqq	8.26%	ee	ττ	2.75%
qq	ZZ(vvvv)	232%	vv	eeqq	40%	μμ		2.61%
ee		370%	μμ	vvqq	7.32%	qq		0.95%
μμ		245%	ZH bkg contribution		19.4%	vv		2.66%

Updates since CDR

Analysis Updated

($5.6ab^{-1}$)	CEPC 2018.11	2019.4
$\sigma(ZH)$	**0.50%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mathrm{bb})$	**0.27%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mathrm{cc})$	**3.3%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mathrm{gg})$	**1.3%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mathrm{WW})$	**1.0%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mathrm{ZZ})$	**5.1%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \tau\tau)$	**0.8%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \gamma\gamma)$	**6.8%**	
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to \mu\mu)$	**17%**	12%
$\sigma(\mathrm{vv}H) * \mathrm{Br}(\mathrm{H} \to \mathrm{bb})$	**3.0%**	
$\mathrm{Br}_{\mathrm{upper}}(\mathrm{H} \to \mathrm{inv.})$	**0.41%**	0.26%
$\sigma(ZH) * \mathrm{Br}(\mathrm{H} \to Z\gamma)$	**16%**	
Width	**2.8%**	

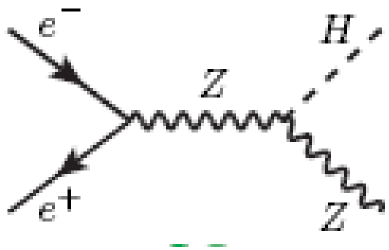


H->invisible	CDR	Now
Z->ee	370%	341%
Z->mm	245%	191%
Z->qq	232%	78%
Combined	153%	71%

CEPC团队、国际顾问委员会部分委员和《CEPC概念设计报告》国际评审委员会成员合影 -- 2018年11月14日

Part 1

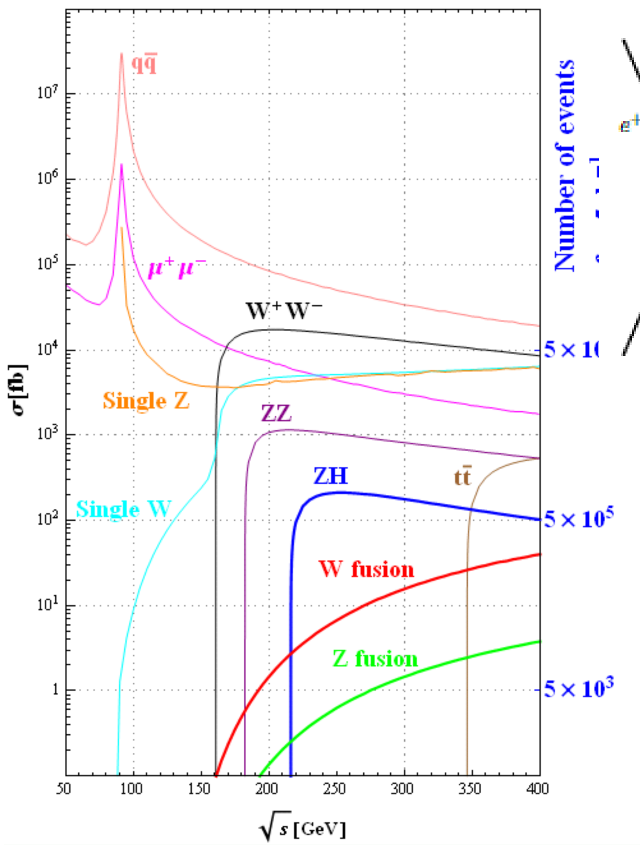
$e^+ e^- \rightarrow HZ$

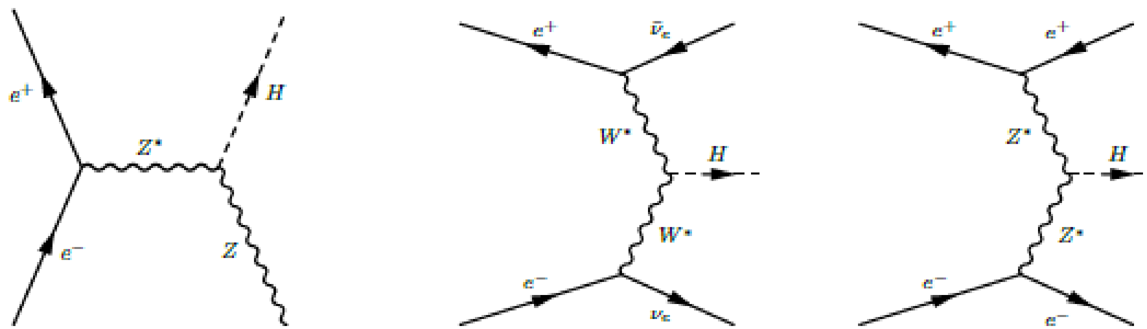


Golden production channel for Higgs at CEPC at 240 GeV

Often referred to as *Higgs-strahlung* process

Higgs-strahlung dominates other production mechanisms when CM energy below 400 GeV





Process	Cross section	Events in 5 ab^{-1}
Higgs boson production, cross section in fb		
$e^+e^- \to ZH$	212	1.06×10^6
$e^+e^- \to \nu\bar{\nu}H$	6.72	3.36×10^4
$e^+e^- \to e^+e^-H$	0.63	3.15×10^3
Total	219	1.10×10^6

Motivation

- CEPC can measure the production rate $\sigma(ZH)$ to an exquisite precision of 0.5%
- Knowing the NLO EW correction (a few percent) is not sufficient to meet experimental precision
- $O(\alpha^2)$ and $O(\alpha\alpha_s)$ corrections need be considered
- We will investigate the latter, which should be more manageable and seemingly more important

Previous work on NLO EW correction for $e^+ e^-$ → H+Z by three German groups

The $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to HZ$ have been calculated independently by three groups:

- J. Fleischer and F. Jegerlehner,
Nucl. Phys. B 216 (1983) 469.

- B. A. Kniehl,
Z. Phys. C 55 (1992) 605.

- A. Denner, J. Kublbeck, R. Mertig
 and M. Bohm,
Z. Phys. C 56 (1992) 261.

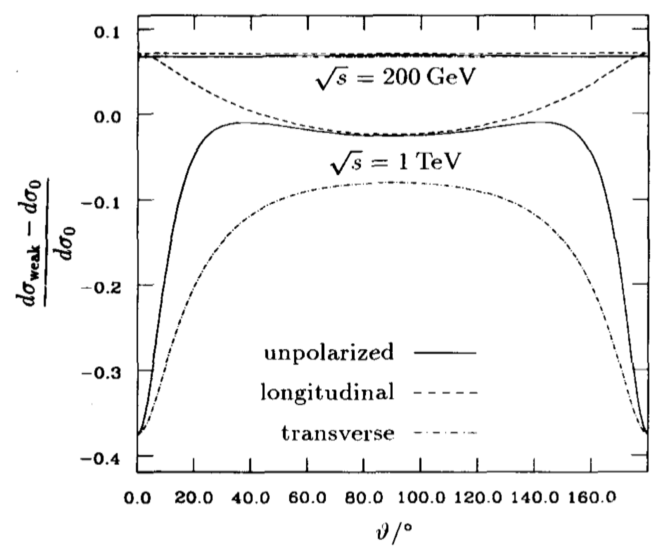


Fig. 15. The relative corrections to the differential cross section for different polarizations of the Z-boson and different CMS energies

Two domestic teams independently accomplished the $O(\alpha\alpha_s)$ corrections simultaneously

PHYSICAL REVIEW D **95**, 093003 (2017)

Mixed QCD-electroweak corrections for Higgs boson production at e^+e^- colliders

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(Received 28 October 2016; published 4 May 2017)

Since the discovery of the Higgs boson at the Large Hadron Collider, a future electron-position collider has been proposed for precisely studying its properties. We investigate the production of the Higgs boson at such an e^+e^- collider associated with a Z boson, and calculate for the first time the mixed QCD-electroweak corrections to the total cross sections. We provide an approximate analytic formula for the cross section and show that it reproduces the exact numeric results rather well for collider energies up to 350 GeV. We also provide numeric results for $\sqrt{s} = 500$ GeV, where the approximate formula is no longer valid. We find that the $\mathcal{O}(\alpha\alpha_s)$ corrections amount to a 1.3% increase of the cross section for a center-of-mass energy around 240 GeV. This is significantly larger than the expected experimental accuracy and has to be included for extracting the properties of the Higgs boson from the measurements of the cross sections in the future.

DOI: 10.1103/PhysRevD.95.093003

Gong, Li, Xu, Yang and Zhao, 1609.03955

RAPID COMMUNICATIONS

PHYSICAL REVIEW D **96**, 051301(R) (2017)

Mixed electroweak-QCD corrections to $e^+e^- \to HZ$ at Higgs factories

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(Received 14 September 2016; published 8 September 2017)

The prospective Higgs factories, exemplified by ILC, FCC-ee and CEPC, plan to conduct precision Higgs measurements at the e^+e^- center-of-mass energy around 250 GeV. The cross sections for the dominant Higgs production channel, the Higgsstrahlung process, can be measured to a (sub)percent accuracy. Merely incorporating the well-known next-to-leading-order (NLO) electroweak corrections appears to be far from sufficient to match the unprecedented experimental precision. In this work, we make an important advancement toward this direction by investigating the mixed electroweak-QCD corrections to $e^+e^- \to HZ$ at next-to-next-to-leading order (NNLO) for both unpolarized and polarized Z bosons. The corrections turn out to reach the 1% level of the Born order results, and thereby must be incorporated in future confrontations with the data.

DOI: 10.1103/PhysRevD.96.051301

Sun, Feng, Jia and Sang, 1609.03995

Typical higher-order Feynman diagrams to the Higgs-strahlung process

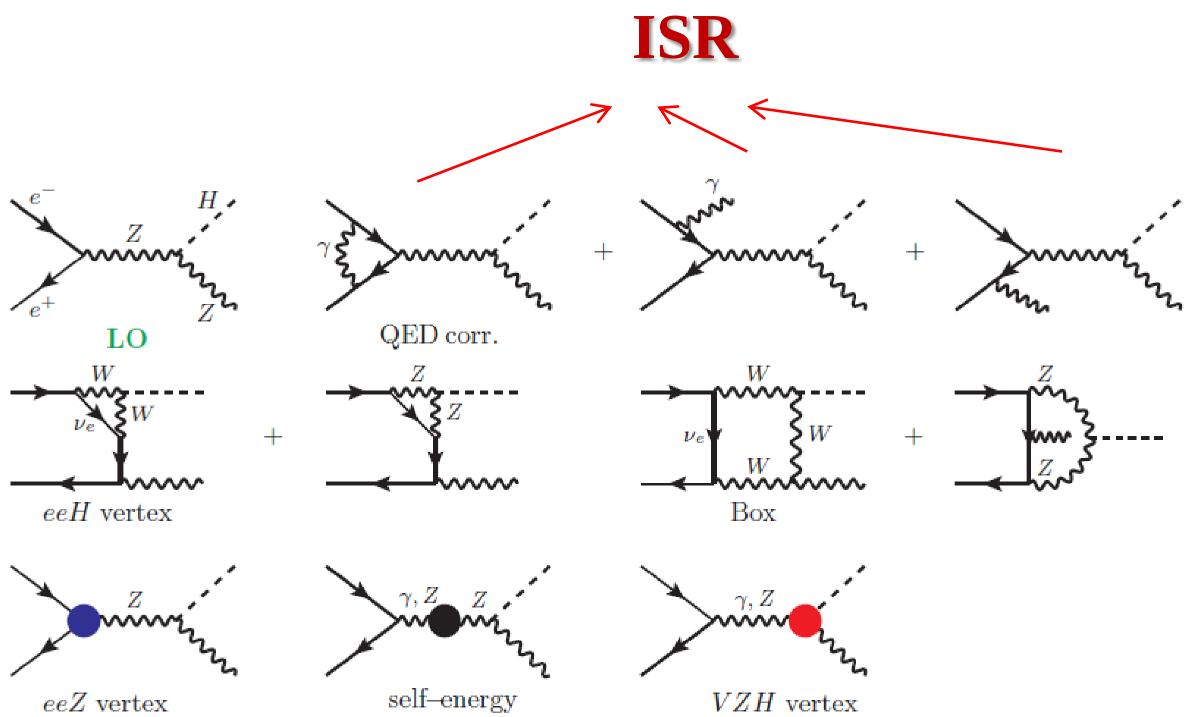


FIG. 1: LO diagram for $e^+e^- \to HZ$, together with some representative higher-order diagrams up to order-$\alpha\alpha_s$.

Typical Feynman diagrams for the **s-channel topologies:** eeZ vertex, self-energy and ZZH vertex.

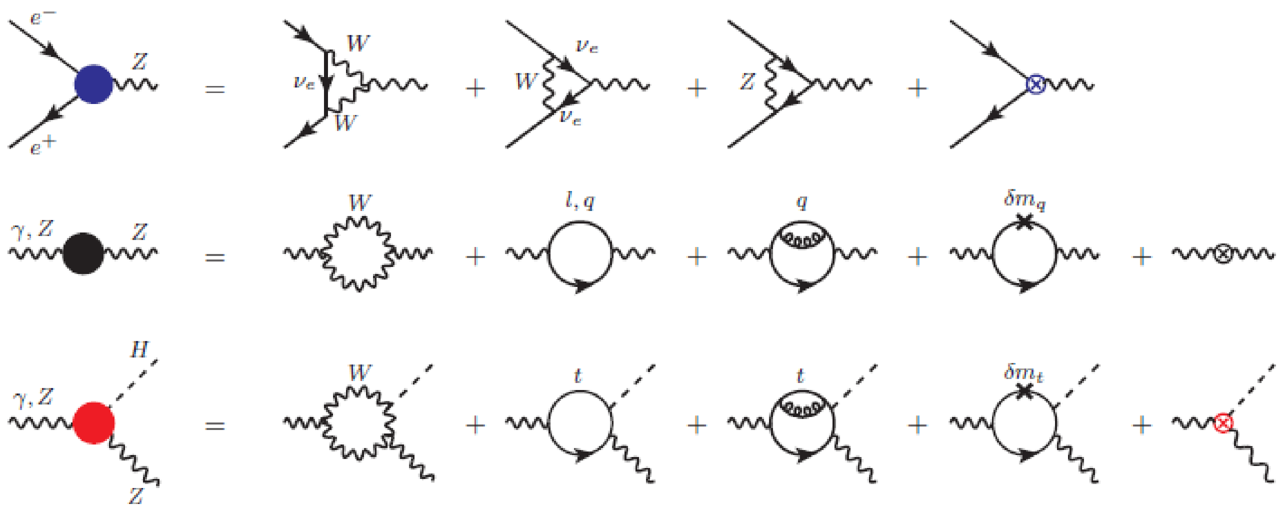


FIG. 2: Representative diagrams for the radiative corrections to the renormalized eeZ vertex, γ/Z self-energy, and VZH vertex, through order-$\alpha\alpha_s$. The cross represents the quark mass counterterm in QCD, cap denotes the electroweak counterterm in on-shell scheme.

Recipe of renormalization

Renormalization is invoked to achieve UV-finite result

We eliminate the **UV divergences** by employing **on-shell renormalization scheme**

- A. Sirlin, Phys. Rev. D22, 971 (1980).
- A. Denner, Fortsch. Phys. 41, 307 (1993).

We calculated numerious electroweak counter-terms, according to the specific renormalization condition, such as

$$\delta Z_e,\ \delta M_z^2,\ \delta M_W^2,\ \delta M_H^2,\ \delta Z_{ZZ},\ \delta Z_{\gamma Z},\ \delta Z_H \ldots$$

To ascertain theoretical uncertainty, we apply three sub-schemes within OS renormalization scheme

$\alpha(0)$ scheme

$$\delta Z_e\big|_{\alpha(0)} = \frac{1}{2}\Delta\alpha^{(5)}_{\rm had}(M_Z^2) + \frac{1}{2}{\rm Re}\,\Pi^{\gamma\gamma(5)}(M_Z^2) + \frac{1}{2}\Pi^{\gamma\gamma}_{\rm rem}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2},$$

$$\Delta\alpha(M_Z^2) = \Pi^{\gamma\gamma}_{f\neq t}(0) - {\rm Re}\,\Pi^{\gamma\gamma}_{f\neq t}(M_Z^2)$$

$\alpha(M_Z)$ scheme

$$\delta Z_e\big|_{\alpha(M_Z^2)} = \delta Z_e\big|_{\alpha(0)} - \tfrac{1}{2}\Delta\alpha(M_Z^2)$$

$$\alpha\left(M_Z^2\right) = \frac{\alpha(0)}{1-\Delta\alpha\left(M_Z^2\right)},$$

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi}G_\mu M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)$$

G_μ scheme

$$\delta Z_e\big|_{G_\mu} = \delta Z_e\big|_{\alpha(0)} - \tfrac{1}{2}\Delta r$$

Consistency check

- By adapting their obsolete input parameters, we confirm Denner et al.'s NLO results
- Our NLO predictions to integrated cross sections are accurate to an exquisite degree, actually we get fully analytic results.
- We also checked against the recent NLO predictions by the automated package GRACE-loop, and found perfect agreement

The only new counterterm relevant at $O(\alpha\alpha_s)$

Just needs to incorporate the one-loop QCD counterterm for top quark mass, also enters the Htt Yukawa vertex

$$\delta m_t = -m_t \Gamma(1+\epsilon)\left(\frac{4\pi\mu^2}{m_t^2}\right)^{\epsilon}\frac{C_F}{4}\frac{\alpha_s}{\pi}\frac{3-2\epsilon}{\epsilon(1-2\epsilon)}$$

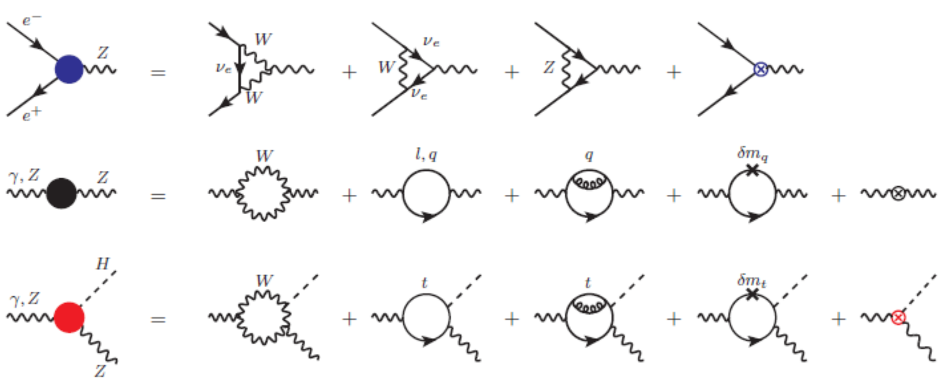


FIG. 2: Representative diagrams for the radiative corrections to the renormalized eeZ vertex, γ/Z self-energy, and VZH vertex, through order-$\alpha\alpha_s$. The cross represents the quark mass counterterm in QCD, cap denotes the electroweak counterterm in on-shell scheme.

Input parameters

Particle Data Group Chin. Phys. C 40, no. 10, 100001 (2016)

Phenomenology. We will take $\sqrt{s} = 240, 250$ GeV as two benchmark energy points at Higgs factory. We adopt the following values for the input parameters: $M_H = 125.09$ GeV, $M_Z = 91.1876$ GeV, $M_W = 80.385$ GeV, $m_t = 174.2$ GeV, $m_e = 0.510998928$ MeV, $m_\mu = 105.6583715$ MeV, $m_\tau = 1.77686$ GeV, $\alpha(0) = 1/137.035999$, $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02764 \pm 0.00013$ [24] and $G_\mu = 1.1663787 \times 10^{-5}$ GeV^2. We take $\alpha(M_Z^2) = 1/128.943$ in the $\alpha(M_Z^2)$ scheme and evaluate the QCD running coupling $\alpha_s(\mu)$ using package RunDec [42].

NNLO predictions in α(0) scheme (including corrections for polarized cross section)

$\sqrt{s}$ (GeV)		LO (fb)	NLO Weak (fb)		NNLO mixed EW-QCD (fb)			
		$\sigma^{(0)}$	$\sigma^{(\alpha)}$	$\sigma^{(0)}+\sigma^{(\alpha)}$	$\sigma_Z^{(\alpha\alpha_s)}$	$\sigma_\gamma^{(\alpha\alpha_s)}$	$\sigma^{(\alpha\alpha_s)}$	$\sigma^{(0)}+\sigma^{(\alpha)}+\sigma^{(\alpha\alpha_s)}$
240	Total	**223.14**	6.64	229.78	2.42	0.008	2.43	232.21
	L	88.67	3.18	91.86	0.96	0.003	0.97	92.82
	T	134.46	3.46	137.92	1.46	0.005	1.46	139.39
250	Total	**223.12**	6.08	229.20	2.42	0.009	2.42	231.63
	L	94.30	3.31	97.61	1.02	0.004	1.02	98.64
	T	128.82	2.77	131.59	1.40	0.005	1.40	132.99

TABLE I: The (un)polarized Higgsstrahlung cross sections at $\sqrt{s} = 240(250)$ GeV in $\alpha(0)$ scheme. We enumerate the NLO weak corrections, together with the NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections. For the latter, we also list individual contribution given in (13).

NLO enhances the LO prediction by about **3.1%**

NNLO mixed EW-QCD correction is sizable, about **1.1%** of the LO prediction!

Exceed the prescribed **0.5% precision of CEPC experiment!**

Angular distribution of the (polarized) Z boson in the Higgs-strahlung process

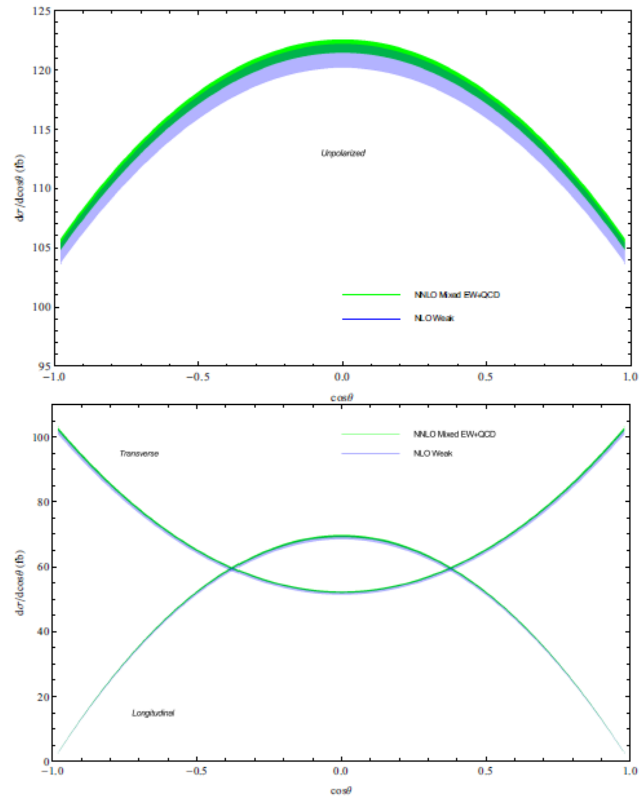


FIG. 3: Differential unpolarized/polarized cross sections for Higgsstrahlung at $\sqrt{s} = 240$ GeV for the NLO $\mathcal{O}(\alpha)$ and NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections. The green band indicates the uncertainties from the input parameters as adopted in Table II and three different schemes.

NNLO predictions for unpolarized cross section in three different sub-schemes **(including QCD scale uncertainty)**

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z^2)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$
250	$\alpha(0)$	223.12 ± 0.47	229.20 ± 0.77	$231.63^{+0.75+0.12}_{-0.75-0.21}$
	$\alpha(M_Z^2)$	252.01 ± 0.60	$227.67^{+0.82}_{-0.81}$	$230.58^{+0.80+0.14}_{-0.79-0.25}$
	G_μ	239.62 ± 0.06	231.82 ± 0.07	$232.65^{+0.07+0.04}_{-0.07-0.07}$

TABLE II: The unpolarized Higgsstrahlung cross sections at $\sqrt{s} = 240(250)$ GeV in three different schemes. To estimate the errors caused by the input parameters, we take $M_W = 80.385 \pm 0.015\,{\rm GeV}$, $m_t = 174.2 \pm 1.4\,{\rm GeV}$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02764 \pm 0.00013$. We also change the strong coupling constant from $\alpha_s(M_Z)$ to $\alpha_s(\sqrt{s})$ with its ceneral value taken as $\alpha_s = \alpha_s(\sqrt{s}/2)$. The remaining parameters are taken the same as in Table I.

We also redo the calculation retaining non-zero bottom quark mass effect too small to include

Observe strong scheme dependence!

The mixed EW-QCD predictions now range from 230 to 233 fb!

Need go to 2-loop EW correction to reduce scheme dependence!

Bad news for the prescribed 0.5% precision at CEPC?

More theoretical work is needed!

Need $O(\alpha^2)$ EW correction to stablize the prediction!

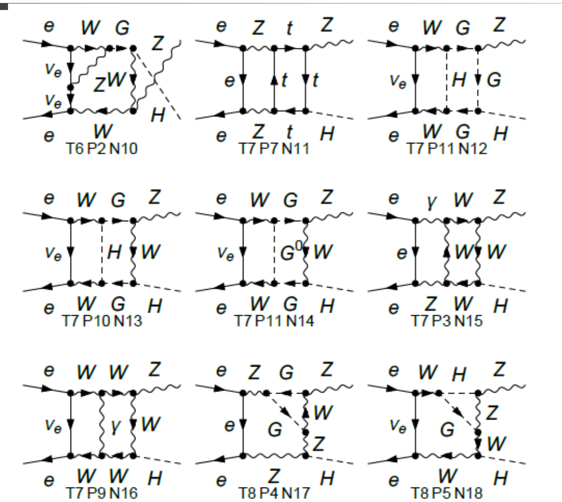
Is it feasible to compute the NNLO pure EW correction?

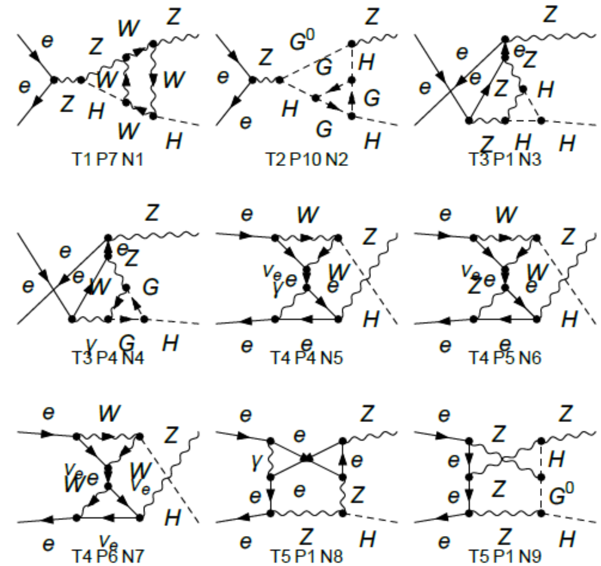
Two-loop Elecroweak corrections, about 124054 Feynman diagrams

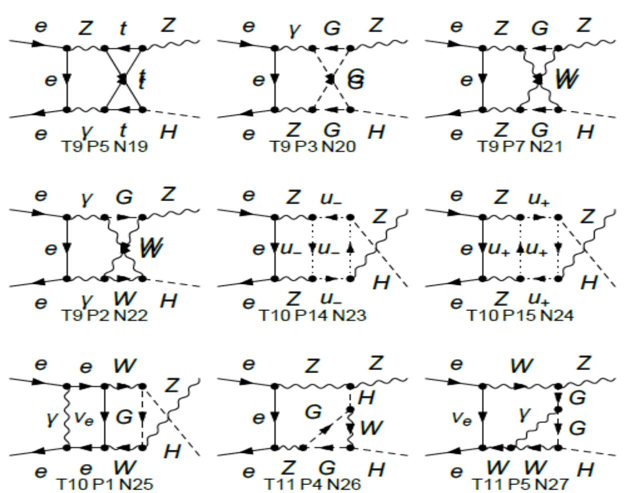
Excluding diagrams including the eeH Yukawa coupling vertex, there remain 64177 Feynman diagrams

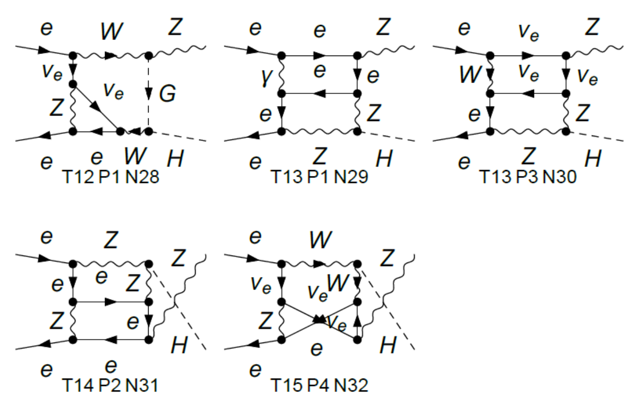
Tensor reduction/IBP difficult, master integrals contain too many scales, formidable job

Here I show some typical two-loop EW diagrams just to frighten you









Part 2

ISR effect for $e^+ e^- \rightarrow HZ$

Incorporating our NNLO mixed EW-QCD corrections

F. Feng, Y.J., X.-H. Liu, W.-L. Sang, in preparation

ISR effect in Higgs-strahlung

Generally, when including ISR, the cross section is

- $$\sigma(s) = \int \mathrm{d}z\, \phi[2\alpha, z]\, \hat{\sigma}(zs)$$

structure function approach

Luminosity or structure function → $\phi[2\alpha, z]$

partonic cross section → $\hat{\sigma}(zs)$

To the Leading logarithmic accuracy, the structure function can be solved analytically

$$\phi_{\mathrm{LL}}[\alpha, z] = \frac{e^{\frac{1}{2}\beta_{\mathrm{LL}}\left(\frac{3}{4}-\gamma_E\right)}}{\Gamma\left(1+\frac{1}{2}\beta_{\mathrm{LL}}\right)} \frac{\beta_{\mathrm{LL}}}{2}(1-z)^{-1+\beta_{\mathrm{LL}}/2}$$

$$\beta_{\mathrm{LL}} = \frac{2\alpha}{\pi} \ln \frac{\mu_F^2}{m_l^2}$$

ISR effect is well understood

We try to go beyond the LL luminosity function, by using the ad hoc exponentiation, which we checked to produce correctly all logs from pure photon to NNLO Nucl. Phys. B 297, 429 (1988)

$$\phi_S[\alpha, z] = e^{\frac{\alpha}{2\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)} \frac{e^{\frac{1}{2}\beta_{\mu_F}\left(\frac{3}{4}-\gamma_E\right)}}{\Gamma\left(1+\frac{1}{2}\beta_{\mu_F}\right)} \frac{\beta_{\mu_F}}{2} (1-z)^{-1+\frac{\beta_{\mu_F}}{2}}$$
$$\times \left(\frac{1}{2}(1+z^2) + \frac{\alpha}{4\pi}\beta_{\mu_F}\left(\frac{3}{32} - \frac{\pi^2}{8} + \frac{3}{2}\zeta[3]\right)\right. \quad (10$$
$$+ \frac{\beta_{\mu_F}}{8}\left(\frac{-1}{2}(1+3z^2)\log(z) - (1-z)^2\right)$$
$$+ \frac{\alpha}{16\pi}\left(-(1+3z^2)\log^2(z) + 4(1+z^2)\left(\text{Li}_2(1-z) + \log(z)\log(1-z)\right)\right.$$
$$\left. + 2(1-z)(3-2z) + 2(3+2z+z^2)\log(z)\right)$$
$$\left. + \frac{\beta_{\mu_F}^2}{32}\left(\frac{1}{2}(3z^2-4z+1)\log(z) + \frac{1}{12}(1+7z^2)\log^2(z) + (1-z^2)\text{Li}_2(1-z) + (1-z)^2\right)\right)$$

$$\beta_{\mu_F} = \frac{2\alpha}{\pi}\left(\log\frac{\mu_F^2}{m_l^2} - 1\right)$$

hep-ph/0203120v1

We also add FO log terms from the electron-pair productions inferred from the full NNLO QED corrections (not shown above)

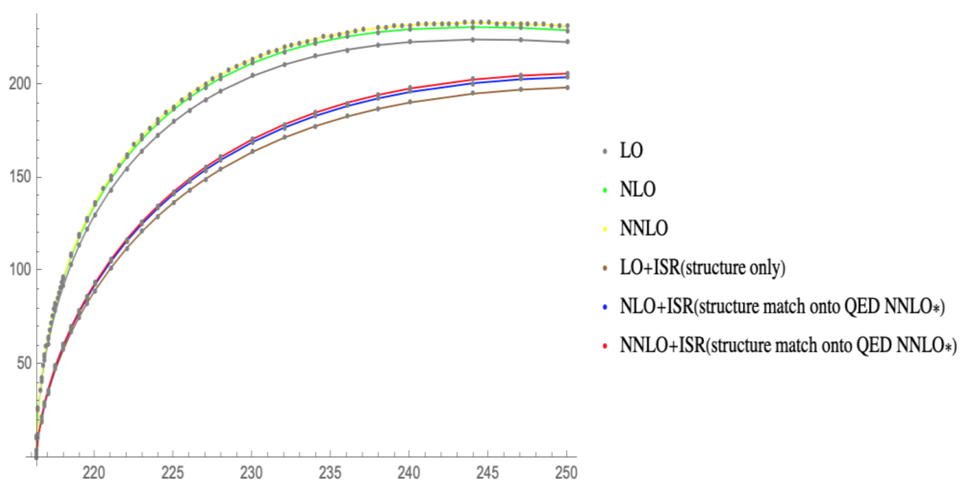
Observed cross sections including ISR effect

$$\sigma(s) = \sigma^{\mathrm{FO}}_{qcd+weak+qed}(s) + \int \mathrm{d}z\, \hat{\sigma}_{qcd+weak}(zs, \mu_F) \left(\phi_S[2\alpha, z, \mu_F] - \phi_S^{\mathrm{exp}}[2\alpha, z, \mu_F] \right)$$

Fixed order with QED

QED logarithms resummed in the structure function

Subtract from fixed order the QED logs included in the structure function



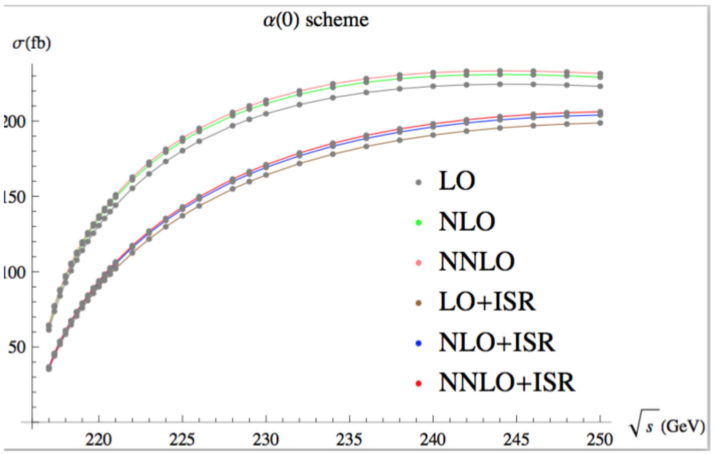
Huge reduction when ISR is turned on

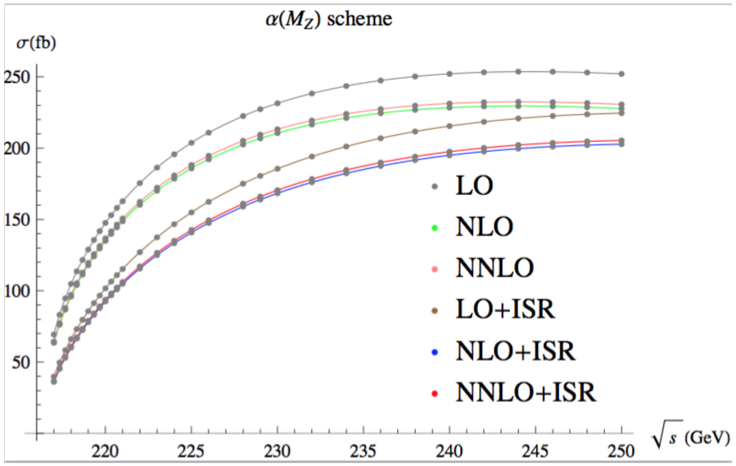
State-of-the-art prediction for σ(ZH) incorporating ISR effect

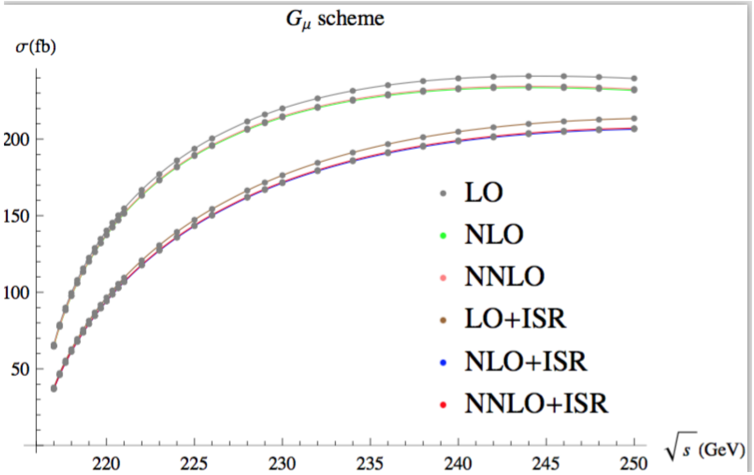
$\sqrt{s}$	schemes	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	$\sigma_{\mathrm{LO}}^{\mathrm{ISR}}$ (fb)	$\sigma_{\mathrm{NLO}}^{\mathrm{ISR}}$ (fb)	$\sigma_{\mathrm{NNLO}}^{\mathrm{ISR}}$ (fb)
240	$\alpha(0)$	223.14	229.78	232.21	190.72	196.14	198.22
	$\alpha(M_Z^2)$	252.03	228.36	231.28	215.41	194.95	197.44
	G_μ	239.64	232.46	233.29	204.82	198.44	199.15
250	$\alpha(0)$	223.12	229.20	231.63	198.77	204.06	206.22
	$\alpha(M_Z^2)$	252.01	227.67	230.58	224.51	202.72	205.32
	G_μ	239.62	231.82	232.65	213.47	206.40	207.14

Very preliminary; will include the uncertainty in input parameters later

Preliminary results for energy dependence *of* σ(ZH) in three different OS sub-schemes







Part 3

Mixed EW-QCD correction to $e^+ e^- \to H+(Z \to) \mu^+\mu^-$

Accounting finite Z width effect

Look for deviation from narrow-width approximation

W. Chen, F. Feng, Y.J., W.-L. Sang, CPC 2019

Part 3

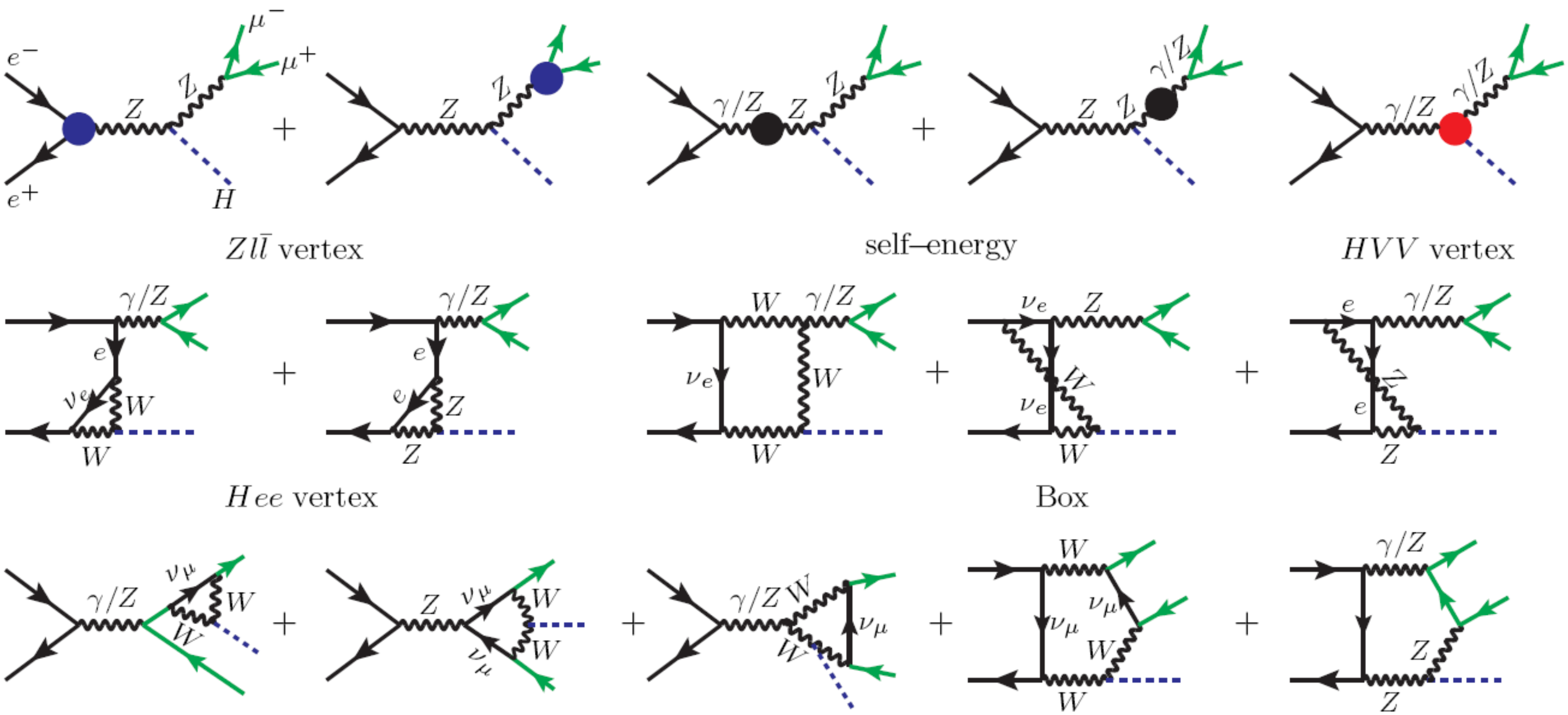
Mixed EW-QCD correction to $e^+ e^- \rightarrow H + (Z \rightarrow) \mu^+ \mu^-$

Accounting finite Z width effect

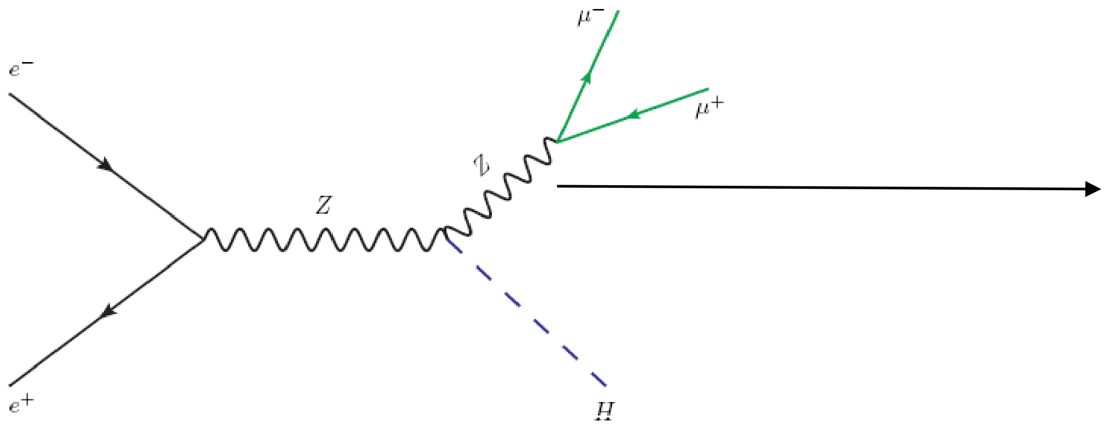
Look for deviation from narrow-width approximation

W. Chen, F. Feng, Y.J., W.-L. Sang, CPC 2019

Sample diagrams for e$^+$ e$^-$ → H $\mu^+\mu^-$



Breit-Wigner approx for e$^+$ e$^-$ → H $\mu^+\mu^-$



$$\frac{s_{12} - M_Z^2}{s_{12} - M_Z^2 + \mathrm{i} M_Z \Gamma_Z}$$

Once beyond LO, naively including the width of unstable particle may ruin gauge invariance and cause double-counting

Treatment of the Z boson width effect

Many treatments exist in the market, e.g., complex mass scheme, Unstable particle EFT, …

For simplicity, we adopt the ``factorization scheme"

A. Denner, S. Dittmaier, M. Roth, and M. M. Weber, Nucl. Phys. B660, 289 (2003)

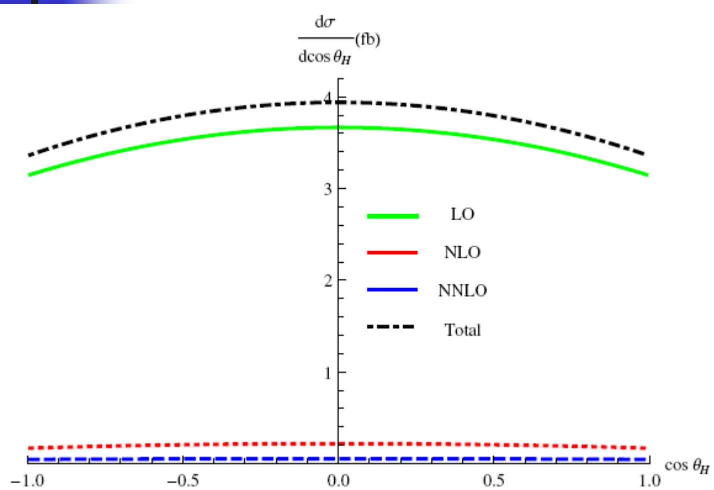
$$\mathcal{M}_{1,\text{fact.}}^{\text{ZH},\sigma} = \frac{s_{12} - M_{\text{Z}}^2}{s_{12} - M_{\text{Z}}^2 + \text{i} M_{\text{Z}} \Gamma_{\text{Z}}} \boxed{\mathcal{M}_1^{\text{ZH},\sigma}(\Gamma_{\text{Z}} = 0)} + \frac{\text{i}\,\text{Im}\{\Sigma_{\text{T}}^{\text{ZZ}}(M_{\text{Z}}^2)\}}{s_{12} - M_{\text{Z}}^2} \boxed{\mathcal{M}_0^{\text{ZH},\sigma}}$$

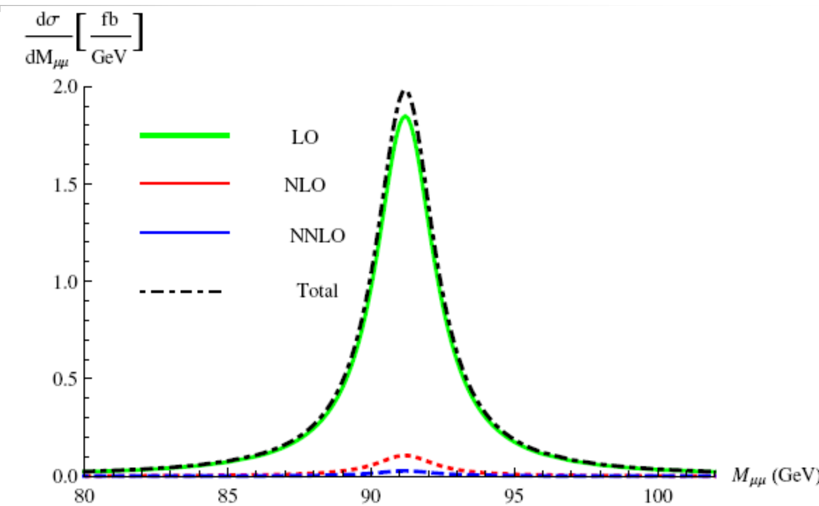
both gauge inv.

$$= \frac{s_{12} - M_{\text{Z}}^2}{s_{12} - M_{\text{Z}}^2 + \text{i} M_{\text{Z}} \Gamma_{\text{Z}}} \left[\mathcal{M}_1^{\text{ZH},\sigma}(\Gamma_{\text{Z}} = 0) - \mathcal{M}_{\text{ZZ-self}}^{\sigma}(\Gamma_{\text{Z}} = 0)\right]$$

$$- \left[\frac{\Sigma_{\text{T}}^{\text{ZZ}}(s) - \text{Re}\{\Sigma_{\text{T}}^{\text{ZZ}}(M_{\text{Z}}^2)\}}{s - M_{\text{Z}}^2} + \frac{\Sigma_{\text{T}}^{\text{ZZ}}(s_{12}) - \Sigma_{\text{T}}^{\text{ZZ}}(M_{\text{Z}}^2)}{s_{12} - M_{\text{Z}}^2} + 2\delta Z_{\text{ZZ}}\right] \mathcal{M}_0^{\text{ZH},\sigma}$$

Differential distributions for $e^+ e^- \to H \mu^+\mu^-$





Differential cross section of Higgs **scattering angle**

Differential cross section of **di-muon invariant mass**.

Our predictions for cross section in three different schemes (including uncertainty)

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	$6.983^{+0.023}_{-0.023}$	$7.385^{+0.037}_{-0.037}$	$7.488^{+0.036+0.004}_{-0.036-0.009}$
	$\alpha(M_Z^2)$	$8.382^{+0.028}_{-0.027}$	$7.317^{+0.037}_{-0.036}$	$7.448^{+0.036+0.005}_{-0.035-0.011}$
	G_μ	$7.772^{+0.004}_{-0.004}$	$7.527^{+0.016}_{-0.017}$	$7.554^{+0.017+0.001}_{-0.017-0.002}$
250	$\alpha(0)$	$7.036^{+0.023}_{-0.023}$	$7.424^{+0.037}_{-0.037}$	$7.527^{+0.037+0.005}_{-0.037-0.009}$
	$\alpha(M_Z^2)$	$8.446^{+0.028}_{-0.028}$	$7.350^{+0.037}_{-0.036}$	$7.481^{+0.037+0.006}_{-0.037-0.011}$
	G_μ	$7.831^{+0.004}_{-0.004}$	$7.564^{+0.017}_{-0.017}$	$7.591^{+0.017+0.001}_{-0.016-0.002}$

Including various sources of uncertainty

Again observe strong scheme dependence!

The NNLO predictions now range from 7.44 to 7.55 fb

Need go to 2-loop EW corrections to reduce Scheme dependence!

Part 4

NLO QCD correction to $e^+ e^- \rightarrow H+\gamma$

A very **rare** Higgs production channel at CEPC

several orders-of-magnitude smaller than HZ production

Loop-induced process, a sensitive channel to seek the footprint of new physics

W.-L. Sang, W. Chen, F. Feng, Y.J., Q. F. Sun, PLB 2017

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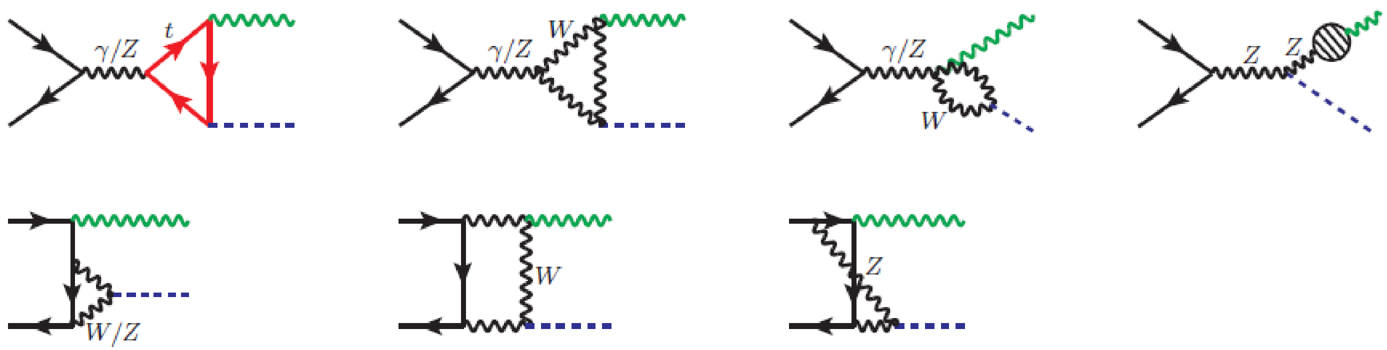
LO results start from one-loop, known in Abbasabadi et al. (95); Gounaris et al. (95); Djouadi et al. (96)



We are investigating the NLO QCD correction

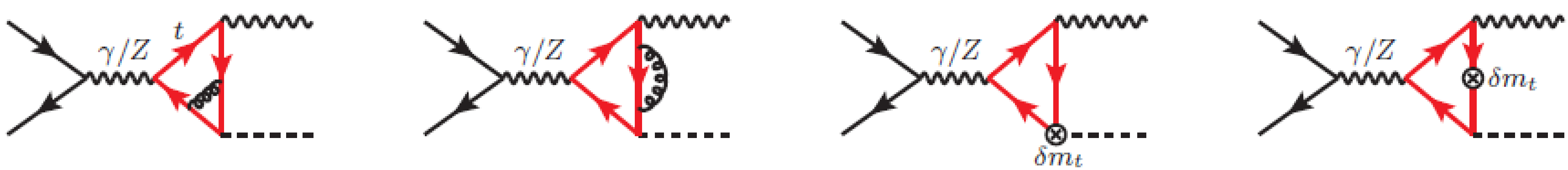


Figure 2: Typical Feynman diagrams for the NLO QCD corrections to $e^+e^- \to H\gamma$. The cap signifies the insertion of the top quark mass counterterm δm_t, as given in (7).

Angular distribution of Higgs

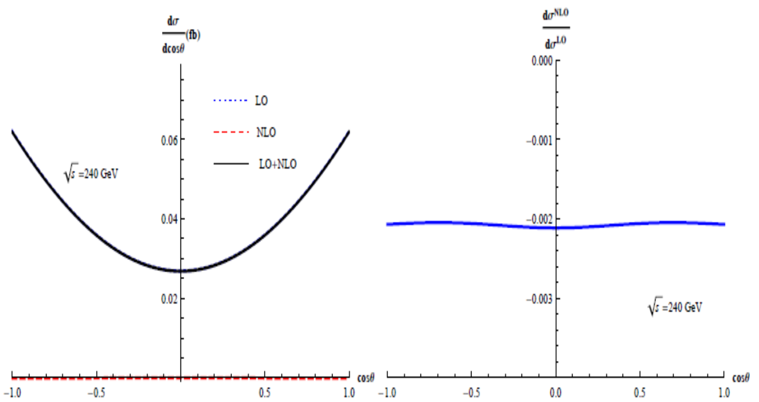


Figure 3: Angular distributions of the Higgs boson in the $e^+e^- \to H\gamma$ process at $\sqrt{s}$ = 240 GeV. The right panel embodies the relative magnitude of the NLO QCD corrections.

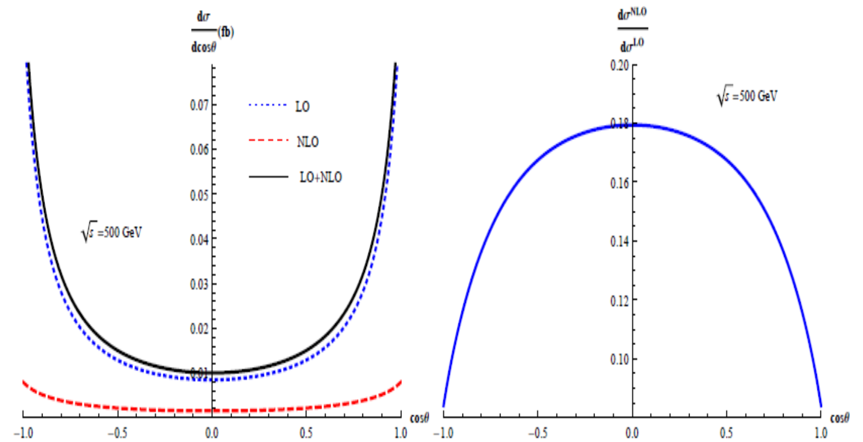


Figure 4: Angular distributions of the Higgs boson in the $e^+e^- \to H\gamma$ process at $\sqrt{s}$ = 500 GeV. The right panel signifies the relative magnitude of the NLO QCD corrections.

CEPC

ILC

The NLO QCD correction is negligible at CEPC energy

Integrated cross section versus CM energy (LO)

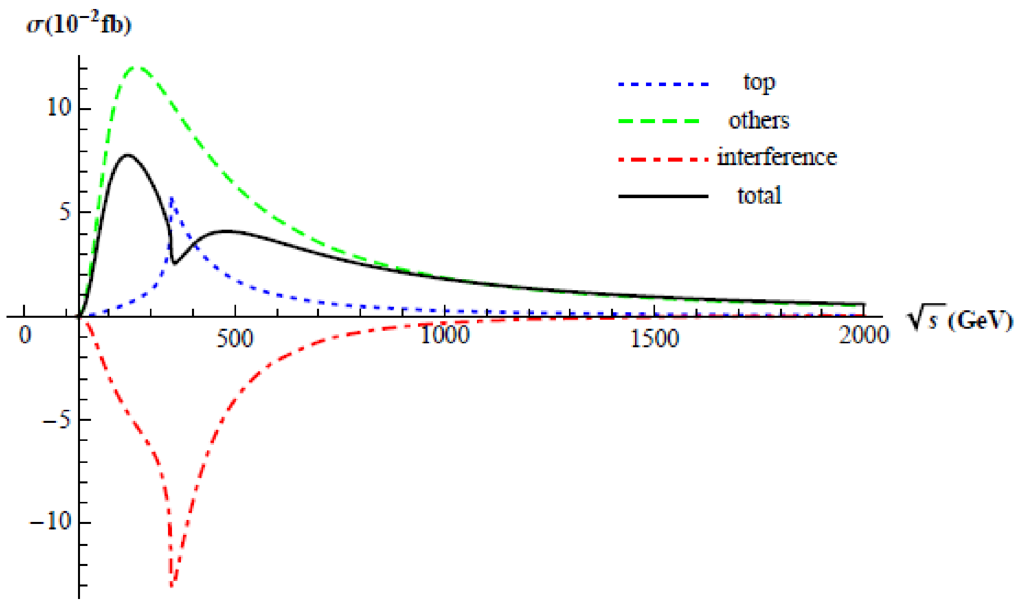


Figure 5: The LO cross section as a function of $\sqrt{s}$ (the solid line). To trace the origin of the nontrivial line shape, we deliberately isolate the contributions from two classes of diagrams. The dotted, dashed and dot-dashed lines represent the contribution from diagrams involving the top quark loop, that from all other diagrams involving weak gauge bosons in the loop, and their interference, respectively.

At asymptotically high energy, **σ _LO ~ 1/s**

Integrated cross section versus CM energy (NLO)

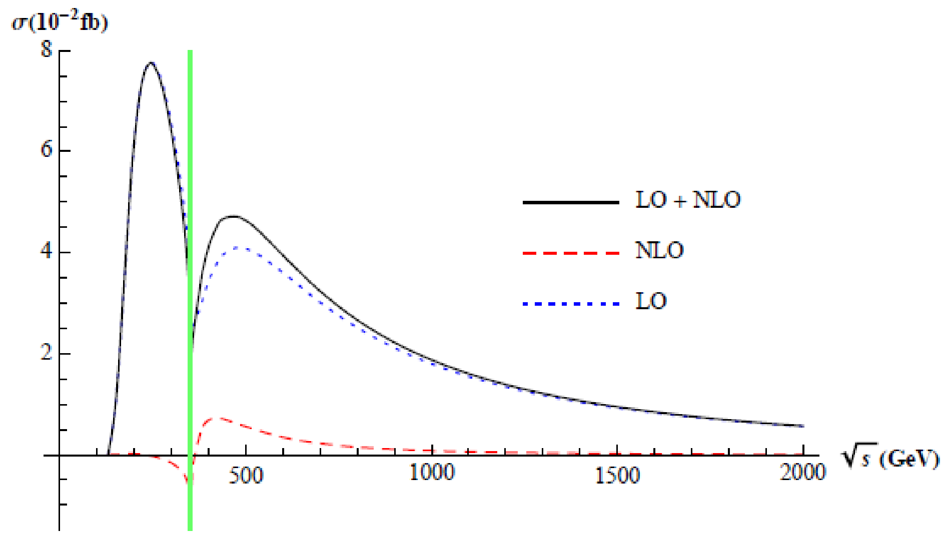


Figure 6: The total cross section as a function of $\sqrt{s}$, both at LO and NLO in α_s. The vertical band with $\sqrt{s} = 2m_t \pm 5$ GeV signifies the threshold region, inside which the perturbative expansion is expected to break down and our fixed-order predictions become invalid.

At asymptotically high energy, **σ _NLO ~ 1/s^2**

Caveat: fixed-order breaks down near ttbar threshold, need resummation of Coulomb gluon to all orders

Summary in part 4

For $e^+ e^- \rightarrow H+\gamma$ (Harbor of new physics),

the QCD correction at CEPC appears to be largely negligible at CEPC energy

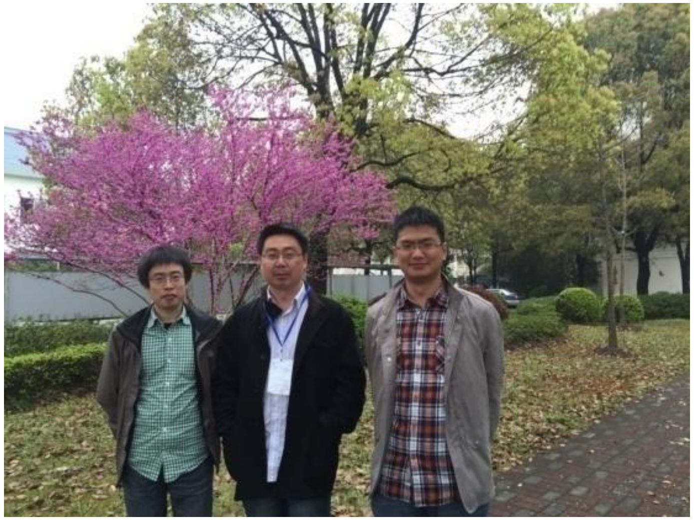
However, the maximum cross section of 0.08 fb can be achieved around the CEPC energy

Search for such rare Higgs production channel at CEPC

Part 5

NNLO QCD correction to $\gamma\gamma^* \rightarrow \eta_{c,\,b}$ form factor at CEPC

Quarkonium production/decay in the low-energy EFT of QCD: the NRQCD factorization approach

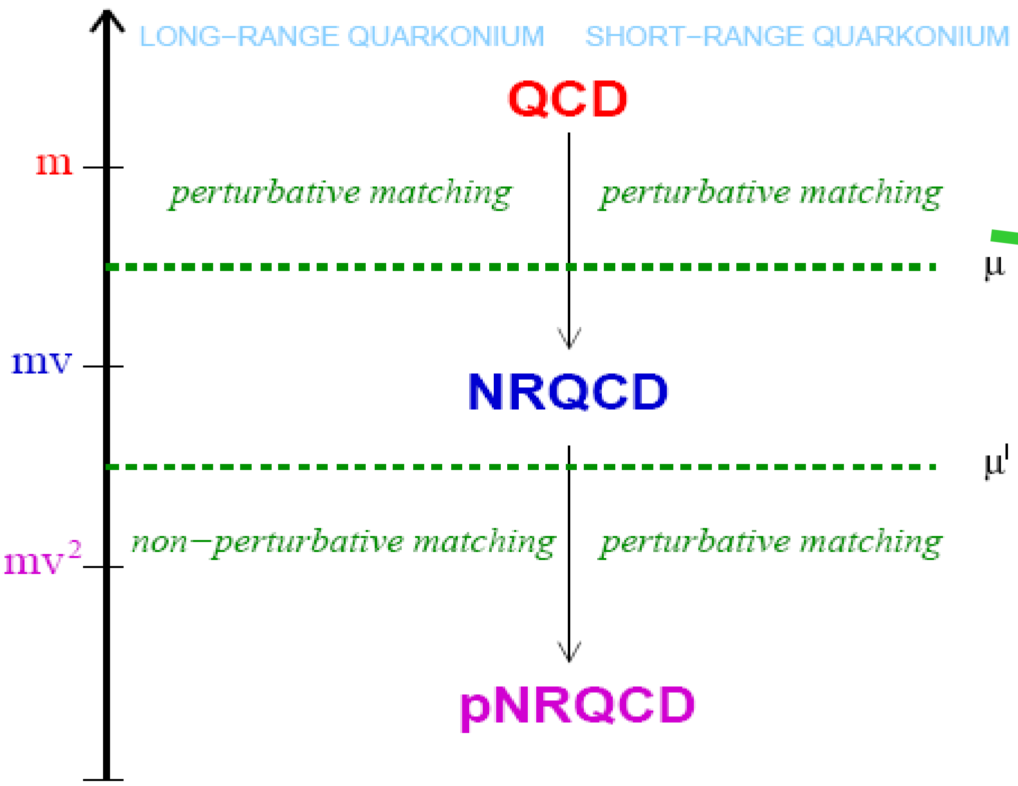


F. Feng, Y.J., W.-L. Sang

Nonrelativistic QCD (NRQCD):

Paradigm of EFT, tailored for describing heavy quarkonium dynamics: exploiting NR nature of quarkonium

Caswell, Lepage (1986); Bodwin, Braaten, Lepage (1995)



NRQCD factorization is viewed as being first principle of QCD

This scale separation is usually referred to as **NRQCD factorization.**

The NRQCD short-dist. coefficients can be computed in perturbation theory, order by order

NRQCD Lagrangian (characterized by velocity (v/c) expansion)

$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\rm light} = -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}\, i\not{D} q,$$

$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

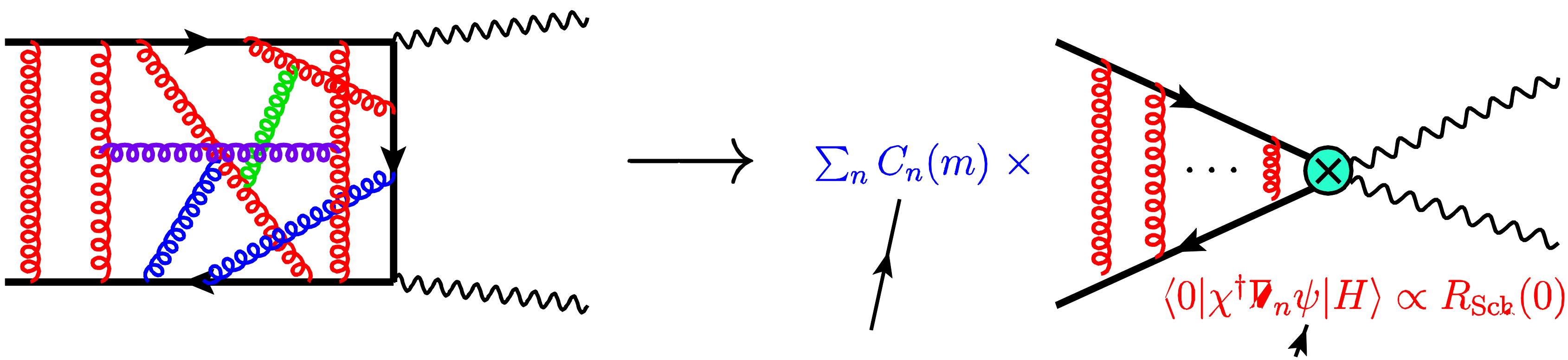
Gauge invariance as guiding principle

$$\begin{aligned}
\delta\mathcal{L}_{\rm bilinear} = & \frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right) \\
& + \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right) \\
& + \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right) \\
& + \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),
\end{aligned}$$

Very similar to HQET, but with different power counting

Physical picture underlying NRQCD factorization

Quarkonium is a QCD bound state involving several distinct scales



Short–distance coefficient — *perturbatively calculable*

wave function at the origin — *nonperturbative yet universal*

Separate the short-distance effect and long-distance dynamics

Asymptotic freedom: $\alpha_s(m) \ll 1$, one can invoke perturbation theory

NRQCD is the mainstream tool in studying quarkonium (see Brambilla et al. EPJC 2011 for a review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes:

Charmonia: $v^2/c^2 \sim 0.3$ not truly non-relativistic to some extent

Bottomonia: $v^2/c^2 \sim 0.1$ a better "non-relativistic" system

Exemplified by

$e^+e^- \to J/\psi + \eta_c$ at B factories **(exclusive charmonium production)**

Unpolarized/polarized J/ψ production at hadron colliders (**inclusive**)

Very active field in recent years (Chao's group, Kniehl's group, Wang's group, Bodwin's group, Qiu's group …) marked by a plenty of PRLs

The strategy of determining the NRQCD short-distance coefficients (NRQCD SDCs)

In principle, NRQCD short-distance coefficients can be computed via the standard perturbative matching procedure:

Computing simultaneously amplitudes in both perturbative QCD and NRQCD, then solve the equations to determine the NRQCD SDCs.

Threshold phenomenon is signaled by four relevant modes: hard ($k^\mu \sim m$), potential ($k^0 \sim mv^2$, $|\mathbf{k}| \sim mv$), soft ($k^\mu \sim mv$), ultrasoft ($k^\mu \sim mv^2$).

Elucidated by the Strategy of region by Beneke & Smirnov 1997

The NRQCD SDCs is associated with the contribution from hard region

Practically, one often directly extract the hard-region contribution in an arbitrary multi-loop diagrams

We then lose track of IR threshold symptom such as Coulomb singularity

The ubiquitous symptom of NRQCD factorization: often plagued with huge QCD radiative correction

Most of the NRQCD successes based on the NLO QCD predictions.

However, the NLO QCD corrections are often large:

$e^+e^- \to J/\psi + \eta_c$	K factor: $1.8 \sim 2.1$	Zhang *et.al.*
$e^+e^- \to J/\psi + J/\psi$	K factor: $-0.31 \sim 0.25$	Gong *et.al.*
$p + p \to J/\psi + X$	K factor: ~ 2	Campbell *et.al.*
$J/\psi \to \gamma\gamma\gamma$	K factor: ≤ 0	Mackenzie *et.al.*

… …

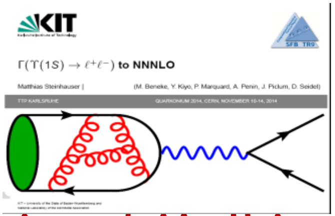
The existing NNLO corrections are rather **few**: all related to S-wave quarkonium **decay**

1. Υ (J/Ψ) → $e^+ e^-$

NNLO corrections were first computed by two groups in 1997:

Czarnecki and Melkinov; Beneke, Smirnov, and Signer;

NNNLO correction available very recently: Steinhausser et al. (2013)



2. η_c → $\gamma\ \gamma$

NNLO correction was computed by Czarnecki and Melkinov (2001) : (neglecting light-by-light)

3. B_c → $l\ \nu$:

NNLO correction computed by Onishchenko, Veretin (2003); Chen and Qiao, (2015)

Perturbative convergence of these decay processes appears to be rather poor

$$\Gamma(J/\psi \to \ell\ell) = \Gamma^{(0)} \left[1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41\, n_f) \left(\frac{\alpha_s}{\pi} \right)^2 \right]^2$$

$$+(-2091 + 120.66\, n_f - 0.82\, n f^2) \left(\frac{\alpha_s}{\pi} \right)^3$$

$$\Gamma(B_c \to \ell\nu) = \Gamma^{(0)} \left[1 - 1.39\frac{\alpha_s}{\pi} - 23.7 \left(\frac{\alpha_s}{\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right]^2$$

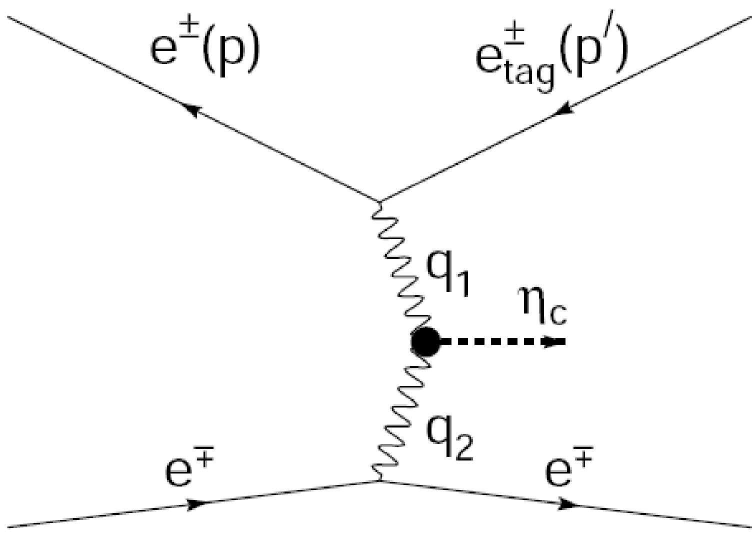
$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma^{(0)} \left[1 - 1.69\frac{\alpha_s}{\pi} - 56.52 \left(\frac{\alpha_s}{\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right]^2$$

So calculating the higher order QCD correction is imperative to test the usefulness of NRQCD factorization!

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Experiment

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



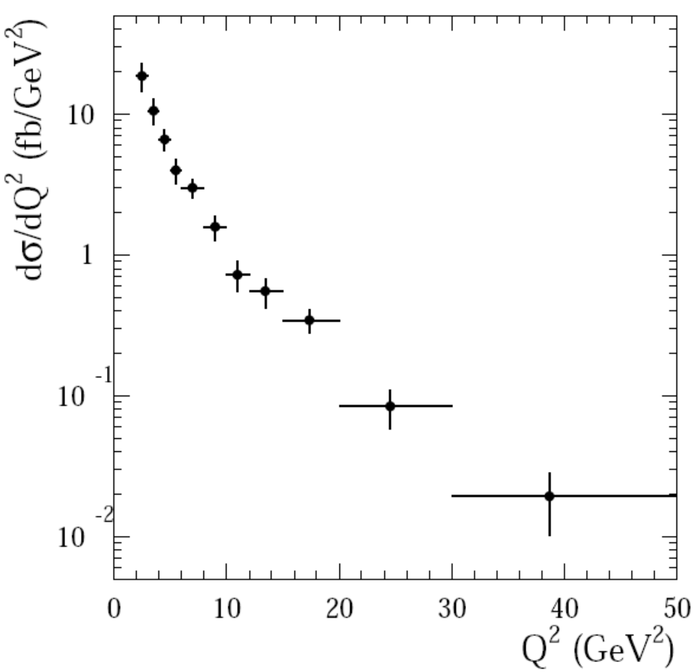
$$q_2^2 \approx 0$$

$$q_1^2 = -Q^2 = (p' - p)^2$$

Babar measures the $\gamma\gamma^* \rightarrow \eta_c$ transition form factor in the momentum transfer range from **2 to 50 GeV^2**.

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor: There also exists BaBar measurements!

BaBar Collaboration: **Phys.Rev. D81 (2010) 052010**



$$\frac{d\sigma(e^+e^-\to\eta_c e^+e^-)}{dQ^2} \times \mathcal{B}(\eta_c \to K\bar{K}\pi)$$

Q^2 interval (GeV^2)	$\overline{Q^2}$ (GeV^2)	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV^2)	$\lvert F(\overline{Q^2})/F(0)\rvert$
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	0.740 ± 0.085
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	0.680 ± 0.073
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	0.629 ± 0.057
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	0.555 ± 0.056
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	0.563 ± 0.043
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	0.490 ± 0.049
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	0.385 ± 0.048
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	0.395 ± 0.047
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	0.385 ± 0.038
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	0.261 ± 0.041
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	0.204 ± 0.049

$F(Q^2)$: $\gamma^*\gamma \to \eta_c$ form factor

$F(0)$: $\eta_c \to \gamma\gamma$ form factor

Investigation on $\gamma\gamma^* \to \eta_c$ form factor

Previous investigation

- $k_\perp$ factorization: Feldmann *et.al.*, Cao *and Huang*
- Lattice QCD: Dudek *et.al.*,
- **J/ψ** -pole-dominance: Lees *et.al.*,
- QCD sum rules: Lucha *et.al.*,
- light-front quark model: Geng *et.al.*,
- Dyson-Schwinger approach: Chang, Chen, Ding, Liu, Roberts, 2016

All yield predictions compatible with the data, at least in the small Q^2 range.

So far, so good. Unlike $\gamma\gamma^* \to \pi^0$, there is no open puzzle here

The first NNLO calculation for (exclusive) quarkonium production process

Feng, Jia, Sang, PRL 115, 222001 (2017)

PRL 115, 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

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Unlike the bewildering situation in the $\gamma\gamma^* \to \pi$ form factor, a widespread view is that perturbative QCD can decently account for the recent *BABAR* measurement of the $\gamma\gamma^* \to \eta_c$ transition form factor. The next-to-next-to-leading-order perturbative correction to the $\gamma\gamma^* \to \eta_{c,b}$ form factor, is investigated in the nonrelativistic QCD (NRQCD) factorization framework for the first time. As a byproduct, we obtain, by far, the most precise order-α_s^2 NRQCD matching coefficient for the $\eta_{c,b} \to \gamma\gamma$ process. After including the substantial negative order-α_s^2 correction, the good agreement between NRQCD prediction and the measured $\gamma\gamma^* \to \eta_c$ form factor is completely ruined over a wide range of momentum transfer squared. This eminent discrepancy casts some doubts on the applicability of the NRQCD approach to hard exclusive reactions involving charmonium.

DOI: 10.1103/PhysRevLett.115.222001 PACS numbers: 13.60.Le, 12.38.Bx, 14.40.Pq

$\gamma\gamma^* \rightarrow \eta_c$ form factor in NRQCD factorization

Definition for form factor:

$$\langle \eta_c(p) | J^\mu | \gamma(k,\varepsilon) \rangle = i e^2 \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

NRQCD factorization demands:

Factorization scale

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda) \frac{\langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

Short-distance coefficient (SDC)
We are going to compute it to NNLO

$$\overline{R_{\eta_c}}(\Lambda) \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \psi(\Lambda) | \eta_c \rangle,$$

$$\overline{R_\psi}(\Lambda)\, \epsilon \equiv \sqrt{\frac{2\pi}{N_c}} \langle 0 | \chi^\dagger \boldsymbol{\sigma} \psi(\Lambda) | \psi(\epsilon) \rangle\ ,$$

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Perturbative series for NRQCD SDCs

Upon general consideration, the SDC can be written as

$$C(Q,m,\mu_R,\mu_\Lambda) = C^{(0)}(Q,m)\Bigg\{1 + C_F\frac{\alpha_s(\mu_R)}{\pi}f^{(1)}(\tau) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2+m^2}C_F f^{(1)}(\tau) - \pi^2 C_F\left(C_F + \frac{C_A}{2}\right)\times\ln\frac{\mu_\Lambda}{m} + f^{(2)}(\tau)\right] + \mathcal{O}(\alpha_s^3)\Bigg\},$$

RG invariance

IR pole matches **anomalous dimension** of NRQCD pseudo-scalar density

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Theoretical calculation

$$C^{(0)}(Q,m) = \frac{4e_c^2}{Q^2+4m^2}$$

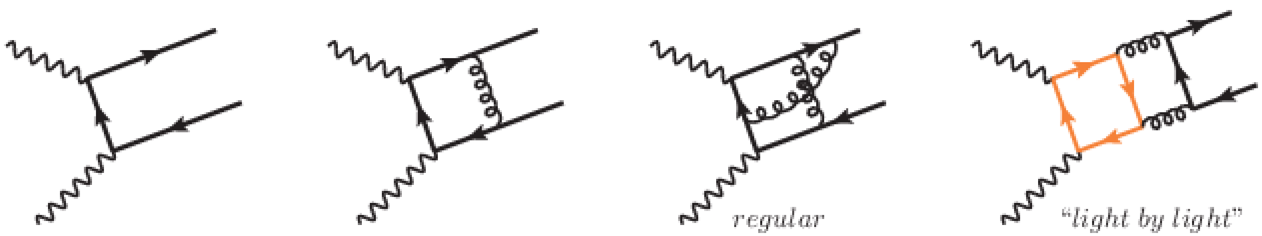
Tree-level SDC

$$f^{(1)}(\tau) = \frac{\pi^2(3-\tau)}{6(4+\tau)} - \frac{20+9\tau}{4(2+\tau)} - \frac{\tau(8+3\tau)}{4(2+\tau)^2}\ln\frac{4+\tau}{2} + 3\sqrt{\frac{\tau}{4+\tau}}\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}} + \frac{2-\tau}{4+\tau}\left(\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}\right)^2 - \frac{\tau}{2(4+\tau)}\mathrm{Li}_2\left(-\frac{2+\tau}{2}\right),$$

$$\tau \equiv \frac{Q^2}{m^2}$$

NLO QCD correction

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor
Feynman diagrams



LO NLO NNLO

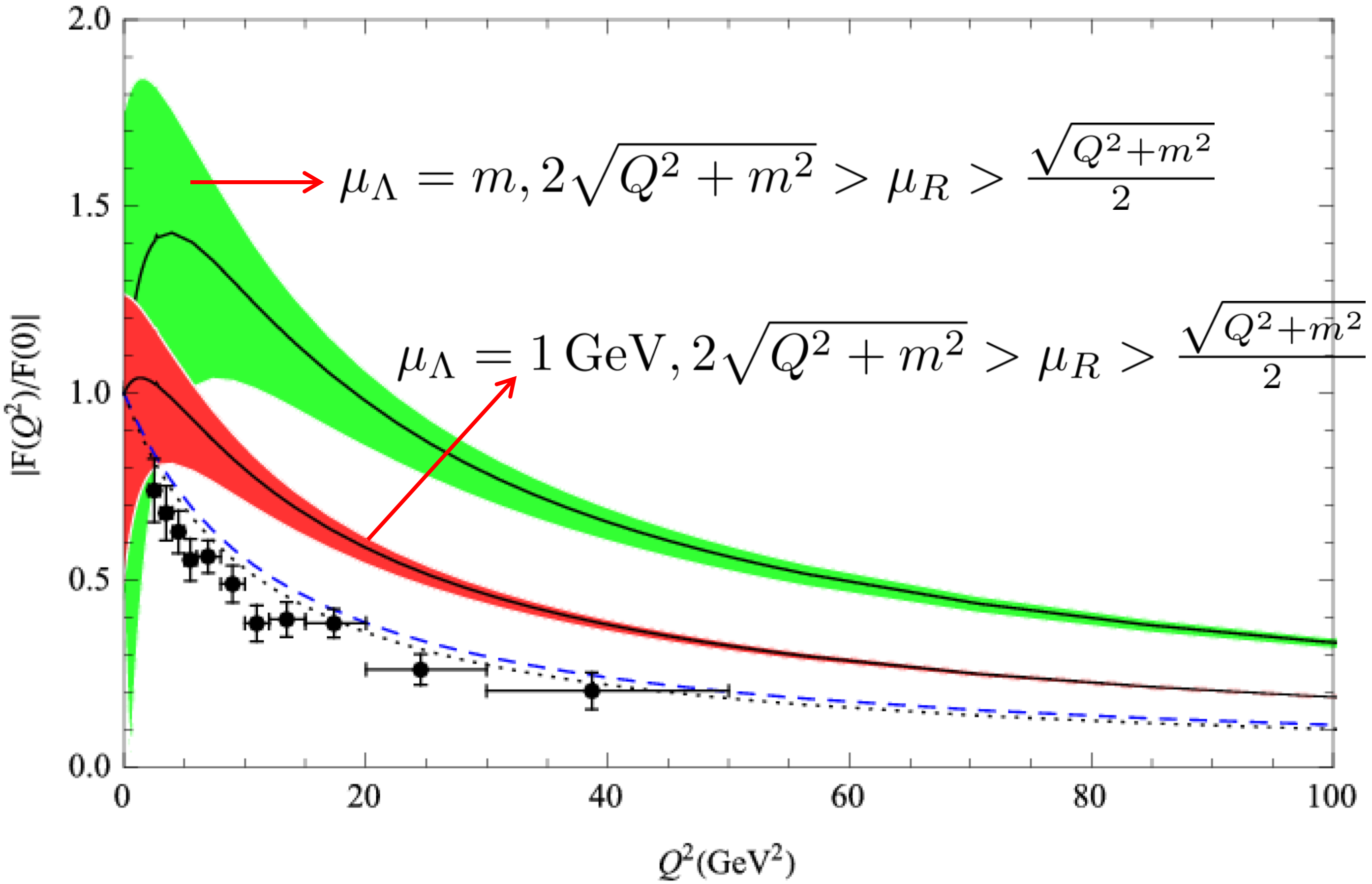
Numer of diagrams

LO	NLO	NNLO (regular)	NNLO ("light by light")
2	**8**	**108**	**12**

Investigation on $\gamma\gamma^* \rightarrow \eta_c$ form factor

Huge discrepancy between NRQCD prediction and experiment

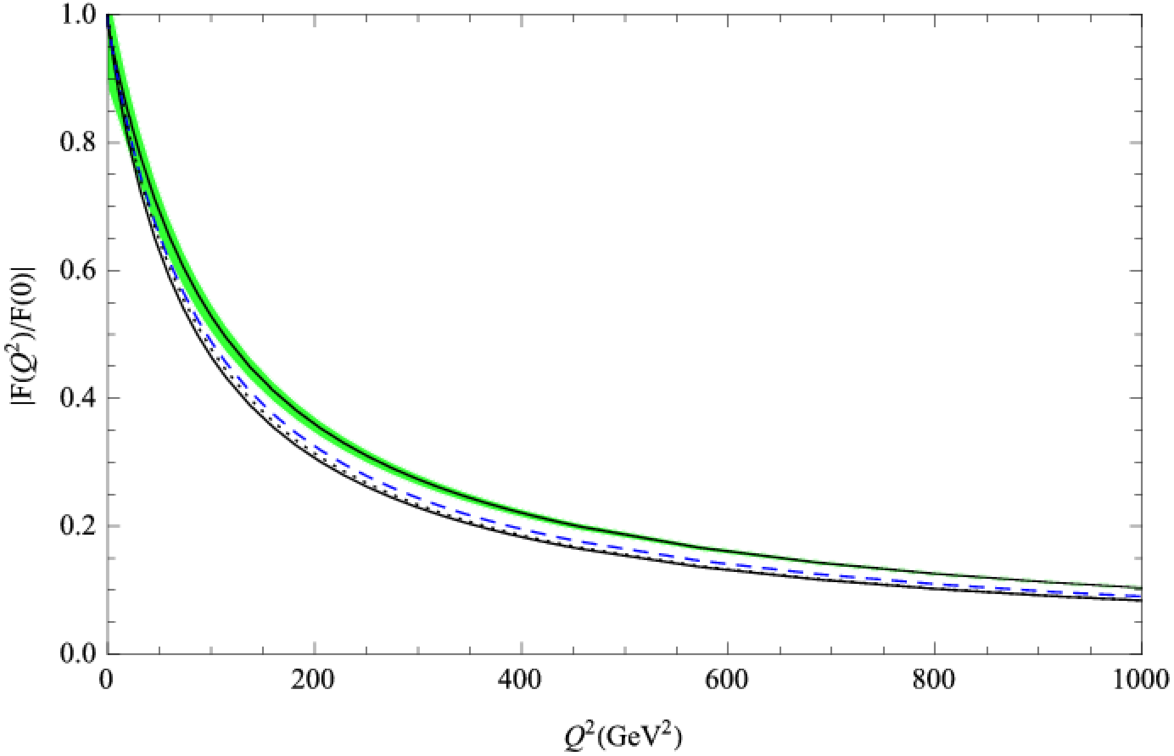
Our Prediction is free of nonperturbative parameters!



$\gamma\gamma^* \rightarrow \eta_c$ **: NNLO predictions seriously fails to describe data!**

Prediction to $\gamma\gamma^* \rightarrow \eta_b$ form factor

Await **CEPC** to test our predictions



Convergence of perturbation series is reasonably well.

Complete NNLO correction to η_c → light hadrons (first NNLO calculation for inclusive process involving quarkonium)

Feng, Jia, Sang, PRL 119, 252001 (2017)

PRL **119,** 252001 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Next-to-Next-to-Leading-Order QCD Corrections to the Hadronic Width of Pseudoscalar Quarkonium

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We compute the next-to-next-to-leading-order QCD corrections to the hadronic decay rates of the pseudoscalar quarkonia, at the lowest order in velocity expansion. The validity of nonrelativistic QCD (NRQCD) factorization for inclusive quarkonium decay process, for the first time, is verified to relative order α_s^2. As a by-product, the renormalization group equation of the leading NRQCD four-fermion operator $\mathcal{O}_1(^1S_0)$ is also deduced to this perturbative order. By incorporating this new piece of correction together with available relativistic corrections, we find that there exists severe tension between the state-of-the-art NRQCD predictions and the measured η_c hadronic width and, in particular, the branching fraction of $\eta_c \to \gamma\gamma$. NRQCD appears to be capable of accounting for η_b hadronic decay to a satisfactory degree, and our most refined prediction is $\mathrm{Br}(\eta_b \to \gamma\gamma) = (4.8 \pm 0.7) \times 10^{-5}$.

DOI: 10.1103/PhysRevLett.119.252001

NLO perturbative corr. 1979/1980

[7] R. Barbieri, E. d'Emilio, G. Curci and E. Remiddi, Nucl. Phys. B **154**, 535 (1979).

[8] K. Hagiwara, C. B. Kim and T. Yoshino, Nucl. Phys. B **177**, 461 (1981).

40 years lapsed from NLO to NNLO;

Another ??? years to transition into NNNLO QCD corrections?

Promising only if Alpha-Loop takes over?

NRQCD factorization for $\eta_c \rightarrow$ light hadrons – up to relative order-v^4 corrections

Bodwin, Petrelli PRD (2002)

$$\Gamma({}^1S_0 \to \mathrm{LH}) = \frac{F_1({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_1({}^1S_0)|{}^1S_0\rangle + \frac{G_1({}^1S_0)}{m^4}\langle {}^1S_0|\mathcal{P}_1({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^3S_1)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^3S_1)|{}^1S_0\rangle + \frac{F_8({}^1S_0)}{m^2}\langle {}^1S_0|\mathcal{O}_8({}^1S_0)|{}^1S_0\rangle + \frac{F_8({}^1P_1)}{m^4}\langle {}^1S_0|\mathcal{O}_8({}^1P_1)|{}^1S_0\rangle + \frac{H_1^1({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^1({}^1S_0)|{}^1S_0\rangle + \frac{H_1^2({}^1S_0)}{m^6}\langle {}^1S_0|\mathcal{Q}_1^2({}^1S_0)|{}^1S_0\rangle.$$

$$\mathcal{O}_1({}^1S_0) = \psi^\dagger\chi\chi^\dagger\psi, \tag{2.2a}$$

$$\mathcal{P}_1({}^1S_0) = \frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi + \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\psi\right], \tag{2.2b}$$

$$\mathcal{O}_8({}^3S_1) = \psi^\dagger\boldsymbol{\sigma}T_a\chi\cdot\chi^\dagger\boldsymbol{\sigma}T_a\psi, \tag{2.2c}$$

$$\mathcal{O}_8({}^1S_0) = \psi^\dagger T_a\chi\chi^\dagger T_a\psi, \tag{2.2d}$$

$$\mathcal{O}_8({}^1P_1) = \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\chi\cdot\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)T_a\psi, \tag{2.2e}$$

$$\mathcal{Q}_1^1({}^1S_0) = \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi, \tag{2.2f}$$

$$\mathcal{Q}_1^2({}^1S_0) = \frac{1}{2}\left[\psi^\dagger\chi\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\psi + \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^4\chi\chi^\dagger\psi\right], \tag{2.2g}$$

$$\mathcal{Q}_1^3({}^1S_0) = \frac{1}{2}[\psi^\dagger\chi\chi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E} + g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\psi - \psi^\dagger(\overleftrightarrow{\mathbf{D}}\cdot g\mathbf{E} + g\mathbf{E}\cdot\overleftrightarrow{\mathbf{D}})\chi\chi^\dagger\psi], \tag{2.2h}$$

Our calculation of short-distance coefficient utilizes **Method of Region (Beneke and Smirnov 1998)** to directly extract the hard region contribution from multi-loop diagrams

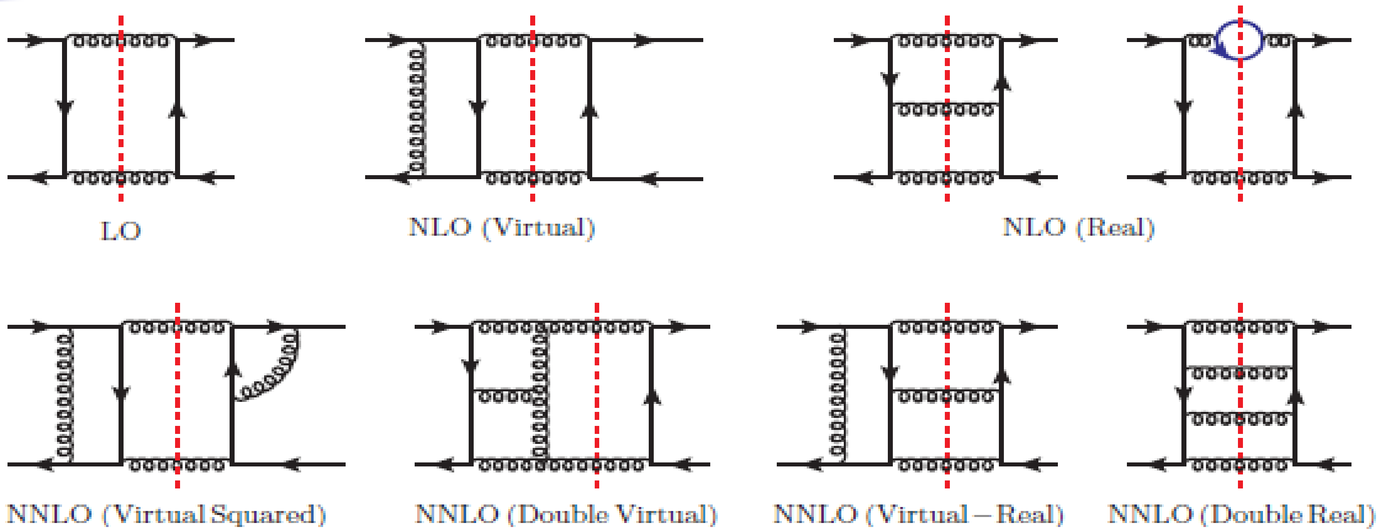
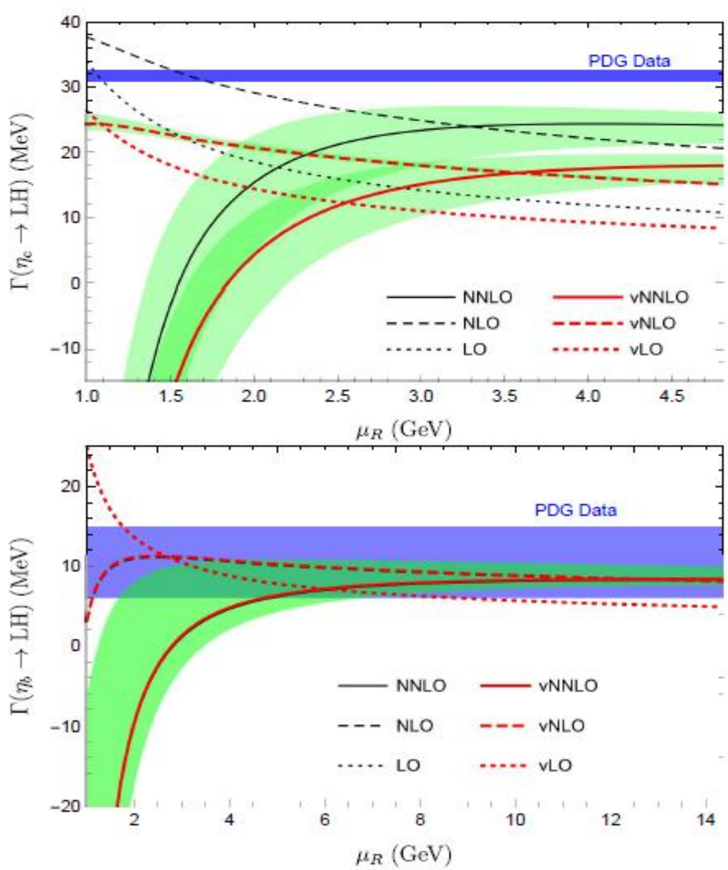


FIG. 1: Representative cut Feynman diagrams responsible for the quark reaction $c\bar{c}({}^1S_0^{(1)}) \to c\bar{c}({}^1S_0^{(1)})$ through NNLO in α_s. The vertical dashed line denotes the Cutkosky cut.

Roughly 1700 3-loop forward-scattering diagrams, divided into 4 distinct cut topologies; Cutkosky rule is imposed

Phenomenological study: hadronic width



Input parameters:

$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_c} = 0.470\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_c} = \frac{0.430\,\mathrm{GeV}^2}{m_c^2},$$

$$\langle \mathcal{O}_1(^1S_0)\rangle_{\eta_b} = 3.069\,\mathrm{GeV}^3,\ \langle v^2\rangle_{\eta_b} = -0.009. \qquad (9)$$

PDG values:

$$\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8\ \mathrm{MeV},$$

$$\Gamma_{\mathrm{had}}(\eta_b) = 10^{+5}_{-4}\ \mathrm{MeV}$$

FIG. 2: The predicted hadronic widths of η_c (top) and η_b (bottom) as functions of μ_R, at various level of accuracy in α_s and v expansion. The horizontal blue bands correspond to the measured hadronic widths taken from PDG 2016 [4], with $\Gamma_{\mathrm{had}}(\eta_c) = 31.8 \pm 0.8$ MeV and $\Gamma_{\mathrm{had}}(\eta_b) = 10^{-4}_{+5}$ MeV. The label "LO" represents the NRQCD prediction at the lowest-order α_s and v, and the label "NLO" denotes the "LO" prediction plus the $\mathcal{O}(\alpha_s)$ perturbative correction, while the label "NNLO" signifies the "NLO" prediction plus the $\mathcal{O}(\alpha_s^2)$ perturbative correction. The label "vLO" represents the "LO" prediction together with the tree-level order-v^2 correction, and the label "vNLO" designates the "vLO" prediction supplemented with the relative order-α_s and order-$\alpha_s v^2$ correction, while the label "vNNLO" refers to the "vNLO" prediction further supplemented with the order-α_s^2 correction. The green bands are obtained by varying μ_Λ from 1 GeV to twice heavy quark mass, and the central curve inside the bands are obtained by setting μ_Λ equal to heavy quark mass.

Phenomenological study of Br($\eta_{c,b} \to \gamma\gamma$), Non-Perturbative matrix elements cancel out

For η_c more than 10σ discrepancy !

$$\mathrm{Br}(\eta_c \to \gamma\gamma) = \frac{8\alpha^2}{9\alpha_s^2}\Bigg\{1 - \frac{\alpha_s}{\pi}\left[4.17\ln\frac{\mu_R^2}{4m_c^2} + 14.00\right] + \frac{\alpha_s^2}{\pi^2}\left[4.34\ln^2\frac{\mu_R^2}{4m_c^2} + 22.75\ln\frac{\mu_R^2}{4m_c^2} + 78.8\right] + 2.24\langle v^2\rangle_{\eta_c}\frac{\alpha_s}{\pi}\Bigg\}, \quad (10a)$$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = \frac{\alpha^2}{18\alpha_s^2}\Bigg\{1 - \frac{\alpha_s}{\pi}\left[3.83\ln\frac{\mu_R^2}{4m_b^2} + 13.11\right] + \frac{\alpha_s^2}{\pi^2}\left[3.67\ln^2\frac{\mu_R^2}{4m_b^2} + 20.30\ln\frac{\mu_R^2}{4m_b^2} + 85.5\right] + 1.91\langle v^2\rangle_{\eta_b}\frac{\alpha_s}{\pi}\Bigg\}. \quad (10b)$$

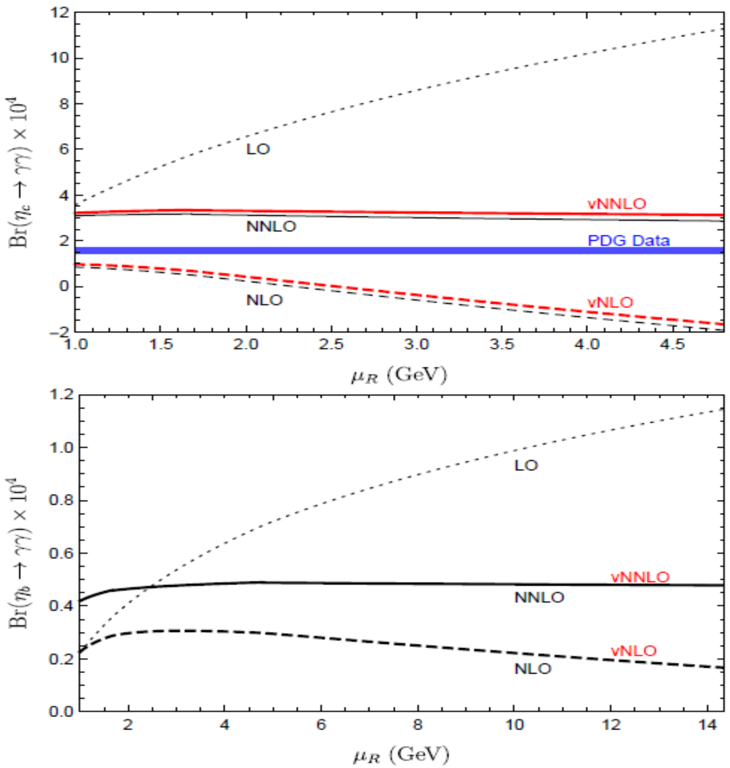


FIG. 3: The predicted branching fractions of $\eta_c \to \gamma\gamma$ (top) and $\eta_b \to \gamma\gamma$ (bottom) as functions of μ_R, at various level of accuracy in α_s and v. The blue band corresponds to the measured branching ratio for $\eta_c \to \gamma\gamma$ taken from PDG 2016 [4], with $\mathrm{Br}(\eta_c \to \gamma\gamma) = (1.59 \pm 0.13) \times 10^{-4}$. The labels characterizing different curves are the same as in Fig. 2.

To date most refined predictior for $\eta_b \to \gamma\gamma$

$$\mathrm{Br}(\eta_b \to \gamma\gamma) = (4.8 \pm 0.7) \times 10^{-5},$$

A famous puzzle since 2002: exclusive double charmonium production: $e^+ e^- \rightarrow J/\Psi + \eta_c$ at B factories (F. Feng, Y. J., W.-L.Sang, arXiv:1901.08447[hep-ph]

Next-to-next-to-leading-order QCD corrections to $e^+e^- \to J/\psi + \eta_c$ at B factories

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(Dated: January 25, 2019)

Within the nonrelativistic QCD (NRQCD) factorization framework, we compute the long-awaited $\mathcal{O}(\alpha_s^2)$ correction for the exclusive double charmonium production process at B factories, *i.e.*, $e^+e^- \to J/\psi + \eta_c$ at $\sqrt{s} = 10.58$ GeV. For the first time, we confirm that NRQCD factorization does hold at next-to-next-to-leading-order (NNLO) for exclusive double charmonium production. It is found that including the NNLO QCD correction greatly reduces the renormalization scale dependence, and also implies the reasonable perturbative convergence behavior for this process. Our state-of-the-art prediction is consistent with the BaBar measurement.

PACS numbers:

A biggest puzzle in SM in the beginning of this century

4. Phenomenology. The production rate initially measured by BELLE is $\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9$ fb [1], later shifted to $\sigma[J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb [44], where $\mathcal{B}_{>n}$ denotes the branching fraction for the η_c into n charged tracks. An indepednent measurement by BABAR in 2005 yields $\sigma[J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb [45].

The LO NRQCD predictions by three groups are smaller
Than Belle measurements by an order of magnitude!

E. Braaten, J. Lee, PRD 2003
K. Y. Liu, Z. G. He, K. T. Chao, PLB 2003 — LO NRQCD factorization
K. Hagiwara, E. Kou, C. F. Qiao, PLB 2003

J. P. Ma, Z. G. Si, PRD 2004 — LO light-cone approach

A crucial progress is the large NLO perturbative correction

Very significant NLO correction comes as a surprise

$$e^+e^- \to J/\psi + \eta_c \qquad \text{K factor: } 1.8 \sim 2.1$$

Y. J. Zhang, Y. J. Gao and K.-T. Chao, PRL 2006
B. Gong, J.-X. Wang, PRD 2008

One may naturally wonder: how about the size of the NNLO QCD corrections? We have to wait for 14 years…

Two-loop, 5 point amplitude is the frontier, especially massive quark!

One influential 2011 review article claims that "The calculation of … is perhaps beyond the current state of the art"

NRQCD factorization formula for exclusive double-charmonium production

$$\langle J/\psi(P_1,\lambda)+\eta_c(P_2)|J^{\mu}_{\rm EM}|0\rangle = i\,F(s)\,\epsilon^{\mu\nu\rho\sigma}P_{1\nu}P_{2\rho}\varepsilon^*_\sigma(\lambda),$$

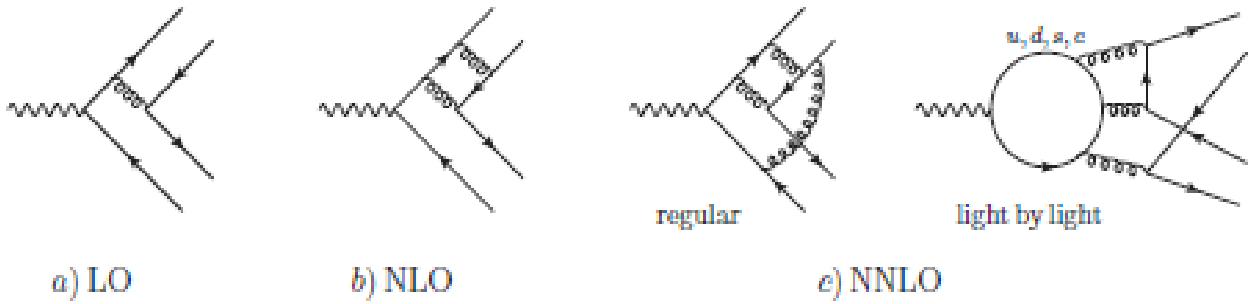
$$F(s) = \sqrt{4M_{J/\psi}M_{\eta_c}}\langle J/\psi|\psi^\dagger\boldsymbol{\sigma}\cdot\epsilon\chi|0\rangle\langle\eta_c|\psi^\dagger\chi|0\rangle \times \left[f+g_{J/\psi}\langle v^2\rangle_{J/\psi}+g_{\eta_c}\langle v^2\rangle_{\eta_c}+\cdots\right],$$

$$\sigma[e^+e^-\to J/\psi+\eta_c] = \frac{4\pi\alpha^2}{3}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)^3|F(s)|^2 = \sigma_0+\sigma_2+\mathcal{O}(\sigma_0 v^4),$$

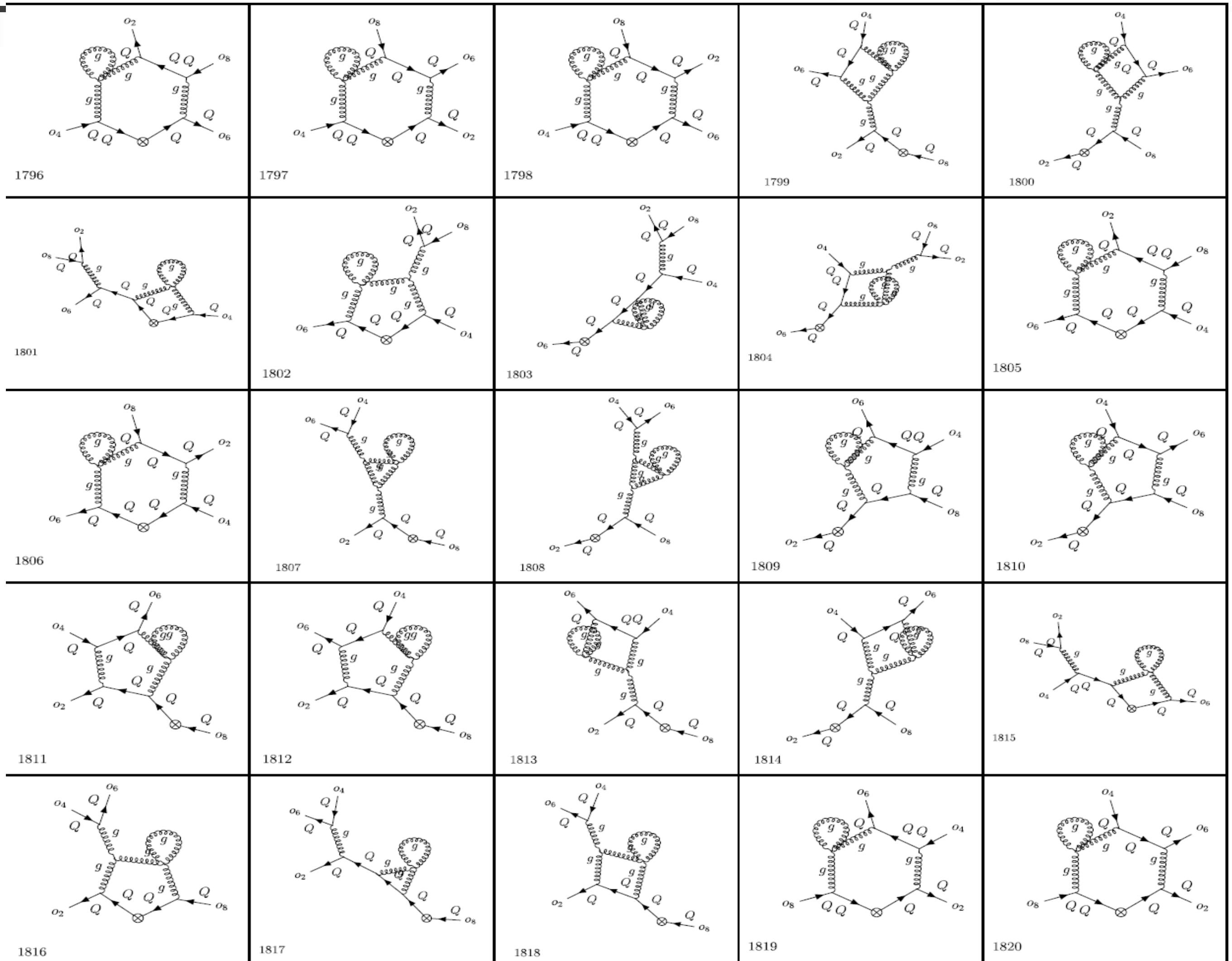
$$f = f^{(0)}+\frac{\alpha_s}{\pi}f^{(1)}+\frac{\alpha_s^2}{\pi^2}f^{(2)}+\cdots,$$

$$g_H = g_H^{(0)}+\frac{\alpha_s}{\pi}g_H^{(1)}+\cdots.$$

$$|f|^2 = |f^{(0)}|^2+\frac{\alpha_s}{\pi}2{\rm Re}(f^{(0)}f^{(1)*}) + \left(\frac{\alpha_s}{\pi}\right)^2\left[2{\rm Re}(f^{(0)}f^{(2)*})+|f^{(1)}|^2\right],$$



About 2000 two-loop diagrams; Cutting-edge NNLO calculation, 1->4 topology



700 master integrals; most complex-valued; Year-long hard efforts in computing them

$$f^{(2)} = f^{(0)}\left\{\frac{\beta_0^2}{16}\ln^2\frac{s}{4\mu_R^2} - \left(\frac{\beta_1}{16}+\frac{1}{2}\beta_0\hat{f}^{(1)}\right)\ln\frac{s}{4\mu_R^2} + (\gamma_{J/\psi}+\gamma_{\eta_c})\ln\frac{\mu_\Lambda^2}{m^2} + \mathsf{F}(r)\right\},$$

log(muR) dictated
By RG invariance

Specific form of single IR pole in hard region

$$\gamma_{J/\psi} = -\frac{\pi^2}{12}C_F\left(2C_F+3C_A\right),$$

$$\gamma_{\eta_c} = -\frac{\pi^2}{4}C_F\left(2C_F+C_A\right).$$

Required by the validity of NRQCD factorization

$$\mathrm{Re}\,\mathsf{F}(r=0.0700) = -25\pm 4,$$

$$\mathrm{Re}\,\mathsf{F}(r=0.1009) = -21\pm 5.$$

This is the main result!

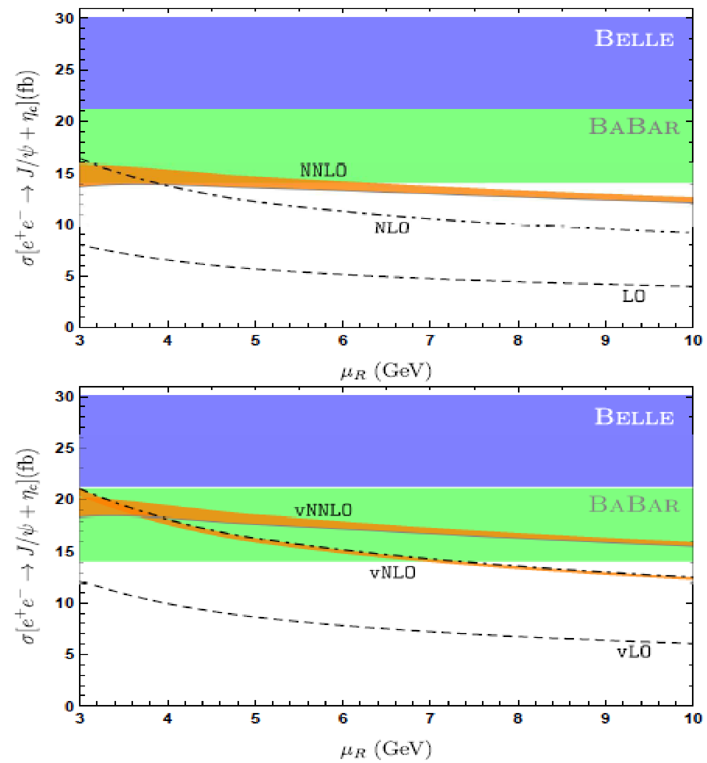
Phenomenology: our state-of-the-art predictions

TABLE I: Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ at $\sqrt{s} = 10.58$ GeV. Each column is labeled by the powers of α_s and v, and given in units of fb. We fix $\mu_\Lambda = m$, and consider $\mu_R = 2m$ and $\sqrt{s}/2$. The two upper rows and the two lower rows correspond to $m = 1.4$ GeV and $m = 1.68$ GeV, respectively.

μ_R	LO	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s v^2)$	$\mathcal{O}(\alpha_s^2)$	Total
$2m$	8.48	4.36	8.64	0.34	−3.7(5)	18.1(5)
$\frac{\sqrt{s}}{2}$	5.52	2.84	6.48	1.18	1.6(2)	17.6(2)
$2m$	5.59	1.44	4.71	−0.33	−1.4(4)	10.0(4)
$\frac{\sqrt{s}}{2}$	4.16	1.07	4.08	0.06	0.7(2)	10.1(2)

$$\sigma = \sigma_{\rm LO}\left[1 + \frac{\sigma^{(v^2)}}{\sigma_{\rm LO}} + \frac{\sigma^{(\alpha_s)}}{\sigma_{\rm LO}} + \frac{\sigma^{(\alpha_s v^2)}}{\sigma_{\rm LO}} + \frac{\sigma^{(\alpha_s^2)}}{\sigma_{\rm LO}}\right].$$

$$\sigma = 8.48\,\text{fb}\,[1 + 0.51 + 1.02 + 0.04 - 0.44(6)],$$
$$\sigma = 5.52\,\text{fb}\,[1 + 0.51 + 1.17 + 0.21 + 0.28(4)],$$
$$\sigma = 5.59\,\text{fb}\,[1 + 0.26 + 0.84 - 0.06 - 0.25(6)],$$
$$\sigma = 4.16\,\text{fb}\,[1 + 0.26 + 0.98 + 0.01 + 0.16(5)],$$



New NNLO piece!

Conclusion of 1901.08447

- Reducing renormalization scale dependence
- See decent perturbative convergence behavior
- Agree with BaBar data, yet not Belle

  Call for Belle 2 re-measurement of this channel

CEPC not only a Higgs machine… lots of QCD research can be conducted as well

Jet physics

Energy-energy correlation (see H.-X. Zhu's talk)

Inclusive hadron production/fragmentation function, especially measured at Z pole

Exclusive hadron production

……

Summary and Outlook

1. Mixed EW-QCD correction for the Higgs-strahlung process appears to be significant, about 1% of LO cross sections

2. Strong α-scheme dependence observed, also sizable uncertainty arising from input parameters. So far, cannot meet the 0.5% precision of CEPC experiment
   What we can do to improve?
   Compute NNLO EW correction??
   Technically feasible?? Computational and human resources sufficient?

3. Perturbative expansion seems to have poor convergence behavior for charmonium in NRQCD factorization

4. Perturbative expansion bears much better behavior for bottomonium
   Wait for CEPC to test our prediction for $\gamma\gamma^* \to \eta_b$ form factor

Thanks for your attention!