Energy Correlators in QCD

and TMD Physics

Moult, HXZ, 1801.02627, JHEP;
Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 1801.03219, PRL;
A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation

朱华星 （Hua Xing Zhu）

浙江大学

TeV物理学工作组研讨会

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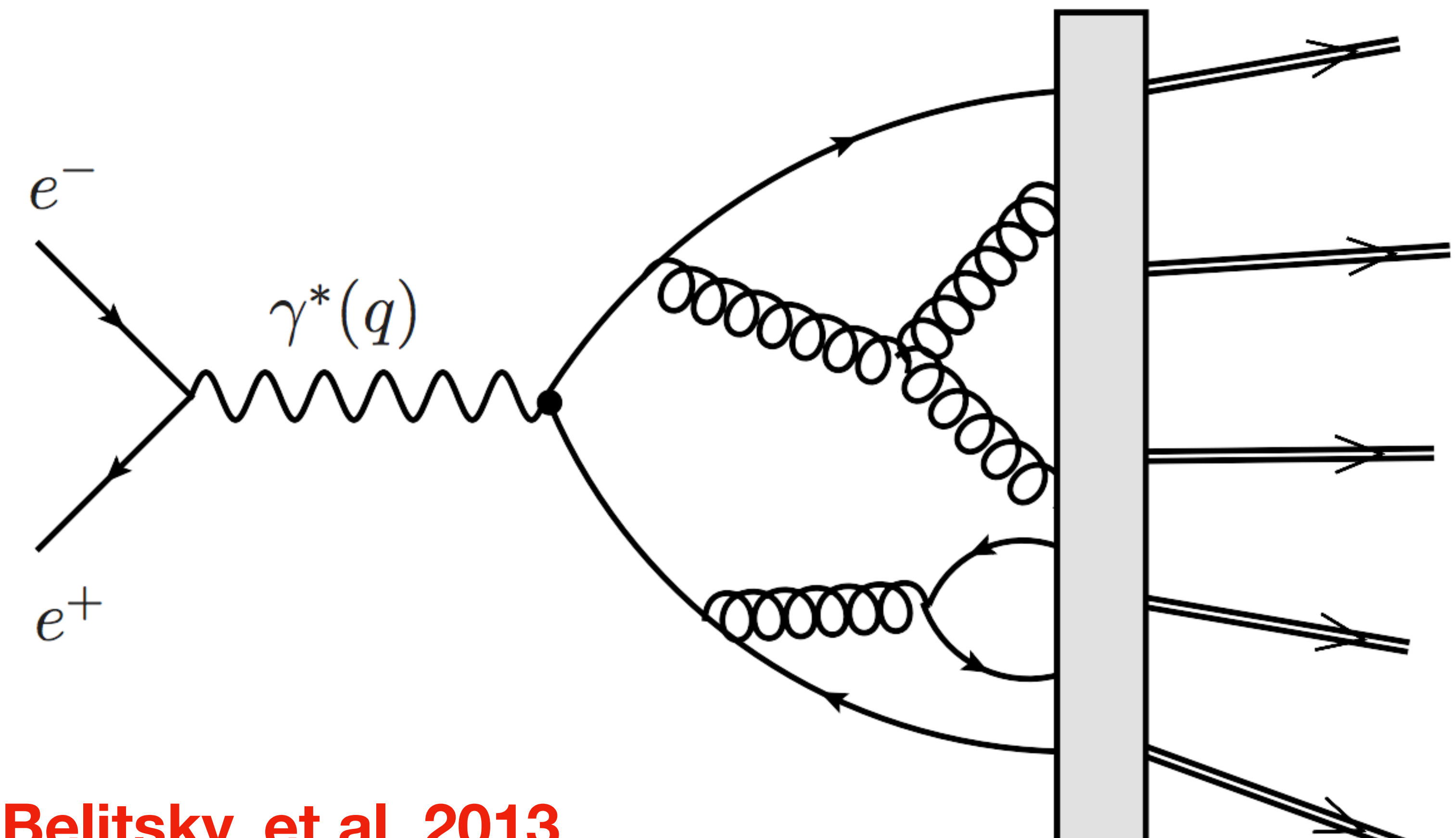
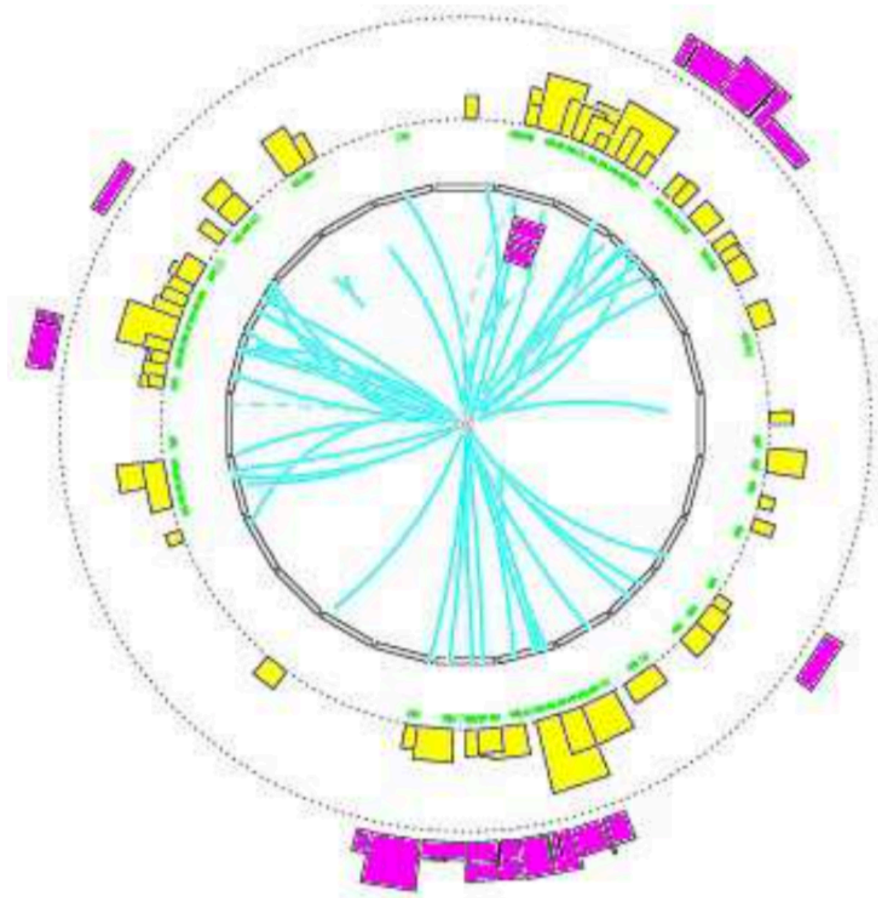
Correlation functions

- Correlations are the most basic observables in physics
- Perhaps the most well-known example is the spin-spin correlation in Ising model
- Scattering amplitudes in QFT are related to correlation functions through LSZ reduction
- But amplitudes in gauge theory are not directly observable beyond tree-level, due to the appearance of IR divergences
- Energy Correlators are a class of observables, which are both physical, and infrared finite

- Consider e+e- to hadrons

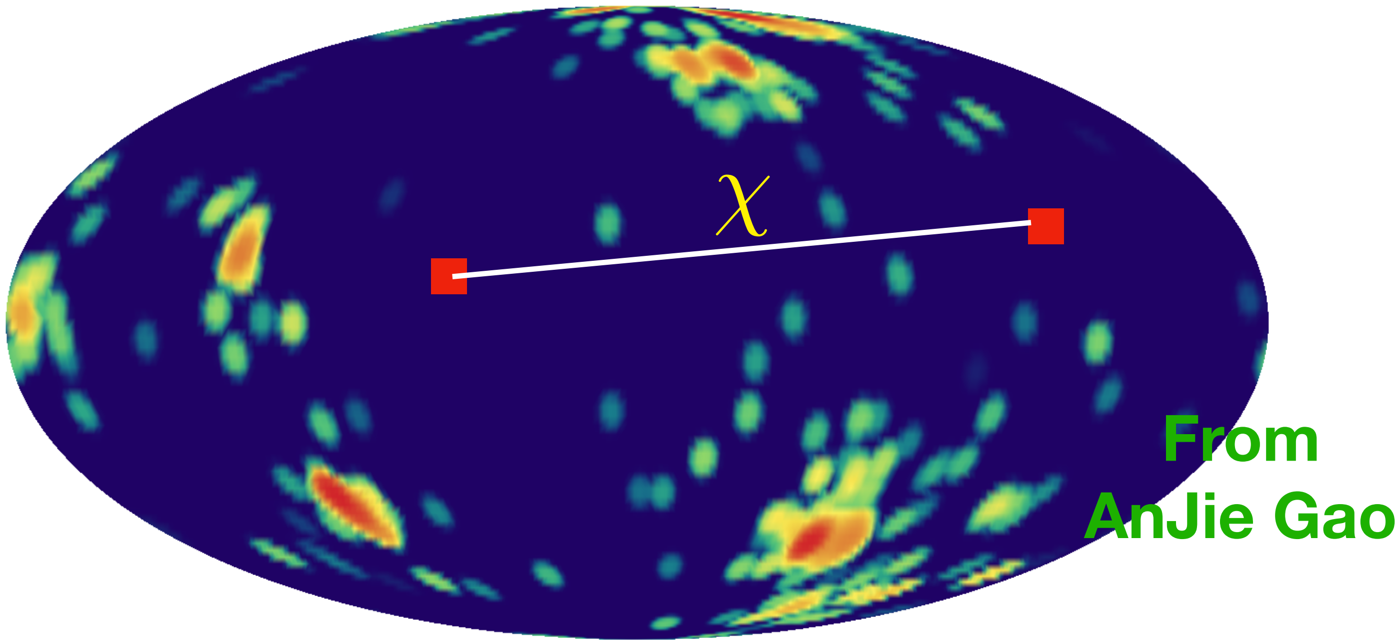




Belitsky, et al, 2013

Energy-Energy Correlator

- Mollweide projection of the energy deposition at the detector



- Integrate over the two calorimeter position
- Average over different event (at the same sqrt(S))
- The simplest energy correlator on the sphere is the two-point energy correlator (EEC)

EEC as four-point Wightman correlator

- EEC is a special jet observable in the sense that it has a simple operator definition

Hofman, Maldacena, 2008

$$\langle \mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)\rangle_Q = \sigma^{-1}\int \mathrm{d}^4x\, e^{ix\cdot Q}\langle 0|O^\dagger(x)\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)O(0)|0\rangle$$

$$\mathcal{E}(\boldsymbol{n}) = \int_0^\infty \mathrm{d}t \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\boldsymbol{n}) \qquad O^\mu(x) = \bar{\psi}\gamma^\mu\psi(x)$$

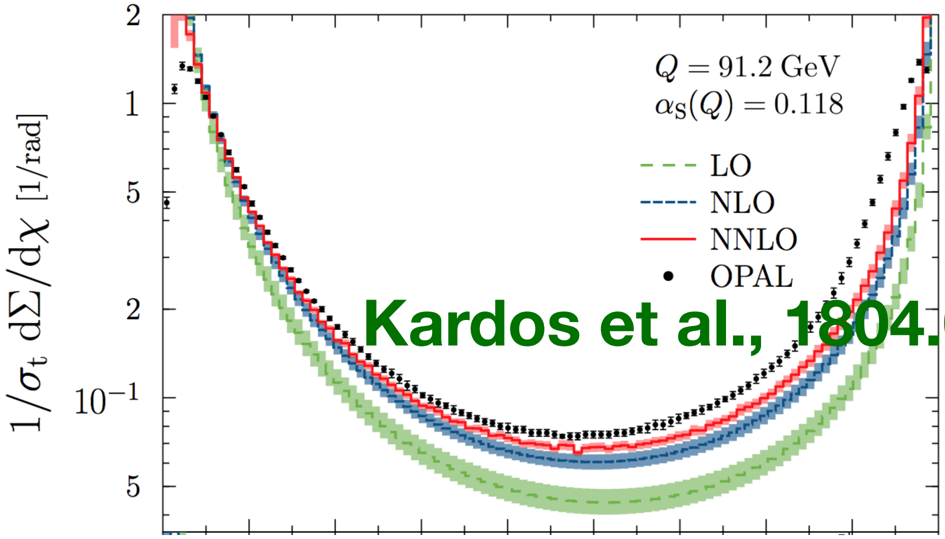
- Tμν is the stress tensor. E.g., for Dirac Fermion

$$T_{\mu\nu} = i\bar{\psi}\gamma_\mu\partial_\nu\psi - g_{\mu\nu}\mathcal{L}$$

- ε(n) is the so-called light-ray operator. see e.g. **Balitsky, Braun, 1989**

Why this observable is interesting?

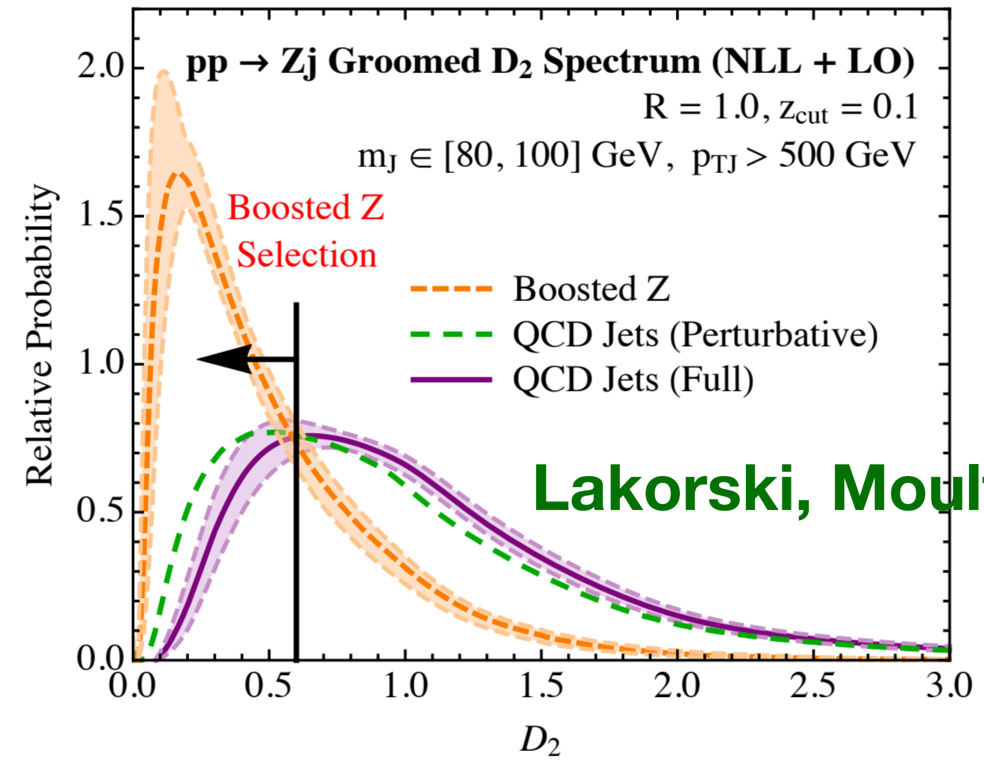
- Precision test of QCD



Kardos et al., 1804.09146

$$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$$

One of the best extraction of alpha_s from jets!



Lakorski, Moult, Neill

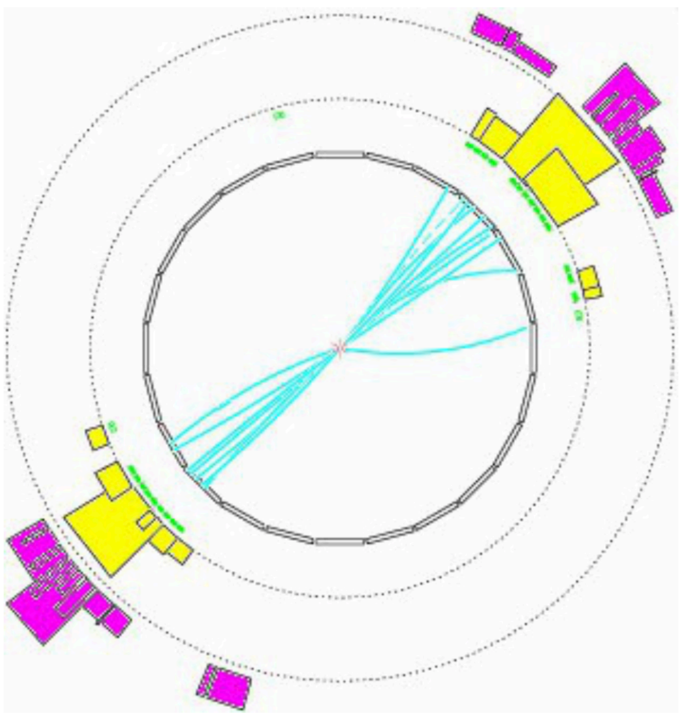
Also important for new physics search through jet substructure!

- Strong interest from conformal field theory community
  - Bounding operator anomalous dimension. e.g., ANEC
  - OPE of light-ray operator
  - Data from integrability => collider predictions
  - ……

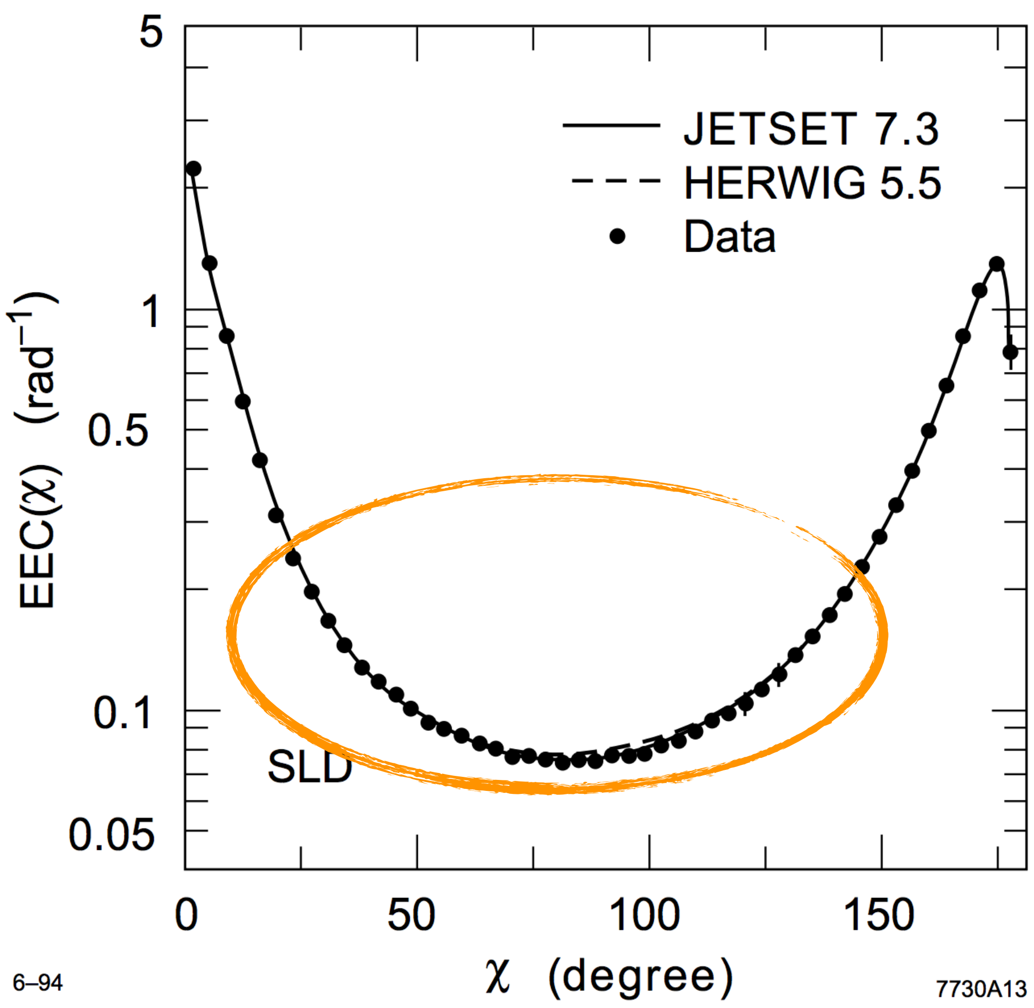
$$\frac{d\sigma}{dz}(z, \ln\frac{Q^2}{\mu^2}, \mu) = \sum_{i,j} \int \frac{E_i E_j}{Q^2} d\sigma \delta \left(z - \frac{1-\cos\chi_{ij}}{2} \right)$$

χ_{ij} **angle between p_i and p_j**

- EEC was proposed 40 years ago **Basham, Brown, Ellis, Love, 1978**
- Since then dedicated theoretical and experimental study



A typical dijet event



PHYSICAL REVIEW LETTERS **120,** 102001 (2018)

Editors' Suggestion

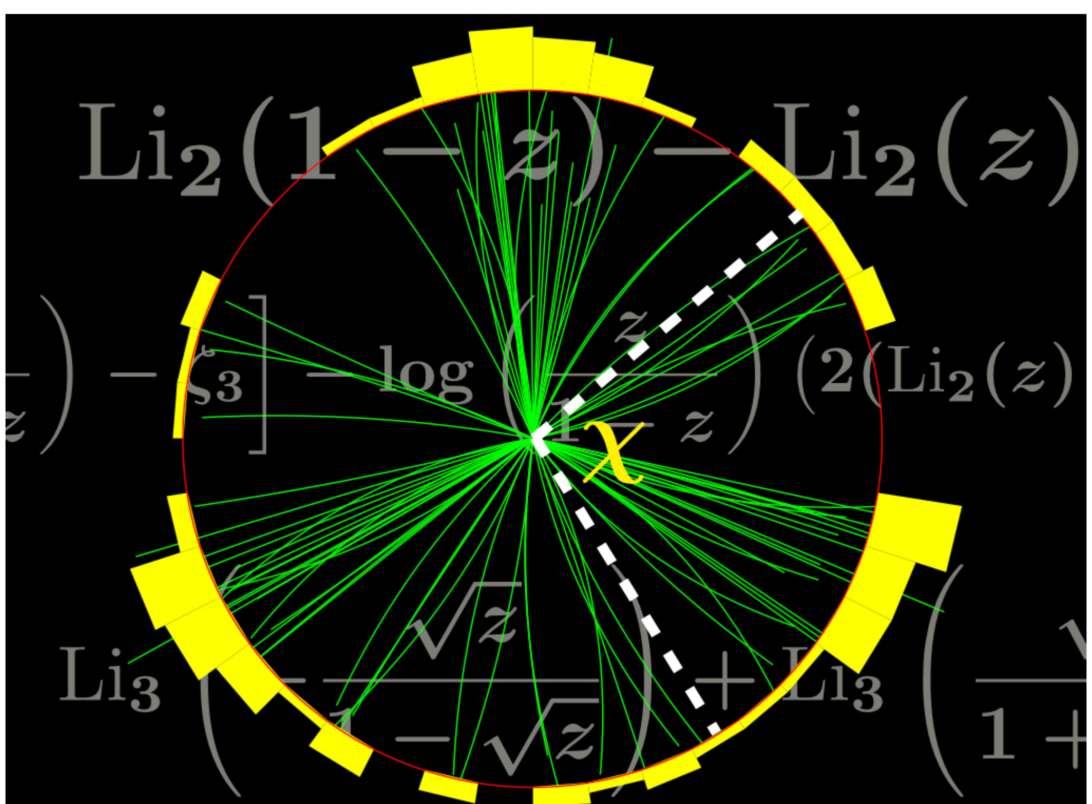
Analytical Computation of Energy-Energy Correlation at Next-to-Leading Order in QCD

Lance J. Dixon,[1,*] Ming-xing Luo,[2,†] Vladyslav Shtabovenko,[2,‡] Tong-Zhi Yang,[2,§] and Hua Xing Zhu[2,∥]
[1] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*
[2] *Zhejiang Institute of Modern Physics, Department of Physics, Zhejiang University, Hangzhou 310027, China*

(Received 17 January 2018; published 9 March 2018)

The energy-energy correlation (EEC) between two detectors in e^+e^- annihilation was computed analytically at leading order in QCD almost 40 years ago, and numerically at next-to-leading order (NLO) starting in the 1980s. We present the first analytical result for the EEC at NLO, which is remarkably simple, and facilitates analytical study of the perturbative structure of the EEC. We provide the expansion of the EEC in the collinear and back-to-back regions through next-to-leading power, information which should aid resummation in these regions.

- **First ever event shape in analytical form at NLO!**



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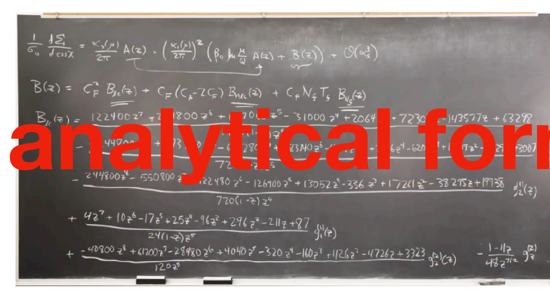
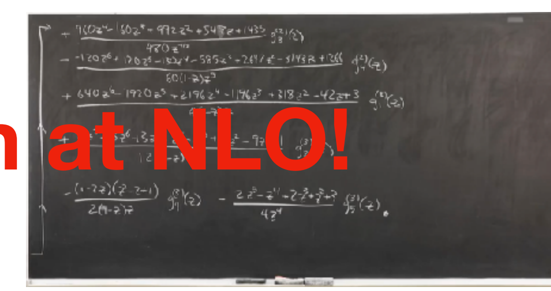

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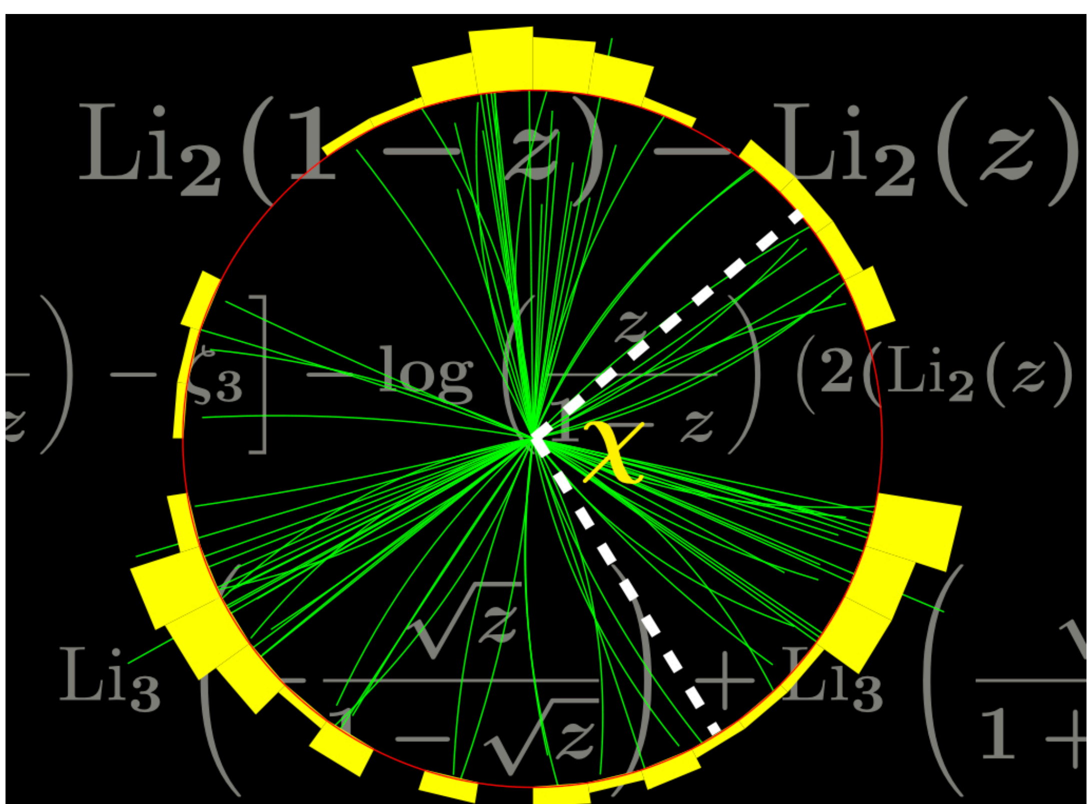
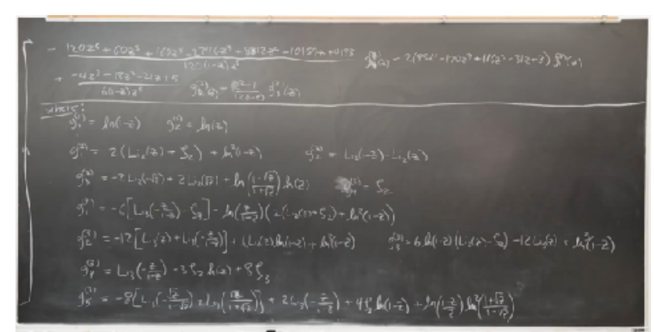
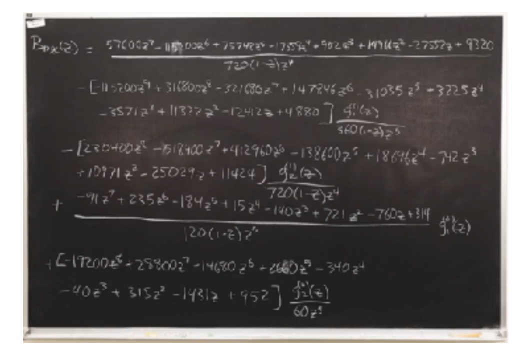
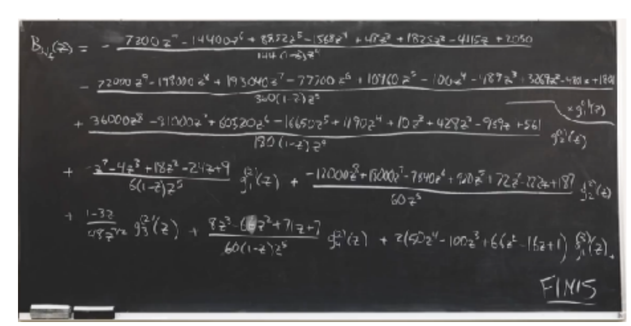

$$\frac{d\sigma}{dz}(z, \ln\frac{Q^2}{\mu^2}, \mu) = \sum_{i,j} \int \frac{E_i E_j}{Q^2} d\sigma \delta \left(z - \frac{1-\cos\chi_{ij}}{2} \right)$$

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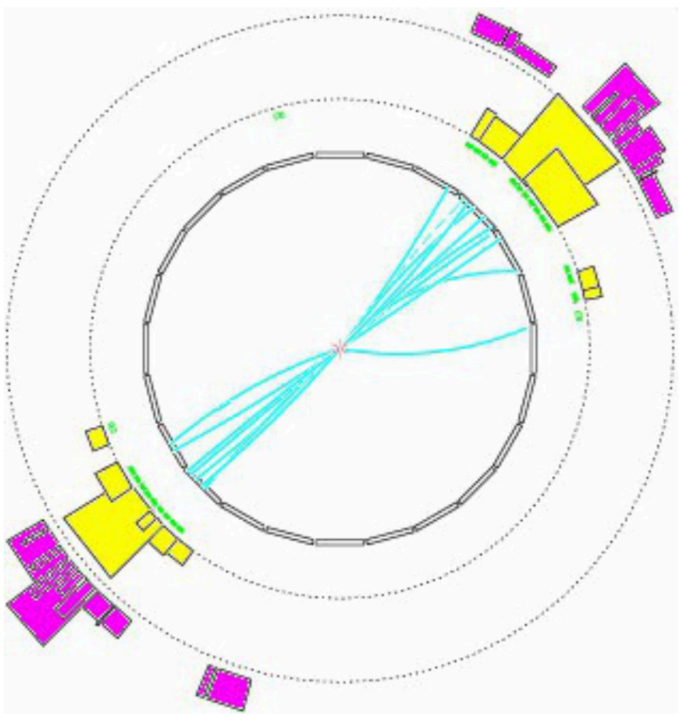
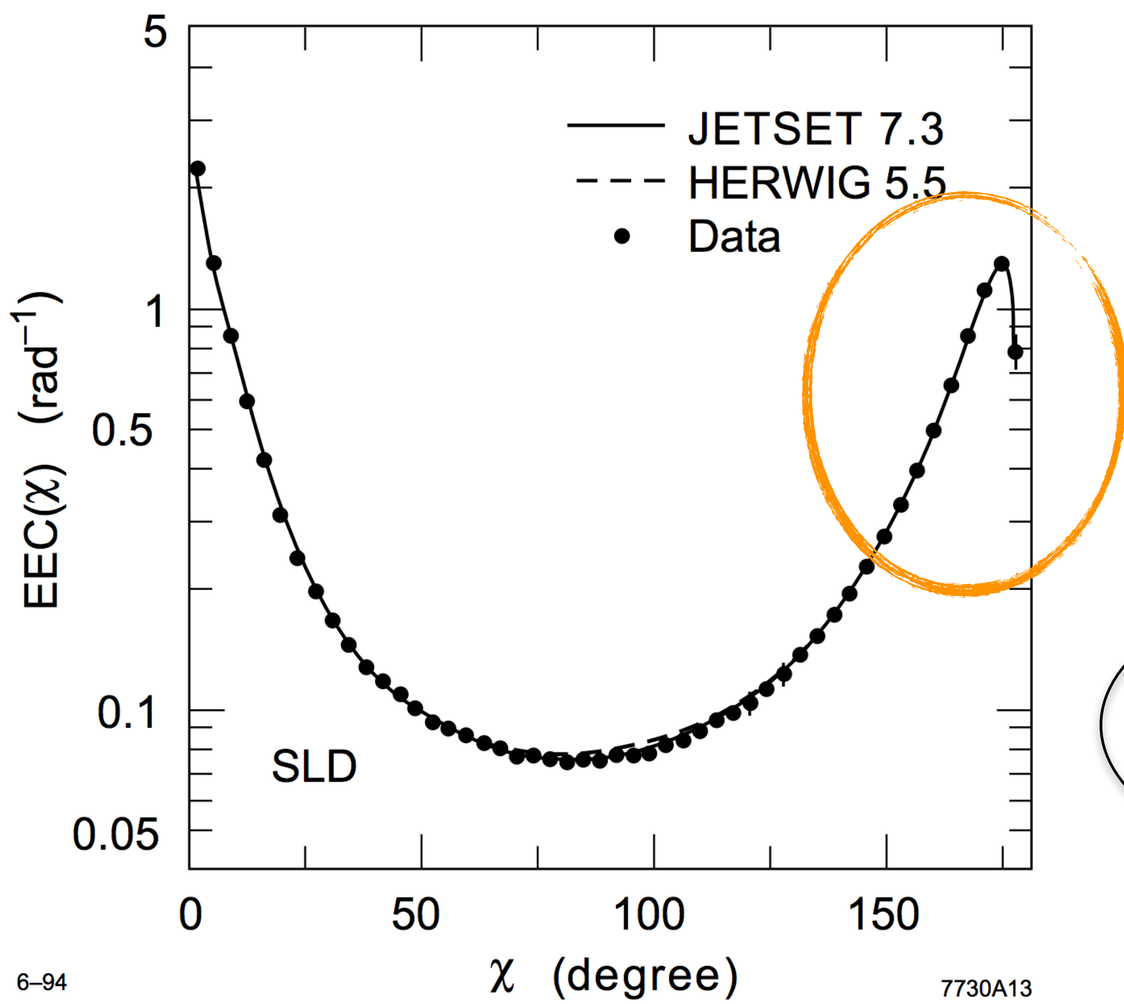
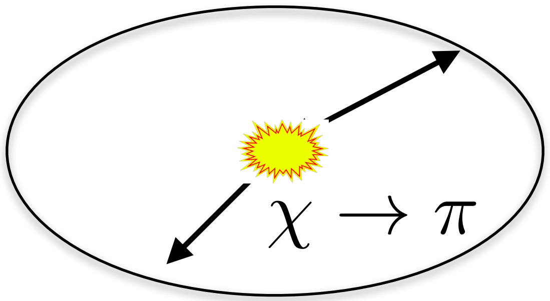


A typical dijet event



Probing TMD physics in jet process



Sudakov double logarithms

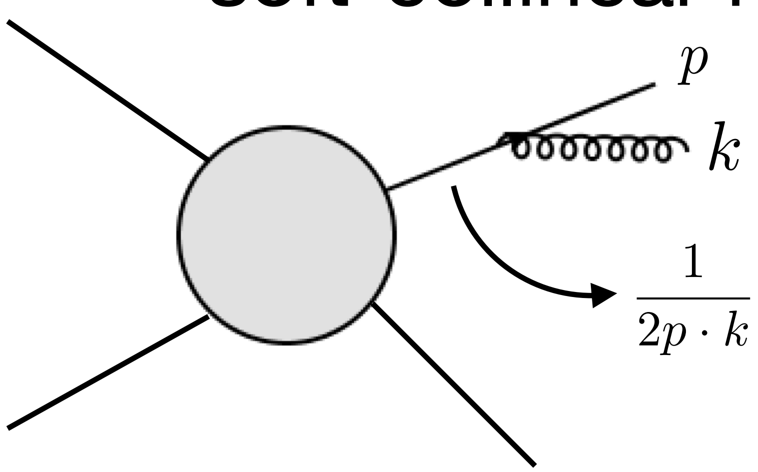
- The back-to-back region of EEC is characterized by large Sudakov logarithms

$$\frac{1}{\sigma_0}\frac{d\sigma}{dz} = \frac{\alpha_s}{4\pi}C_F\left(-\frac{\ln(1-z)}{1-z}+\dots\right) + (\frac{\alpha_s}{4\pi})^2 C_F^2\left(4\frac{\ln^3(1-z)}{1-z}+\dots\right)+\dots$$

$$z = \frac{1-\cos\chi}{2}$$

$$\int_0^z dz'\frac{\ln(1-z')}{1-z'} \sim \ln^2(1-z)$$

- They are Sudakov double logarithms. They arise from soft-collinear radiation of massless gauge theory



$$\frac{1}{2p\cdot k} = \frac{1}{2E_q E_g(1-\cos\theta)}$$

$$\int_{E_0}\frac{dE_g}{E_g}\int_{\theta_0}\frac{d\theta}{\theta} \sim \ln E_0 \ln\theta_0$$

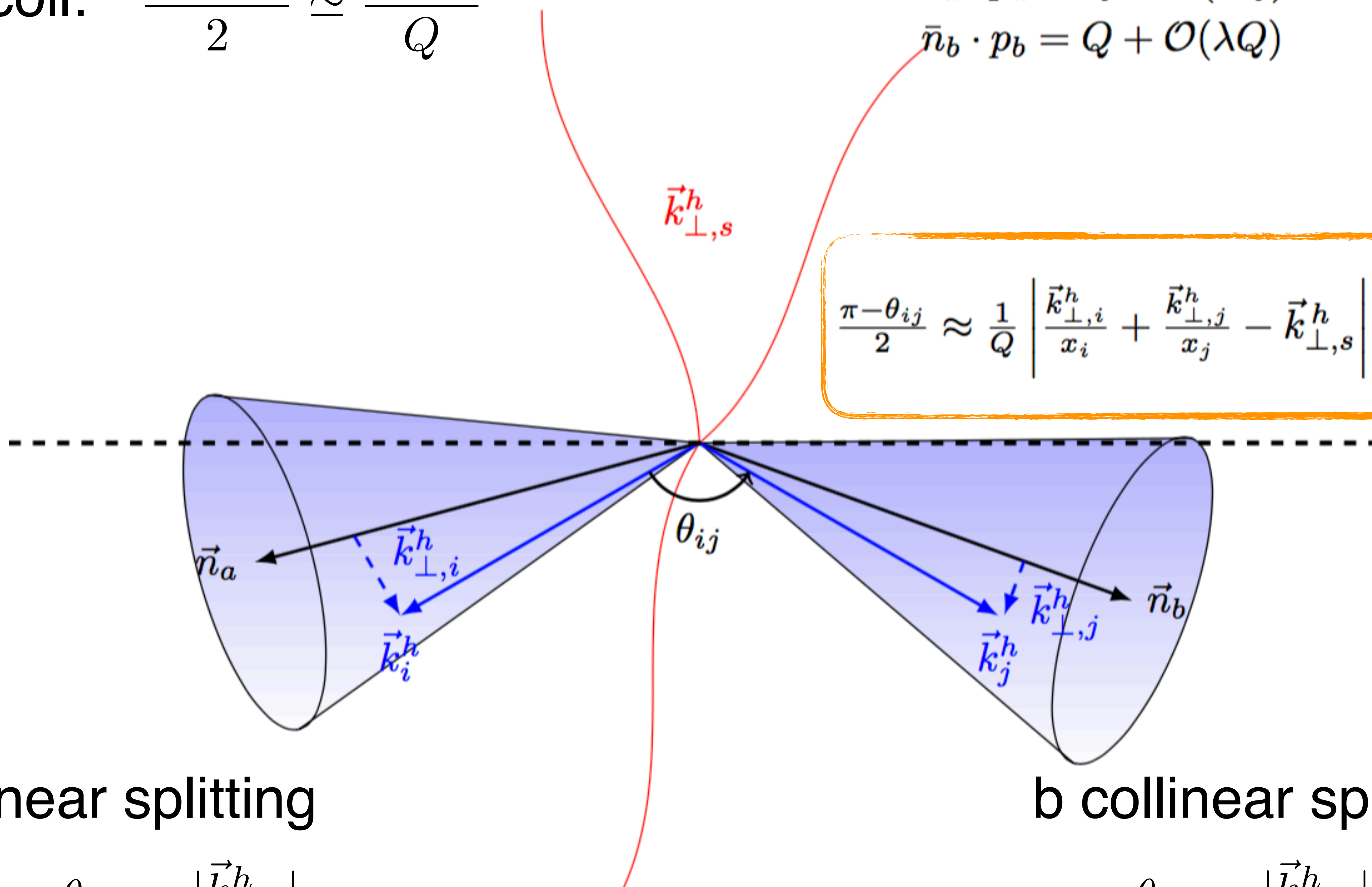
- We will resum the Sudakov logs in Soft-Collinear Effective Theory (SCET)

Factorization in kinematics

soft recoil: $\dfrac{\pi - \theta_{ij}}{2} \simeq \dfrac{|\vec{k}^h_{\perp,s}|}{Q}$

$$\bar{n}_a \cdot p_a = Q + \mathcal{O}(\lambda Q)$$
$$\bar{n}_b \cdot p_b = Q + \mathcal{O}(\lambda Q)$$

$$\frac{\pi - \theta_{ij}}{2} \approx \frac{1}{Q}\left|\frac{\vec{k}^h_{\perp,i}}{x_i} + \frac{\vec{k}^h_{\perp,j}}{x_j} - \vec{k}^h_{\perp,s}\right|$$



a collinear splitting

$$\frac{\pi - \theta_{ij}}{2} \simeq \frac{|\vec{k}^h_{\perp,i}|}{x_i Q}$$

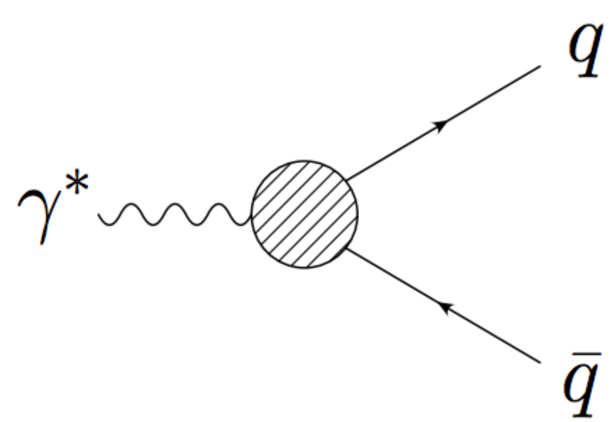
b collinear splitting

$$\frac{\pi - \theta_{ij}}{2} \simeq \frac{|\vec{k}^h_{\perp,j}|}{x_j Q}$$

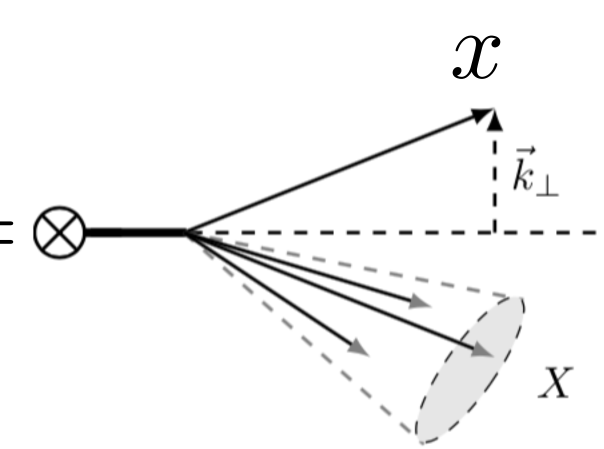
Factorization formula

$$\frac{d\sigma}{dz} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J^q_{\rm EEC}(\vec{b}_\perp,\mu,\nu) J^{\bar{q}}_{\rm EEC}(\vec{b}_\perp,\mu,\nu) S_{\rm EEC}(\vec{b}_\perp,\mu,\nu)\,\delta\left(1-z-\frac{\vec{k}_\perp^2}{Q^2}\right)$$

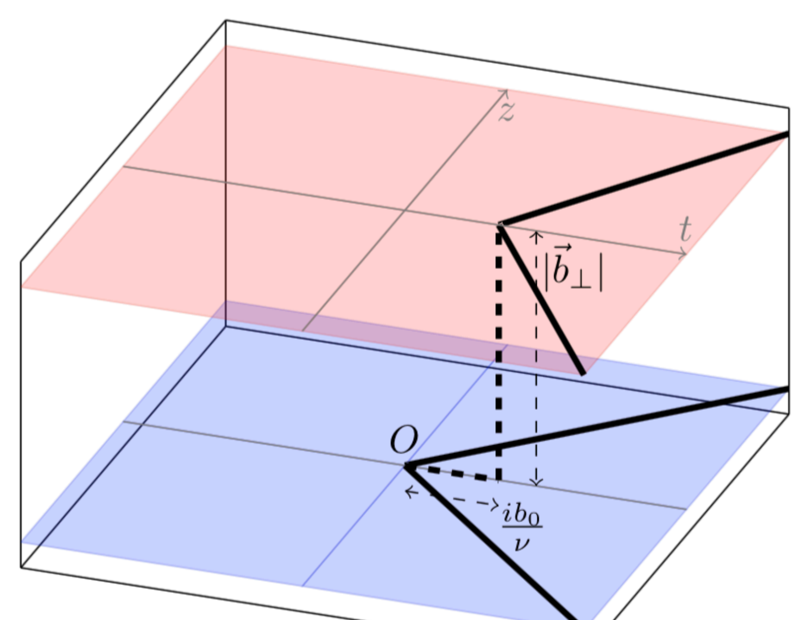
$$H(Q,\mu) =$$



$$J^q_{\rm EEC}(\vec{b}_\perp) = \sum_i \int_0^1 dx\; x\, \mathcal{I}_{qi}\left(\frac{\vec{b}_\perp}{x}, x\right) \qquad \mathcal{I}_{qi}(\vec{b}_\perp, x) = \otimes$$



$$S_{\rm EEC}(\vec{b}_\perp,\mu,\nu) =$$



Resummation from RG evolution

- Virtuality evolution

$$\mu \frac{d}{d\mu} H(Q,\mu) = 2\left[\Gamma^{\text{cusp}}(\alpha_s) \ln \frac{Q^2}{\mu^2} + \gamma^H(\alpha_s)\right] H(Q,\mu)$$

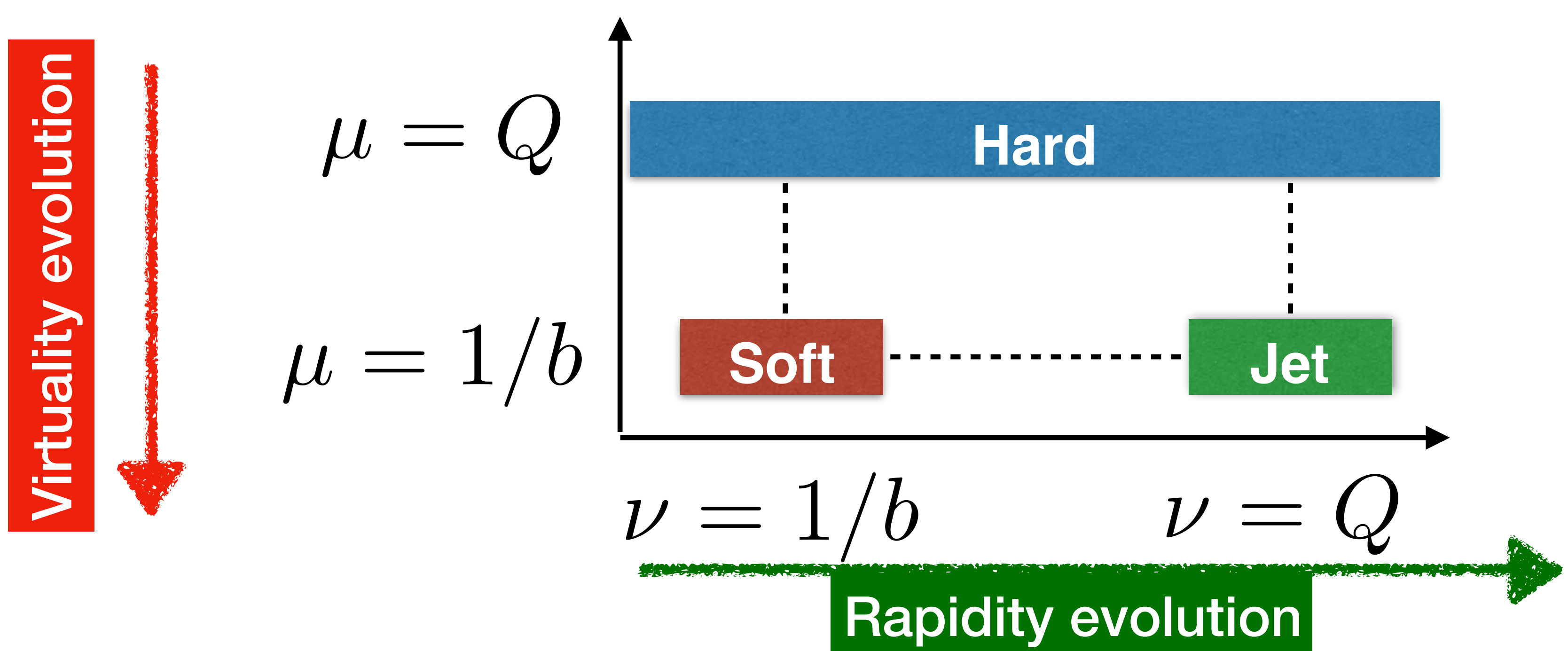
$$\mu \frac{dS_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)}{d\mu} = \left[2\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{\nu^2} - 2\gamma^s_{\text{EEC}}(\alpha_s)\right] S_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)$$

$$\mu \frac{dJ^q_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)}{d\mu} = \left[-\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{Q^2}{\nu^2} + 2\gamma^J_{\text{EEC}}(\alpha_s)\right] J^q_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)$$

- Rapidity evolution

$$\nu \frac{dS_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)}{d\nu} = 2\left[-\int_{b_0^2/\vec{b}_\perp^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Gamma_{\text{cusp}}(\alpha_s(\bar{\mu})) + \gamma^r_{\text{EEC}}(\alpha_s(b_0/|\vec{b}_\perp|))\right] S_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)$$

$$\nu \frac{dJ^q_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)}{d\nu} = \left[\int_{b_0^2/\vec{b}_\perp^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Gamma_{\text{cusp}}(\alpha_s(\bar{\mu})) - \gamma^r_{\text{EEC}}(\alpha_s(b_0/|\vec{b}_\perp|))\right] J^q_{\text{EEC}}(\vec{b}_\perp,\mu,\nu)$$



Counting of logarithms

$$
\begin{aligned}
\ln \sigma(b) \sim\ & \alpha_s \left[\ln^2 b + \ln b + 1\right] \\
& + \alpha_s^2 \left[\ln^3 b + \ln^2 b + \ln b + 1\right] \\
& + \alpha_s^3 \left[\ln^4 b + \ln^3 b + \ln^2 b + \ln b + 1\right] \\
& + \ \ldots
\end{aligned}
$$

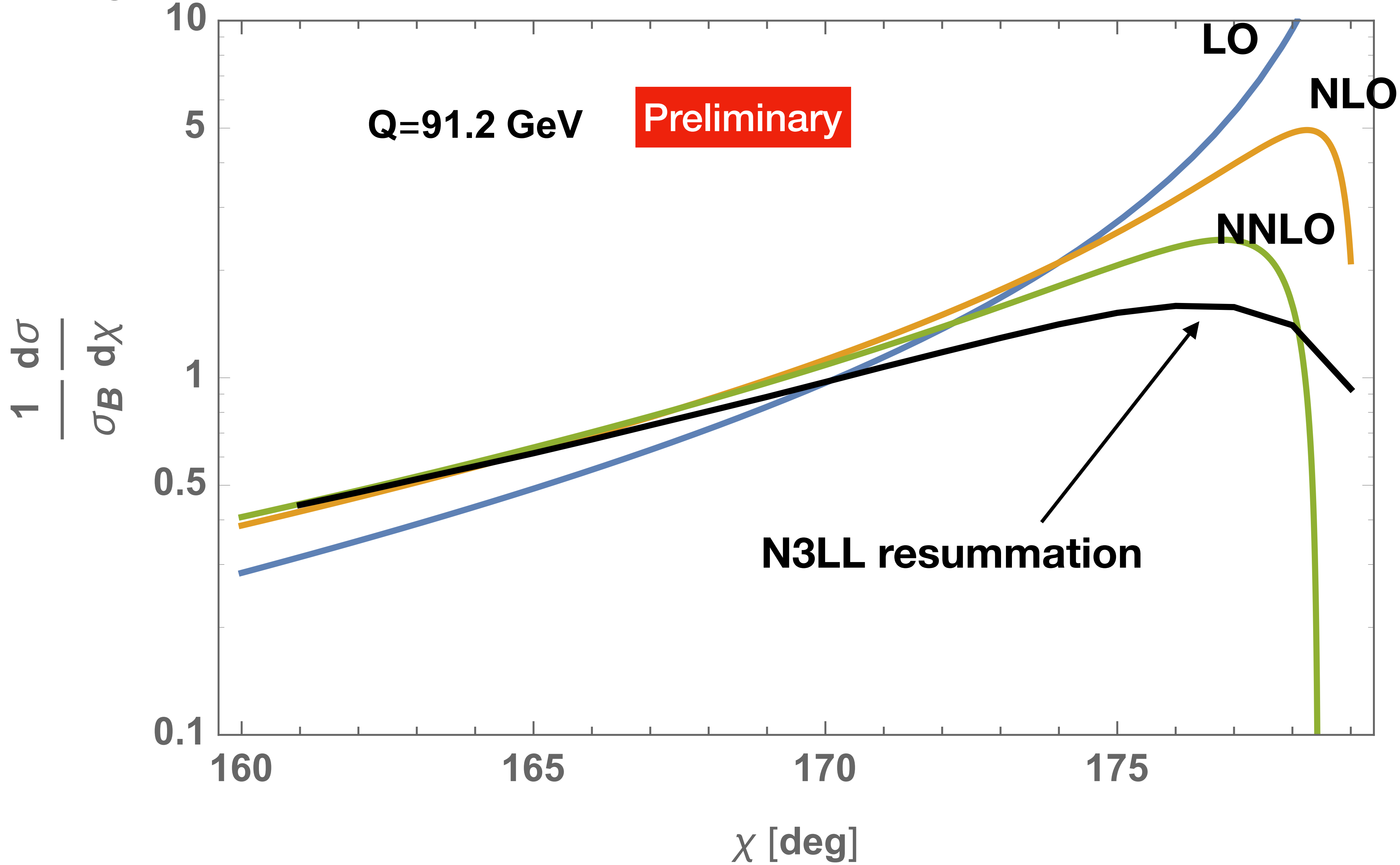
LL NLL NNLL N3LL N3LL’

Grazzini, de Florian, 2004 (→ NNLL)

Moult, HXZ, 2018 (→ N3LL)

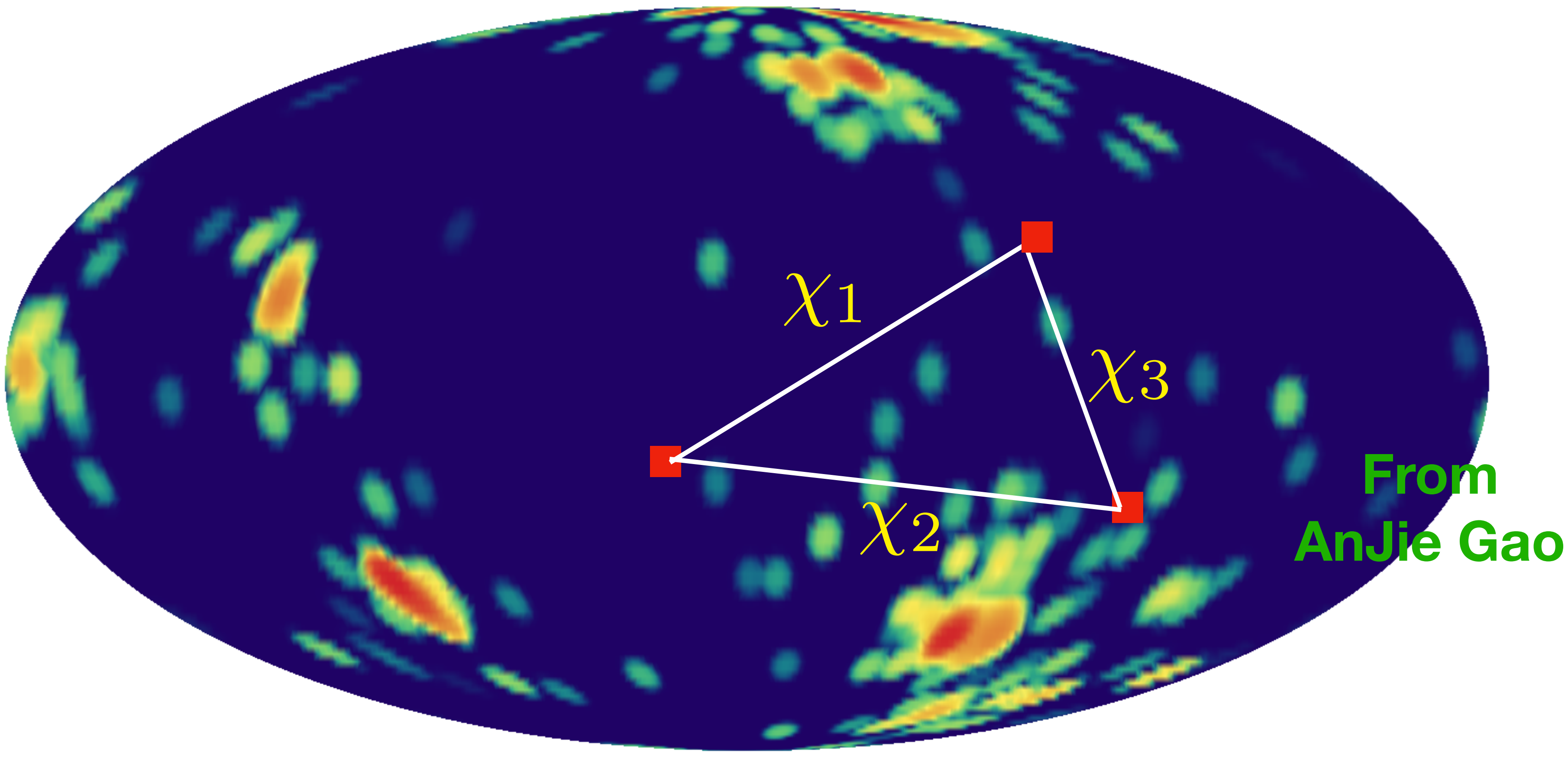
Numerical results

- Resummation is necessary for comparing theory and date
- Provide precision constraint on α_s and TMDFF (work in progress)



Going beyond two-point correlation

- Can we compute three-point energy correlators?



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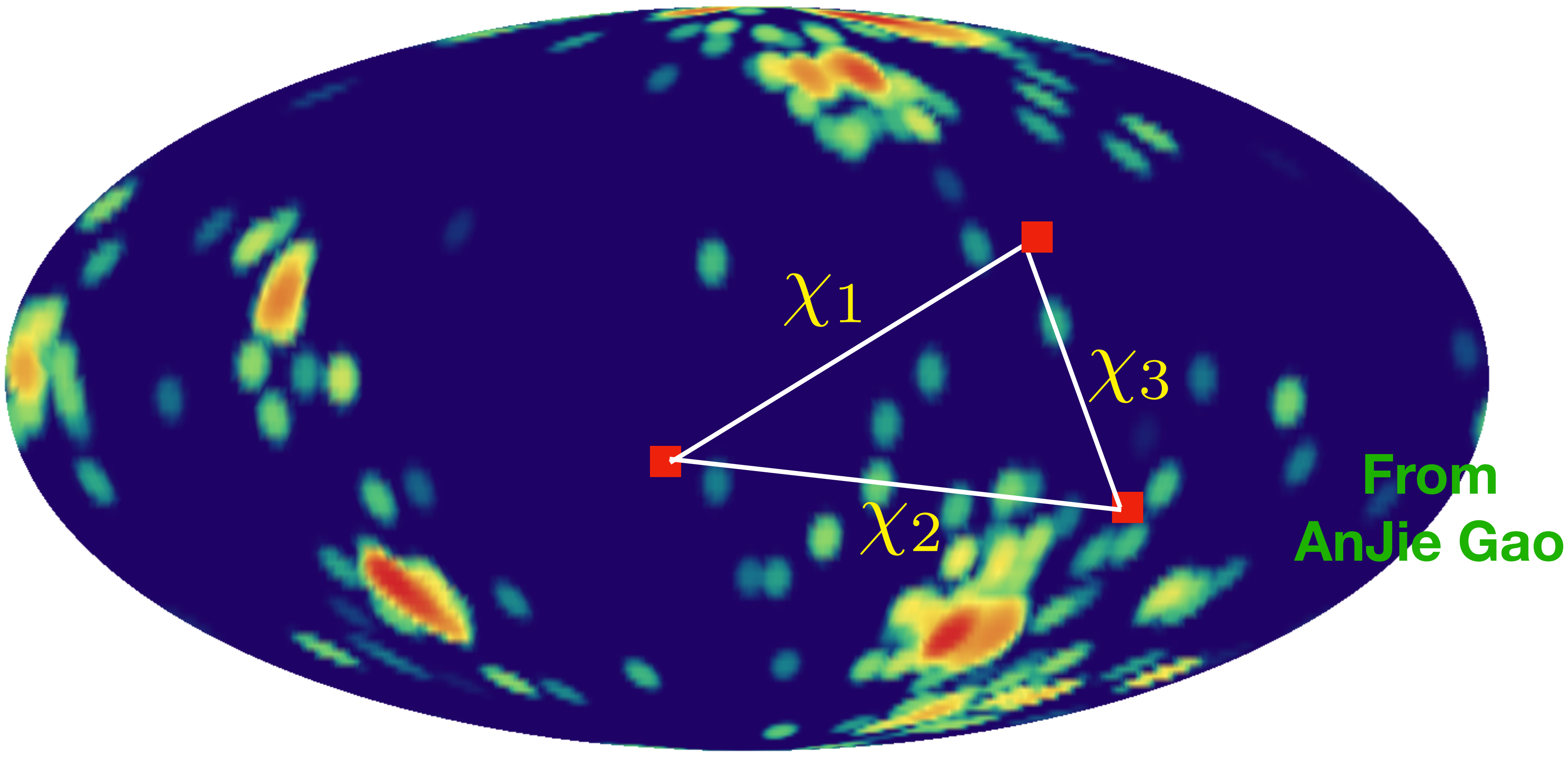
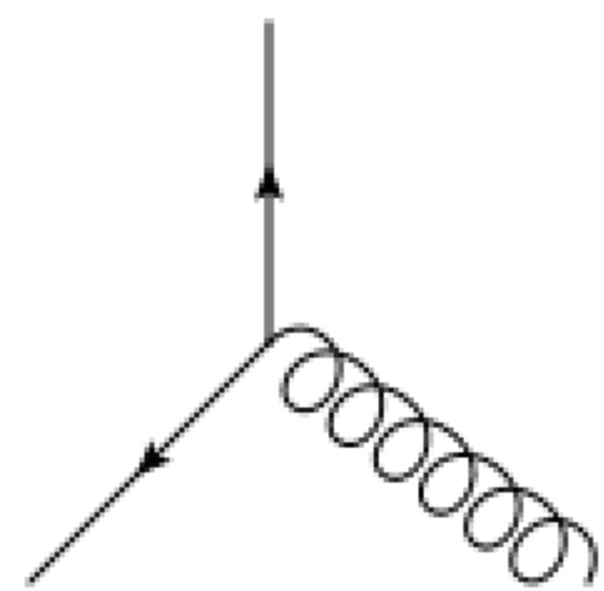


- Yes, in the coplanar limit!

- Three point correlation requires at least three particles in the final state

- We are interested in event where there is three well separated jet

- Momentum conservation implies that three jet production at LO in e+e- collision is coplanar



- Soft and collinear radiation will violate the coplanarity, and leads to large double logarithms

Energy Triple-Production Correlator

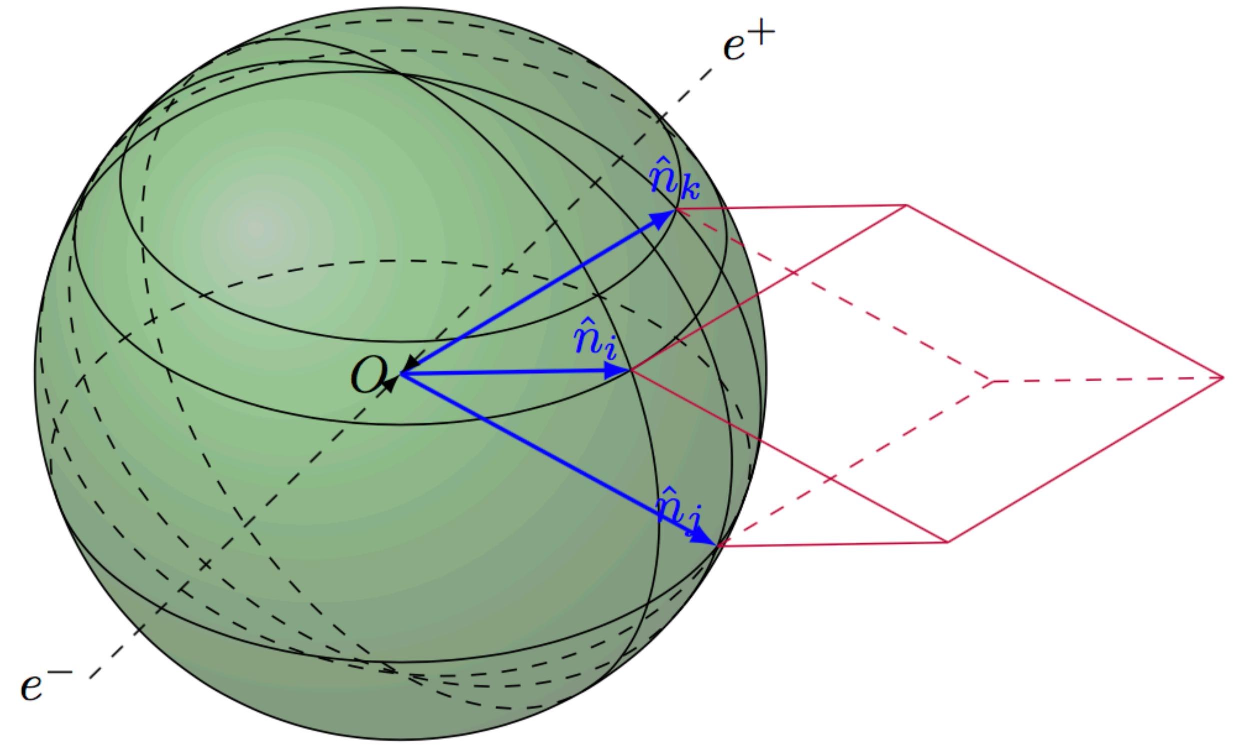
A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation

- We define a new observable ETPC, which measures the deviation from coplanarity

$$ETPC = \frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma}{d\tau_p} = \sum_{ijk}\int d\sigma \frac{E_i E_j E_k}{Q^3 \sigma_{\text{tot}}}\delta\left(\tau_p - \tau_{ijk}\right)$$

$$\tau_p = \left|\vec{n}_i \cdot \left(\vec{n}_j \times \vec{n}_k\right)\right|$$

- i, j, k runs over all final state particles
- τ_p is the volume of the parallelepiped
- τ_p -> 0 is the coplanar limit



Constraint to three-jet event

A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation

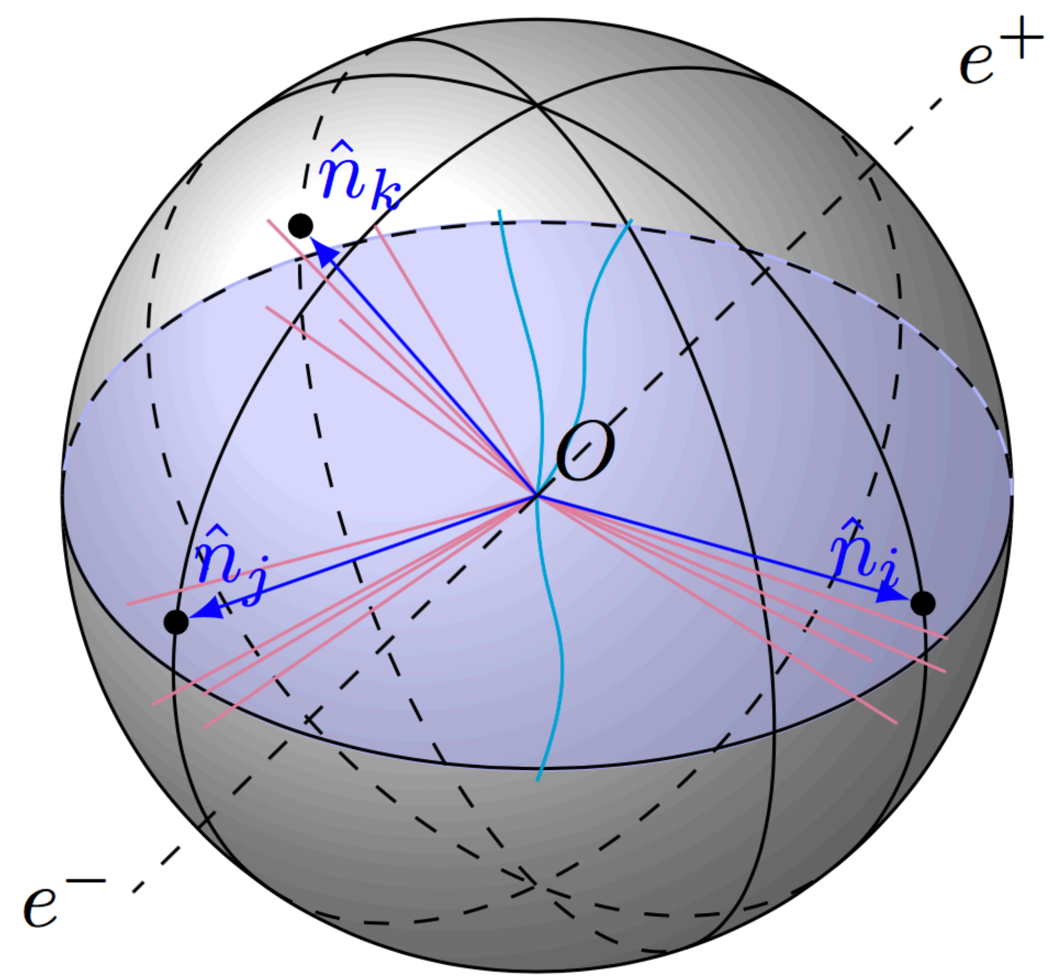
- We are most interested in event with three well-separated jets, and i, j, k are from three different jets

- Two ways to impose this constraint

  - a cut:

$$\sin\theta_{ij} > a_{\text{cut}}\,,\quad \sin\theta_{jk} > a_{\text{cut}}\,,\quad \sin\theta_{ki} > a_{\text{cut}}$$

  - y cut: use k_T algorithm with some ycut to select three-jet event, and i, j, k must not be in the same jet



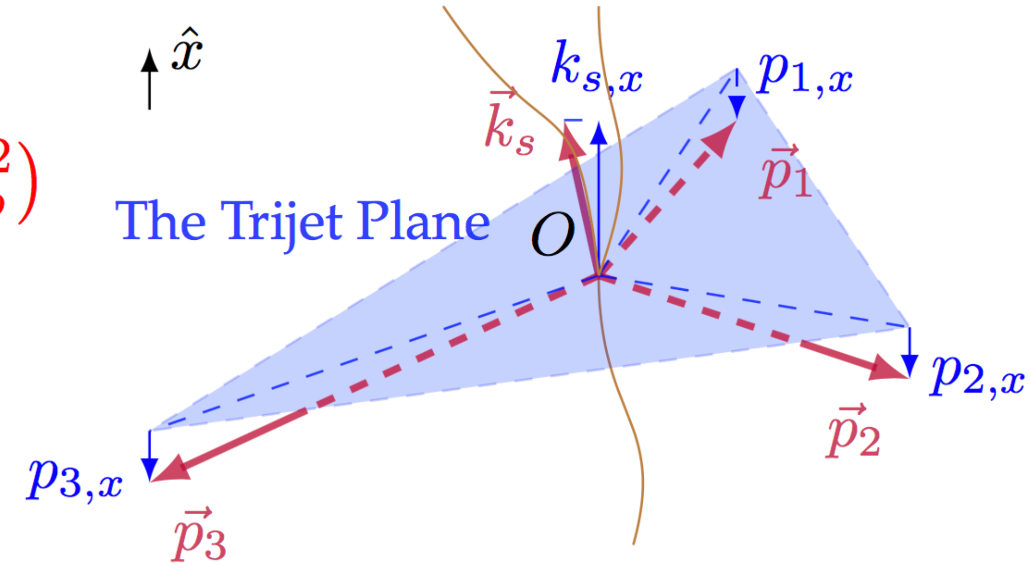
Factorization formula in the coplanar limit

A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation

- In the coplanar limit, ETPC reduced essentially to the off plane transverse momentum

$$\tau_{ijk} = \frac{|\vec{p}_1 \times \vec{p}_2|}{E_1 E_2 E_3} \left| \frac{k_{i,x}^h}{z_i^h} + \frac{k_{j,x}^h}{z_j^h} + \frac{k_{k,x}^h}{z_k^h} - k_{s,x} \right| + \mathcal{O}\left(\tau_p^2\right)$$

area of the triangle span by the jets



$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_p} = \int_D dx_1 dx_2 H(x_1, x_2, \mu) \int_{-\infty}^{\infty} \frac{db}{2\pi\xi} \cos(b\tau_p/\xi)$$

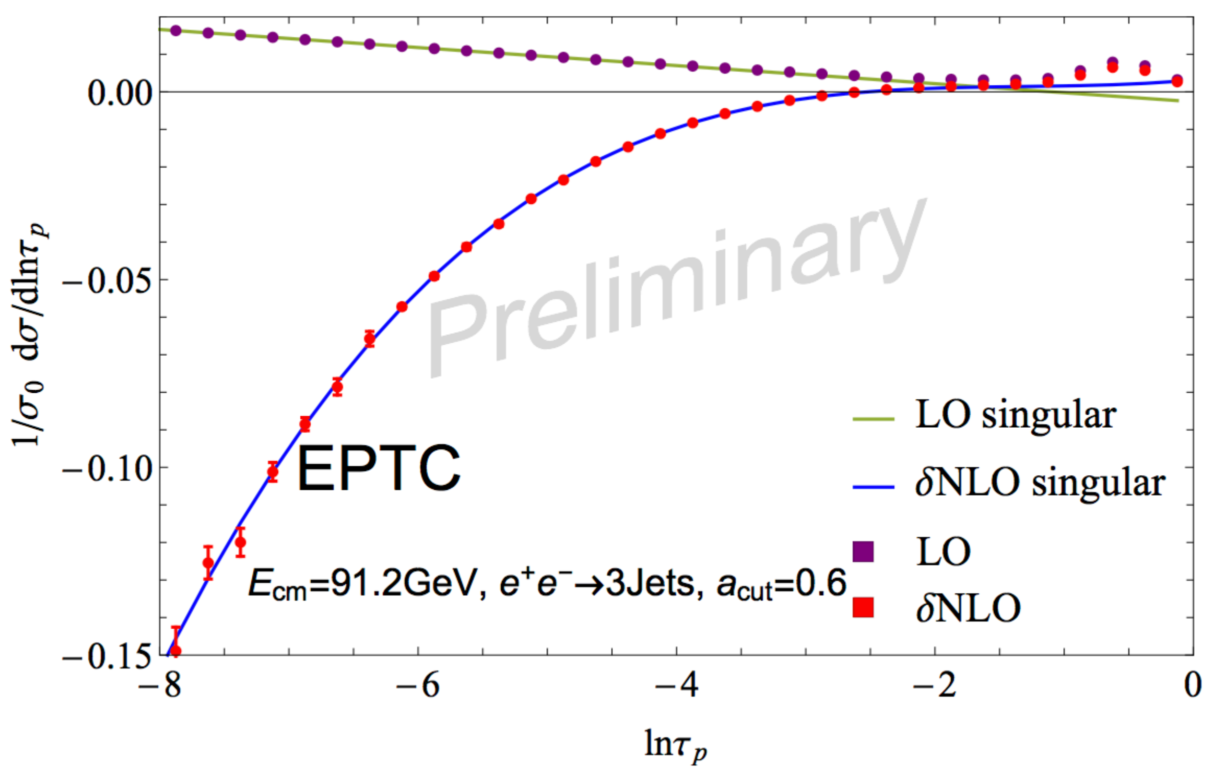
$$S(b,\mu,\nu) J_q(b,\mu,nu) J_{\bar{q}}(b,\mu,\nu) J_g(b,\mu,\nu)$$

Also sensitive to gluon TMDFF!

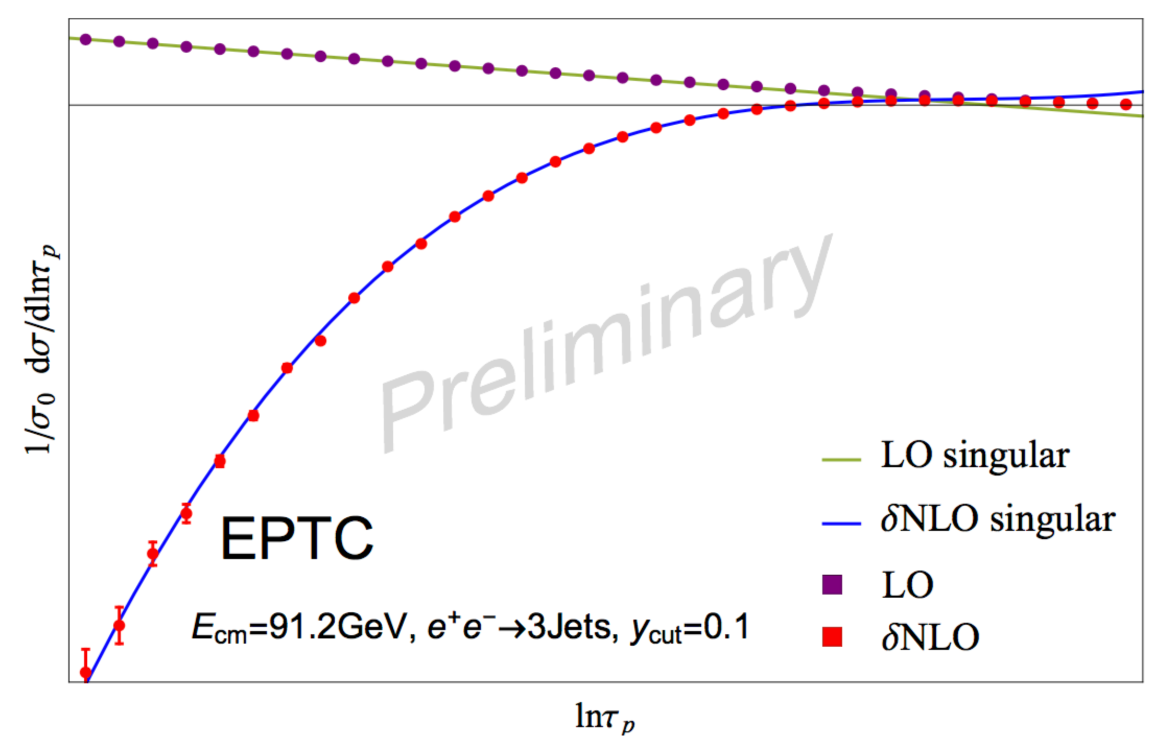
Cross check with NLOJET++

A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation

- Factorization formula predict the leading power behavior at $\tau_p \to 0$
- Can compare with 4 jet production at LO and NLO (using NLOjet++)



$a_{\text{cut}} = 0.6$

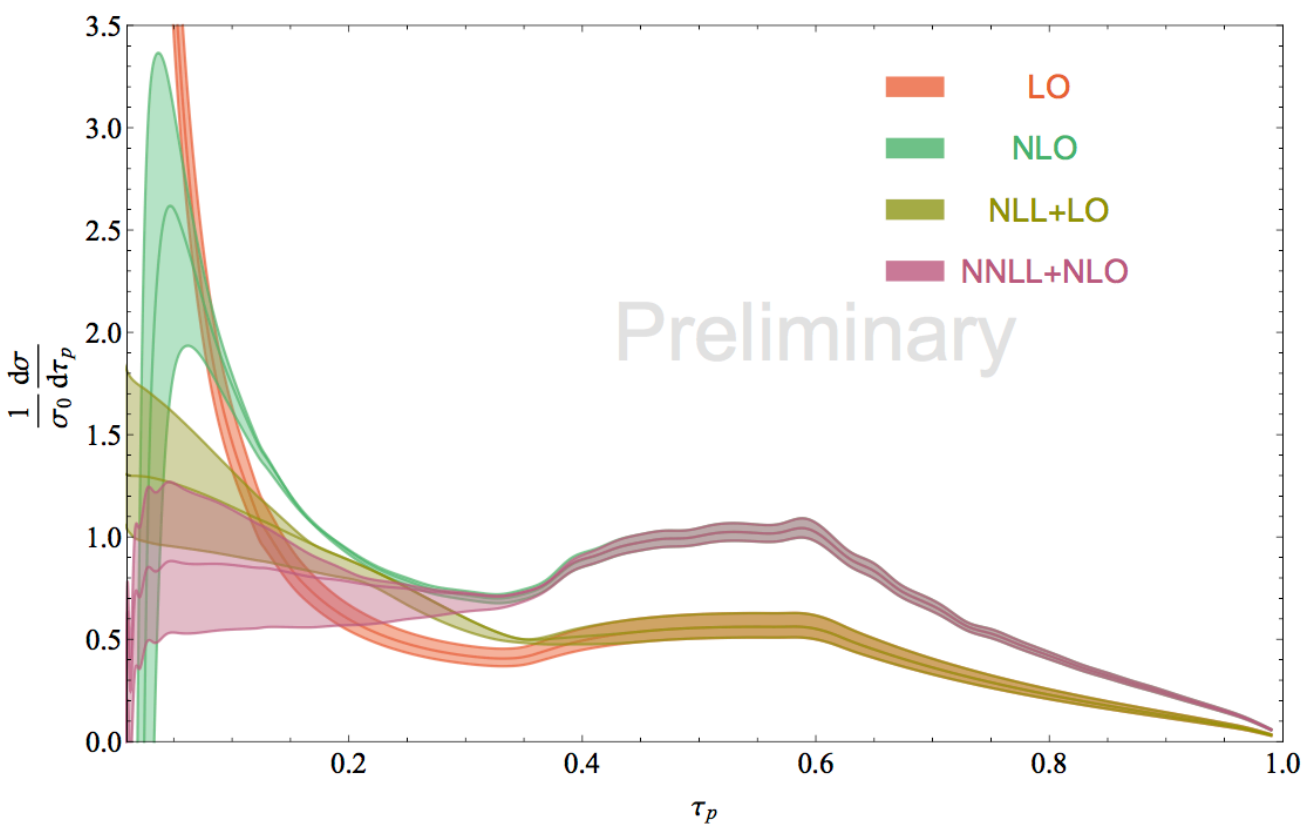


$y_{\text{cut}} = 0.1$

Highly non-trivial agreement!

Resummation prediction

A.J. Gao, M.X. Luo, T.Z. Yang, HXZ, in preparation



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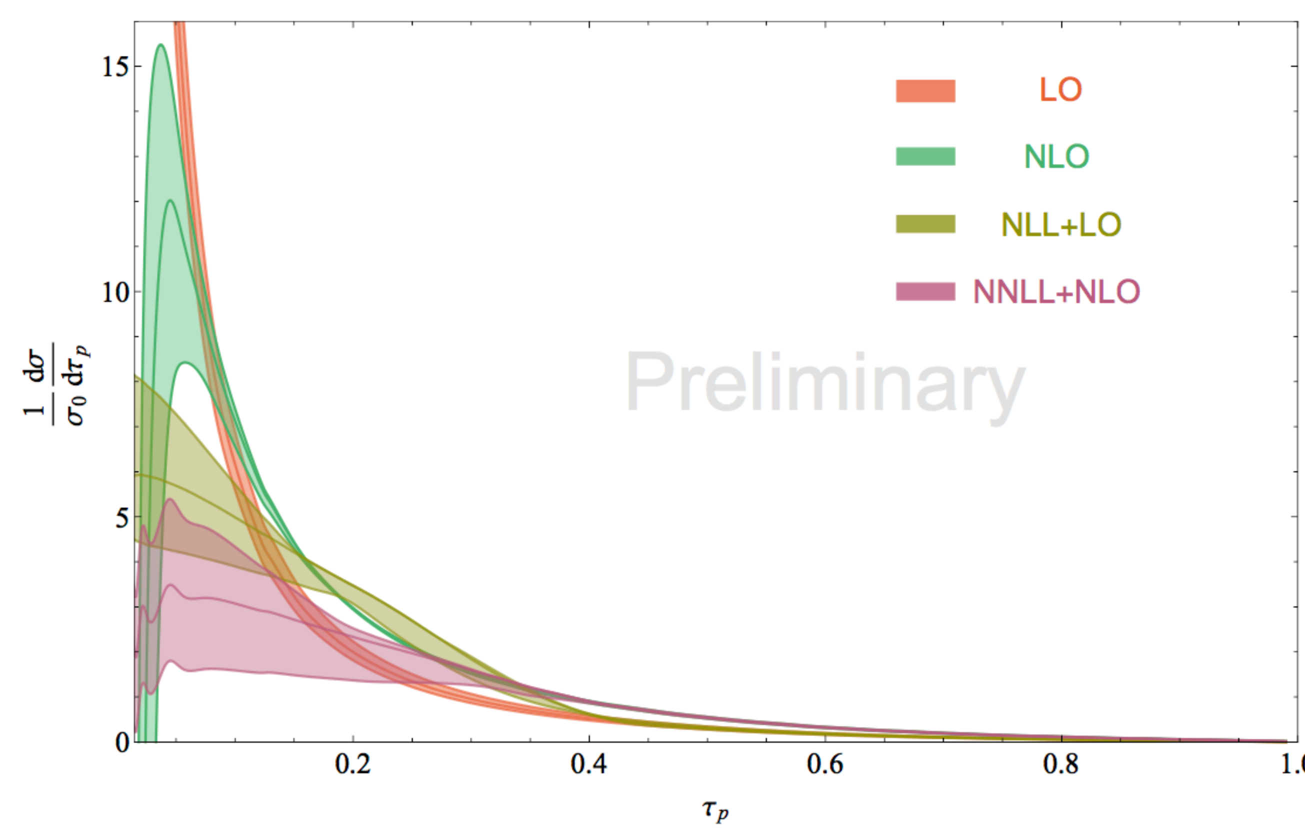

$y_{\mathrm{cut}} = 0.1$

- First NNLL+NLO resummation for three-jet event shape
- Important resummation effects at small τ_p
- Opportunity to constraint gluon TMDFF in e+e-!

Summary

- Very rich physics in energy correlators
- Energy-Energy Correlator and Energy Triple-Product Correlator
- Closely related to TMD physics
- not mentioned in this talk
    - Fixed-order analytical calculation [Henn, Sokatchev, K. Yan, Zhibodove, 2019]
    - EEC in the collinear limit [Dixon, Moult, HXZ, to appear]
- Many interesting things to explore. Stay tuned!

Thank you for your attention!