Hidden Relations in Two Loop Higgs Amplitudes

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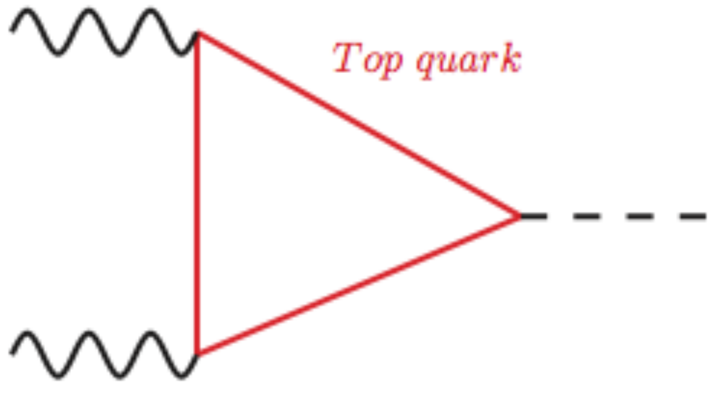
Based on 1904.07260 in collaboration with Gang Yang

Content

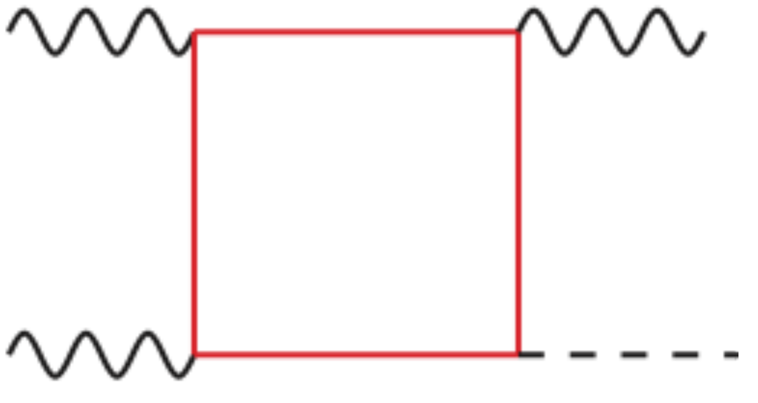
- **Motivations**
- Computations
- Results
- Summary and outlook

LHC and analytical Higgs amplitude

- After the discovery of Higgs particle in 2012, the next very important task in LHC is establishing the properties (coupling, spin and CP) of Higgs particles.
- In order to interpret the high precision data in LHC, the analytical computation of Higgs amplitudes is desirable.
- Gluon fusion is the main channel of Higgs production.

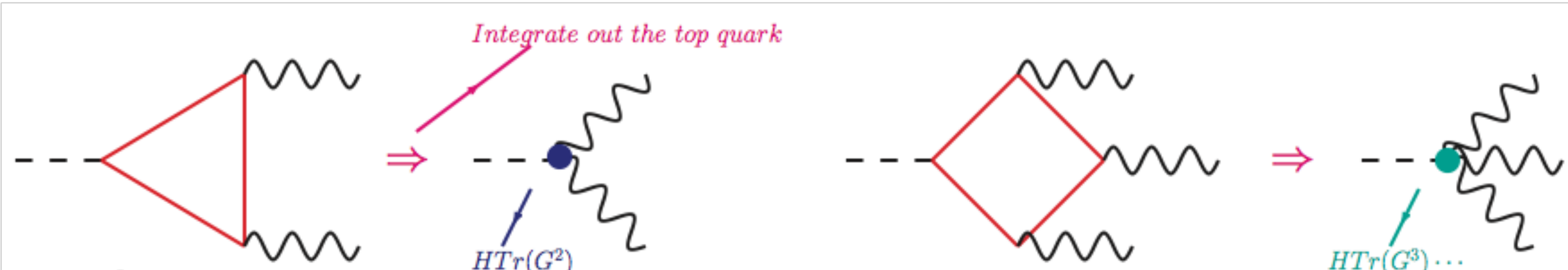


$gg \to H$



$gg \to H + jet$

Higgs effective field theory (HEFT)



- When the transverse momentum of Higgs particle is less than the top quark mass, the HEFT [*Wilczek 1977*] is good approximation.

$$\mathcal{L}_{\rm eff} = \hat{C}_0 O_0 + \frac{1}{m_t^2}\sum_{i=1}^{4}\hat{C}_i O_i + \mathcal{O}\left(\frac{1}{m_t^4}\right)$$

$$O_0 = H\mathrm{Tr}(G^2)$$

$$O_1 = H\mathrm{Tr}(G_\mu{}^\nu G_\nu{}^\rho G_\rho{}^\mu)\,,\quad O_2 = H\mathrm{Tr}(D_\rho G_{\mu\nu} D^\rho G^{\mu\nu})\,,$$

$$O_3 = H\mathrm{Tr}(D^\rho G_{\rho\mu} D_\sigma G^{\sigma\mu})\,, O_4 = H\mathrm{Tr}(G_{\mu\rho} D^\rho D_\sigma G^{\sigma\mu})\,.$$

- The leading (dimension 5) contribution of Higgs to 3 parton amplitude was computed in [*Gehrmann, Jaquier, Glover, Koukoutsakis 2011*].

- The contributions of dimension 7 operators. [QJ, Yang 2018] and [QJ, Yang 2019].

The maximally transcendental principle

$$\mathcal{N}=4\ SYM \stackrel{?}{=} QCD$$

The maximally transcendental principle: the maximal transcendental part of N=4 SYM and QCD amplitudes are the same.

- Classify the amplitude by the transcendental degree.

  Degree 0: $2-\sqrt{3},\ \frac{uv}{w^2}$

  Degree 1: $\pi\ ,log(\frac{u}{v})$

  Degree k: $\zeta_k,\ Li_k(u)$

The maximally transcendental principle

$$\mathcal{N}=4\ SYM \overset{?}{=} QCD$$

- Anomalous dimensions of twist-two operators [*Kotikov, Lipatov, Onishchenko, Velizhanin 2004*]
- The dimension 5 contribution of Higgs to 3 gluon ($H \to 3g$) amplitude in HEFT [*Brandhuber, Travaglini, Yang 2012*].
- The dimension 7 contribution of $H \to 3g$ amplitude in HEFT [QJ, Yang 2018].
- Higgs to 2 quark 1 gluon ($H \to q\bar{q}g$)amplitudes [QJ, Yang 2019].

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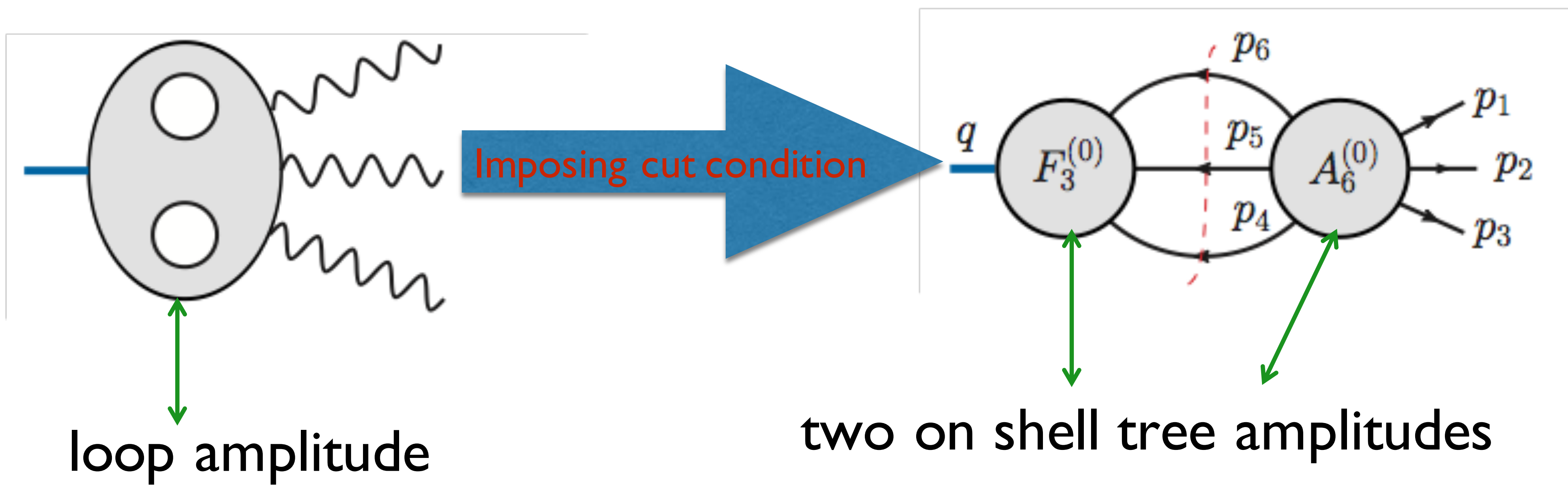
Feynman diagram and unitarity cut



- The $H \to 3g$ amplitudes are computed indendently using unitarity cut method and Feynman diagram approach, and the results are consistent.
- The $H \to q\bar{q}g$ amplitudes are computed in Feynman diagram approach.

Integration by parts (IBP) reduction

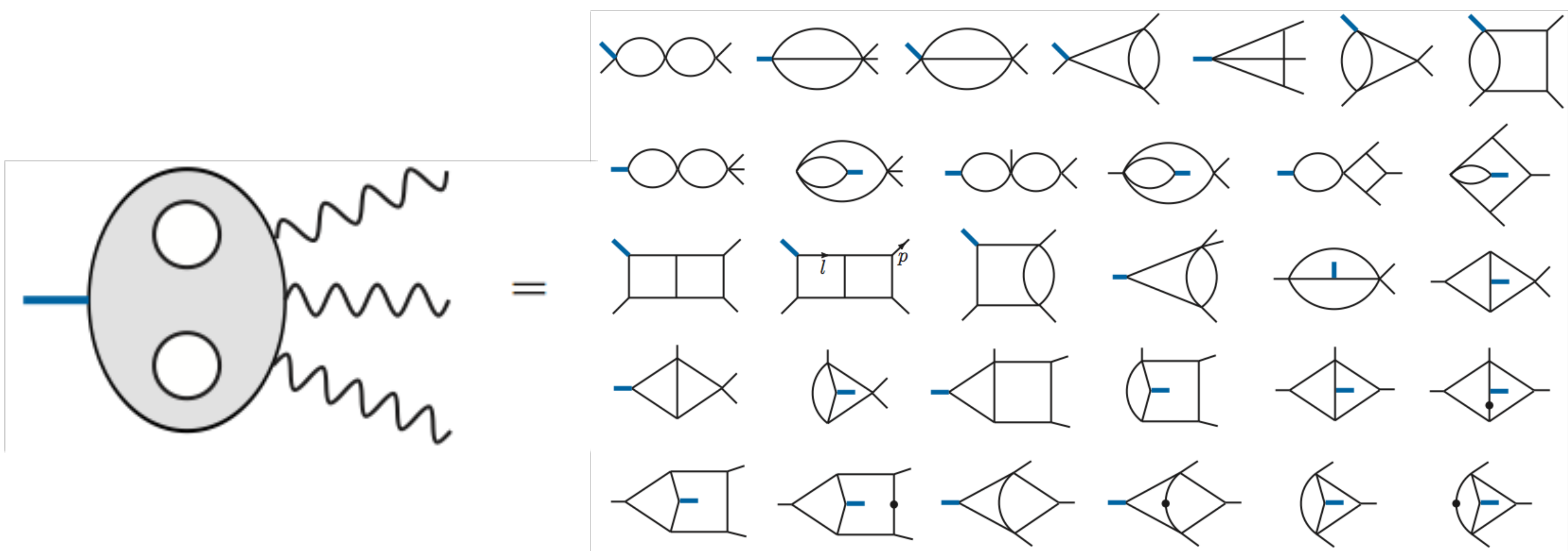
- Loop integrand can be reduced using integration by parts (IBP) relations. The complicated integrand is reduced to only a few master integrals.

$$A = \sum_i c_i M_i$$

Coefficients which does not depend on loop momenta (c_i)

master integrals (M_i)

- 31 master integrals for Higgs to 3 gluon 2 loop amplitude:



Simplify the multi-polylog functions using Symbol

$$
\begin{aligned}
A_\alpha^{(2)} = \Big[&\frac12\Big(-G(1-z,-z,1-z,0,y) - G(1-z,-z,0,1-z,y) + G(1-z,1-z,0,0,y)\\
&+ G(1-z,0,-z,1-z,y) + G(1-z,0,1-z,0,y) - G(1-z,0,1,0,y)\\
&+ G(1-z,0,0,1-z,y) + H(1,1,0,0,z) + H(1,0,1,0,z) + H(1,0,0,1,z)\\
&+ H(1,0,0,z)G(1-z,y) - H(1,0,z)G(-z,0,y) + H(1,0,z)G(1-z,-z,y)\\
&- G(1,0,y)H(1,0,z) + H(1,z)G(1-z,-z,0,y) + H(1,z)G(1-z,0,-z,y)\\
&- H(1,z)G(1-z,0,0,y) - G(0,-z,1-z,y) + G(0,-z,y)H(1,0,z)\\
&- G(0,1-z,-z,1-z,y) + G(0,1-z,1-z,0,y) + G(0,1-z,0,1-z,y)\\
&+ G(0,1-z,-z,y)H(1,z) - G(0,-z,1-z,y) + G(0,1-z,1-z,y)H(0,z)\\
&- G(0,1-z,1,0,y) - G(0,1-z,0,y)H(1,z) - G(0,1-z,0,y)H(0,z)\\
&- H(0,z)G(1-z,0,1-z,y) - H(0,z)G(1-z,0,0,y) - G(0,y)H(1,1,0,z)\\
&- G(0,y)H(1,0,1,z) + G(0,y)H(1,0,0,z) - G(0,1,1-z,0,y) + H(0,1,1,0,z)\\
&- H(0,1,1,z)G(0,y) - G(0,1,0,1-z,y) + H(0,1,0,1,z)\\
&+ H(0,1,0,z)G(1-z,y) + H(0,1,0,z)G(0,y) + G(0,1,0,y)H(1,z)\\
&- G(0,1,0,y)H(0,z) - H(0,1,z)G(-z,0,y) - H(0,1,z)G(1,0,y)\\
&+ H(0,1,z)G(0,-z,y) + G(0,0,1-z,1-z,y) - G(0,0,1-z,y)H(1,z)\\
&- G(0,0,1-z,y)H(0,z) + H(0,0,1,1,z) + H(0,0,1,z)G(0,y)\\
&+ H(0,0,z)G(1-z,1-z,y) + H(0,0,z)G(1-z,0,y) + H(0,0,z)G(0,1-z,y)\\
&+ G(0,0,y)H(1,1,z) - G(0,0,y)H(1,0,z) - G(0,0,y)H(0,1,z)\\
&+ G(0,0,y)H(0,0,z) + H(0,z)G(-z,1-z,0,y) + H(0,z)G(-z,0,1-z,y)\\
&- H(0,z)G(1-z,-z,1-z,y) - H(0,z)G(1-z,1-z,0,y)\\
&- H(0,z)G(1-z,1,0,y) + H(0,z)G(1,1-z,0,y) + H(0,z)G(1,0,1-z,y)\Big)\\
&+\Big(-G(-z,1-z,1-z,0,y) - G(-z,1-z,0,1-z,y)\\
&- G(-z,0,1-z,1-z,y) - G(1-z,1-z,-z,1-z,y) + G(1-z,1-z,1,0,y)\\
&+ G(1-z,1,1-z,0,y) + G(1-z,1,0,1-z,y) - G(1-z,1,0,0,y)\\
&- G(1-z,0,-z,1-z,y) - G(1,1-z,0,0,y) - H(1,1,0,z)G(-z,y)\\
&- H(1,1,z)G(-z,0,y) - G(1,0,1-z,0,y) + H(1,0,1,z)G(-z,y)\\
&- G(1,0,0,1-z,y) + G(1,0,0,y)H(1,z) - H(1,0,z)G(-z,1-z,y)\\
&- H(1,0,z)G(1-z,1-z,y) + H(1,0,z)G(1-z,0,y) + H(1,z)G(-z,1-z,0,y)\\
&+ H(1,z)G(-z,0,1-z,y) + H(1,z)G(1-z,1-z,-z,y)\\
&- H(1,z)G(1-z,1,0,y) - G(0,-z,1-z,1-z,y) + G(0,-z,1-z,0,y)\\
&+ G(0,-z,0,1-z,y) + G(0,-z,1-z,y) - G(0,-z,y)H(1,1,z)\\
&+ G(0,-z,1-z,y)H(0,z) - G(0,-z,0,y)H(1,z) + G(0,-z,1-z,y)H(1,z)\\
&+ H(0,1,1,z)G(-z,y) - H(0,1,z)G(-z,1-z,y) + H(0,1,z)G(1-z,1-z,y)\\
&+ H(0,1,z)G(1-z,0,y) + G(0,0,-z,1-z,y) - G(0,0,-z,y)H(1,z)\\
&- G(0,0,1,0,y) - H(0,z)G(-z,1-z,1-z,y) + H(0,z)G(1-z,0,0,y)\\
&- H(0,z)G(1,0,0,y)\Big)\\
&+\frac32\Big(-G(0,-z,1-z,y)H(0,z) - H(0,1,z)G(1-z,-z,y)\\
&- H(0,0,1,z)G(1-z,y)\Big)\\
&+2\Big(-G(-z,-z,-z,1-z,y) + G(-z,-z,1-z,1-z,y)\\
&+ G(-z,1-z,-z,1-z,y) + G(1-z,-z,-z,1-z,y) - G(1,1,1,0,y)\\
&+ G(1,1,0,0,y) + H(1,1,0,z)G(1-z,y) + H(1,1,z)G(-z,-z,y) + G(1,0,1,0,y)\\
&+ H(1,z)G(-z,-z,-z,y) - H(1,z)G(-z,-z,1-z,y)\\
&- H(1,z)G(-z,1-z,-z,y) - H(1,z)G(1-z,-z,-z,y) + G(0,1,1,0,y)\\
&+ H(0,1,z)G(-z,-z,y) + H(0,0,1,z)G(-z,y)\Big)\\
&+\frac{11}{24}\Big(-G(0,1-z,y)H(0,z) + H(0,1,z)G(0,y) - H(0,z)G(1-z,0,y)\\
&+ G(0,y)H(1,0,z)\Big)\\
&+\frac{11}{6}\Big(-G(-z,-z,1-z,y) + G(1-z,1-z,0,y) + G(1-z,0,1-z,y)\\
&+ G(1-z,0,0,y) - G(1,1,0,y) + H(1,0,0,z) - H(1,0,z)G(1-z,y)\\
&+ H(1,z)G(-z,-z,y) - H(1,z)G(1-z,0,y) + G(0,1-z,1-z,y)\\
&- G(0,1-z,y)H(1,z) + G(0,1-z,0,y) - H(0,1,1,z) + H(0,1,z)G(-z,y)\\
&+ G(0,0,1-z,y) + H(0,0,z)G(1-z,y) + H(0,0,z)G(0,y) - G(0,0,y)H(1,z)\\
&+ G(0,0,y)H(0,z) + H(0,z)G(1-z,1-z,y) + G(0,y)H(1,1,z)\Big)\\
&+\frac{11}{4}\Big(-H(1,0,1,z) + H(0,1,z)G(1-z,y) + H(0,1,0,z)\Big)\\
&+\frac{11}{3}\Big(-G(1,0,0,y) - G(-z,1-z,1-z,y) - H(1,1,z)G(-z,y)\\
&+ H(1,z)G(-z,1-z,y)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{55}{12}\Big(-G(1-z,-z,1-z,y) + H(1,z)G(1-z,-z,y) - G(0,1,0,y)\Big)\\
&+\frac{\pi^2}{3}\Big(G(1,1,y) - G(0,1,y) + H(1,1,z) - G(1,0,y) + H(0,0,z) + G(0,0,y)\Big)\\
&+\Big(\frac{\zeta_3}{4} - \frac{33\pi^2}{8}\Big)\Big(-G(1-z,y) + H(1,z) - H(0,z) - G(0,y)\Big)\\
&+\Big(-\frac{121}{48} + \frac{\pi^2}{3}\Big)\Big(-G(1-z,1-z,y) - H(1,1,z) + H(1,z)G(1-z,y)\\
&- H(0,0,z) - G(0,0,y)\Big)\\
&+\Big(\frac{49}{48} - \frac{\pi^2}{8}\Big)\Big(-G(1-z,0,y) - G(0,1-z,y) - H(0,z)G(1-z,y)\\
&+ G(0,y)H(1,z) - G(0,y)H(0,z)\Big)\\
&+\Big(\frac{67}{18} + \frac{\pi^2}{12}\Big)\Big(G(-z,1-z,y) + G(1,0,y) - H(1,z)G(-z,y)\Big)\\
&-\Big(\frac{245}{144} - \frac{\pi^2}{8}\Big)H(1,0,z) - \Big(\frac{389}{144} - \frac{\pi^2}{8}\Big)H(0,1,z)\\
&-\Big(\frac{13}{8} + \frac{451\pi^2}{96}\Big)G(1-z,y) + \Big(\frac{13}{8} + \frac{1265\pi^2}{288}\Big)H(1,z)\\
&+\Big(\frac{13}{8} + \frac{1133\pi^2}{288}\Big)\Big(-H(0,z) - G(0,y)\Big) + \frac{11\pi^2 G(1,y)}{36}\\
&-\frac{1}{36}\Big(\frac{5029\pi^2}{24} - 72\zeta_4 + \frac{99\zeta_3}{4} + \frac{3\pi^4}{16} - \frac{1321}{6}\Big)\Big]\\
&+\frac16\Big((y+z)(1-y) - z^2\Big)\Big[+G(1,0,y) + G(-z,1-z,y) - H(1,z)G(-z,y)\\
&+ G(0,y)H(1,z) - G(0,y)H(0,z)\\
&+\frac12\Big(-G(1-z,0,y) - H(1,0,z) - G(0,1-z,y) - H(0,1,z)\\
&- H(0,z)G(1-z,y) - G(0,y)H(1,z) + G(0,y)H(0,z)\Big)\\
&-\frac{41}{12}\Big(G(1-z,y) - H(1,z)\Big) + \frac{19}{12}H(0,z) - \frac{3\pi^2}{2} + \frac{247}{18}\Big]\\
&+\Big(\frac{25z}{12}\big(-1+\frac{1}{1-y-z}\big) - \frac{15z^2}{4(1-y-z)} - \frac{yz}{6} + \frac{5z^2}{6}\big(1+\frac{2z}{1-y-z}\\
&+\frac{1}{(1-y-z)^2}(1-2z+z^2)\big)\Big)\Big[G(0,y)H(0,z) + H(1,0,z) - G(1,0,y)\Big]\\
&+\Big(\frac{25y}{12z} - \frac{9y}{4} + \frac{yz}{6} - \frac{15y^2}{4z} + y^2 + \frac{5y^2}{6}\big(\frac{2y}{z} + \frac{1}{z^2}(1-2y+y^2)\big)\Big)\times\\
&\Big[G(1-z,0,y) - G(1,0,y) - G(-z,1-z,y) + H(1,z)G(-z,y) + G(0,1-z,y)\\
&+ H(0,1,z) - G(0,y)H(1,z)\Big]\\
&+\Big(\frac{25z}{12y} - \frac{9z}{4} + \frac{yz}{6} - \frac{15z^2}{4y} + z^2 + \frac{5z^2}{6}\big(\frac{2z}{y} + \frac{1}{y^2}(1-2z+z^2)\big)\Big)\times\\
&\Big[H(0,z)G(1-z,y) - G(-z,1-z,y) + H(1,z)G(-z,y)\Big]\\
&+\frac{1}{36}\Big(63 - 93(y+z) + 4yz + \frac{30z}{y}(1-2z+z^2) + \frac{30y}{z}(1-2y+y^2)\\
&+ 30(y^2+z^2)\Big)\Big[G(1-z,y) - H(1,z)\Big]\\
&-\frac{1}{36}\Big(-63z + 60z^2 - 30z^2(1-z)\big(\frac1y + \frac{1}{1-y-z}\big) + 26y(1-y-z)\Big)H(0,z)\\
&-\frac{1}{36}\Big(\frac{93z(1-z)}{2} - \frac{145y}{2} + \frac{27yz}{2} + \frac{79y^2}{2} + \frac{30z}{1-y-z}(-1+2z-z^2)\\
&-\frac{30y^2(1-y)}{z}\Big)G(0,y)\\
&+\frac{\pi^2}{2}\Big(\frac{25z}{36(1-y-z)} - \frac{2z}{9} + \frac{5z^2}{18(1-y-z)}\big(2z - \frac92 + \frac{1}{1-y-z}(1-2z+z^2)\big)\\
&-\frac{7z^2}{36} - \frac{19yz}{36} + \frac{17y(1-y)}{36}\Big)\\
&+i\pi\Big[\frac{55}{24}\Big(-H(0,z)G(1-z,y) - H(0,z)G(0,y) - H(0,1,z)\\
&-2H(1,z)G(-z,y) + H(1,z)G(0,y) - H(1,0,z) - G(1-z,0,y)\\
&+2G(-z,1-z,y) - G(0,1-z,y) + 2G(1,0,y) + \frac{11}{6}(-H(0,z) + H(1,z)\\
&- G(1-z,y) - G(0,y)) + \frac13(y(1-y-z) + z(1-z))\Big) - \frac{77\pi^2}{288} + \frac{3\zeta_3}{4} + \frac{185}{24}\Big]
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)}_{\mathrm{L2};4} =& -2\left[J_4\left(-\frac{uv}{w}\right) + J_4\left(-\frac{vw}{u}\right) + J_4\left(-\frac{wu}{v}\right)\right]\\
&- 8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1-\frac{1}{u_i}\right) + \frac{\log^4 u_i}{4!}\right] - 2\left[\sum_{i=1}^{3}\mathrm{Li}_2\left(1-\frac{1}{u_i}\right)\right]^2\\
&+ \frac12\left[\sum_{i=1}^{3}\log^2 u_i\right]^2 + 2(J_2^2 - \zeta_2 J_2) - \frac{\log^4(uvw)}{4!}\\
&- \zeta_3\log(uvw) - \frac{123}{8}\zeta_4, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
J_4(x) =& \mathrm{Li}_4(x) - \log(-x)\mathrm{Li}_3(x) + \frac{\log^2(-x)}{2!}\mathrm{Li}_2(x)\\
&- \frac{\log^3(-x)}{3!}\mathrm{Li}_1(x) - \frac{\log^4(-x)}{48}, \qquad (15)
\end{aligned}
$$

$$
J_2 = \sum_{i=1}^{3}\left(\mathrm{Li}_2(1-u_i) + \frac12\log(u_i)\log(u_{i+1})\right), \qquad (16)
$$

$$
\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \to 3g
$$

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The dimension 7 contribution of the $H \to 3g$ amplitude

$$A_{QCD} = \sum_{i=0}^{4} A_{QCD;i}$$

$$\begin{aligned} A_{QCD;4} = & -\frac{3}{2}\mathrm{Li}_4(u) + \frac{3}{4}\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\mathrm{Li}_3\left(-\frac{u}{v}\right) \\ & + \frac{\log^2(u)}{32}\left[\log^2(u) + \log^2(v) + \log^2(w) - 4\log(v)\log(w)\right] \\ & + \frac{\zeta_2}{8}\left[5\log^2(u) - 2\log(v)\log(w)\right] - \frac{1}{4}\zeta_4 - \frac{1}{2}\zeta_3\log(-q^2) + \mathrm{perms}(u,v,w)\,. \end{aligned}$$

$$u = \frac{s_{12}}{q^2}\,, \quad v = \frac{s_{23}}{q^2}\,, \quad w = \frac{s_{13}}{q^2}\,, \qquad q^2 = s_{123}\,.$$

$$A_{QCD;4} = A_{\mathcal{N}=4;4}$$

A new example of maximally transcendental principle [QJ, Yang 2018]

$$\mathcal{N} = 4\ SYM \overset{\checkmark}{=} QCD$$

The $H \to q\bar{q}g$ amplitude

- The leading operator $O_0 = \mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$
- The maximal transcdental part can be decomposed to 3 pieces with different color factors:

$$\mathcal{R}^{(2)}_{\mathcal{O};4} = C_A^2\,\mathcal{R}^{(2),C_A^2}_{\mathcal{O};4} + C_A C_F \mathcal{R}^{(2),C_A C_F}_{\mathcal{O};4} + C_F^2 \mathcal{R}^{(2),C_F^2}_{\mathcal{O};4}$$

- C_F and C_A are the quadratic Casimir of SU(N) group in the fundamental and adjoint representation, respectively.

The new maximal transcendentality principle

- The $H \to q\bar{q}g$ amplitude:

$$\mathcal{R}^{(2)}_{\mathcal{O};4} = C_A^2\, \mathcal{R}^{(2),C_A^2}_{\mathcal{O};4} + C_A C_F \mathcal{R}^{(2),C_A C_F}_{\mathcal{O};4} + C_F^2 \mathcal{R}^{(2),C_F^2}_{\mathcal{O};4}$$

- Set $C_F \to C_A$
- We found the $H \to 3g$ amplitude!
- Hidden relation between two amplitudes with different external states.

The other operators in Higgs EFT?

- The subleading operator $O_1 = \mathrm{tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)$
- The correspondence is still satisfied!

$$O_1 \to q\bar{q}g \xrightarrow{C_F \to C_A} O_1 \to 3g$$

- Another operator $O_4 = \mathrm{tr}(F_{\mu\rho} D^\rho D_\sigma F^{\sigma\mu})$

$$O_4 \to q\bar{q}g \xrightarrow{C_F \to C_A} O_1 \to 3g$$

Operators not in Higgs EFT?

- The simplest quark operator $\bar{\psi}\psi$
- The correspondence is again satisfied!

$$\bar{\psi}\psi \to q\bar{q}g \xrightarrow{C_F \to C_A} \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \to 3g$$

- A more complicated operator $F^a_{\mu\nu}(\bar{\psi}\gamma^{\mu\nu}T^a\psi)$

$$F^a_{\mu\nu}(\bar{\psi}\gamma^{\mu\nu}T^a\psi) \to q\bar{q}g \longrightarrow O_4 \to q\bar{q}g$$

Does the spin matter?

- Consider the scalar Yang-Mills theory, which can be obtained by replacing quarks by scalars in QCD.
- The maximal transcendental part is exactly the same in QCD and scalar Yang-Mills theory!

$$\bar{\psi}\psi \to q\bar{q}g \quad \longrightarrow \quad \bar{\phi}\phi \to \phi\bar{\phi}g$$

$$\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \to q\bar{q}g \quad \longrightarrow \quad \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \to \phi\bar{\phi}g$$

The maximal transcendental principle is **insensitive** to the spin!

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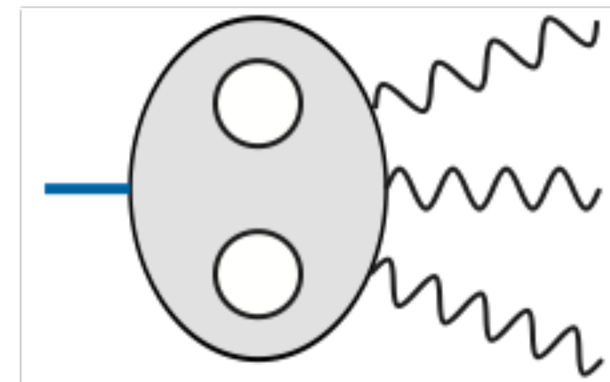
Summary



- Analytical Higgs to 3 parton amplitude at two loops.
- Extend the maximally transcendental principle to $H \to q\bar{q}g$ amplitude.
- Various examples showing the universality of maximally transcendental principle. It is insensitive to the exact form and type of the operators, or the spin of the external particles.

Outlook

- Test the maximal transcendental principle in Higgs to 4 parton amplitude.
- Even more constraints than maximal transcendentality principle. Towards the determination of scattering amplitude by physical constraints.

Thank you!