双圈图和muon的反常磁矩

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1、 muon 的反常磁矩

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$g-2$ of the muon: status report

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$$a_\ell = (g-2)_\ell/2,$$

$$a_\mu^{\mathrm{EXP}} = 1\ 165\ 920\ 89(63) \times 10^{-11}$$

The anomalous magnetic moment receives contributions from all sectors of the SM, and possibly from New Physics (NP): $a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$.

$$a_\mu^{\mathrm{QED}} = 116\ 584\ 718.951\ (0.009)(0.019)(0.007)(.077) \times 10^{-11}$$

where the uncertainties come from the lepton masses, the four- and five-loop contributions and the input value for α obtained from measurements using ^{87}Rb atoms [8], respectively.

$$a_\mu^{\mathrm{EW}} = (153.6 \pm 1.0) \times 10^{-11}\ [17].$$

Their uncertainty is large compared to the uncertainty of a_μ^{QED} but small compared to the hadronic uncertainties.

The story is less straightforward for the hadronic contributions. They are divided into hadronic vacuum polarisation (VP) and so-called hadronic light-by-light (HLbL) scattering contributions. Both classes are dominated by contributions from the low mass spectrum of hadronic resonances and can not be calculated within perturbative QCD (pQCD).

Several groups have made estimates which are, by and large, compatible, and recent estimates mostly used are $a_\mu^{\mathrm{HLbL}} = (105 \pm 26) \times 10^{-11}$ (the 'Glasgow consensus', following a conference on $g-2$ in Glasgow) [19] and $a_\mu^{\mathrm{HLbL}} = (116 \pm 39) \times 10^{-11}$ [20], see also [21].

However, unlike in the HLbL case, there are dispersion relations which allow the direct calculation of a_μ^{HVP} at LO, NLO and NNLO.

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM}$$
$$= 24.8(8.7)(4.8) \times 10^{-10}.$$

The discrepancy $3 - 4\,\sigma$

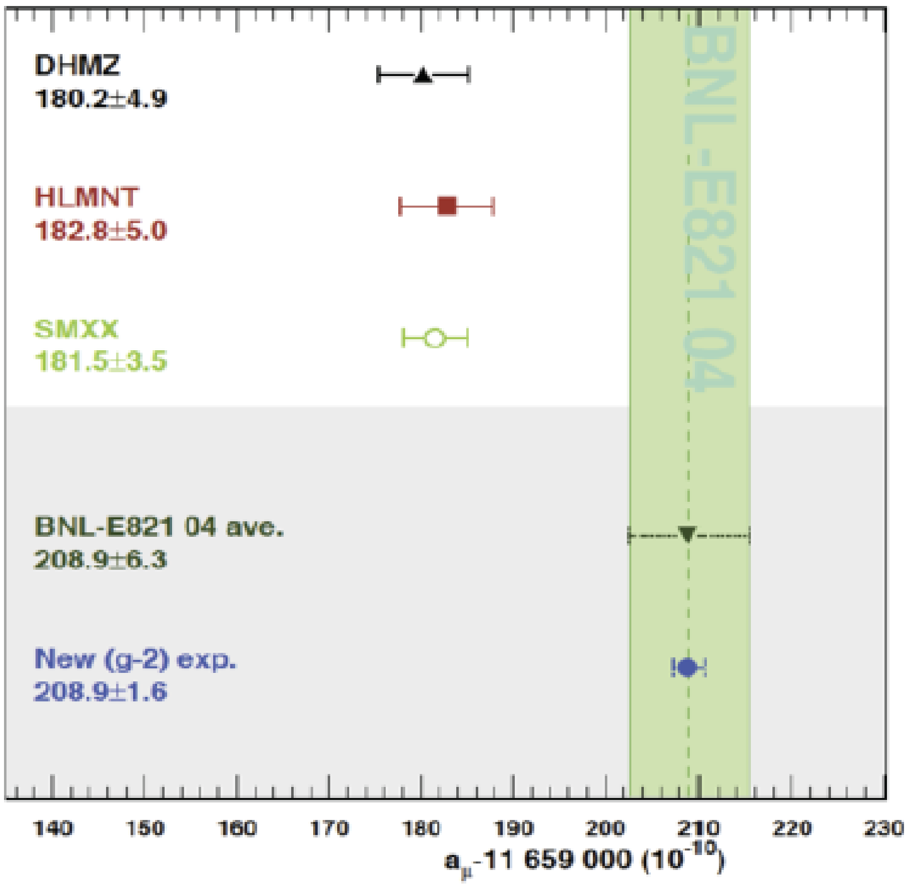


Figure 1: Comparison of recent SM predictions of g–2 with the current experimental average. The marker labelled 'SMXX' indicates an anticipated improvement in the SM prediction, while 'New (g-2) exp.' assumes no change in the mean value but a four-fold improvement in the error of the experimental value, as planned for E989 at Fermilab. See [4] for more details. (Figure from [4].)

2、有效算符

We use the effective Lagrangian method

$$\mathcal{O}_1^{\mp} = \frac{1}{(4\pi)^2}\bar{l}(i\not{\mathcal{D}})^3\omega_{\mp}l, \qquad \mathcal{O}_4^{\mp} = \frac{eQ_f}{(4\pi)^2}\bar{l}(\partial^{\mu}F_{\mu\nu})\gamma^{\nu}\omega_{\mp}l,$$

$$\mathcal{O}_2^{\mp} = \frac{eQ_f}{(4\pi)^2}\overline{(i\mathcal{D}_{\mu}l)}\gamma^{\mu}F\cdot\sigma\omega_{\mp}l, \qquad \mathcal{O}_5^{\mp} = \frac{m_l}{(4\pi)^2}\bar{l}(i\not{\mathcal{D}})^2\omega_{\mp}l,$$

$$\mathcal{O}_3^{\mp} = \frac{eQ_f}{(4\pi)^2}\bar{l}F\cdot\sigma\gamma^{\mu}\omega_{\mp}(i\mathcal{D}_{\mu}l), \qquad \mathcal{O}_6^{\mp} = \frac{eQ_f m_l}{(4\pi)^2}\bar{l}F\cdot\sigma\omega_{\mp}l.$$

with $\mathcal{D}_{\mu} = \partial_{\mu} + ieA_{\mu}$ and $\omega_{\mp} = \frac{1\mp\gamma_5}{2}$.

The lepton MDM is the combination of the Wilson coefficients $C_{2,3,6}^{\mp}$ and can be obtained from the following effective Lagrangian

$$\mathcal{L}_{\mathrm{MDM}} = \frac{e}{4m_l}\, a_l\, \bar{l}\sigma^{\mu\nu}l\, F_{\mu\nu}.$$

MDM是CP守恒的， EDM是CP破坏的

To obtain the non-relativistic limit, look at the wavefunction in the rest frame of the particle, which has a Dirac spinor of the form

$$u(p_0) = \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$\begin{aligned} &\overline{\psi}(x)\sigma^{\mu\nu}F_{\mu\nu}\psi(x) \\ &= \; 2\begin{pmatrix} \xi^\dagger & \xi^\dagger \end{pmatrix}\begin{pmatrix} \mathbf{E}\cdot\sigma - i\mathbf{B}\cdot\sigma & 0 \\ 0 & -\mathbf{E}\cdot\sigma - i\mathbf{B}\cdot\sigma \end{pmatrix}\begin{pmatrix} \xi \\ \xi \end{pmatrix} \\ &= \; -4i\xi^\dagger\mathbf{B}\cdot\sigma\xi \end{aligned}$$

$$\begin{aligned} &\overline{\psi}\gamma_5\sigma^{\mu\nu}F_{\mu\nu}\psi \\ &= \; 2\overline{\psi}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbf{E}\cdot\sigma - i\mathbf{B}\cdot\sigma & 0 \\ 0 & -\mathbf{E}\cdot\sigma - i\mathbf{B}\cdot\sigma \end{pmatrix}\psi \\ &\xrightarrow{\text{NRlimit}} \; -4\xi^\dagger\mathbf{E}\cdot\sigma\xi \end{aligned}$$

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}$$

$$\text{EDM} = \overline{\psi}\gamma_5\sigma^{\mu\nu}F_{\mu\nu}\psi$$

$$\begin{aligned}\mathbf{E} &= -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A}\end{aligned}$$

the spin

$$\sigma \xrightarrow{\text{CP}} \sigma$$

However, the fields transform as

$$\begin{aligned}\mathbf{E} &\xrightarrow{\text{CP}} -\left[(-\nabla)(\phi)\right] - \frac{\partial(-\mathbf{A})}{\partial t} = -\mathbf{E} \\ \mathbf{B} &\xrightarrow{\text{CP}} -\nabla \times (-\mathbf{A}) = \mathbf{B}\end{aligned}$$

The overall effect of CP on the nonrelativistic limits is given by

$$\begin{aligned}\sigma \cdot \mathbf{E} &\xrightarrow{\text{CP}} -\sigma \cdot \mathbf{E} \\ \sigma \cdot \mathbf{B} &\xrightarrow{\text{CP}} \sigma \cdot \mathbf{B}\end{aligned}$$

3、单圈图

Ward identity required by the QED gauge invariance

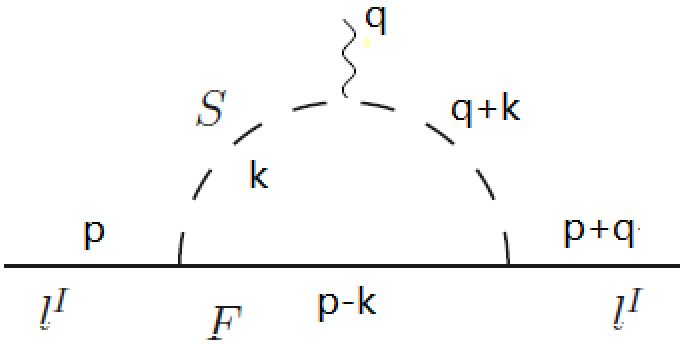
$$k^\rho \mathcal{A}_{\mathrm{ww},\rho}(p,k) = e[\Sigma_{\mathrm{ww}}(p+k) - \Sigma_{\mathrm{ww}}(p)]\ ,$$

计算有效算符之前，先验证三角图的振幅满足word恒等式！

将外线动量视为小量，进行泰勒展开，保留所需要的阶数

$$\frac{1}{(p+k)^2-m^2} = \frac{1}{k^2-m^2}\left(1-\frac{2k.p+p^2}{k^2-m^2}+\frac{4(k.p)^2}{(k^2-m^2)^2}+\ldots\right)$$

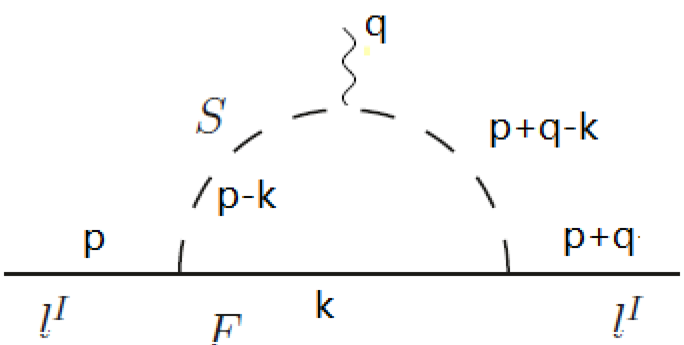
p代表外线动量，k代表内线虚动量



这种标动量的方法，必
须用平移不变性公式！
有许多平移不变性公式

$$\int \frac{d^Dq}{(2\pi)^D}\frac{q^2}{(q^2-m^2)^2}-\frac{D}{2}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-m^2}\equiv 0;$$

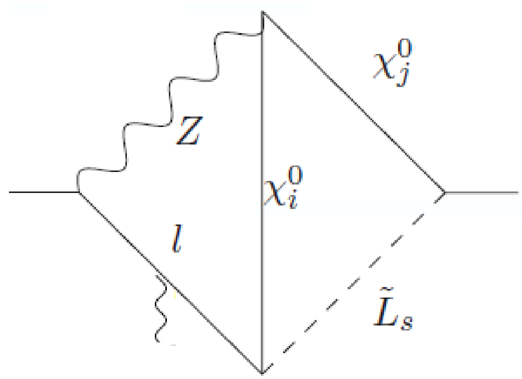
$$\int \frac{d^Dq}{(2\pi)^D}\frac{(q^2)^2}{(q^2-m^2)^2}-\frac{D+2}{2}\int\frac{d^Dq}{(2\pi)^D}\frac{q^2}{q^2-m^2}\equiv 0;$$



这种标动量的方法，不
需要平移不变性公式！
计算简洁很多。

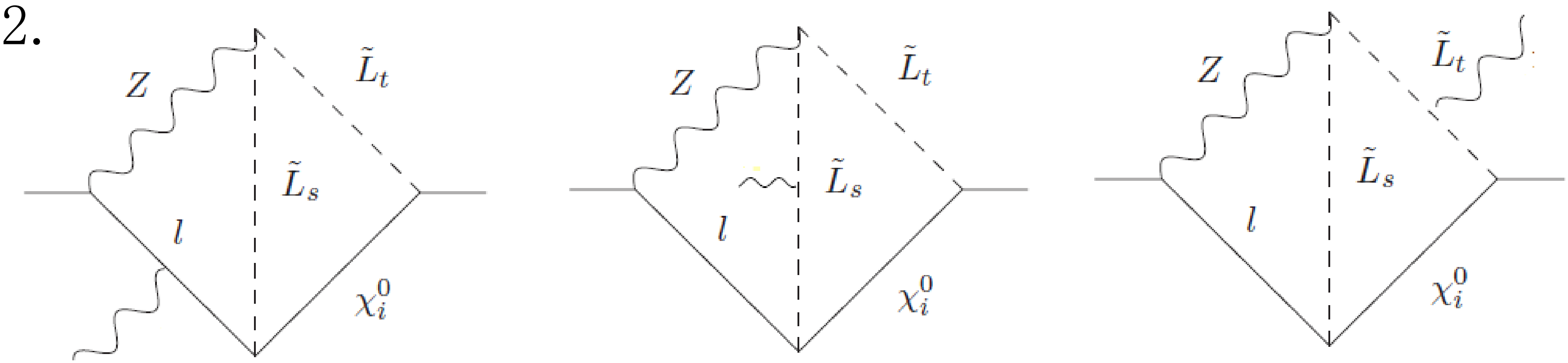
4、双圈图

1.



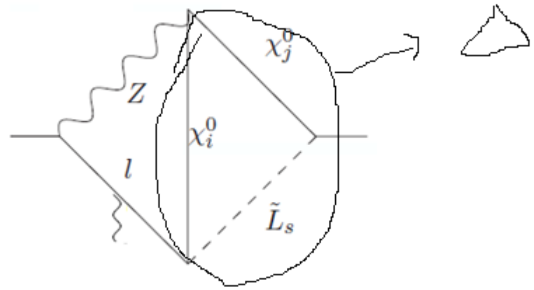
这两组图都不用平移不变性的公式就行

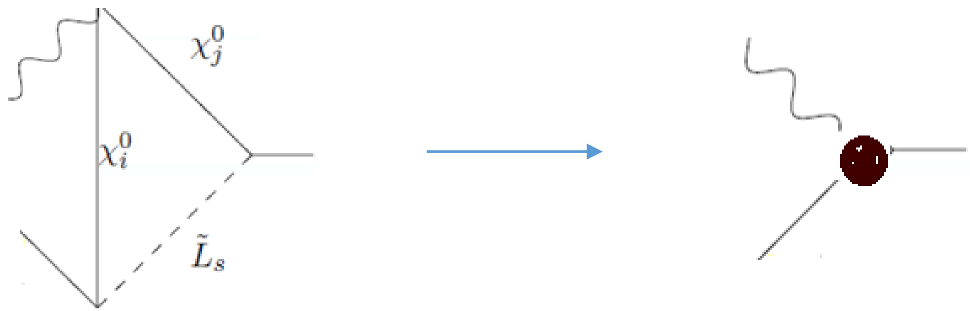
2.

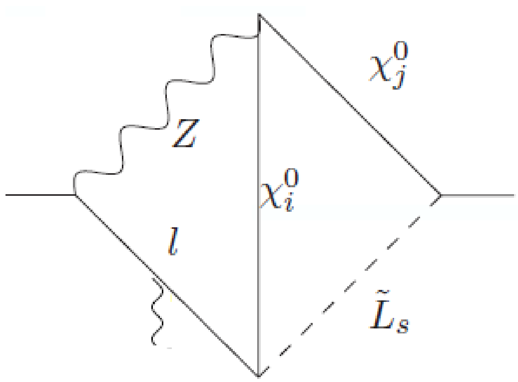


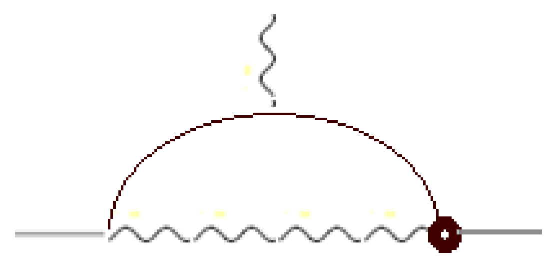
发散　　　　收敛　　　　发散

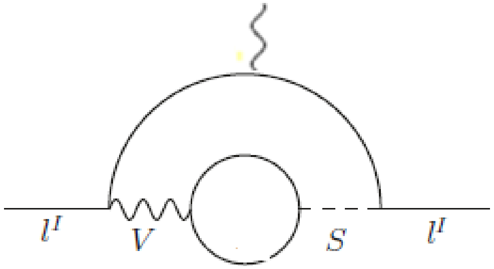
发散源于子图


Z
l
χ_i^0
χ_j^0
$\tilde{L}_s$

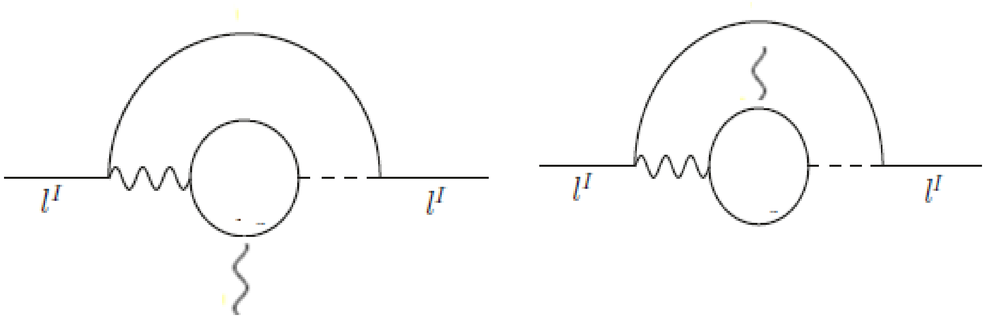

χ_i^0
χ_j^0
$\tilde{L}_s$


Z
l
χ_i^0
χ_j^0
$\tilde{L}_s$

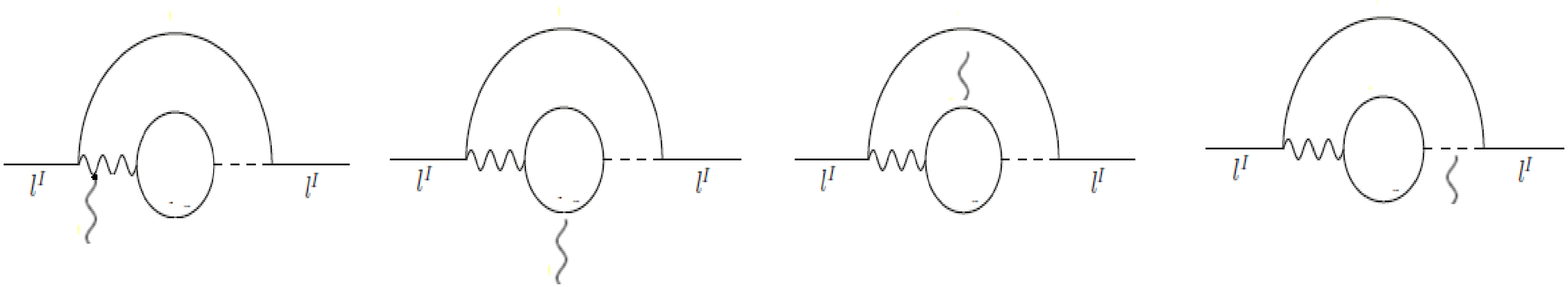




单独满足ward 恒等式，
不用平移不变性公式



V 和 S不带电，两个图之和满足ward 恒等式，必须利用平移不变性公式



V 和 S带电，四个图之和满足ward 恒等式，
必须利用平移不变性公式

Barr-zee 图是收敛的，不需要低消项。

平移不变性公式，有很多。

$$\int\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu}}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} \equiv 0 \ .$$

$$q_1 \to q_1, \ q_2 \to q_2 - a \text{ with } a_\rho \to 0 \ (\rho = 0, 1, \cdots, D),$$

$$\begin{aligned}
&\int\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu}}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} \\
= &\int\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu}}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} \\
&\times \left\{1 + \frac{2q_2 \cdot a}{q_2^2 - m_2^2} + \frac{2(q_2 - q_1) \cdot a}{(q_2 - q_1)^2 - m_0^2} + \cdots \right\} .
\end{aligned}$$

This result implies

$$\begin{aligned}
&\int\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_1 \cdot q_2}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)^2((q_2 - q_1)^2 - m_0^2)} \\
= &\int\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_1^2 - q_1 \cdot q_2}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)^2} \ .
\end{aligned}$$

平移不变性公式 with $\mathcal{D}_0 = ((q_2-q_1)^2-m_0^2)(q_1^2-m_\alpha^2)(q_2^2-m_\alpha^2)$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{\frac{q_1^2q_1\cdot(q_2-q_1)}{(q_2-q_1)^2-m_0^2}+\frac{q_1^2q_1\cdot q_2}{q_2^2-m_\alpha^2}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{\frac{2}{(q_2-q_1)^2-m_0^2}\frac{(q_1\cdot q_2)^2-q_1^2q_2^2}{D(D-1)}+\frac{q_1\cdot q_2}{D}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{-\frac{q_1\cdot(q_2-q_1)}{D}+\frac{2}{q_2^2-m_\alpha^2}\frac{(q_1\cdot q_2)^2-q_1^2q_2^2}{D(D-1)}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{\frac{1}{(q_2-q_1)^2-m_0^2}\Big[\frac{D(q_1\cdot q_2)^2-q_1^2q_2^2}{D(D-1)}-\frac{q_1^2q_1\cdot q_2}{D}\Big]$$
$$+\frac{1}{q_2^2-m_\alpha^2}\frac{D(q_1\cdot q_2)^2-q_1^2q_2^2}{D(D-1)}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{-q_1^2+\frac{2}{D}\frac{q_1^2q_2\cdot(q_2-q_1)}{(q_2-q_1)^2-m_0^2}+\frac{2}{D}\frac{q_1^2q_2^2}{q_2^2-m_\alpha^2}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{-q_1\cdot q_2+\frac{q_2^2q_1\cdot(q_2-q_1)}{(q_2-q_1)^2-m_0^2}+\frac{q_2^2q_1\cdot q_1}{q_2^2-m_\alpha^2}\Big\}\equiv 0\,,$$

$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\Big\{-\frac{2+D}{2}q_2^2+\frac{q_2^2q_2\cdot(q_2-q_1)}{(q_2-q_1)^2-m_0^2}+\frac{q_2^4}{q_2^2-m_\alpha^2}\Big\}\equiv 0\,,$$

积分变量的约化公式，2次和4次

$$\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu} q_{1\nu},\ q_{1\mu} q_{2\nu}}{\mathcal{D}_0} \longrightarrow \frac{g_{\mu\nu}}{D} \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_1^2,\ q_1 \cdot q_2}{\mathcal{D}_0} \,,$$

$$\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu} q_{1\nu} q_{1\rho} q_{1\sigma},\ q_{1\mu} q_{1\nu} q_{1\rho} q_{2\sigma}}{\mathcal{D}_0}$$

$$\longrightarrow \frac{T_{\mu\nu\rho\sigma}}{D(D+2)} \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_1^4,\ q_1^2 (q_1 \cdot q_2)}{\mathcal{D}_0} \,,$$

$$\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_{1\mu} q_{1\nu} q_{2\rho} q_{2\sigma}}{\mathcal{D}_0}$$

$$\longrightarrow \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{\mathcal{D}_0} \Big(\frac{D (q_1 \cdot q_2)^2 - q_1^2 q_2^2}{D(D-1)(D+2)} T_{\mu\nu\rho\sigma} - \frac{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}{D(D-1)} g_{\mu\nu} g_{\rho\sigma} \Big) \,,$$

$$T_{\mu\nu\rho\sigma} = g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\rho\nu}$$

$$\mathcal{D}_0 = ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_a^2)(q_2^2 - m_\alpha^2).$$

积分变量的约化公式，6次

$$\int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{q_{1\mu}q_{1\nu}q_{1\rho}q_{1\sigma}q_{1\alpha}q_{1\beta},\ q_{1\mu}q_{1\nu}q_{1\rho}q_{1\sigma}q_{1\alpha}q_{2\beta}}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)}$$

$$\longrightarrow \frac{S_{\mu\nu\rho\sigma\alpha\beta}}{D(D+2)(D+4)}\int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{(q_1)^3,\ (q_1)^2q_1\cdot q_2}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)},$$

$$\int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{q_{1\mu}q_{1\nu}q_{1\rho}q_{1\sigma}q_{2\alpha}q_{2\beta}}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)}$$

$$\longrightarrow \int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)}$$

$$\times\Big[\frac{Dq_1^2(q_1\cdot q_2)^2-(q_1^2)^2q_2^2}{D(D-1)(D+2)(D+4)}S_{\mu\nu\rho\sigma\alpha\beta}-\frac{q_1^2(q_1\cdot q_2)^2-(q_1^2)^2q_2^2}{D(D-1)(D+2)}T_{\mu\nu\rho\sigma}g_{\alpha\beta}\Big],$$

$$\int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{q_{1\mu}q_{1\nu}q_{1\rho}q_{2\alpha}q_{2\beta}q_{2\delta}}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)}$$

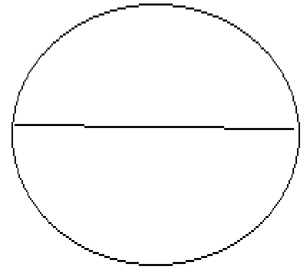
$$\longrightarrow \int \frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{((q_2-q_1)^2-m_0^2)(q_1^2-m_1^2)(q_2^2-m_2^2)}$$

$$\times\Big[\frac{(D+1)q_1^2q_1\cdot q_2q_2^2-2(q_1\cdot q_2)^3}{D(D-1)(D+2)(D+4)}S_{\mu\nu\rho\alpha\beta\delta}+\frac{(q_1\cdot q_2)^3-q_1^2q_1\cdot q_2q_2^2}{D(D-1)(D+2)}\Big(g_{\mu\alpha}(g_{\nu\beta}g_{\rho\delta}$$

$$+g_{\nu\delta}g_{\rho\beta})+g_{\mu\beta}(g_{\nu\alpha}g_{\rho\delta}+g_{\nu\delta}g_{\rho\alpha})+g_{\mu\delta}(g_{\nu\alpha}g_{\rho\beta}+g_{\nu\beta}g_{\rho\alpha})\Big)\Big],$$

$$S_{\mu\nu\rho\sigma\alpha\beta}=g_{\mu\nu}T_{\rho\sigma\alpha\beta}+g_{\mu\rho}T_{\nu\sigma\alpha\beta}+g_{\mu\sigma}T_{\nu\rho\alpha\beta}+g_{\mu\alpha}T_{\nu\rho\sigma\beta}+g_{\mu\beta}T_{\nu\rho\sigma\alpha}$$

双圈真空函数



$$\Lambda_{\rm RE}^{4\epsilon}\int\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{(q_1^2-m_1^2)(q_2^2-m_2^2)((q_2-q_1)^2-m_0^2)}$$

$$=\frac{\Lambda^2}{2(4\pi)^4}\frac{\Gamma^2(1+\epsilon)}{(1-\epsilon)^2}\Big(4\pi x_{_R}\Big)^{2\epsilon}\Big\{-\frac{1}{\epsilon^2}\Big(x_0+x_1+x_2\Big)$$

$$+\frac{1}{\epsilon}\Big(2(x_0\ln x_0+x_1\ln x_1+x_2\ln x_2)-x_0-x_1-x_2\Big)$$

$$-2(x_0+x_1+x_2)+2(x_0\ln x_0+x_1\ln x_1+x_2\ln x_2)$$

$$-x_0\ln^2x_0-x_1\ln^2x_1-x_2\ln^2x_2-\Phi(x_0,x_1,x_2)\Big\}$$

$$\Phi(x,y,z)=(x+y-z)\ln x\ln y+(x-y+z)\ln x\ln z$$

$$+(-x+y+z)\ln y\ln z+{\rm sign}(\lambda^2)\sqrt{|\lambda^2|}\Psi(x,y,z)\;,$$

$$\lambda^2 = x_0^2 + x_1^2 + x_2^2 - 2x_0x_1 - 2x_0x_2 - 2x_1x_2$$

The definition of $\Psi(x, y, z)$ is written as:

- $\lambda^2 > 0,\ \sqrt{y} + \sqrt{z} < \sqrt{x}$:

$$\Psi(x,y,z) = 2\ln\Big(\frac{x+y-z-\lambda}{2x}\Big)\ln\Big(\frac{x-y+z-\lambda}{2x}\Big) - \ln\frac{y}{x}\ln\frac{z}{x}$$
$$-2L_{i_2}\Big(\frac{x+y-z-\lambda}{2x}\Big) - 2L_{i_2}\Big(\frac{x-y+z-\lambda}{2x}\Big) + \frac{\pi^2}{3}\ ,$$

where $L_{i_2}(x)$ is the spence function;

- $\lambda^2 > 0,\ \sqrt{x} + \sqrt{z} < \sqrt{y}$: $\quad \Psi(x, y, z) = \text{Eq.(B1)}(x \leftrightarrow y)$;
- $\lambda^2 > 0,\ \sqrt{x} + \sqrt{y} < \sqrt{z}$: $\quad \Psi(x, y, z) = \text{Eq.(B1)}(x \leftrightarrow z)$;
- $\lambda^2 < 0$:

$$\Psi(x,y,z) = 2\Big\{Cl_2\Big(2\arccos\big(\frac{-x+y+z}{2\sqrt{yz}}\big)\Big) + Cl_2\Big(2\arccos\big(\frac{x-y+z}{2\sqrt{xz}}\big)\Big)$$
$$+Cl_2\Big(2\arccos\big(\frac{x+y-z}{2\sqrt{xy}}\big)\Big)\Big\}\ ,$$

where $Cl_2(x)$ denotes the Clausen function.

利用简化公式

$$\frac{1}{k^2-m_1^2}\frac{1}{k^2-m_2^2}=\frac{1}{m_1^2-m_2^2}\left(\frac{1}{k^2-m_1^2}-\frac{1}{k^2-m_2^2}\right),$$

$$\frac{1}{(k^2-m^2)^n}=\frac{1}{(n-1)!}\left(\frac{\partial}{\partial m^2}\right)^{n-1}\frac{1}{k^2-m^2},$$

例如：

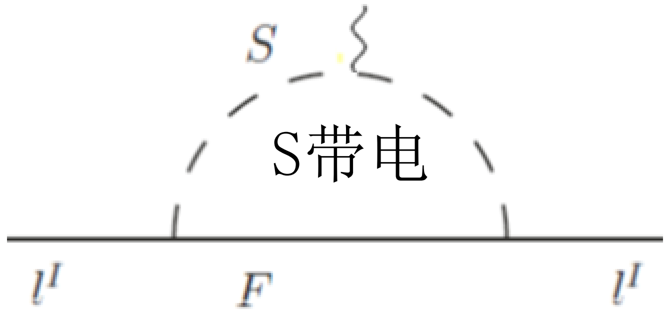
$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\frac{q_1^2}{(q_1^2-m_{\tilde{s}_i}^2)^2}=$$

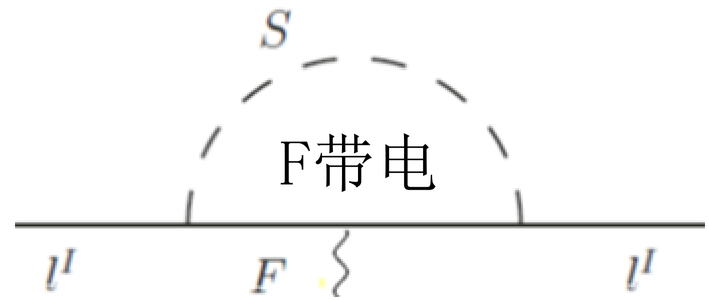
$$\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\frac{1}{(q_1^2-m_{\tilde{s}_i}^2)}+\int\frac{d^Dq_1}{(2\pi)^D}\frac{d^Dq_2}{(2\pi)^D}\frac{1}{\mathcal{D}_0}\frac{m_{\tilde{s}_i}^2}{(q_1^2-m_{\tilde{s}_i}^2)^2}$$

这样都能拆成双圈真空函数与两个单圈函数乘积的线性组合

5. BLMSSM模型中的muon反常磁矩

单圈图
的贡献





$$a_1^{\tilde{L}\chi^0} = -\frac{e^2}{2s_W^2}\sum_{i=1}^{6}\sum_{j=1}^{4}\Big[\mathrm{Re}[(\mathcal{S}_1)_{ij}^I(\mathcal{S}_2)_{ij}^{I*}]\sqrt{x_{\chi_j^0}x_{m_{l^I}}}\,x_{\tilde{L}_i}\frac{\partial^2\mathcal{B}(x_{\chi_j^0},x_{\tilde{L}_i})}{\partial x_{\tilde{L}_i}^2}$$
$$+\frac{1}{3}(|(\mathcal{S}_1)_{ij}^I|^2+|(\mathcal{S}_2)_{ij}^I|^2)x_{\tilde{L}_i}x_{m_{l^I}}\frac{\partial\mathcal{B}_1(x_{\chi_j^0},x_{\tilde{L}_i})}{\partial x_{\tilde{L}_i}}\Big],$$

where the couplings $(\mathcal{S}_1)_{ij}^I$, $(\mathcal{S}_2)_{ij}^I$ are shown as

$$(\mathcal{S}_1)_{ij}^I = \frac{1}{c_W}Z_{\tilde{L}}^{Ii*}(Z_N^{1j}s_W+Z_N^{2j}c_W)-\frac{m_{l^I}}{\cos\beta m_W}Z_{\tilde{L}}^{(I+3)i*}Z_N^{3j},$$
$$(\mathcal{S}_2)_{ij}^I = -2\frac{s_W}{c_W}Z_{\tilde{L}}^{(I+3)i*}Z_N^{1j*}-\frac{m_{l^I}}{\cos\beta m_W}Z_{\tilde{L}}^{Ii*}Z_N^{3j*}.$$

$$a_1^{\tilde{\nu}\chi^\pm} = \sum_{J=1}^{3}\sum_{i,j=1}^{2}\frac{e^2}{s_W^2}\Big[\sqrt{2}\frac{m_{l^I}}{m_W}\mathrm{Re}[Z_+^{1j}Z_-^{2j}]|Z_{\tilde{\nu}^{IJ}}^{1i}|^2\sqrt{x_{\chi_j^\pm}x_{l^I}}\mathcal{B}_1(x_{\tilde{\nu}^{Ji}}, x_{\chi_j^\pm})$$
$$+\frac{1}{3}(|Z_+^{1j}Z_{\tilde{\nu}^{IJ}}^{1i*}|^2 + \frac{m_{l^I}^2}{2m_W^2}|Z_-^{2j*}Z_{\tilde{\nu}^{IJ}}^{1i*}|^2)x_{\chi_j^\pm}x_{l^I}\frac{\partial\mathcal{B}_1(x_{\tilde{\nu}^{Ji}}, x_{\chi_j^\pm})}{\partial x_{\chi_j^\pm}}\Big].$$

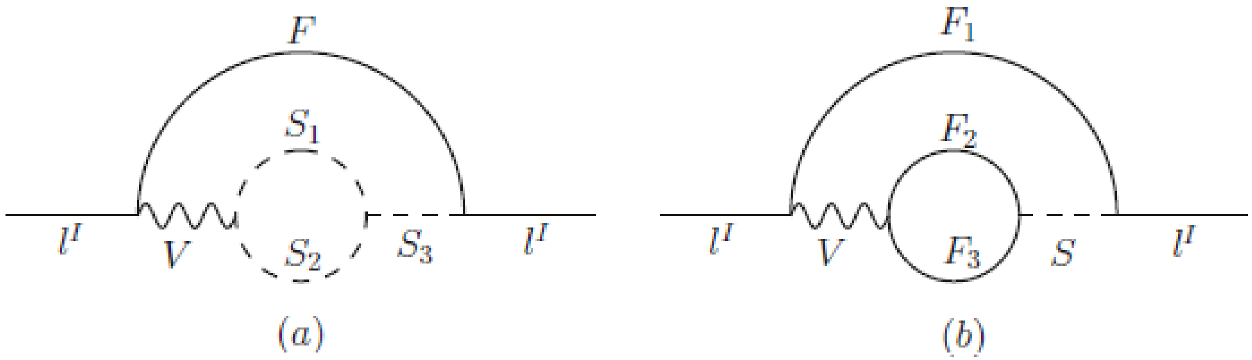
单圈函数

$$\mathcal{B}(x,y) = \frac{1}{16\pi^2}\Big(\frac{x\ln x}{y-x} + \frac{y\ln y}{x-y}\Big),$$

$$\mathcal{B}_1(x,y) = (\frac{\partial}{\partial y} + \frac{y}{2}\frac{\partial^2}{\partial y^2})\mathcal{B}(x,y).$$

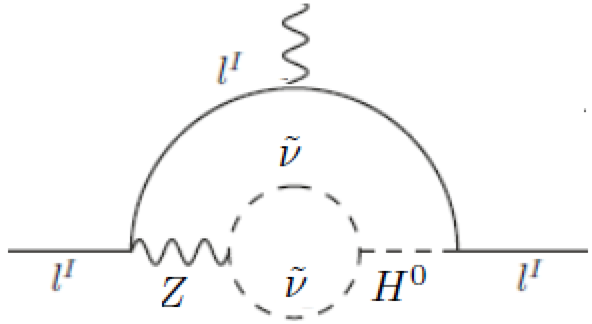
中性Higgs和带电Higgs的贡献都很小，被忽略了

V可以是W， Z 和光子



The two loop Barr-Zee type diagrams with sub Fermion loop and sub scalar loop.

双圈三角图是从自能图的带电粒子内线上拉出一个光子线，
属于一个自能图的所有双圈三角图之和满足ward恒等式。



最简单的一个双圈三角图，
且单独满足ward恒等式

$$a_2^{\tilde{\nu},H^0}(Z) = -\frac{e^3(1-4s_W^2)x_{l^I}}{4s_W^3c_W^2\sqrt{x_W}} \sum_{S=\tilde{\nu},\tilde{N}'_{4,5}} \sum_{i,j,k=1}^{2} \mathrm{Re}[(Z_S^\dagger)^{i1} Z_S^{1j} \frac{H_{H_k^0 S_i S_j}}{M_{NP}} Z_R^{1k}]$$
$$\times\Big(4\mathcal{P}_1 + x_{l^I}\mathcal{P}_2\Big)(x_Z, x_{H_k^0}, x_{l^I}, x_{S_i}, x_{S_j}),$$

$$H_{H_1^0\tilde{\nu}_i\tilde{\nu}_j} = (N_M^u)_{ij}\sin\alpha + (N_M^d)_{ij}\cos\alpha; \qquad H_{H_2^0\tilde{\nu}_i\tilde{\nu}_j} = (N_M^u)_{ij}\cos\alpha - (N_M^d)_{ij}\sin\alpha,$$
$$(N_M^u)_{ij} = V_{EW}\sin\beta\Big(\frac{e^2}{4s_W^2c_W^2}(Z_{\tilde{\nu}}^\dagger)^{i1}Z_{\tilde{\nu}}^{1j} - |Y_\nu|^2\delta_{ij}\Big) + \lambda_{\nu_c}^*\bar{v}_L(Z_{\tilde{\nu}}^\dagger)^{i2}Z_{\tilde{\nu}}^{1j} - \frac{A_N}{\sqrt{2}}(Z_{\tilde{\nu}}^\dagger)^{i2}Z_{\tilde{\nu}}^{1j},$$
$$(N_M^d)_{ij} = -\frac{e^2}{4s_W^2c_W^2}V_{EW}\cos\beta(Z_{\tilde{\nu}}^\dagger)^{i1}Z_{\tilde{\nu}}^{1j} - \frac{\mu^*}{\sqrt{2}}Y_\nu.$$

The concrete forms of the functions $\mathcal{P}_1, \mathcal{P}_2$ are

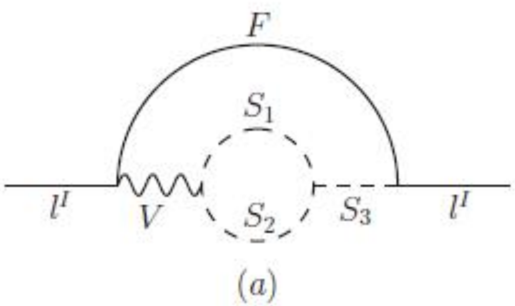
$$\mathcal{P}_1(v,t,f,s,w) = \frac{f(s-w)}{8}\frac{\partial^2}{\partial f^2}\Big(\mathcal{B}(s,w)\mathcal{C}(v,t,f) + \mathcal{F}(v,t,f,s,w) + \frac{\mathcal{C}_1(v,t,f)}{16\pi^2}\Big),$$

$$\mathcal{P}_2(v,t,f,s,w) = -\frac{4}{3}(2 + f\frac{\partial}{\partial f})\frac{1}{f}\mathcal{P}_1(v,t,f,s,w).$$

The functions $\mathcal{C}, \mathcal{C}_1, \mathcal{F}$ are collected in the appendix.

$$\begin{aligned}
\mathcal{C}(x,y,z) &= -\frac{1}{16\pi^2}\Big(\frac{x\log(x)}{(x-y)(x-z)} + \frac{y\log(y)}{(y-x)(y-z)} + \frac{z\log(z)}{(x-z)(y-z)}\Big) \\
\mathcal{C}_1(x,y,z) &= \frac{1}{32\pi^2}\Big(\frac{x\log^2(x)}{(x-y)(x-z)} + \frac{y\log^2(y)}{(x-y)(z-y)} + \frac{z\log^2(z)}{(z-x)(z-y)}\Big) \\
\mathcal{F}(v,f,t,s,w) &= \frac{1}{512\pi^4}\Big(\frac{\mathcal{G}(f,s,w)}{(f-t)(f-v)} + \frac{\mathcal{G}(t,s,w)}{(f-t)(v-t)} + \frac{\mathcal{G}(v,s,w)}{(v-f)(v-t)}\Big) \\
\mathcal{G}(x,y,z) &= -\Phi(x,y,z) - 2(x+y+z) + 2(x\log(x) + y\log(y) + z\log(z)) \\
&\quad -x\log^2(x) - y\log^2(y) - z\log^2(z).
\end{aligned}$$

When the vector is photon, contributions from the figure 2(a) are just produced by the neutral CP even Higgs. That is to say the corresponding CP odd Higgs' contribution is zero.



$$a_2^{S,H^0}(\gamma) = -\frac{8Q_S e^3 x_{l^I}}{s_W\sqrt{x_W}}\sum_{i=1}^{2}\Big(\sum_{S=\tilde{L},\tilde{U},\tilde{D},\tilde{L}',\tilde{\mathcal{U}},\tilde{\mathcal{D}},H^\pm}\Big)\mathrm{Re}[Z_R^{1i}\frac{H_{H_i^0SS}}{M_{NP}}]$$
$$\times\Big((Q_S\mathcal{W}_1+\mathcal{P}_1)+x_{l^I}(Q_S\mathcal{W}_2+\frac{1}{4}\mathcal{P}_2)\Big)(0,x_{l^I},x_{H_i^0},x_S,x_S).$$

For the figure 2(a), with vector Z, both CP even and CP odd Higgs give corrections to the lepton MDM, and their results are obtained here.

$$a_2^{S,H^0}(Z) = \frac{e^3 x_{l^I}}{s_W^3 c_W^2\sqrt{x_W}}\sum_{i=1}^{2}\Big(\sum_{S_1,S_2=\tilde{L},\tilde{U},\tilde{D},\tilde{L}',\tilde{\mathcal{U}},\tilde{\mathcal{D}},H^\pm}\Big)\Big[\mathrm{Re}[\frac{H_{H^0S_1S_2}}{M_{NP}}G_{ZS_1S_2}Z_R^{1i}]$$
$$\times(1-4s_W^2)\Big((Q_S\mathcal{W}_1+\mathcal{P}_1)+x_{l^I}(Q_S\mathcal{W}_2+\frac{\mathcal{P}_2}{4})\Big)(x_Z,x_{l^I},x_{H_i^0},x_{S_1},x_{S_2})$$
$$+\mathrm{Re}[\frac{H_{A^0S_1S_2}}{M_{NP}}G_{ZS_1S_2}Z_H^{1i}]\Big((Q_S\mathcal{W}_1+\mathcal{P}_1)-x_{l^I}(Q_S\mathcal{W}_2+\frac{\mathcal{P}_2}{4})\Big)(x_Z,x_{l^I},x_{A_i^0},x_{S_1},x_{S_2})\Big],$$

The used parameters in BLMSSM are collected here.

$$\begin{aligned}
\tan\beta_B &= \tan\beta_L = 2, \quad B_4 = L_4 = \frac{3}{2},\ \tan\beta = 15, \\
m_{\tilde{Q}_3} &= m_{\tilde{U}_3} = m_{\tilde{D}_3} = 1.4\text{TeV}, \quad m_{Z_B} = m_{Z_L} = 1\text{TeV}, \\
m_{\tilde{U}_4} &= m_{\tilde{D}_4} = m_{\tilde{Q}_5} = m_{\tilde{U}_5} = m_{\tilde{D}_5} = 1\text{TeV}, \ \ m_{\tilde{Q}_4} = 790\text{GeV}, \\
m_{\tilde{L}_4} &= m_{\tilde{\nu}_4} = m_{\tilde{E}_4} = m_{\tilde{L}_5} = m_{\tilde{\nu}_5} = m_{\tilde{E}_5} = 1.4\text{TeV}\ , \\
A_{u_4} &= A_{u_5} = A_{d_4} = A_{d_5} = 550\text{GeV}\ , A_b = A_t = -1\text{TeV}, \\
\upsilon_{B_t} &= \sqrt{\upsilon_B^2 + \overline{\upsilon}_B^2} = 3\text{TeV}\ , \ \ \upsilon_{L_t} = \sqrt{\upsilon_L^2 + \overline{\upsilon}_L^2} = 3\text{TeV}\ , \\
m_1 &= 1\text{TeV},\ m_2 = 750\text{GeV},\ \mu_H = -800\text{GeV}. \\
Y_{u_4} &= 0.8Y_t,\ Y_{d_4} = 0.7Y_b,\ Y_{u_5} = 0.7Y_b,\ Y_{d_5} = 0.1Y_t, \\
A'_e &= A'_\mu = A'_\tau = 130\text{GeV}, \lambda_{\nu^c} = 1,\ A_{\nu_4} = A_{\nu_5} = 550\text{GeV}, \\
A'_u &= A'_c = A'_t = A'_d = A'_s = A'_b = 500\text{GeV}, \\
Y_{\nu_4} &= 0.6, Y_{\nu_5} = 1.1, Y_{e_4} = 1.3, Y_{e_5} = 0.6, \mu_L = 500\text{GeV} \\
A_{\nu_e} &= A_{\nu_\mu} = A_{\nu_\tau} = A_{\nu_e^c} = A_{\nu_\mu^c} = A_{\nu_\tau^c} = -500\text{GeV}.
\end{aligned}$$

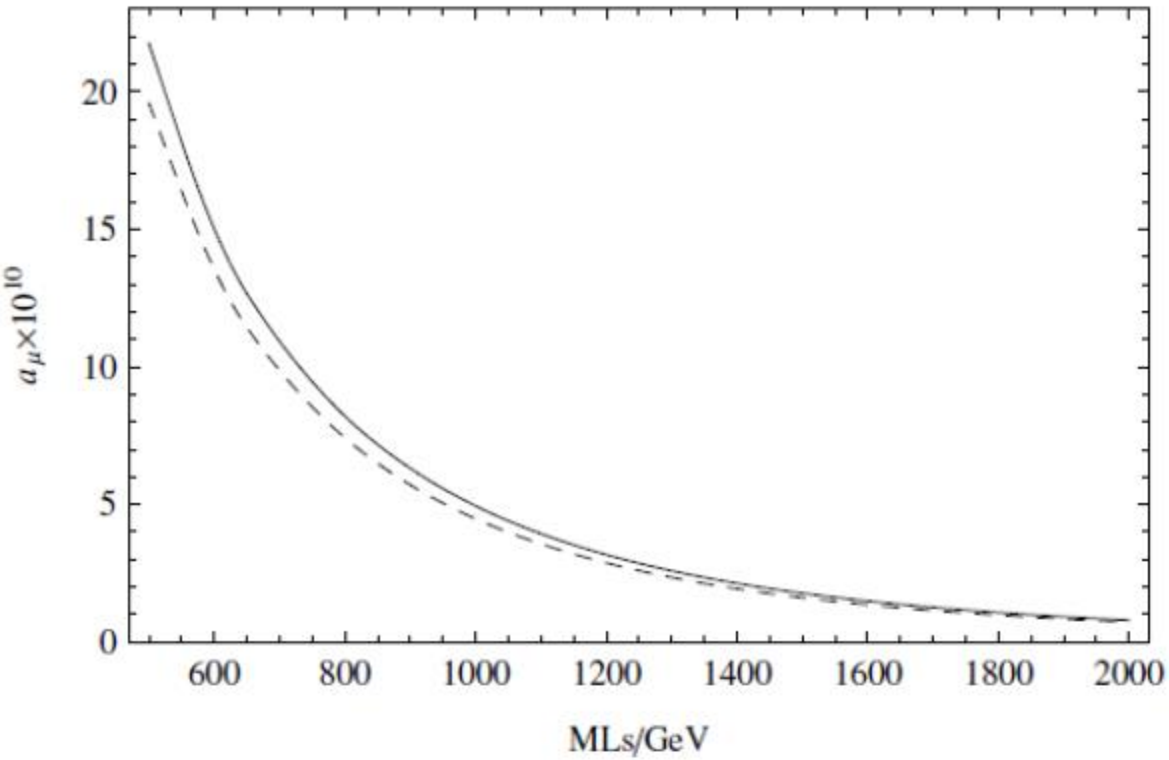
We suppose the following relations in the numerical discussion, then the numerical discussion is simplified.

$$A_e = A_\mu = A_\tau = A_L, \ \ A_c = A_s = A_{cs},$$
$$m_{\tilde{e}} = m_{\tilde{\mu}} = m_{\tilde{\tau}} = m_{\tilde{\nu}_e} = m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_\tau} = ML_s$$
$$A_{e_4} = A_{e_5} = AE45, \ m_{\tilde{Q}_2} = m_{\tilde{U}_2} = m_{\tilde{D}_2} = MQ2;$$

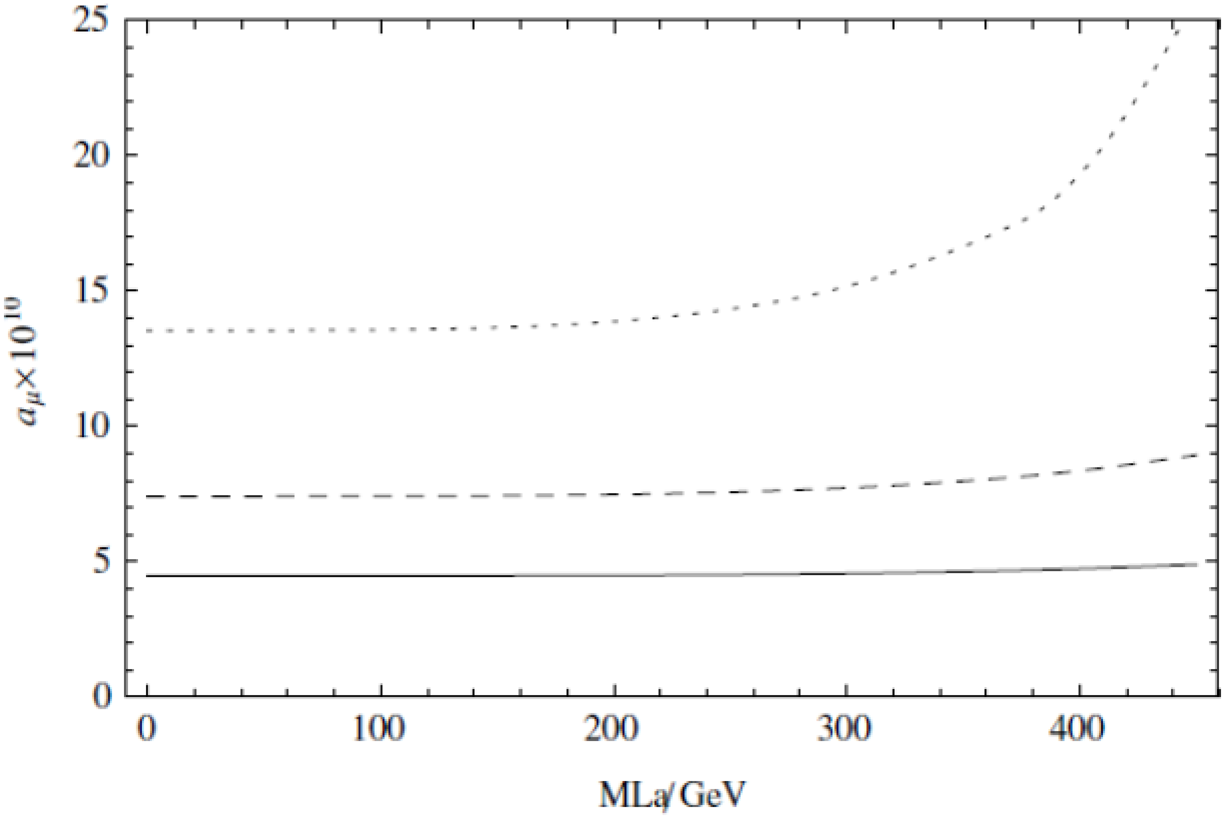
In order to reflect the flavor mixing obviously and simplify the discussion, we define the off-diagonal elements in the following form.

$$(m^2_{\tilde{\nu}^c})_{ij} = (m^2_L)_{ij} = (m^2_R)_{ij} = MLa^2, \ (AL)_{ij} = ALa, \ \texttt{with} \ i \neq j, (i,j = 1,2,3).$$

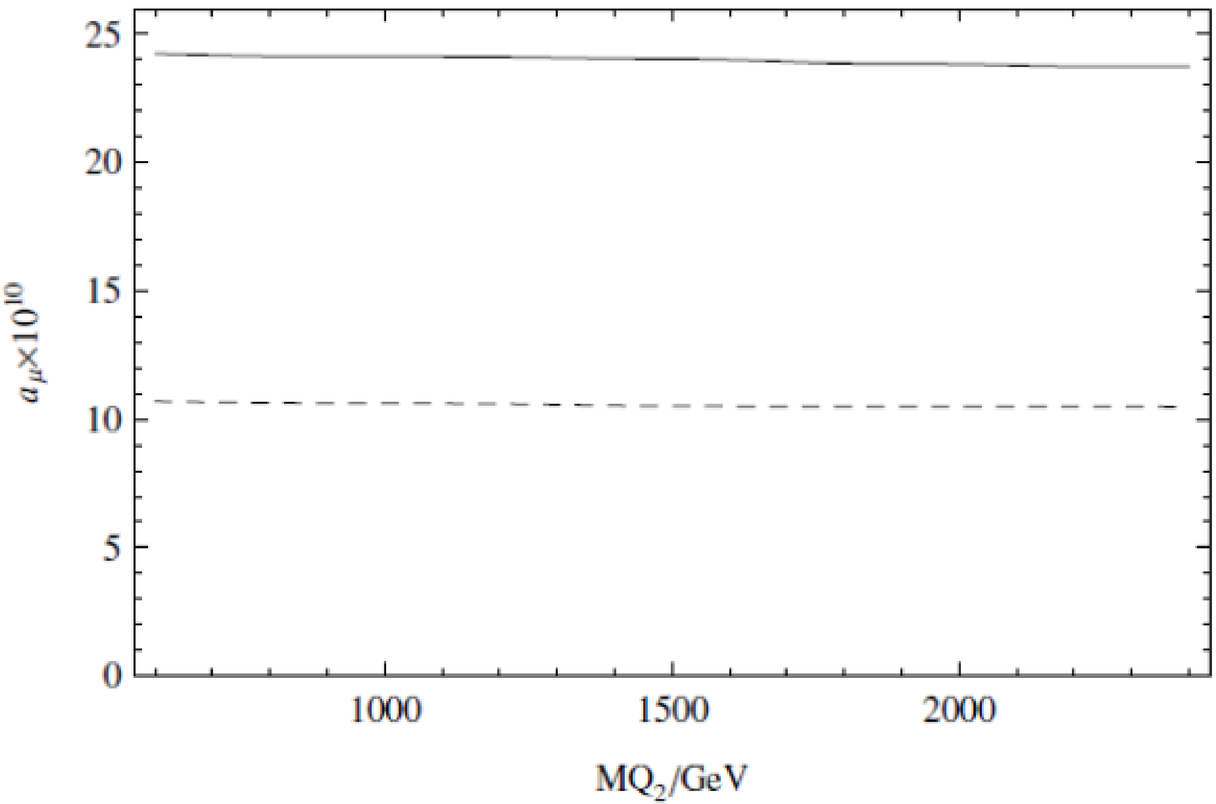
When $MLa = 0, ALa = 0$ there is no flavor mixing for the scalar leptons.



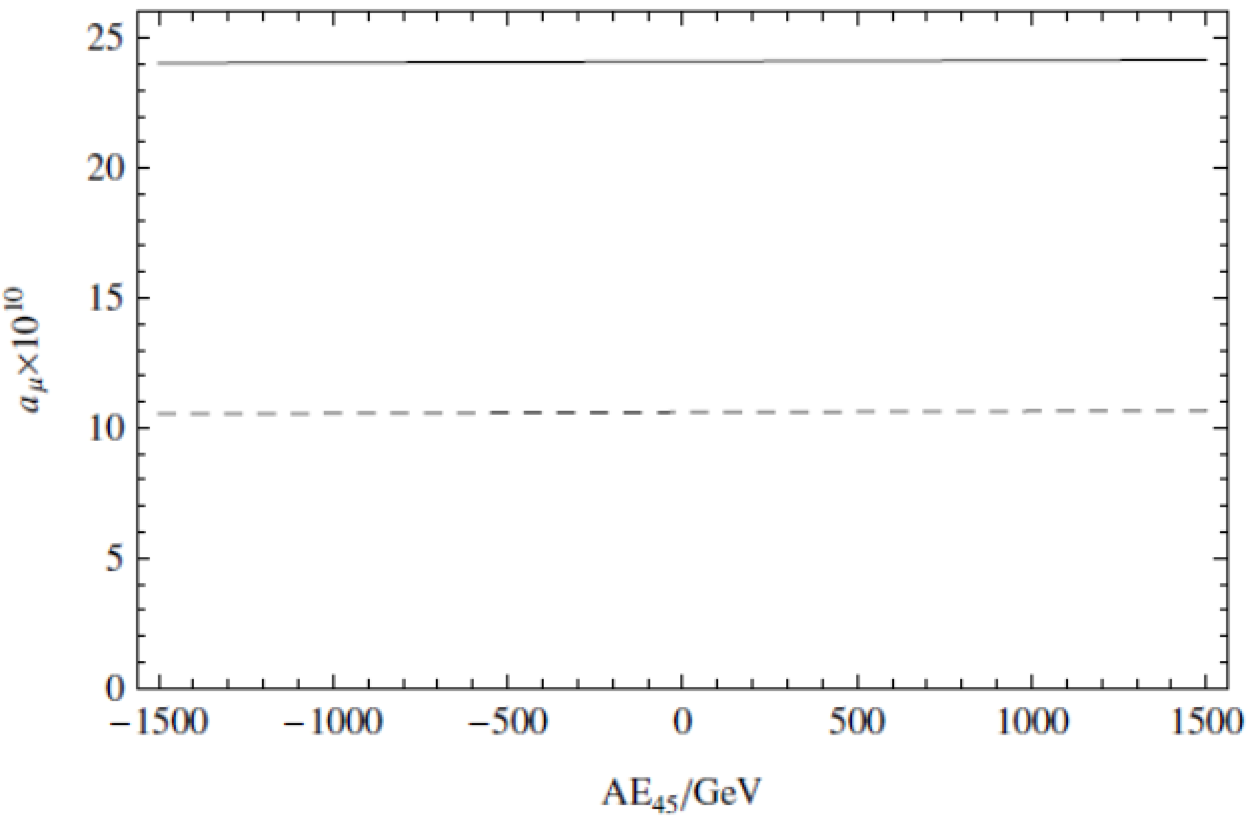
The one loop scalar lepton and neutralino contributions to muon MDM, the dashed-line and solid-line represent muon MDM varying with ML_s, for $A_L = -500\text{GeV}$ and $A_L = -800\text{GeV}$ respectively.



The one loop scalar lepton-neutralino contributions to muon MDM, the dotted-line, dashed-line and solid-line represent muon MDM varying with ML_a, for $MLs = 600\text{GeV}$, $MLs = 800\text{GeV}$ and $MLs = 1000\text{GeV}$ respectively.



As $AE_{45} = 550\text{GeV}$ and $A_{cs} = -500\text{GeV}$, $AL = -800\text{GeV}$, the solid-line and dashed-line represent muon MDM varying with MQ_2, for $ML_s = 500\text{GeV}$ and $ML_s = 800\text{GeV}$ respectively.



As $A_L = -800\text{GeV}, A_{cs} = -500\text{GeV}, MQ_2 = 1000\text{GeV}$, the solid-line and dashed-line represent muon MDM varying with AE_{45}, for $ML_s = 500\text{GeV}$ and $ML_s = 800\text{GeV}$ respectively.

6.总结

1、采用泰勒展开传播子近似，用软件实现了双圈图的解析推导，使理论推导又快又准，效率比用手推导提高十倍以上。

2、双圈图的贡献是单圈图贡献的5%以下，极端参数可以达到10%。

3、双圈图的类型非常多，没有足够的理由说明所研究的双圈图是所有双圈图中的主要贡献者，除非对所有的双圈图都进行量级分析或者估算或者计算。

4、双圈图的贡献也受新物理参数空间取值的影响，一般规律是新物理粒子的质量越重，贡献越小。耦合强度越大贡献越大。


谢谢