imagine the impossible

Leptoquark interpretations of $R_{D^{(*)}}$ and top quark FCNC

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arXiv: 1812.08484, under JHEP revision

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"14th Workshop on TeV Physics"
Nanjing Normal University
April 19-22, 2019

Outline

- Anomalies in rare B decays
- Leptoquark explanations of $R_{D^{(*)}}$
  - SU(2) singlet: $S_1, \quad U_1$
- Implications on top FCNC
  - tree level
  - 1-loop level
- Collider prospects
- Summary

Moriond, 2019

Anomalies in rare B decays, lepton Non-universality

$\boxed{R^{\text{exp}}_{D^{(*)}} > R^{\text{SM}}_{D^{(*)}}}$ ~ 3.8 σ

$$R_{D^{(*)}} = \left.\frac{Br(B \to D^{(*)}\tau\bar{\nu})}{Br(B \to D^{(*)}l\bar{\nu})}\right|_{l\in\{e,\mu\}}$$

$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

$$R_D^{\text{SM}} = 0.300(8)\,, \qquad R_{D^*}^{\text{SM}} = 0.257(3).$$

$\boxed{R^{\text{exp}}_{K^{(*)}} < R^{\text{SM}}_{K^{(*)}}}$ ~ 2.5 σ

$$R^{[q_1^2,q_2^2]}_{K^{(*)}} = \frac{\partial\left(Br(B \to K^{(*)}\mu\mu)\right)/\partial q^2}{\partial\left(Br(B \to K^{(*)}ee)\right)/\partial q^2}$$

$\boxed{q^2 = (p_{l^+} + p_{l^-})^2}$

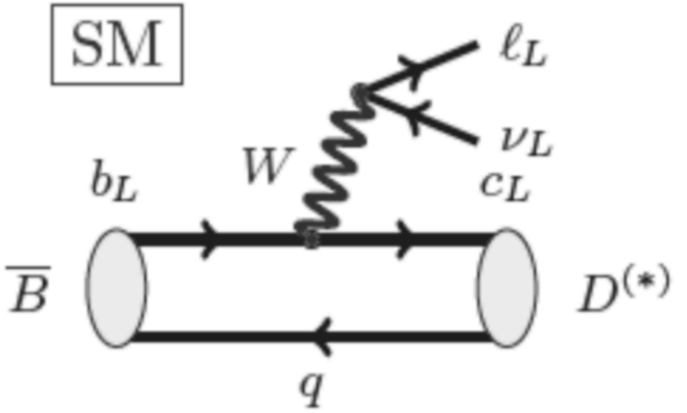
SM ~ 1

$$R_K \equiv R^{[1,6]}_{K^+} = 0.846^{+0.060+0.016}_{-0.054-0.014}\ \text{(LHCb)},$$

$$R_{K^*} \equiv R^{[1.1,6]}_{K^{*0}} = 0.96^{+0.45}_{-0.29} \pm 0.11\ \text{(Belle)}.$$

$$R^{[0.045,1.1]}_{K^{*0}} = 0.52^{+0.36}_{-0.26} \pm 0.05\ \text{(Belle)}$$

Low-energy EFT, $R_{D^{(*)}}$



$$-L_{\rm eff} = (C_{\rm SM}\delta_{l\tau} + C^l_{V_1})O^l_{V_1} + C^l_{V_2}O^l_{V_2} + C^l_{S_1}O^l_{S_1} + C^l_{S_2}O^l_{S_2} + C^l_T O^l_T$$

$\boxed{l = 1,2,3 \text{ being the neutrino generation index}}$ $\boxed{C_{\rm SM} = 2\sqrt{2}G_F V_{cb}}$

$$O^l_{V_1} = (\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_{lL})\,, \quad O^l_{V_2} = (\bar{c}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma_\mu\nu_{lL})$$

$$O^l_{S_1} = (\bar{c}_L b_R)(\bar{\tau}_R\nu_{lL})\,, \qquad O^l_{S_2} = (\bar{c}_R b_L)(\bar{\tau}_R\nu_{lL})\,,$$

$$O^l_T = (\bar{c}_R\sigma^{\mu\nu}b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_{lL})\,.$$

LQ contributions

Dumont *et al*, 1603.05248

$$\mathcal{L}^{\mathrm{LQ}} = \mathcal{L}^{\mathrm{LQ}}_{F=0} + \mathcal{L}^{\mathrm{LQ}}_{F=-2}\,,$$

$$\mathcal{L}^{\mathrm{LQ}}_{F=0} = \boxed{\left(h_{1L}^{ij}\,\bar{Q}_L^i\gamma_\mu L_L^j + h_{1R}^{ij}\,\bar{d}_R^i\gamma_\mu \ell_R^j\right) U_1^\mu} + h_{3L}^{ij}\,\bar{Q}_L^i\boldsymbol{\sigma}\gamma_\mu L_L^j \boldsymbol{U}_3^\mu$$
$$+ \left(h_{2L}^{ij}\,\bar{u}_R^i L_L^j + h_{2R}^{ij}\,\bar{Q}_L^i i\sigma_2 \ell_R^j\right) R_2 + \mathrm{h.c.}\,,$$

$$\mathcal{L}^{\mathrm{LQ}}_{F=-2} = \boxed{\left(g_{1L}^{ij}\,\bar{Q}_L^{c,j} i\sigma_2 L_L^j + g_{1R}^{ij}\,\bar{u}_R^{c,i}\ell_R^j\right) S_1} + g_{3L}^{ij}\,\bar{Q}_L^{c,i} i\sigma_2\boldsymbol{\sigma} L_L^j \boldsymbol{S}_3$$
$$+ \left(g_{2L}^{ij}\,\bar{d}_R^{c,i}\gamma_\mu L_L^j + g_{2R}^{ij}\,\bar{Q}_L^{c,i}\gamma_\mu \ell_R^j\right) V_2^\mu + \mathrm{h.c.}\,,$$

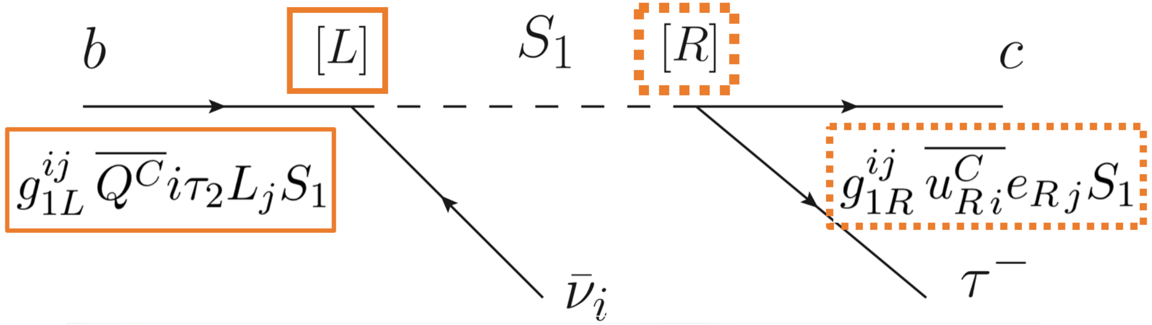
$$C^l_{\mathcal{V}_1} = \sum_{k=1}^{3} V_{k3}\left[\frac{g_{1L}^{kl}g_{1L}^{23*}}{2M_{S_1}^2} - \frac{g_{3L}^{kl}g_{3L}^{23*}}{2M_{\boldsymbol{S}_3}^2} + \frac{h_{1L}^{2l}h_{1L}^{k3*}}{M_{U_1}^2} - \frac{h_{3L}^{2l}h_{3L}^{k3*}}{M_{\boldsymbol{U}_3}^2}\right]$$

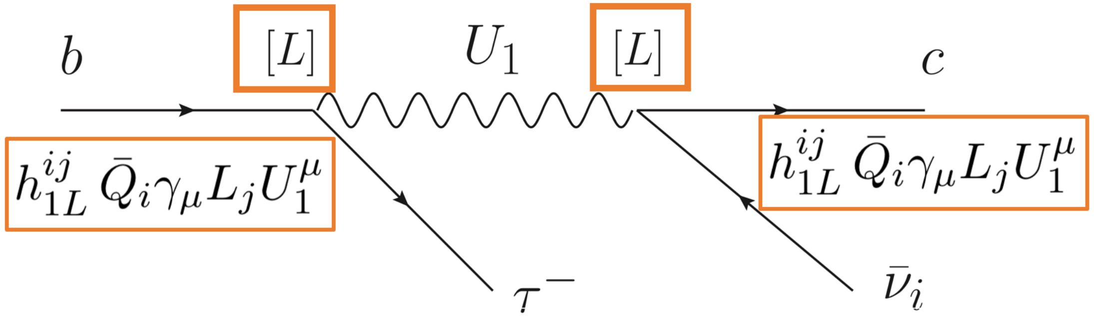
$$C^l_{\mathcal{V}_2} = 0\,,$$

$$C^l_{\mathcal{S}_1} = \sum_{k=1}^{3} V_{k3}\left[-\frac{2g_{2L}^{kl}g_{2R}^{23*}}{M_{V_2}^2} - \frac{2h_{1L}^{2l}h_{1R}^{k3*}}{M_{U_1}^2}\right],$$

$$C^l_{\mathcal{S}_2} = \sum_{k=1}^{3} V_{k3}\left[-\frac{g_{1L}^{kl}g_{1R}^{23*}}{2M_{S_1}^2} - \frac{h_{2L}^{2l}h_{2R}^{k3*}}{2M_{R_2}^2}\right],$$

$$C^l_{\mathcal{T}} = \sum_{k=1}^{3} V_{k3}\left[\frac{g_{1L}^{kl}g_{1R}^{23*}}{8M_{S_1}^2} - \frac{h_{2L}^{2l}h_{2R}^{k3*}}{8M_{R_2}^2}\right],$$





focus of this talk: $R_{D^{(*)}}$

LQ contributions

$$\mathcal{L}^{\rm LQ} = \mathcal{L}^{\rm LQ}_{F=0} + \mathcal{L}^{\rm LQ}_{F=-2}\,,$$

$$\mathcal{L}^{\rm LQ}_{F=0} = \boxed{\left(h_{1L}^{ij}\,\bar{Q}_L^i\gamma_\mu L_L^j + h_{1R}^{ij}\,\bar{d}_R^i\gamma_\mu \ell_R^j\right) U_1^\mu} + h_{3L}^{ij}\,\bar{Q}_L^i\boldsymbol{\sigma}\gamma_\mu L_L^j \boldsymbol{U}_3^\mu$$
$$+ \left(h_{2L}^{ij}\,\bar{u}_R^i L_L^j + h_{2R}^{ij}\,\bar{Q}_L^i i\sigma_2 \ell_R^j\right) R_2 + \text{h.c.}\,,$$

$$\mathcal{L}^{\rm LQ}_{F=-2} = \boxed{\left(g_{1L}^{ij}\,\bar{Q}_L^{c,j} i\sigma_2 L_L^j + g_{1R}^{ij}\,\bar{u}_R^{c,i}\ell_R^j\right) S_1} + g_{3L}^{ij}\,\bar{Q}_L^{c,i} i\sigma_2\boldsymbol{\sigma} L_L^j \boldsymbol{S}_3$$
$$+ \left(g_{2L}^{ij}\,\bar{d}_R^{c,i}\gamma_\mu L_L^j + g_{2R}^{ij}\,\bar{Q}_L^{c,i}\gamma_\mu \ell_R^j\right) V_2^\mu + \text{h.c.}\,,$$

Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$ & $R_{D^{(*)}}$
S_1	✗*	✓	✗*
R_2	✗*	✓	✗
$\widetilde{R}_2$	✗	✗	✗
S_3	✓	✗	✗
U_1	✓	✓	✓
U_3	✓	✗	✗

Angelescu *et al*, 1808.08179

Numerical fit

$m_{LQ} = 1\ \text{TeV}$

Dumont *et al*, 1603.05248

Leptoquark	$\bar{B} \to D^{(*)}\tau\bar{\nu}$	$\bar{B} \to X_s\nu\bar{\nu}$
S_1	$-0.87 < g_{1L}^{33} g_{1R}^{23*} < -0.54$ $1.64 < \lvert g_{1L}^{3i} g_{1R}^{23*}\rvert < 1.81 \quad (i = 1, 2)$ $0.19 < g_{1L}^{33} g_{1L}^{23*} < 0.48, \quad -5.59 < g_{1L}^{33} g_{1L}^{23*} < -5.87$ $1.04 < \lvert g_{1L}^{3i} g_{1L}^{23*}\rvert < 1.67 \quad (i = 1, 2)$	$\lvert g_{1L}^{3i} g_{1L}^{2j*}\rvert \lesssim 0.15$
$\boldsymbol{S}_3$	$0.19 < g_{3L}^{33} g_{3L}^{23*} < 0.48, \quad -5.59 < g_{3L}^{33} g_{3L}^{23*} < -5.87$ $1.04 < \lvert g_{3L}^{3i} g_{1L}^{23*}\rvert < 1.67 \quad (i = 1, 2)$	$\lvert g_{3L}^{3i} g_{3L}^{2j*}\rvert \lesssim 0.15$
R_2	$1.64 < \left\lvert \mathrm{Im}(h_{2L}^{2i} h_{2R}^{33*})\right\rvert < 1.81$	-
V_2	$g_{2L}^{3i} g_{2R}^{23*}$: no region within 2σ	$\lvert g_{2L}^{3i} g_{2L}^{2j*}\rvert \lesssim 0.07$
U_1	$0.10 < h_{1L}^{23} h_{1L}^{33*} < 0.24, \quad -2.94 < h_{1L}^{23} h_{1L}^{33*} < -2.80$ $0.52 < \lvert h_{1L}^{2i} h_{1L}^{33*}\rvert < 0.84 \quad (i = 1, 2)$ $h_{1L}^{2i} h_{1R}^{33*}$: no region within 2σ	-
$\boldsymbol{U}_3$	$0.10 < h_{3L}^{23} h_{3L}^{33*} < 0.24, \quad -2.94 < h_{3L}^{23} h_{3L}^{33*} < -2.80$ $0.52 < \lvert h_{3L}^{2i} h_{3L}^{33*}\rvert < 0.84 \quad (i = 1, 2)$	$\lvert h_{3L}^{2i} h_{3L}^{3j*}\rvert \lesssim 0.04$

Numerical fit

$\boxed{m_{LQ} = 1\ \text{TeV}}$

Dumont *et al*, 1603.05248

Leptoquark	$\bar{B} \to D^{(*)}\tau\bar{\nu}$	$\bar{B} \to X_s \nu\bar{\nu}$
	$-0.87 < g_{1L}^{33} g_{1R}^{23*} < -0.54$	
	$1.64 < \lvert g_{1L}^{3i} g_{1R}^{23*}\rvert < 1.81 \quad (i = 1, 2)$	

Leptoquark	2σ range for $\bar{B} \to D^{(*)}\tau\bar{\nu}$
S_1	$\boxed{g_{1L}^{3l}}\,\boxed{g_{1R}^{23*}} \in \left(\frac{M_{S_1}}{1\ \text{TeV}}\right)^2 \times \begin{cases} (1.64, 1.81) & l = 1, 2 \\ (-0.87, -0.54) & l = 3 \end{cases}$
U_1	$\boxed{h_{1L}^{2l}}\,\boxed{h_{1L}^{33*}} \in \left(\frac{M_{U_1}}{1\ \text{TeV}}\right)^2 \times \begin{cases} (0.52, 0.84) & l = 1, 2 \\ (-2.94, -2.80) & l = 3 \end{cases}$

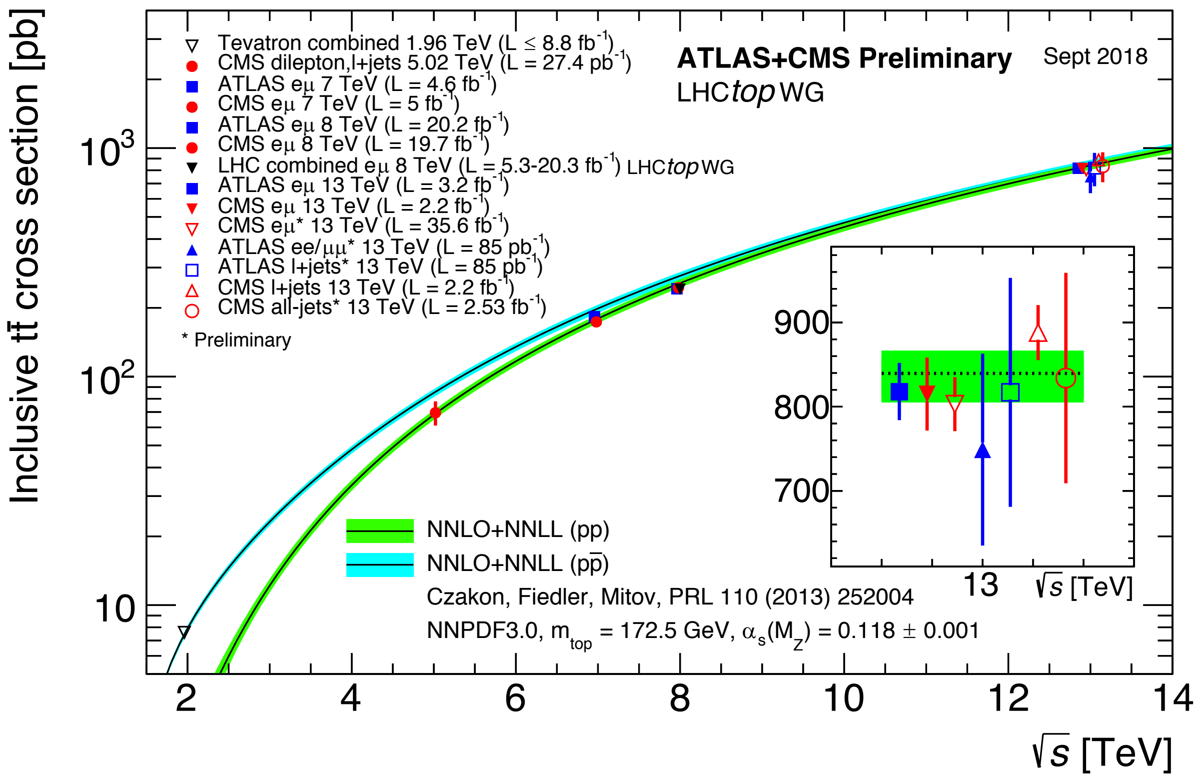
	$h_{1L}^{2i} h_{1R}^{33*}$: no region within 2σ	
$\boldsymbol{U_3}$	$0.10 < h_{3L}^{23} h_{3L}^{33*} < 0.24, \quad -2.94 < h_{3L}^{23} h_{3L}^{33*} < -2.80$ $0.52 < \lvert h_{3L}^{2i} h_{3L}^{33*}\rvert < 0.84 \quad (i = 1, 2)$	$\lvert h_{3L}^{2i} h_{3L}^{3j*}\rvert \lesssim 0.04$

Sept. 2018 | LHCTopWG Summary

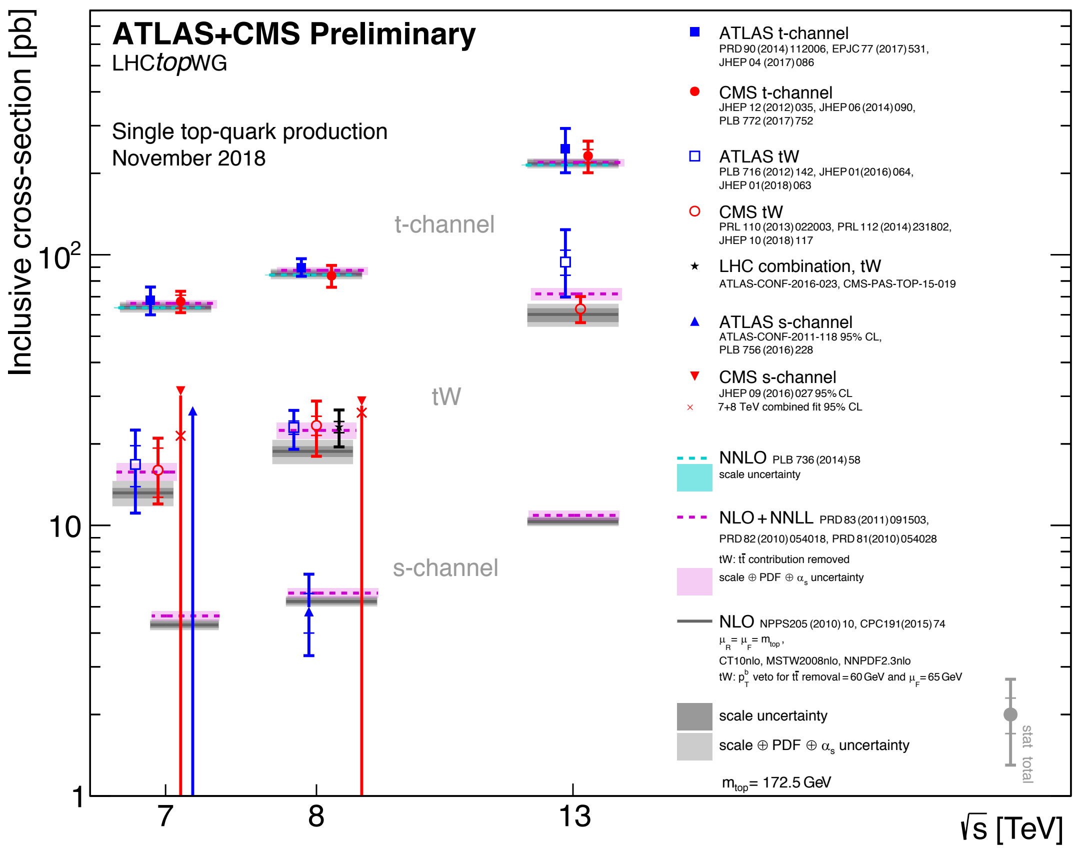
Top quark at LHC

- fairly good agreements for cross section
- within theoretical uncertainties: scales of Ren. & Fact., PDF, α_S

$t\bar{t}$ cross section (pb)



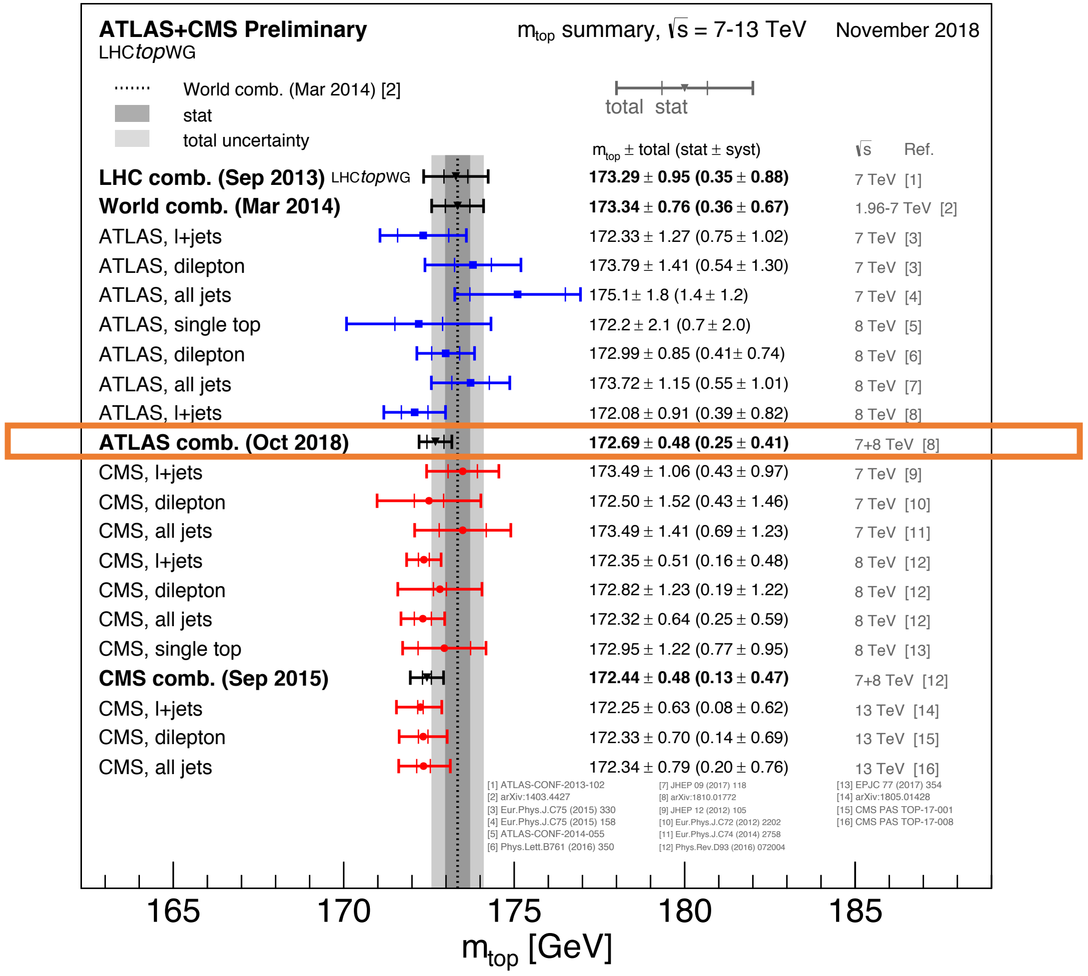
single t cross section (pb)



Sept. 2018 LHCTopWG Summary

Top quark at LHC

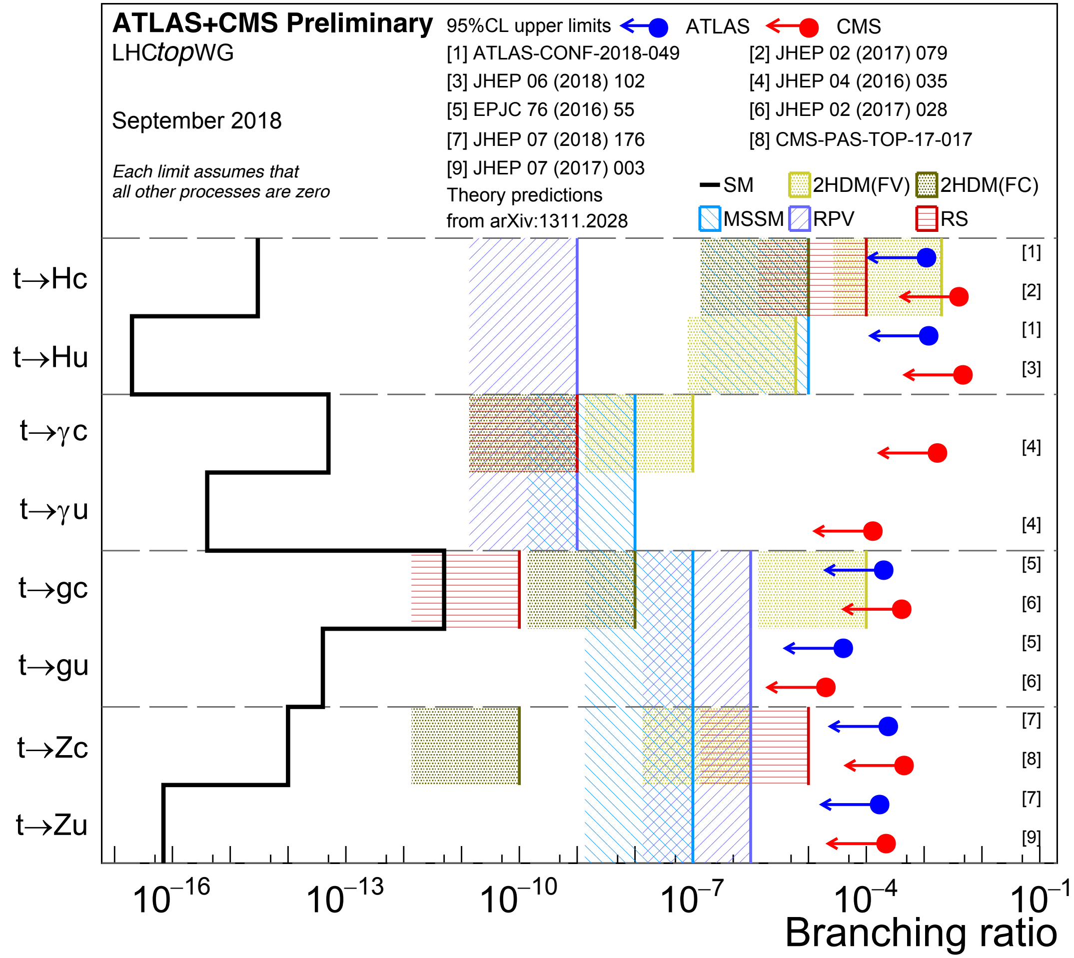
$$m_t = 172.69 \pm 0.48 \text{ GeV}$$



$q = c$
$V = \{g, Z, \gamma, H\} \sim \{10^{-4}, 10^{-4}, 10^{-3}, 10^{-3}\}$

$q = u$, slightly stronger limits, but same order

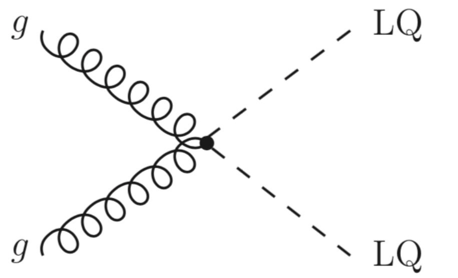
$$Br(t \to qV)$$

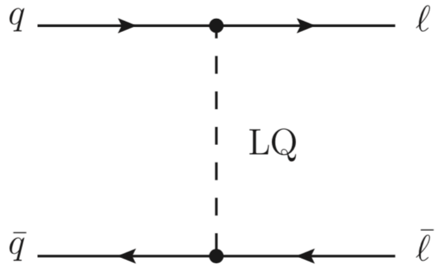


LQ limits at LHC

- to set limits, specific LQ decay modes are assumed
- currently, LQ mass bounds are 1 ~ 2 TeV, stronger for Vector LQ

Decays	LQs	Scalar LQ limits	Vector LQ limits	$\mathcal{L}_{\rm int}$ / Ref.
$jj\,\tau\bar{\tau}$	S_1, R_2, S_3, U_1, U_3	–	–	–
$b\bar{b}\,\tau\bar{\tau}$	R_2, S_3, U_1, U_3	850 (550) GeV	1550 (1290) GeV	12.9 fb^{-1} [52]
$t\bar{t}\,\tau\bar{\tau}$	S_1, R_2, S_3, U_3	900 (560) GeV	1440 (1220) GeV	35.9 fb^{-1} [53]
$jj\,\mu\bar{\mu}$	S_1, R_2, S_3, U_1, U_3	1530 (1275) GeV	2110 (1860) GeV	35.9 fb^{-1} [54]
$b\bar{b}\,\mu\bar{\mu}$	R_2, U_1, U_3	1400 (–) GeV	1900 (1700) GeV	36.1 fb^{-1} [55]
$t\bar{t}\,\mu\bar{\mu}$	S_1, R_2, S_3, U_3	1420 (950) GeV	1780 (1560) GeV	36.1 fb^{-1} [56, 57]
$jj\,\nu\bar{\nu}$	R_2, S_3, U_1, U_3	980 (640) GeV	1790 (1500) GeV	35.9 fb^{-1} [58]
$b\bar{b}\,\nu\bar{\nu}$	S_1, R_2, S_3, U_3	1100 (800) GeV	1810 (1540) GeV	35.9 fb^{-1} [58]
$t\bar{t}\,\nu\bar{\nu}$	R_2, S_3, U_1, U_3	1020 (820) GeV	1780 (1530) GeV	35.9 fb^{-1} [58]





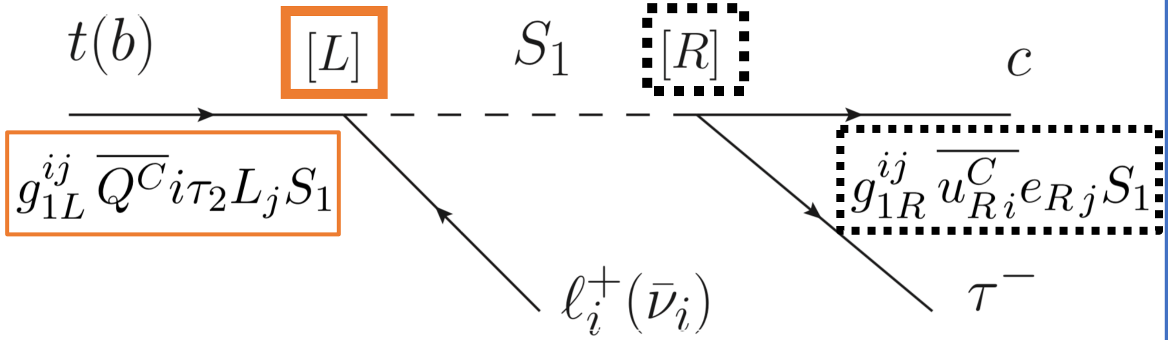
Angelescu *et al*, 1808.08179

Tree

Top FCNC at tree-level, 3-body

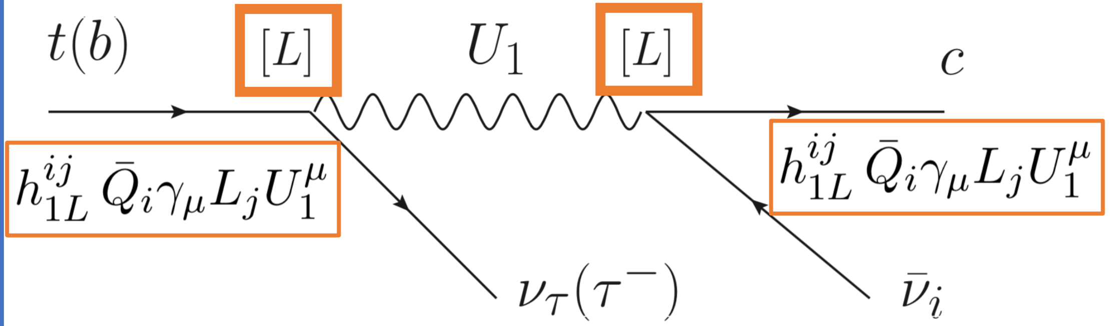
$$t \to c\tau^- \ell_i^+$$

scalar S_1



$$t \to c\nu_\tau\bar{\nu}_i$$

vector U_1

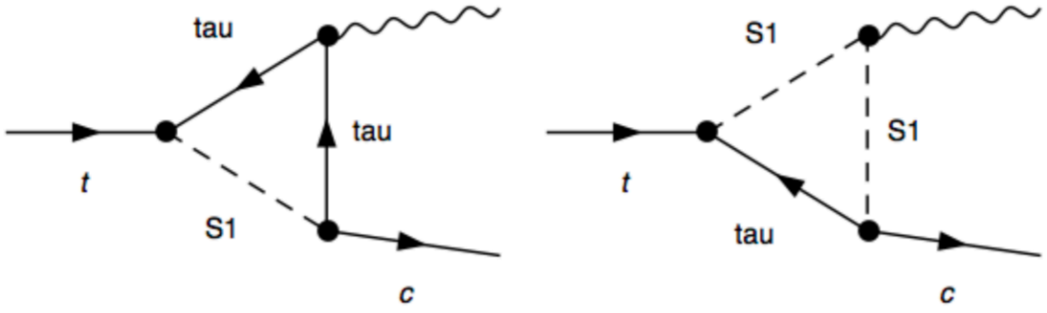


$\boxed{\text{1-loop}}$ Top FCNC at one-loop, 2-body

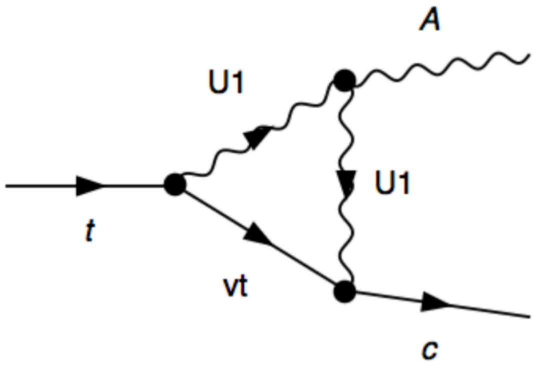
- we only consider loop for scalar S_1
- loop of vector U_1 needs UV-completion of m_{U_1} generation
  - SM-GIM-like mechanism needed to cancel divergence
  - more complete flavor structure needed

$$t \to cV$$

$\boxed{\text{scalar } S_1}$



$\boxed{\text{vector } U_1}$

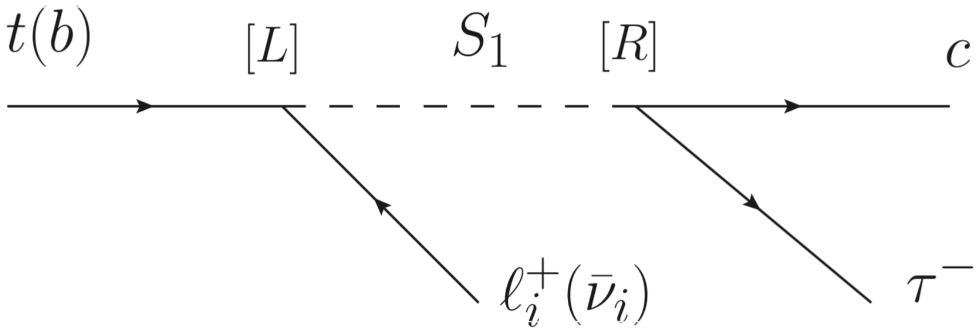


+ external leg diagrams

Tree-level: $R_{D^{(*)}}$ fit of $\frac{g_L g_R}{m_{LQ}^2}$ naturally implies Br_t^{FCNC}

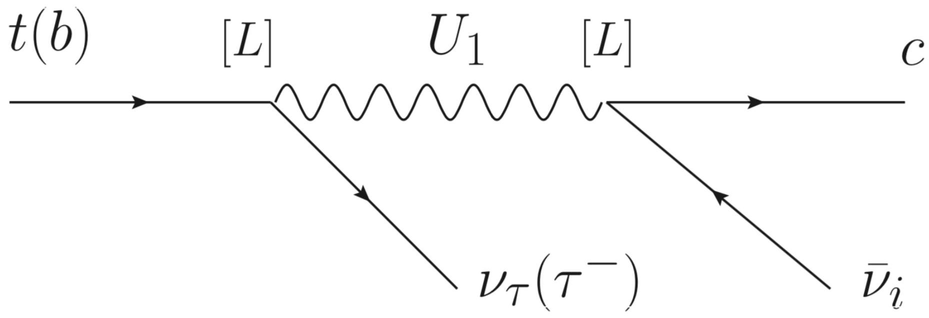
scalar S_1

$$t \to c\tau^- \ell_i^+$$



vector U_1

$$t \to c\nu_\tau \bar{\nu}_i$$

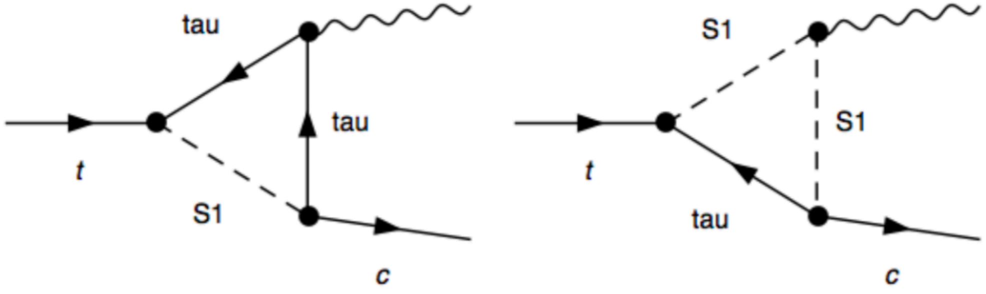


$$S_1 : Br(t \to c\tau^- \ell_l^+) \approx \frac{1}{\Gamma_{t,SM}} \left(\frac{m_t^5}{6144\pi^3}\right) \left|\frac{g_{1L}^{3l} g_{1R}^{23*}}{M_{S_1}^2}\right|^2 = 10^{-6} \times \begin{cases} 1.4 \sim 1.8 & l = 1,2 \\ 0.16 \sim 0.41 & l = 3 \end{cases}$$

$$U_1 : Br(t \to c\nu_\tau \bar{\nu}_l) \approx \frac{1}{\Gamma_{t,SM}} \left(\frac{m_t^5}{1536\pi^3}\right) \left|\frac{h_{1L}^{33} h_{1L}^{2l*}}{M_{U_1}^2}\right|^2 = 10^{-6} \times \begin{cases} 0.58 \sim 1.5 & l = 1,2 \\ 17 \sim 19 & l = 3 \end{cases}$$

Loop-level:

+ external leg corrections



Leptoquark	
S_1	$g_{1L}^{3l} g_{1R}^{23*}$

Given the coupling chirality we choose, only left-handed part appears in dipole

$$x_\tau = \frac{m_\tau^2}{M_{S_1}^2}$$
$$x_t = \frac{m_t^2}{M_{S_1}^2}$$

$$f_{TR}^g = f_{TR}^\gamma = f_{TR}^Z = 0$$

$$f_{TL}^g \simeq g_{1L}^{33} g_{1R}^{23*} \frac{1}{16\pi^2} g_s m_\tau \times \frac{1}{M_{S_1}^2} \frac{1}{12} \big(-6\,(22 x_\tau x_t + 3 x_t + 16 x_\tau + 6) \log x_\tau - 49 x_t - 48 \big)$$

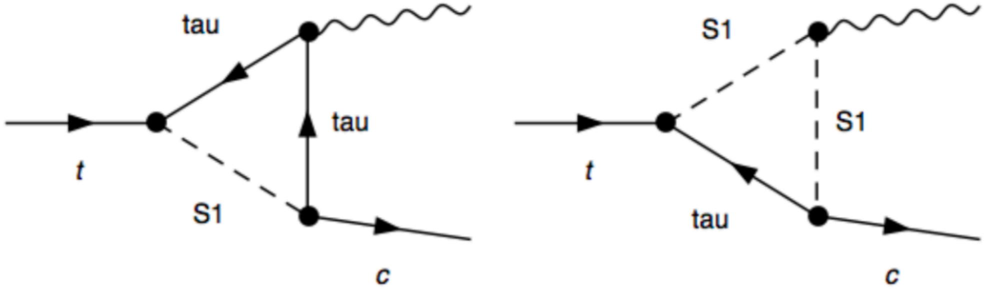
$$f_{TL}^\gamma \simeq -g_{1L}^{33} g_{1R}^{23*} \frac{1}{48\pi^2} e m_\tau \times \frac{1}{M_{S_1}^2} \big(\frac{1}{6} (14 x_t x_\tau + x_t + 9 x_\tau + 3) + (x_t + 1)\, x_\tau \log x_\tau \big)$$

$$i\mathcal{M}_{tcV} = \bar{u}(p_2)\,\Gamma^\mu u(p_1)\,\epsilon_\mu(k,\lambda)$$

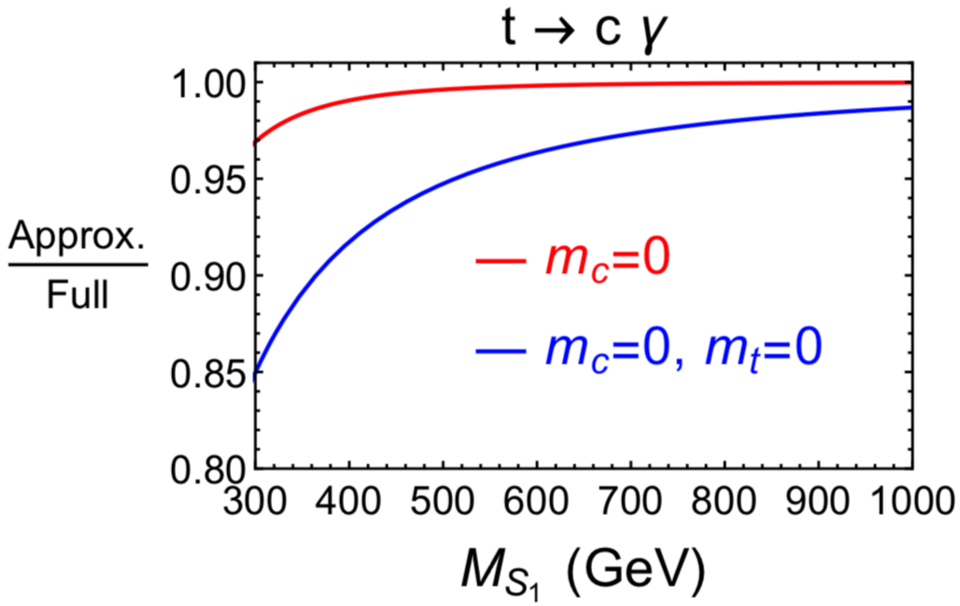
$$\Gamma_{tcZ}^\mu = \gamma^\mu (P_L f_{VL}^Z + P_R f_{VR}^Z) + i\sigma^{\mu\nu} k_\nu (P_L f_{TL}^Z + P_R f_{TR}^Z)\,,$$
$$\Gamma_{tc\gamma}^\mu = i\sigma^{\mu\nu} k_\nu (P_L f_{TL}^\gamma + P_R f_{TR}^\gamma)\,,$$
$$\Gamma_{tcg}^\mu = T^a i\sigma^{\mu\nu} k_\nu (P_L f_{TL}^g + P_R f_{TL}^g)\,,$$

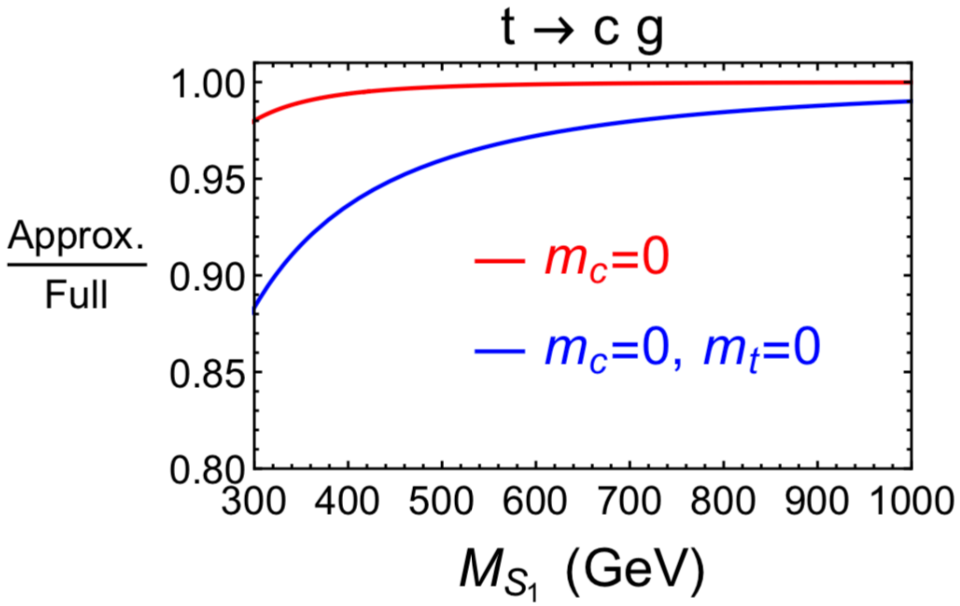
Loop-level:

+ external leg corrections



we kept $O(m_t^1)$ in loop function expansions for $m_V = 0$
unlike $b \to s\gamma,\ \mu \to e\gamma$ where m_b, m_μ are ignored





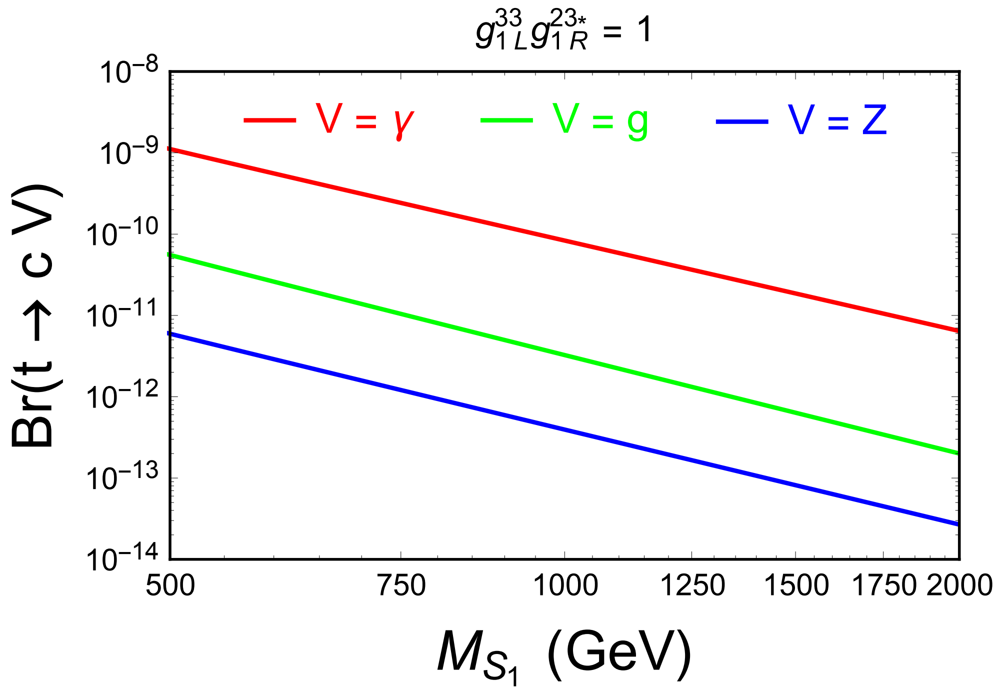
Loop-level:

$$Br(t \to c\gamma) \sim 10^{-10}$$

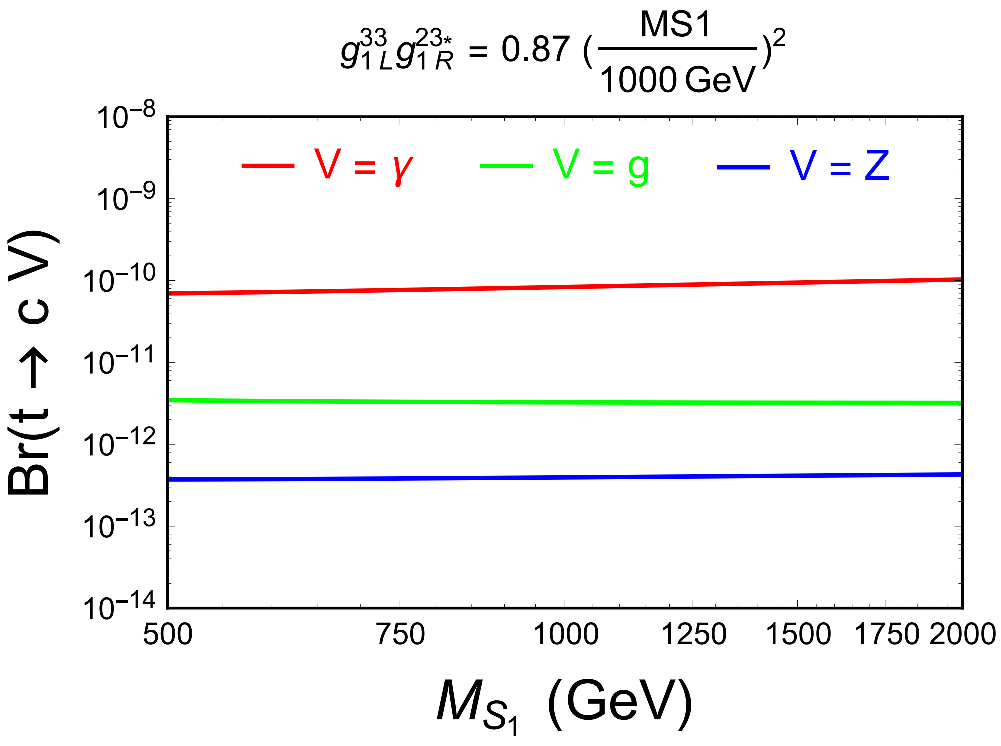
$$Br(t \to cg) \sim 5\times 10^{-12}$$

$$Br(t \to cZ) \sim 5\times 10^{-13}$$

$\boxed{g_{1L}^{33} g_{1R}^{23*} = 1}$

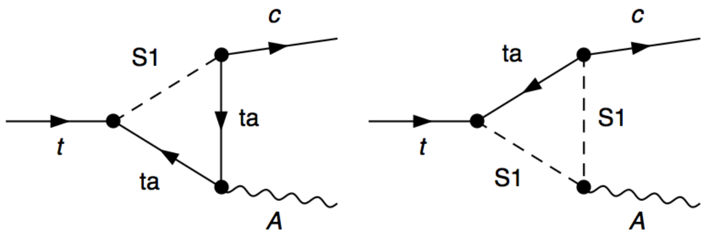


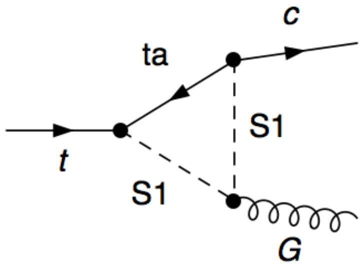
$\boxed{\text{fixing } \frac{g_{1L}^{33} g_{1R}^{23*}}{M_{S_1}^2} \text{ to upper fit value}}$



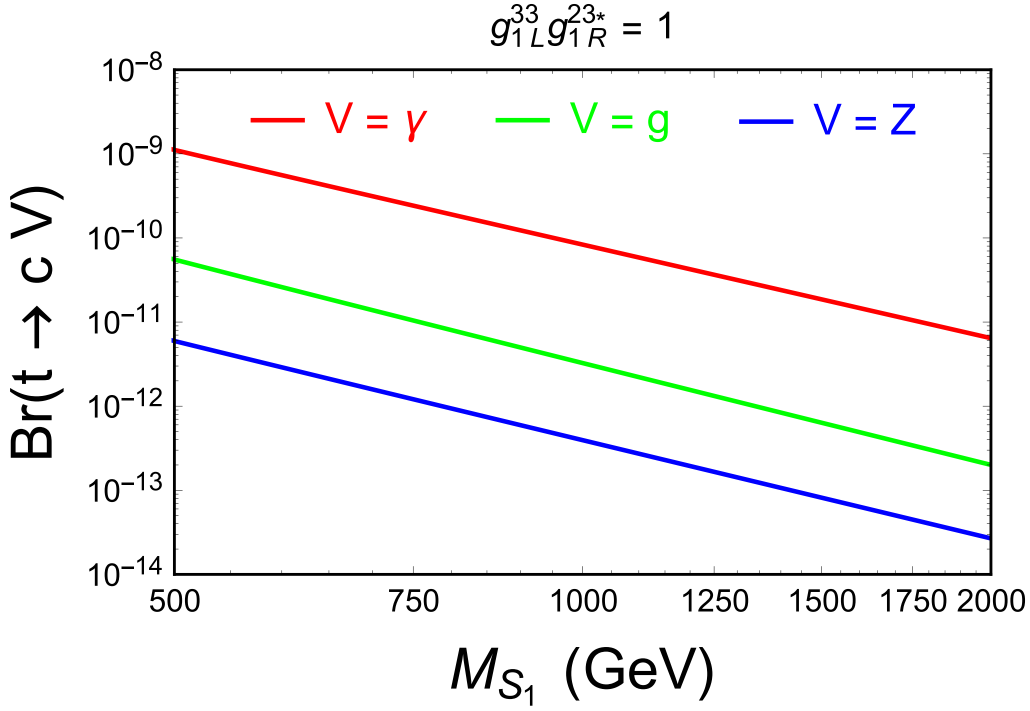
Leptoquark	2σ range for $\bar{B} \to D^{(*)}\tau\bar{\nu}$
S_1	$g_{1L}^{3l} g_{1R}^{23*} \in \left(\frac{M_{S_1}}{1\text{ TeV}}\right)^2 \times \begin{cases} (1.64, 1.81) & l = 1,2 \\ (-0.87, -0.54) & l = 3 \end{cases}$

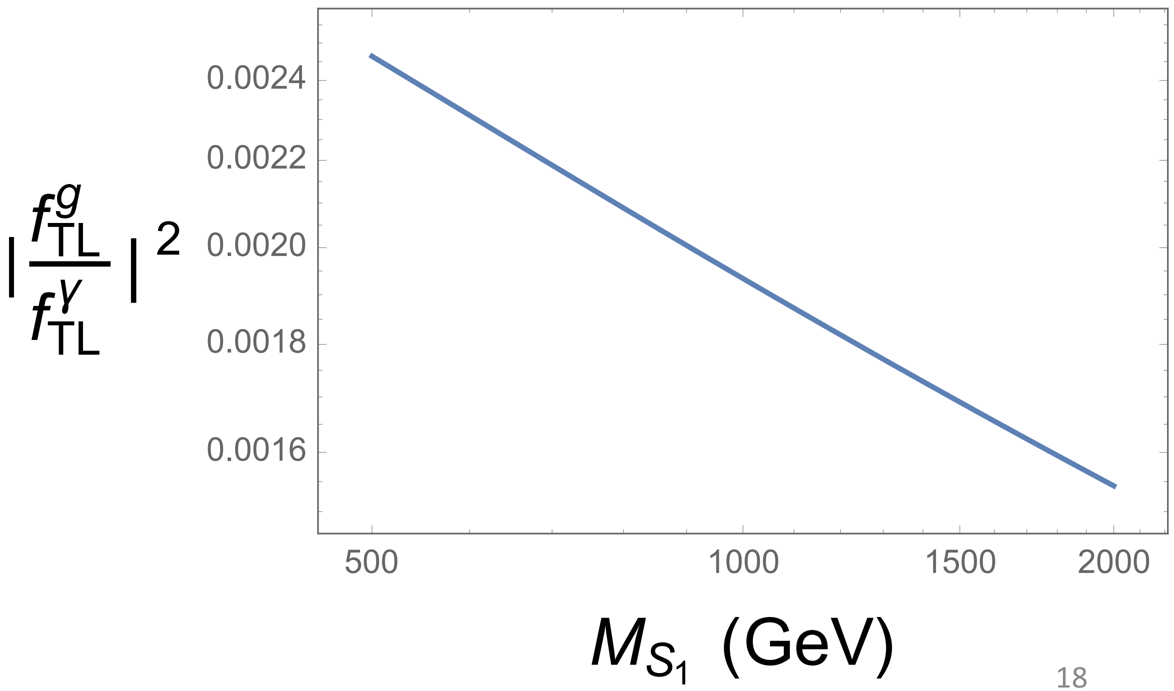
$$\frac{Br(t\to cg)}{Br(t\to c\gamma)} \approx 0.05\text{, why?}$$





- $\frac{\alpha_S}{\alpha_{em}} \frac{N_c^2-1}{2N_c} \simeq 20$
- Scalar S1 $\quad f_T^g \ll f_T^\gamma \quad Br_{tcg}/Br_{tcA} \simeq 20 * 0.002 \simeq 0.05.$
- SM has W boson $\quad |f_T^g \sim O(1) f_T^\gamma| \quad Br_{tcg}/Br_{tcA} \simeq 20 * O(1) \gtrsim O(10)$

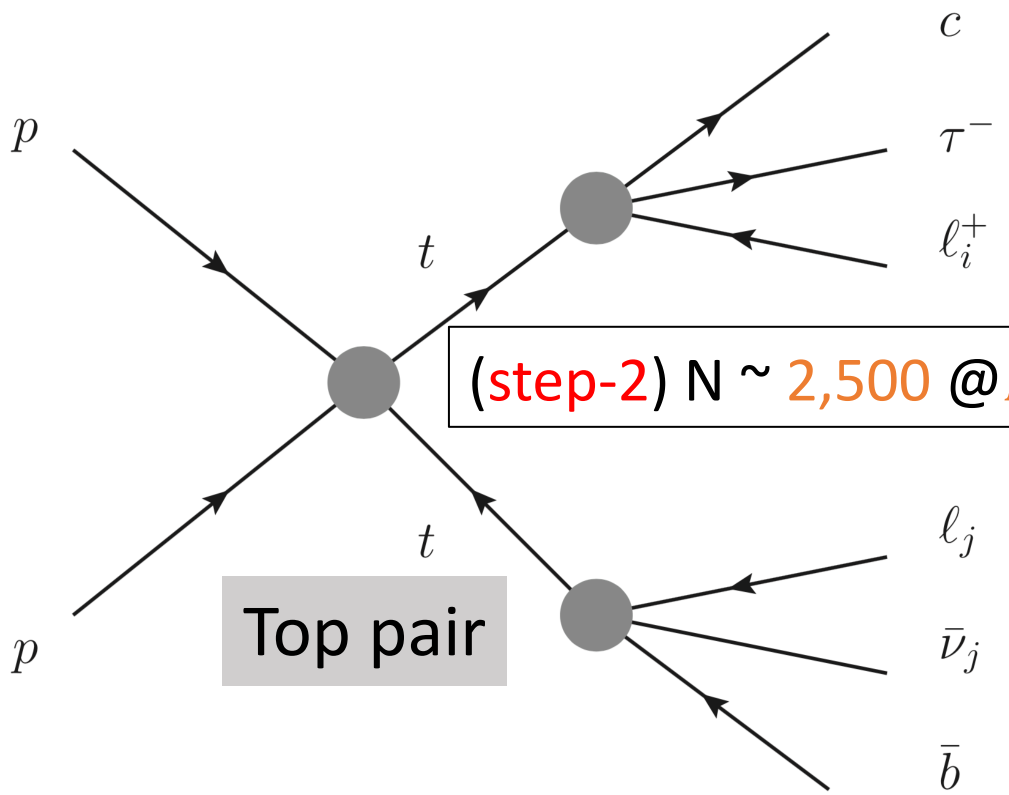




S_1 collider prospects via $t\bar{t}$ production

(step-1) N ~ 2.5×10^9 @ 3 ab^{-1} LHC-13

$t \to c\tau^-\ell_i^+$



(step-2) N ~ 2,500 @$Br_t^{\mathrm{FCNC}}\sim10^{-6}$

(step-3) N ~ 500 @ leptonic top

$t\bar{t}$ estimation

$1\,\ell, \quad \geq 3\, j \ni \{1\, b, 2\, \tau\}$

- **Selection 1**: Exactly one lepton, at least three jets including exactly one b jet and two τ jets.

$2\,\ell \ni \{\mu, \ell\}, \quad \geq 2\, j \ni \{1\, b, 1\, \tau\}$

- **Selection 2**: Exactly two leptons, at least one muon, at least 2 jets including exactly one b jet and one τ jet.

$1\,\ell, \quad \geq 2\, j \ni \{1\, b\}$

- **Selection 3**: Exactly one lepton and more than two jets in the final state, where one of the jet is b-tagged, the missing transverse energy $E_T^{\rm miss} > 80$ GeV.

	VV	DY	W+jet	$t\bar{t}$	$t \to c\mu\tau$	$t \to c\tau\tau$	$t \to c\nu\nu$
Selection 1	9559	108095	-	1189719	28	19	0.3
Selection 2	5433	54047	-	839651	39	5	0.0
Selection 3	296814	594522	16530371	64764862	140	94	102

@ 3 ab^{-1} LHC-13
one leptonic top

- simple cuts give signal/BG ~ 10^{-5}
- advanced techniques needed, e.g. multivariate analysis

Summary

- Anomalies in rare B decays have stimulated active discussions.

- LQ S_1, U_1 interpretations of $R_{D^{(*)}}$ naturally imply top FCNC
  - tree-level: $\sim 10^{-6}$ $\quad S_1 : Br(t \to c\tau^- \ell_l^+) \quad U_1 : Br(t \to c\nu_\tau \bar{\nu}_l)$
  - loop-level: $\sim 10^{-10}$ $\quad S_1 : Br(t \to c\gamma)$

- Collider searches
  - simple analyses of $t\bar{t}$ are challenging
  - advanced/alternative methods needed

Thank you for listening

Back up

WC of 1-loop $t \to cV$ via S_1

For both $m_t \neq 0$ and $m_Z \neq 0$, loop function reductions are tediously long. Thus we do not proceed with further expansion.

$$
\begin{aligned}
f_{TL}^{Z} &= g_{1L}^{33} g_{1R}^{23*} \frac{1}{96\pi^2} \frac{e}{c_W s_W} m_\tau \\
&\times \Big(3(s_W^2 - c_W^2) C_1(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) \\
&+ 2 s_W^2 \big(3 C_2(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) + C_2(m_Z^2, m_t^2, m_c^2, M_{S_1}^2, M_{S_1}^2, m_\tau^2) \big) \Big)
\end{aligned}
$$

$$
\begin{aligned}
f_{VL}^{Z} &= g_{1L}^{33} g_{1R}^{23*} \frac{1}{96\pi^2} \frac{e}{c_W s_W} \frac{m_\tau m_c}{m_c^2 - m_t^2} \\
&\times \Big((3c_W^2 - s_W^2) \big(B_0(m_t^2, m_\tau^2, M_{S_1}^2) - B_0(m_c^2, m_\tau^2, M_{S_1}^2) \big) \\
&+ 2 s_W^2 (m_c^2 - m_t^2) \big(3 C_0(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) + 3 C_1(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) \\
&+ 3 C_2(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) + C_2(m_Z^2, m_t^2, m_c^2, M_{S_1}^2, M_{S_1}^2, m_\tau^2) \big) \Big)
\end{aligned}
$$

$$
\begin{aligned}
f_{VR}^{Z} &= g_{1L}^{33} g_{1R}^{23*} \frac{1}{96\pi^2} \frac{e}{c_W s_W} \frac{m_\tau m_t}{m_c^2 - m_t^2} \\
&\times \Big(4 s_W^2 \big(B_0(m_c^2, m_\tau^2, M_{S_1}^2) - B_0(m_t^2, m_\tau^2, M_{S_1}^2) \big) \\
&+ (m_c^2 - m_t^2) \Big(2 s_W^2 C_2(m_Z^2, m_t^2, m_c^2, M_{S_1}^2, M_{S_1}^2, m_\tau^2) - 3(c_W^2 - s_W^2) \big(C_0(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) \\
&+ C_1(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) + C_2(m_c^2, m_Z^2, m_t^2, M_{S_1}^2, m_\tau^2, m_\tau^2) \big) \Big) \Big)
\end{aligned}
$$