Valence quark distribution inside pion from lattice QCD

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Why pion PDFs ?

- Quarks and gluons in massless NG bosons (PDFs)

The proton mass in chiral limit is produced by gluons and due to the trace anomaly:

$$\langle P(p)|\Theta_0|P(p)\rangle = -p_\mu p_\nu = m_N^2$$

While

$$\langle \pi(k)|\Theta_0|\pi(k)\rangle = -k_\mu k_\nu = m_\pi^2 = 0$$

- Strong versus Higgs-driven mass generation mechanisms (valence-quark distribution of pion and kaon)

- And so on …

Pion and Kaon Structure at the Electron-Ion Collider

White paper arXiv:1907.08218

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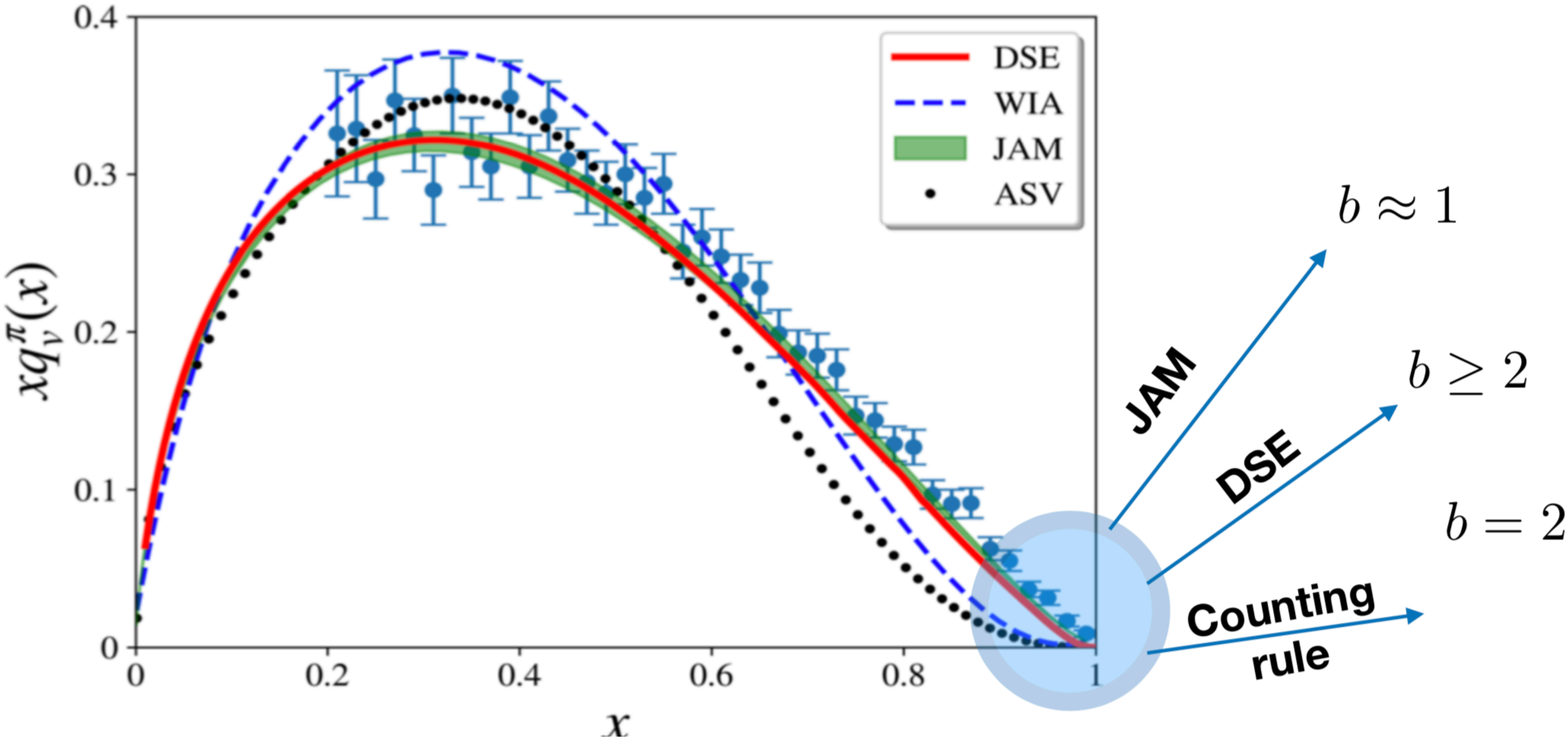
Abstract. Understanding the origin and dynamics of hadron structure and in turn that of atomic nuclei is a central goal of nuclear physics. This challenge entails the questions of how does the roughly 1 GeV mass-scale that characterizes atomic nuclei appear; why does it have the observed value; and, enigmatically, why are the composite Nambu-Goldstone (NG) bosons in quantum chromodynamics (QCD) abnormally light in comparison? In this perspective, we provide an analysis of the mass budget of the pion and proton in QCD; discuss the special role of the kaon, which lies near the boundary between dominance of strong and Higgs mass-generation mechanisms; and explain the need for a coherent effort in QCD phenomenology and continuum calculations, in exa-scale computing as provided by lattice QCD, and in experiments to make progress in understanding the origins of hadron masses and the distribution of that mass within them. We compare the unique capabilities foreseen at the electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA), the only previous electron-proton collider; and describe five key experimental measurements, enabled by the EIC and aimed at delivering fundamental insights that will generate concrete answers to the

Valence PDF of $\pi^+(u\bar{d})$

One of the important physic concerns is x=1 behavior:

$$\lim_{x \to 1} f_v^{\pi}(x) \sim (1-x)^b (1 + subleading\ terms)$$



First principle calculation essential!

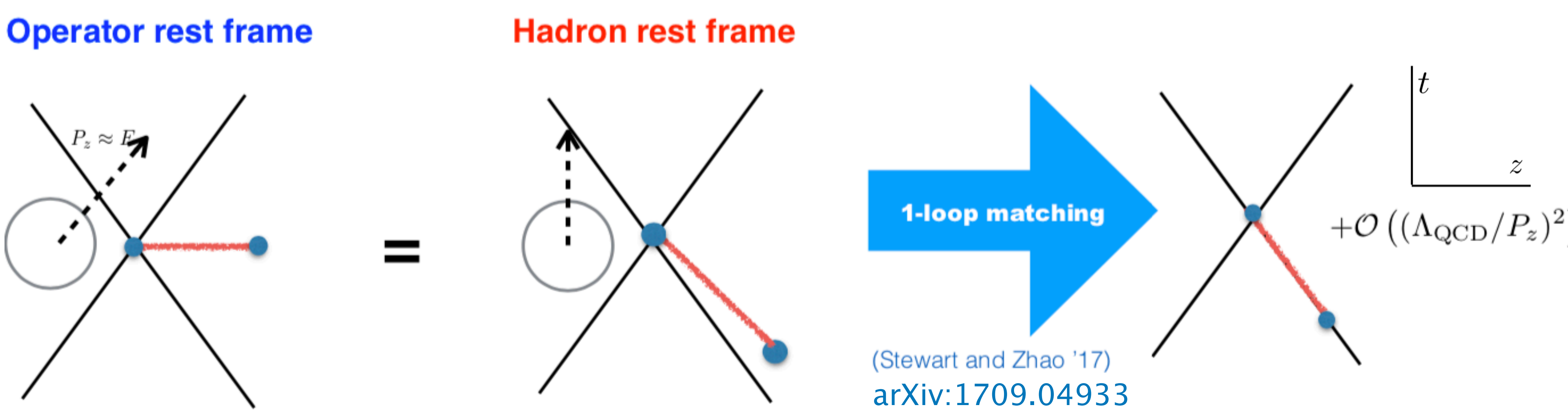
Compute Light-Cone PDFs from lattice

Equal time correlation function can be determined on lattice:

X. Ji, arXiv:1305.1539

$$\tilde{q}(x) = \int \frac{dz}{4\pi} e^{-ixP_z z} \underbrace{\langle H(P_z, E)|\overline{\psi}(0)\gamma_\mu W_{\hat{z}}(0,z)\tau\psi(z)|H(P_z,E)\rangle}_{h(z,P_z)}$$

μ=z or t



quasi-PDF in RI-MOM renormalization scheme

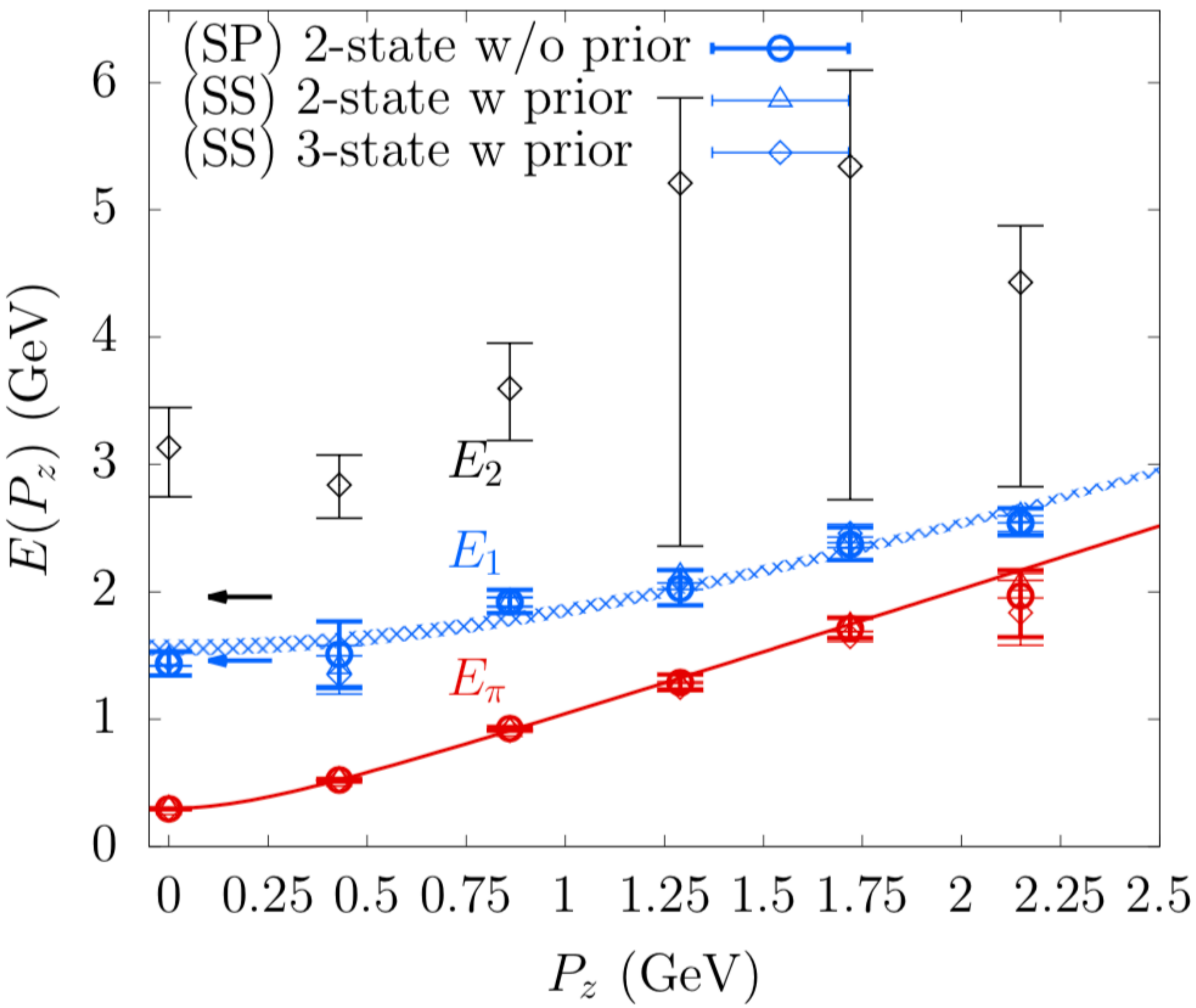
PDF In MS-bar scheme

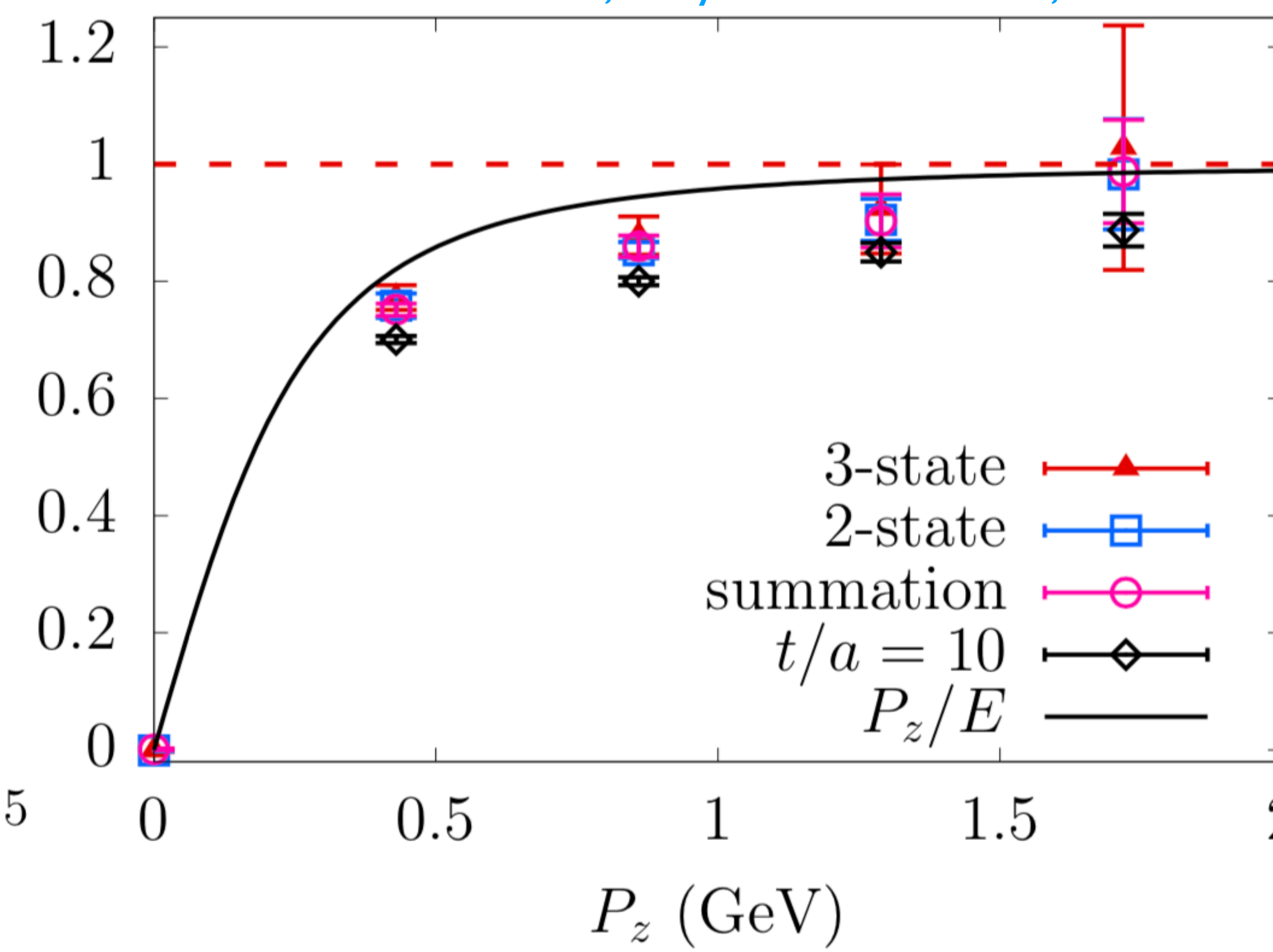
Quasi-PDF Lattice Analysis

- Step 1: Boosted pion on lattice
- Step 2: Extraction of bare quasi-PDF matrix element
- Step 3: Renormalization in a lattice scheme
- Step 4: Perturbative matching between quasi-PDF and PDF

$$\tilde{q}(x; P_z, P^R) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) q(x, \mu) + \mathcal{O}\left(\frac{m_h^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$

Quasi-PDF Analysis: Boosted pion on lattice

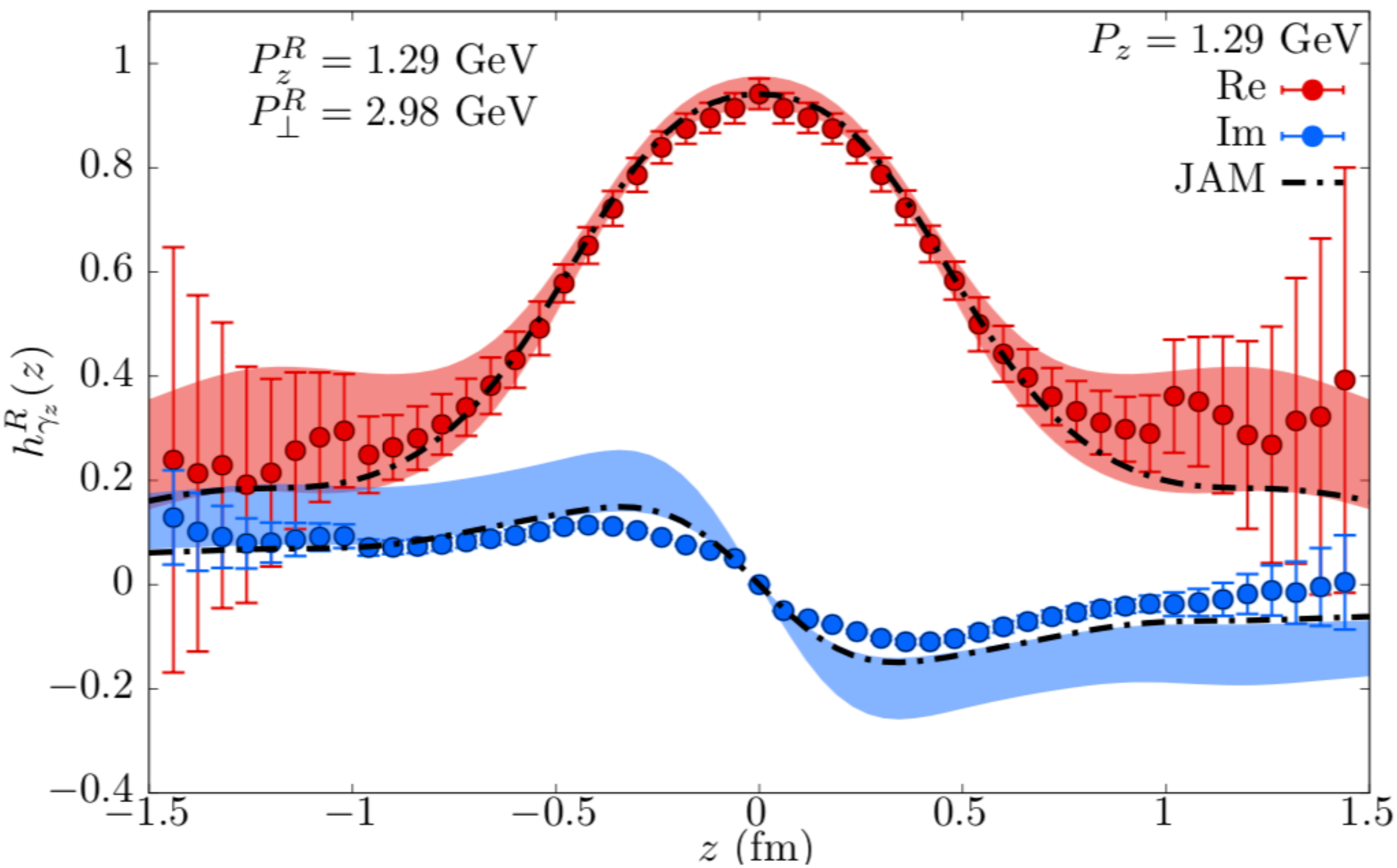




Lattice setup:

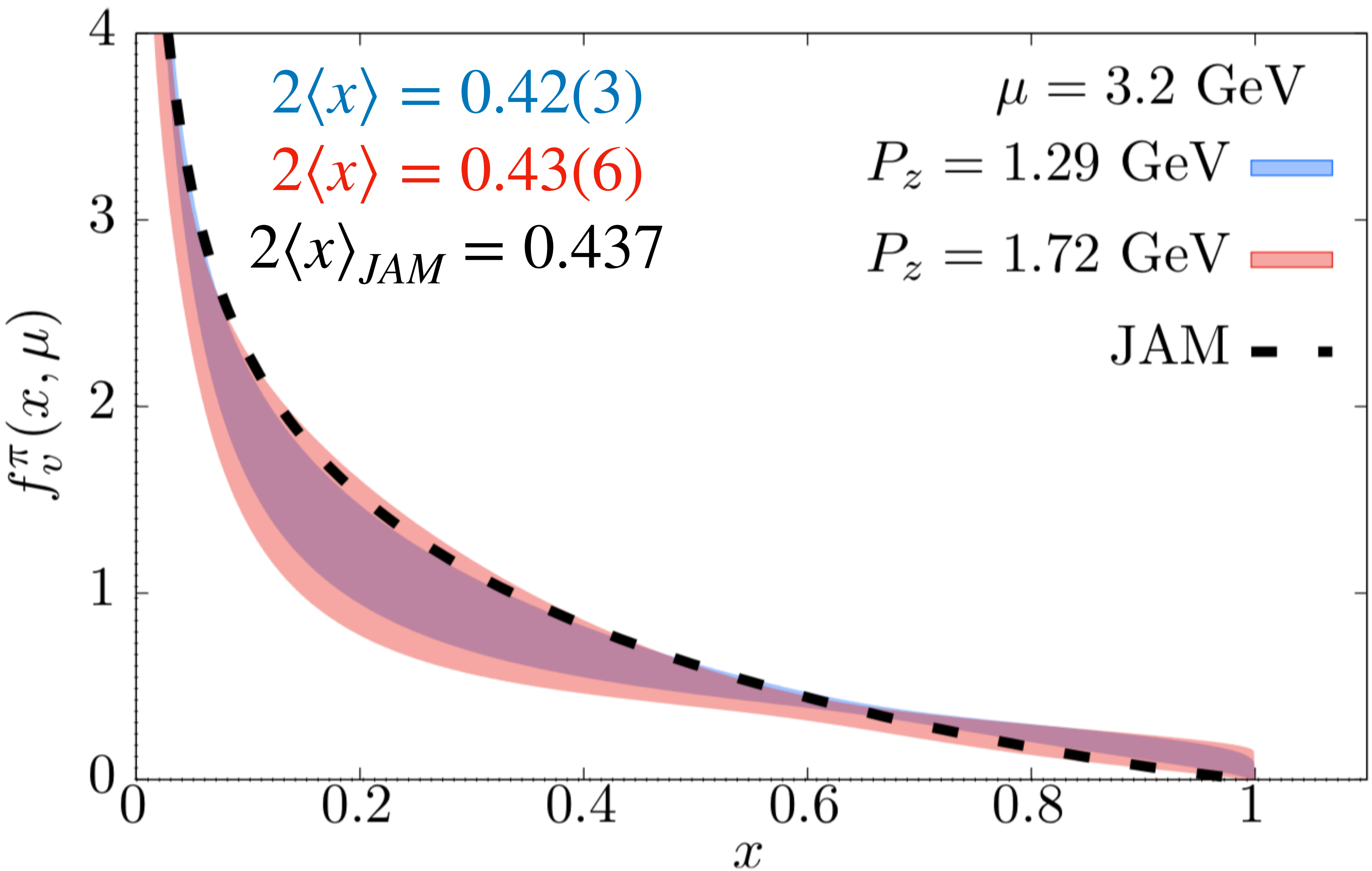
- a=0.06 fm, 48^3 x 64 lattice with HISQ sea quarks and Wilson-Clover valence quarks.
- m_π=300MeV, P_z= 1.29GeV and 1.72GeV are used for **quasi-PDF** analysis.
- Statistics ~216 gauge configurations. All-Mode Averaging using 1 exact and 32 sloppy quark propagators.

Quasi-PDF Analysis: renormalization and matching



$$\tilde{q}(x; P_z, P^R) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) q(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$

Quasi-PDF Analysis: Valence PDF of $\pi^+(u\bar{d})$



$$f_v^\pi(x) = f_u(x) - f_{\bar{u}}(x) = f_u(x) - f_d(x), \quad 0 < x < 1$$

(sea+valence) **(sea)** **(Isospin symmetry)**

Pseudo-PDF & Reduced Ioffe time distribution

Spacial correlator $\tilde{Q}_{\gamma^\mu}$:

$$P^\mu \tilde{Q}_{\gamma^\mu}(\nu, \epsilon) = \frac{1}{2}\langle H(P^z, E)\,|\,\overline{\psi}(0)\gamma^\mu W_{\hat{z}}(0,z)\tau\psi(z)\,|\,H(P^z, E)\rangle$$

μ=z or t

Pseudo-PDF:

$$\mathscr{P}(x, \mu^2 z^2) \equiv \int \frac{d\nu}{2\pi} e^{ix\nu} \tilde{Q}(\nu, \mu^2 z^2) + \mathcal{O}(z^2 m_h^2, z^2 \Lambda_{QCD}^2),\ \ z^2 \to 0 \ \&\ \nu = zP^z$$

Lorentz scalar

Limited by finite Ioffe time $\nu = zP^z$ ($P_z = \frac{2\pi n}{L_s}, n = 0,1,2,3,4,\ldots$), only first few moments visible.

multiplicative renormalized

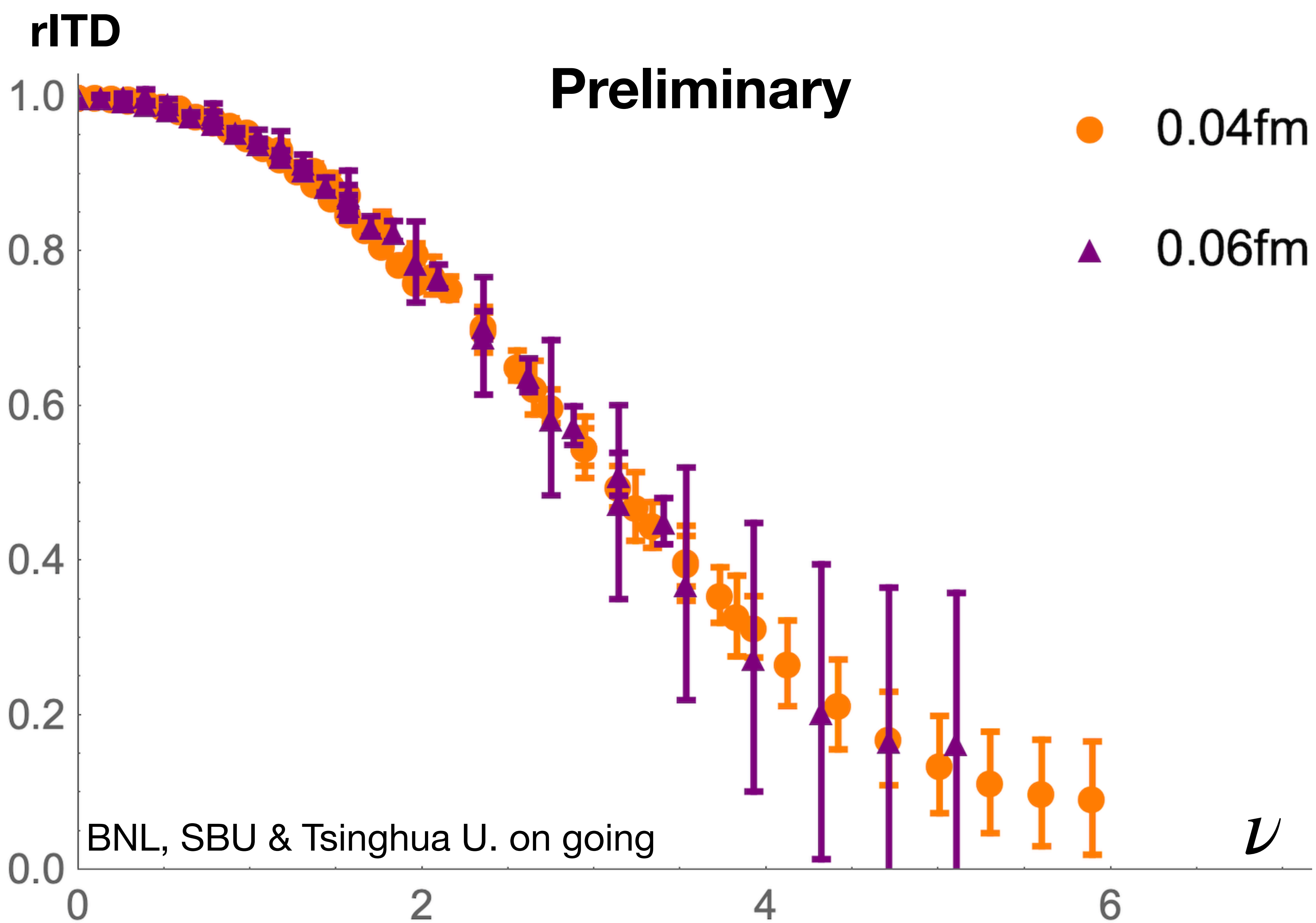
$$rITD(\nu, \mu^2 z^2) = \frac{Z_R \tilde{Q}(\nu, \mu^2 z^2)}{Z_R \tilde{Q}(0, \mu^2 z^2)} = \sum_{n=0} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-i\nu)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(z^2 m_h^2, z^2 \Lambda_{QCD}^2)$$

A. V. Radyushkin arXiv:1705.01488

Don't need renormalization

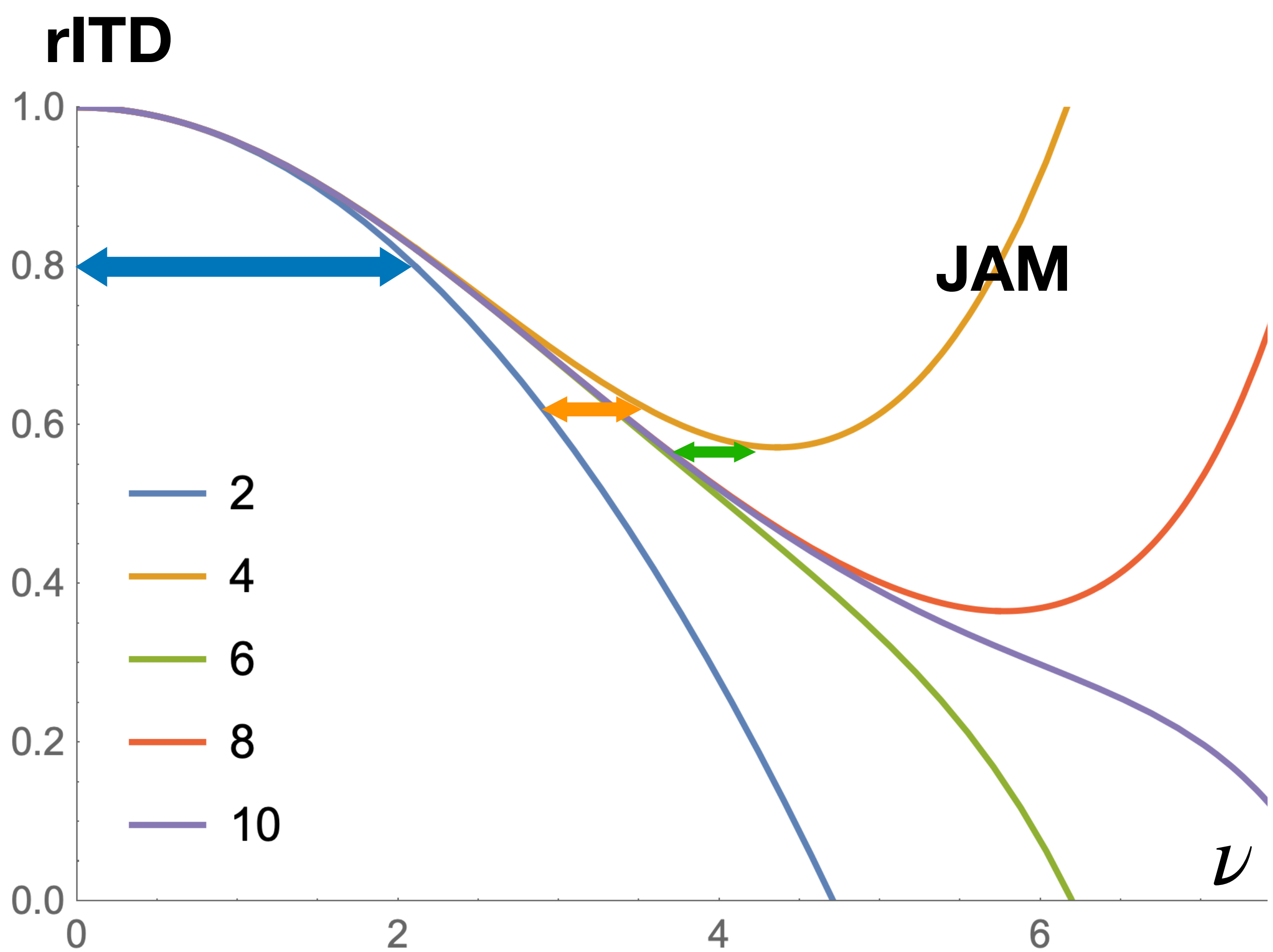
Moments of light-cone PDF

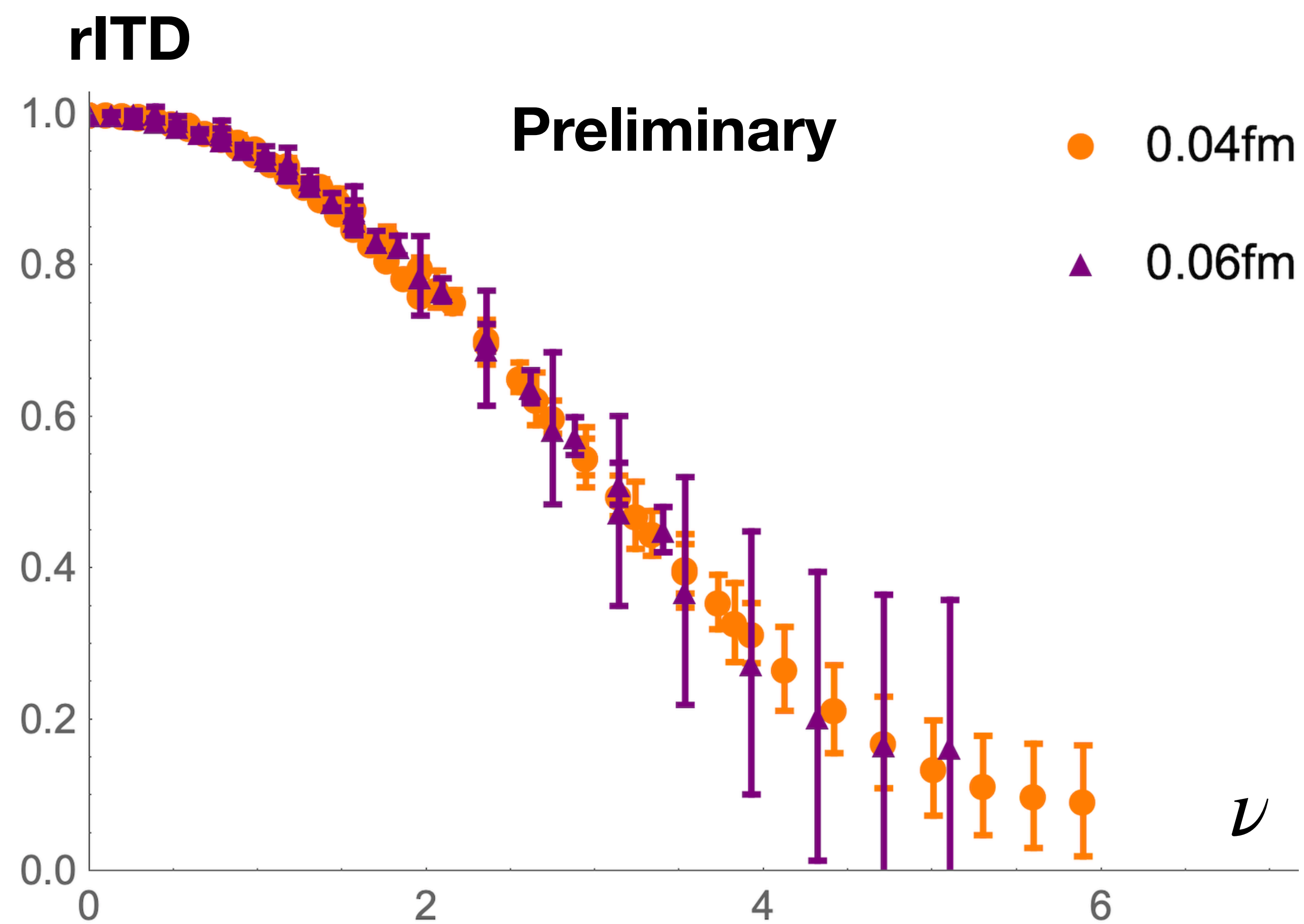
Reduced Ioffe time distribution



- a=0.06fm, $48^3 \times 64$ lattice, m_π=300MeV, $P_z =$ 0 ~ 1.29GeV, Statistics ~ 100 gauge configurations
- a=0.04fm, $64^3 \times 64$ lattice, m_π=300MeV, $P_z =$ 0 ~ 1.48GeV, Statistics ~ 167 gauge configurations
- z < 0.8 fm

Reduced Ioffe time distribution





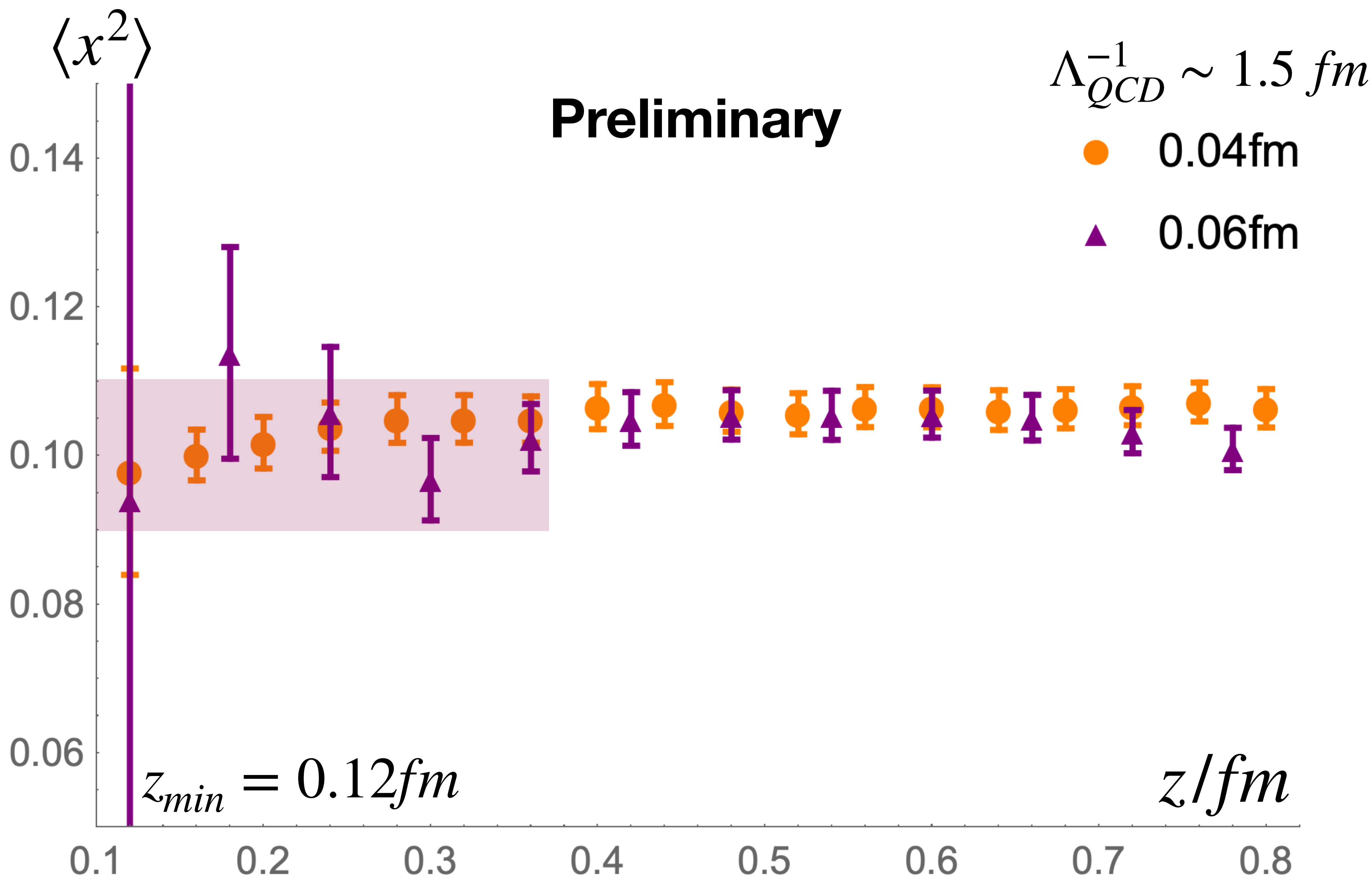
With finite ν, only first few moments visible

- rITD constructed by moments of JAM data.
- z=0.2fm, moments from 2 to 10

Joint fit from z_{min} to z_{max}

$$rITD(zP^z, \mu^2 z^2) = \sum_{n=0} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-izP^z)^n}{n!} \langle x^n \rangle(\mu) + \mathcal{O}(z^2 m_H^2, z^2 \Lambda_{QCD}^2)$$

2nd moment of $f_v^{\pi}(x)$

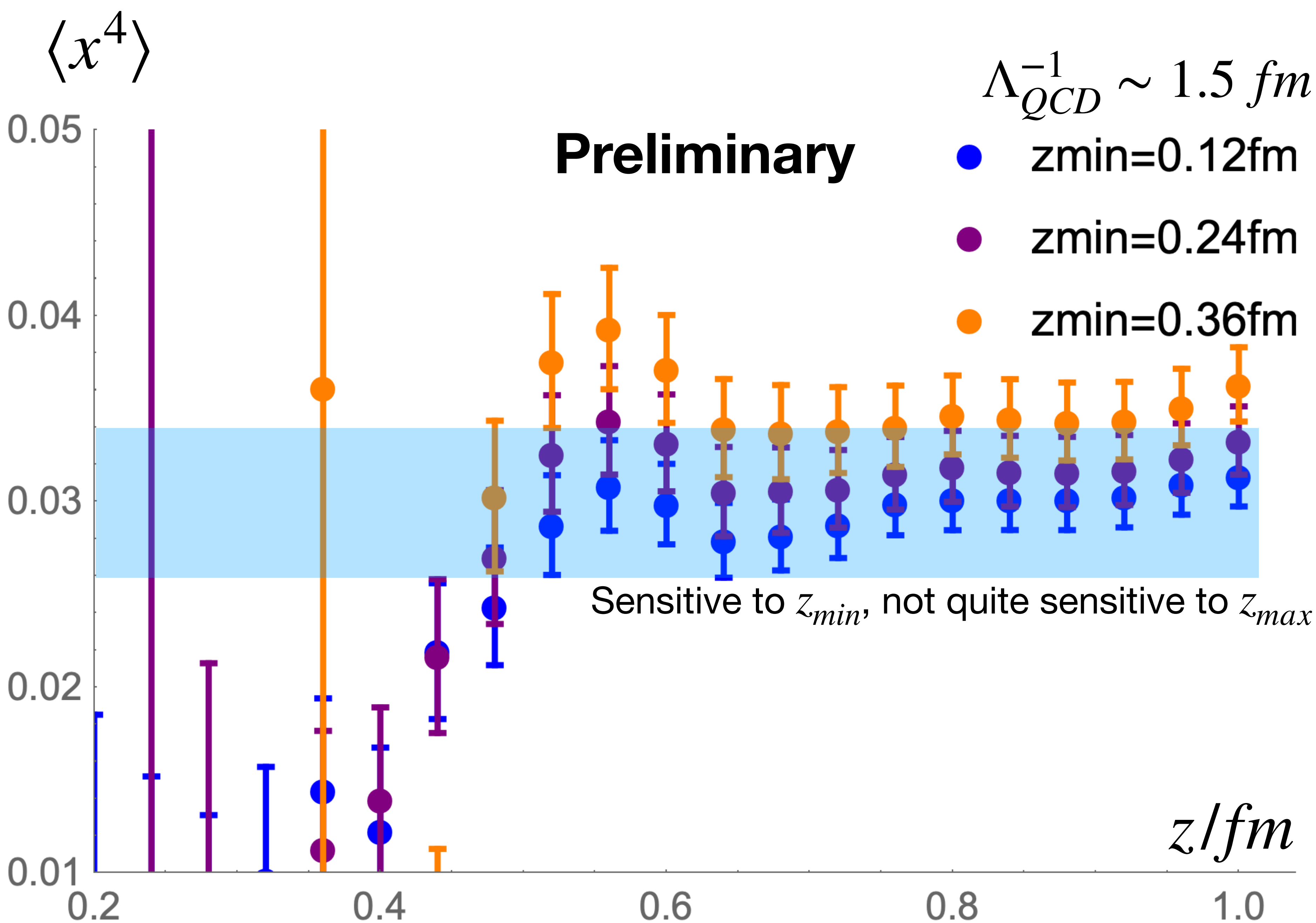


$\langle x^2 \rangle \sim 0.10 + -0.01$

$\langle x^2 \rangle_{JAM} \approx 0.095$

$\mu = 3.2 GeV$

4th moment of $f_v^\pi(x)$

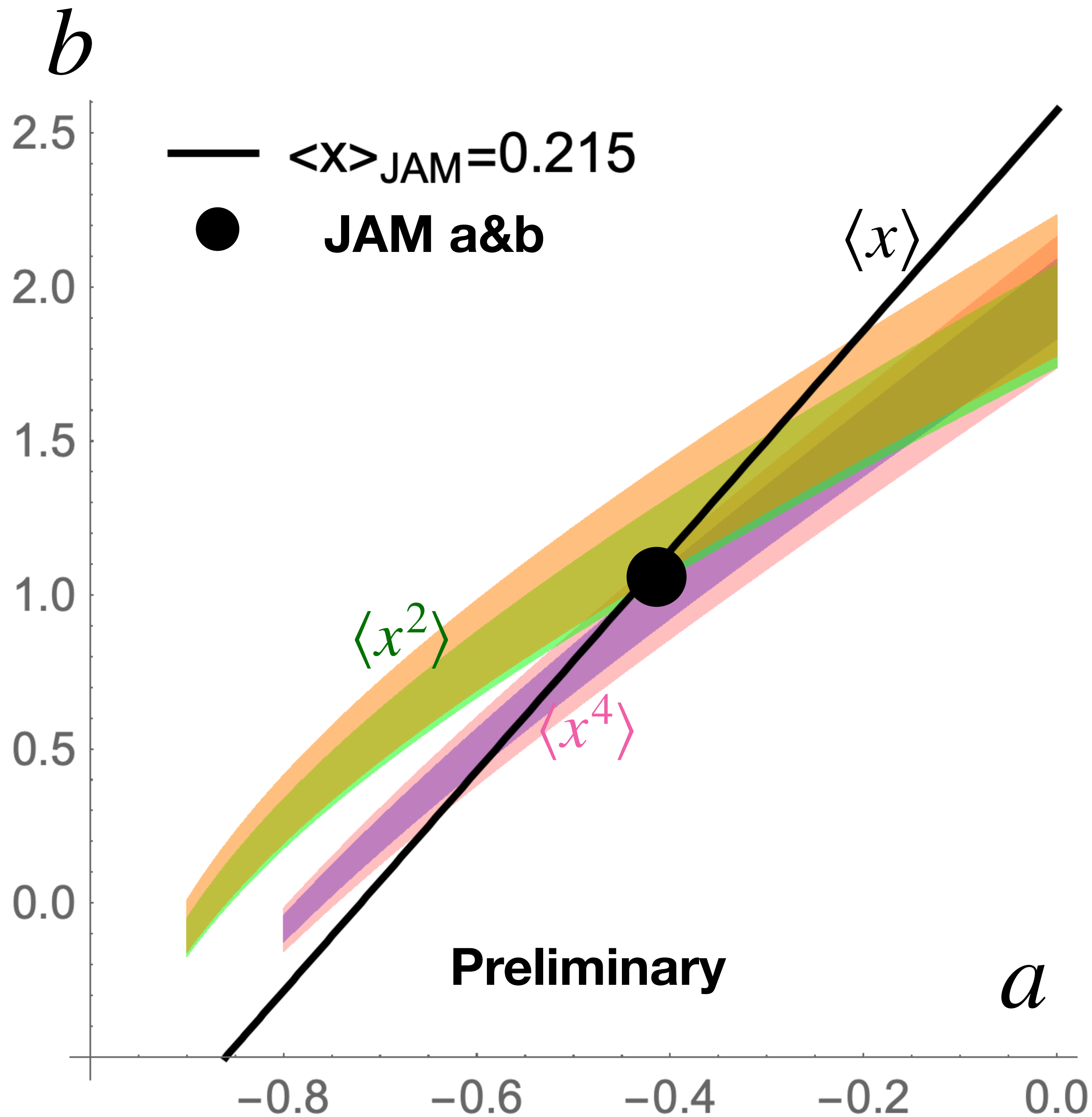


$\langle x^4 \rangle \sim 0.031 + -0.004$

$\langle x^4 \rangle_{JAM} \approx 0.032$

$\mu = 3.2 GeV$

$$f_v^\pi(x,\mu;a,b) = Ax^a(1-x)^b$$



- More precise moments are needed to constrain a&b

Summary

- Quasi-PDF analysis with 0.06fm lattice is studied, and have a good agreement with the JAM data.
- We extracted 2nd and 4th moments from rITD with 0.04fm and 0.06fm lattice, which are also close to the JAM data.
- On going: quasi-PDF analysis of 0.04fm lattice, continuum limit analysis, GPD.