# Study of Low-Lying Baryons with Hamiltonian Effective Field Theory

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# Introduction

# Hadron Physics

Hadron physics is mainly focused on hadron scatterings, spectra, structures, interactions, etc.

- Hadron spectra are obtained from experimental
  Hadron scattering.
- Hadron structures and interactions $\rightleftharpoons$
  Hadron spectra and scattering.

Hadron physics lies in the region of low energies with a large $\alpha_s$, traditional perturbation expansion in series of $(\alpha_s)^n$ cannot work here.

- constituent quark model
- effective field theory—expanded by small momenta
- lattice QCD—discretized QCD
- QCD sum rule—operator product expansion—twist
- large Nc—1/Nc
- ...

# Low-lying Baryons

Much more scattering data on low-lying baryons, $N^*(1440)$, $N^*(1535)$, $\Lambda(1405)$ compared to those for large-mass resonances or charmed hadrons.

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IF: harmonic oscillator form for confinement potential

Then: $E = (2n_r + L + 3/2)\omega$

$N^*(1440)$: $n_r = 1,\ L = 0 \quad \Longrightarrow E = 7/2\omega$

$N^*(1535)$: $n_r = 0,\ L = 1 \quad \Longrightarrow E = 5/2\omega$

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Triquark or pentaquark state?

...

# Lattice QCD

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions
  at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

# Connection between Scattering Data and Lattice QCD Data

## Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

## Lattice QCD Data → Physical Data

- Lüscher Formalisms and extensions:
  Model independent; efficient in single-channel problems
    Spectrum → Phaseshifts; $m_{K_L} - m_{K_S}$ etc.
- Effective Field Theory (EFT), Models, etc
  with low-energy constants fitted by Lattice QCD data

## Physical Data → Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

# Lattice QCD and Effective Field Theory

## Effective field theory deals with extrapolation powerfully.

Guo, Hanhart, Llanes-Estrada, Meißner, *Quark mass dependence of the pion vector form factor*, Phys.Lett.B678:90-96,2009.

## Finite-volume effect can be studied by discretizing the EFT.

Molina, Doring, *Pole structure of the* Λ(1405) *in a recent QCD simulation*, Phys.Rev. D94 (2016) no.5, 056010, Addendum: Phys.Rev. D94 (2016) no.7, 079901

discretize the mass equation (in integral form)
(most of time, potentials are momentum independent.)

Hall, Hsu, Leinweber, Thomas, Young, *Finite-volume matrix Hamiltonian model for a* $\Delta \to N\pi$ *system*, Phys.Rev. D87 (2013) no.9, 094510

discretize the Hamiltonian equation (in differential form)

## Discrete spacing effects can also be studied with EFT.

Ren, Geng, Meng, *Baryon chiral perturbation theory with Wilson fermions up to O(a2) and discretization effects of latest nf=2+1 LQCD octet baryon masses*, Eur.Phys.J. C74 (2014) no.2, 2754

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

# Hamiltonian Effective Field Theory

Hamiltonian Effective Field Theory (HEFT)

analyses both experimental data at infinite volume

and lattice QCD results at finite volume at the same time.

- at infinite volume

  Lagrangian (via 2-particle irreducible diagrams) $\rightarrow$

  potentials (via Betha-Salpeter Equation) $\rightarrow$

  phaseshifts and inelasticities

- at finite volume

  potentials discretized (via Hamiltonian Equation)$\rightarrow$ spectra

  wavefunctions: analyse the structure of the eigenstates on the lattice

- finite-volume and infinite-volume results are connected by the coupling constants etc.

# This Work

We use Hamiltonian effective field theory to analyse the scatterings data at experiment and spectra of lattice QCD which are related to

- $N^*(1535)$
- $N^*(1440)$
- $\Lambda(1405)$

By our analyses, we try to better understand the structures of those resonances and relevant interactions.

# Hamiltonian effective field theory study of the $N^*(1535)$ resonance in lattice QCD

# $N^*(1535)$ **with** $\pi N$ **Scattering**

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions
  among the bare baryon $N_0^*$, $\pi N$ channel, and $\eta N$ channel.

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$$G^2_{\pi N; N_0^*}(k) = \frac{3 g^2_{\pi N; N_0^*}}{4\pi^2 f^2} \omega_\pi(k)$$

$$V^S_{\pi N, \pi N}(k, k') = \frac{3 g^S_{\pi N}}{4\pi^2 f^2} \frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)} \frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')}$$

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$$T_{\alpha,\beta}(k,k';E) = V_{\alpha,\beta}(k,k') + \sum_{\gamma} \int q^2 dq$$
$$V_{\alpha,\gamma}(k,q) \frac{1}{E - \sqrt{m_{\gamma_1}^2 + q^2} - \sqrt{m_{\gamma_2}^2 + q^2} + i\epsilon} T_{\gamma,\beta}(q,k';E)$$

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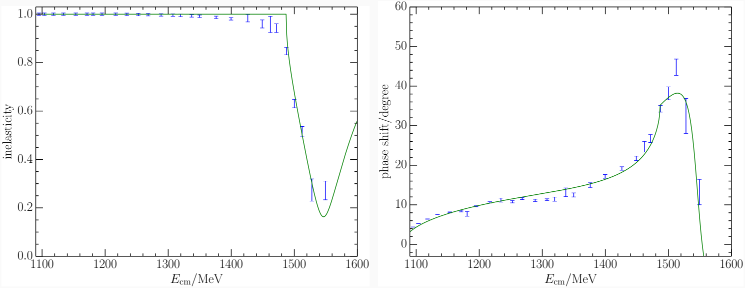


$\pi N$ Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

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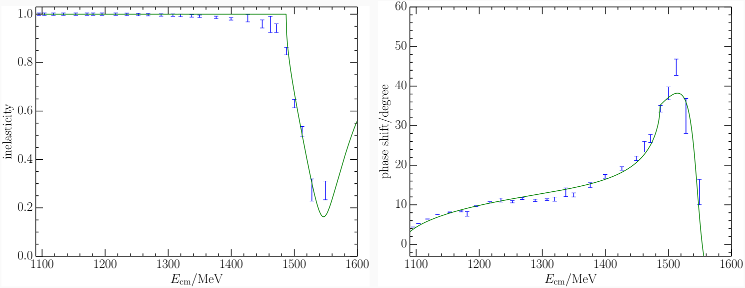


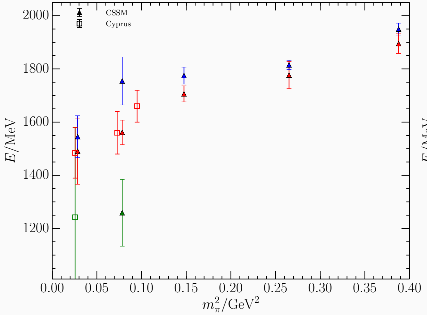

$\pi N$ Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- Pole position for $N^*(1535)$: $1531 \pm 29 - \mathrm{i}\ 88 \pm 2$ MeV.

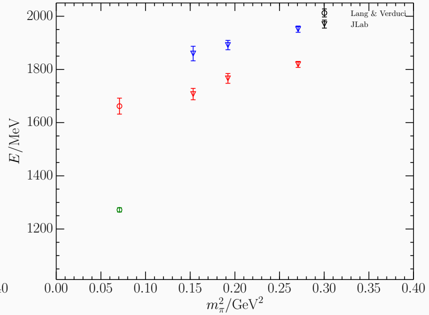
  Particle Data Group (PDG): $1510 \pm 20 - \mathrm{i}\ 85 \pm 40$ MeV.

# Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes




$L \approx 3$ fm



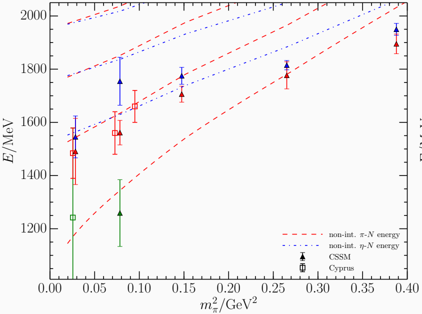

$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

# Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels

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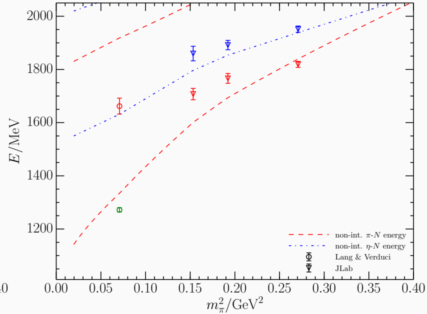


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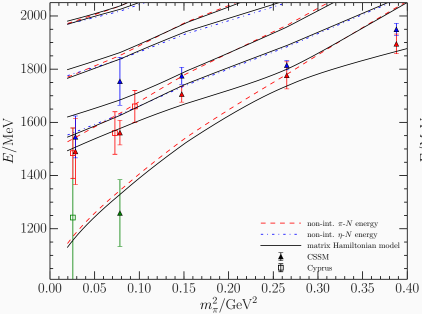
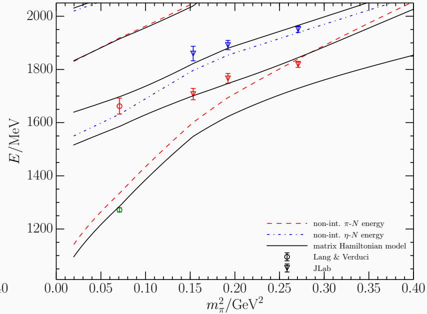
Eigenenergies of Hamiltonian effective field theory




$L \approx 3$ fm

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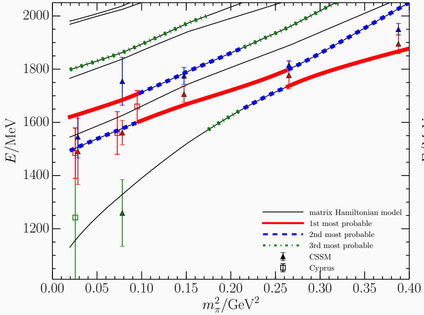
3 sets of lattice data at different pion masses and finite volumes

Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD

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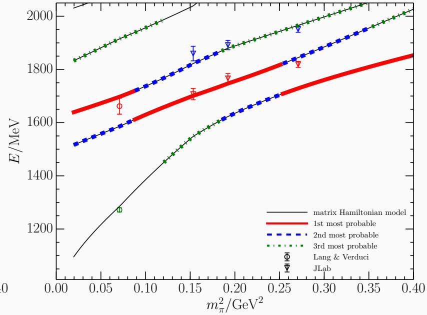


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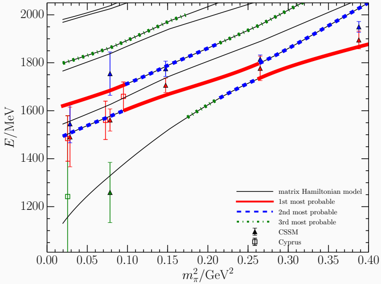
Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

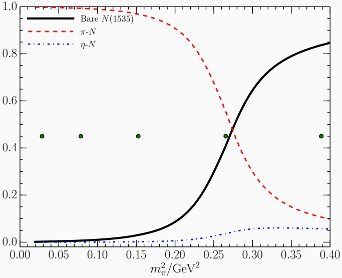
# Components of Eigenstates with $L \approx 3$ fm



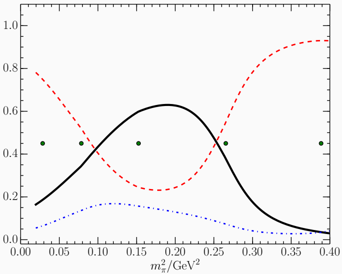

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

- The 1st eigenstate at light quark masses is mainly $\pi N$ scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.
  It contains about 60% bare $N^*(1535)$, 20% $\pi N$ and 20% $\eta N$.

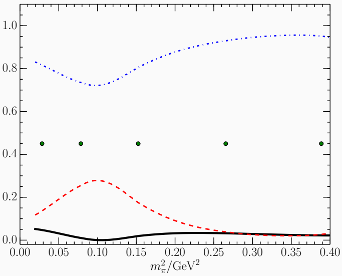
# Components of Eigenstates with $L \approx 3$ fm



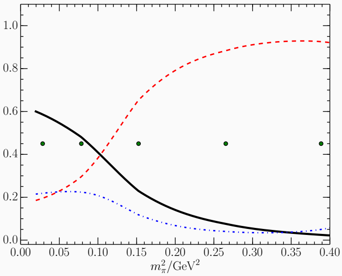

1st eigenstate




2nd eigenstate



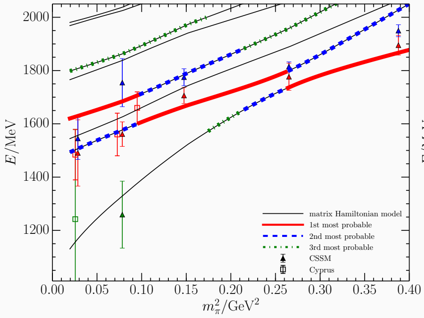

3rd eigenstate




4th eigenstate

# Lattice Results → Experimental Results

- Experimental Data → Lattice QCD Data  We have shown that.
- Lattice QCD Data → Experimental Data  We show it here.

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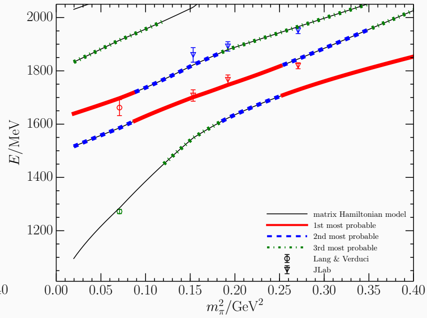


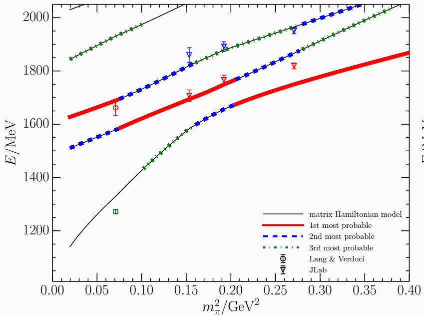

$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and the bare mass is fitted by LQCD data

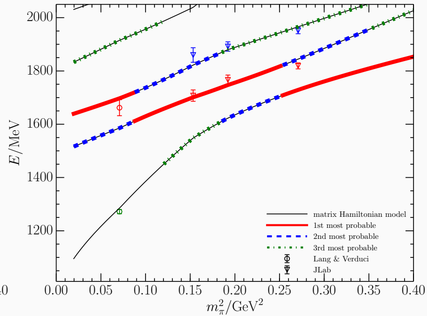
By fitting lattice QCD data, the pole for $N^*(1535)$ at infinite volume lies at $1602 \pm 48 - i\ 88.6^{+0.7}_{-2.8}$ MeV.  PDG: $1510 \pm 20 - i\ 85 \pm 40$.

# Effects of Separable Potentials

fit for lattice QCD data




without separable potential




with separable potential

# Hamiltonian effective field theory study of the $N^*(1440)$ resonance in lattice QCD

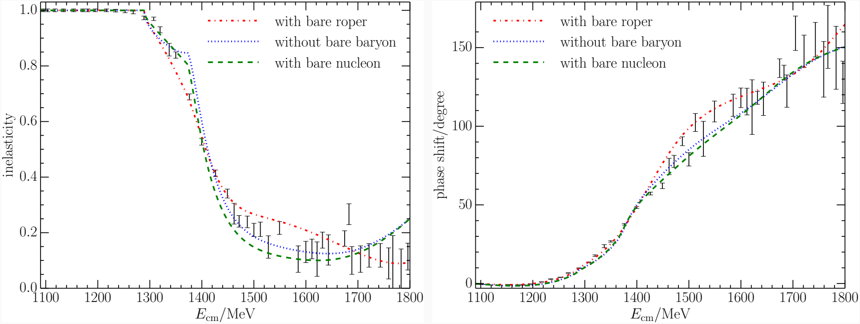
# $N^*(1440)$ Resonance

- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$
  if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

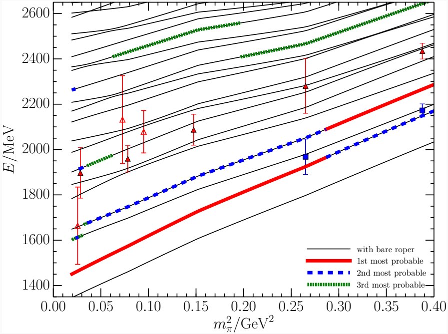
# $N^*(1440)$ Resonance




$\pi N$ scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

# Results of the Model with a Bare Roper

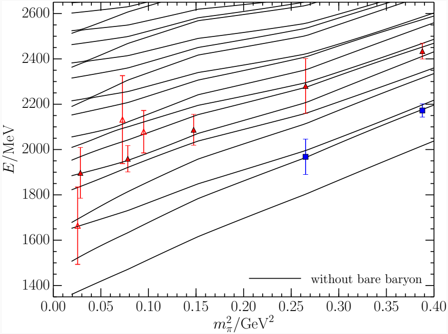



Spectrum given by the scenario with a bare Roper.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

At low pion masses, the 2nd state contains more than 20% bare Roper, so this state should be observed with a 3-quark interpolating operators on the lattice.

But it is not.

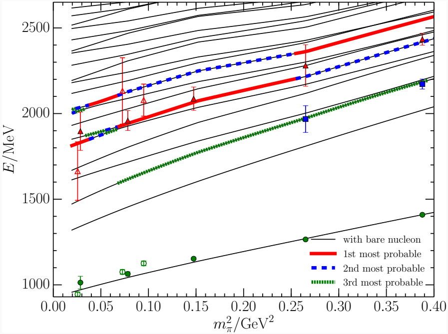
# Results of the Model without Bare Baryons




Spectrum given by the scenario without any bare baryon.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

- The lattice data sit on the eigenenergy spectrum of this model;
- ALTHOUGH it is hard to predict which state is easier to observe on the lattice,
- we notice that lattice QCD prefers to extract eigenstates with non-trivial mixing of scattering states.

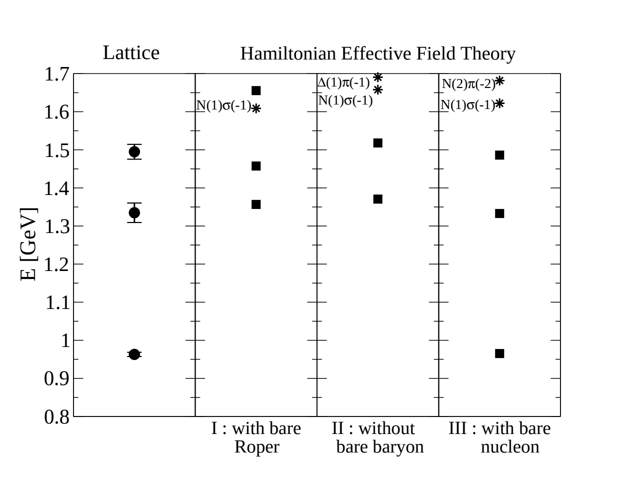
# Including the Effect of the Bare Nucleon




Spectrum given by the scenario with a bare nucleon.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

- The bare nucleon does not affect the spectrum very much compared to the results of the model without any bare baryons;
- We can plot the probability based on the distribution of the bare nucleon;
- It can explain both the experimental data and lattice data.

# Our results are verified




interpolating operators: $N(0)$, $N(0)\sigma(0)$, $N(p)\pi(-p)$, $\Delta(p)\pi(-p)$.

from Lang, Leskovec, Padmanath, Prelovsek, PRD95 (2017) no.1, 014510.

No these two higher states with $N^{-P}(0)\pi(0)$... from CMMS.

# Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory

# $\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

# $\Lambda(1405)$ **with** $K^- p$ **scattering**

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

$$V_{\alpha,\beta}(k,k') = g_{\alpha,\beta}\frac{\omega_{\alpha_M}(k)+\omega_{\beta_M}(k')}{8\pi^2 f^2\sqrt{2\omega_{\alpha_M}(k)}\sqrt{\omega_{\beta_M}(k')}}$$

# $\Lambda(1405)$ with $K^- p$ scattering

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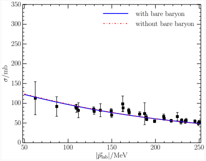
# $\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

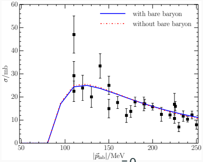
  momentum-dependent, non-separable

- We can fit the cross sections of $K^- p$ well

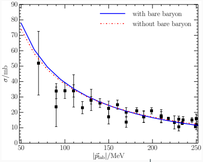
  both with and without a bare baryon.



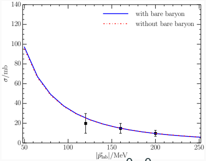

$K^- p \to K^- p$



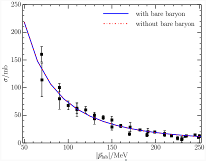

$K^- p \to \bar{K}^0 n$



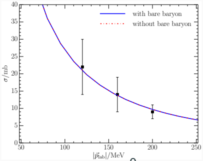

$K^- p \to \pi^- \Sigma^+$




$K^- p \to \pi^0 \Sigma^0$




$K^- p \to \pi^+ \Sigma^-$




$K^- p \to \pi^0 \Lambda$

# $\Lambda(1405)$ with $K^- p$ scattering

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  momentum-dependent, non-separable

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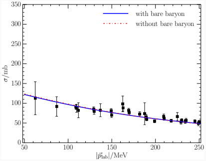
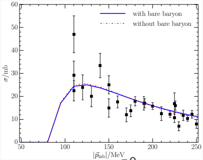
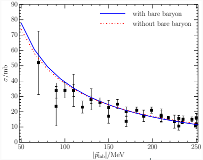


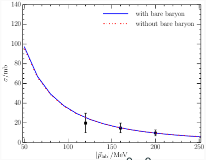

$K^- p \to K^- p$



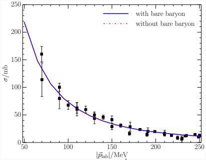

$K^- p \to \bar{K}^0 n$



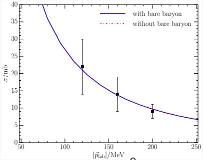

$K^- p \to \pi^- \Sigma^+$




$K^- p \to \pi^0 \Sigma^0$




$K^- p \to \pi^+ \Sigma^-$

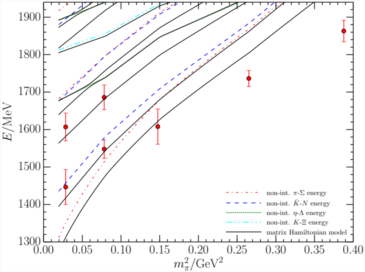



$K^- p \to \pi^0 \Lambda$

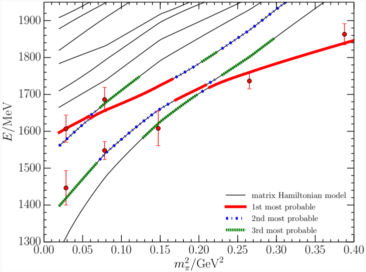
- Two-pole structure of $\Lambda(1405)$

  $1430 - i22$ MeV, $1338 - i89$ MeV

# Spectrum on the Lattice




without a bare baryon




with a bare baryon

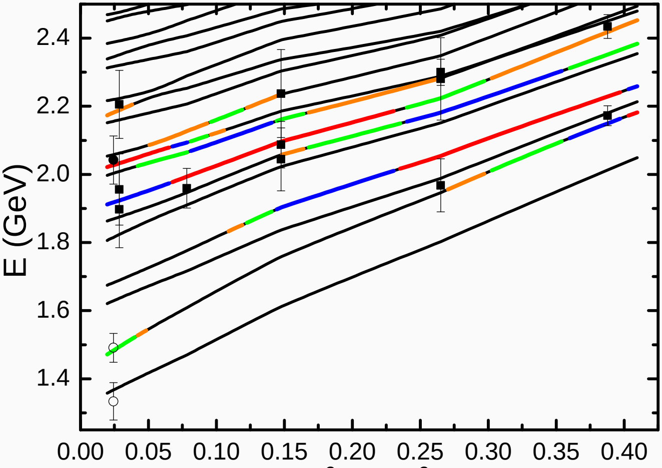
Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.
  The bare state introduces a new pole for Λ(1670) at 1660-30i MeV
- Λ(1405) is mainly a $\bar{K}N$ molecular state
  containing very little of bare baryon at physical pion mass.

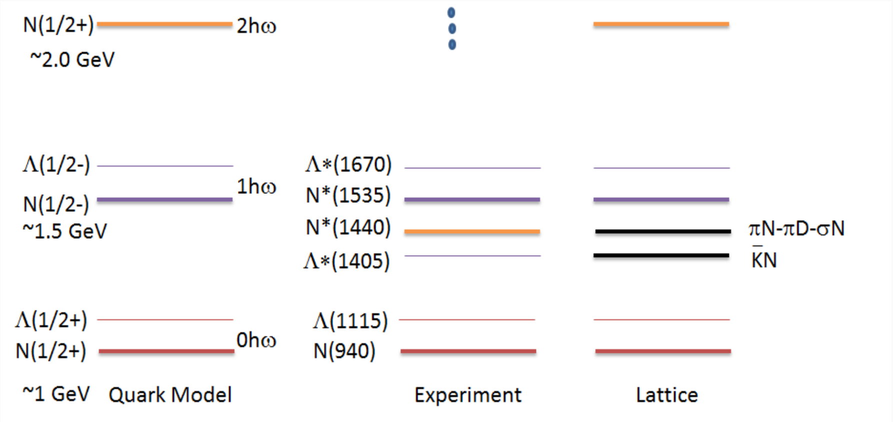
# Summary

## Extension of the Roper work

- effects of a resonance with bare mass/pole around 2 GeV
- constituent quark model w/ harmonic oscillator potential predicts mass of first radially excited nucleon is approximately 2 GeV



# Quark model states + dynamically generated states



# Summary

We have analysed the scattering data at experiment and the lattice spectra on the lattice relevant to $N^*(1440)$, $N^*(1535)$, and $\Lambda(1405)$ with Hamiltonian effective field theory

- $N^*(1535)$ contains a 3-quark core;
- $N^*(1440)$ should contain little of 3-quark consistent;
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state at physical quark mass, while a 3-quark core dominates at large quark masses.

Thanks!

This report is mainly based on the following works:

Phys.Rev. D97 (2018) no.9, 094509
Phys.Rev. D95 (2017) no.1, 014506
Phys.Rev. D95 (2017) no.3, 034034
Phys.Rev.Lett. 116 (2016) no.8, 082004