HYPERON RESONANCES AND MESON-BARYON INTERACTIONS

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More information: Khemchandani, Martinez Torres, Oller, Phys.Rev. C100 (2019), 015208



HIGHLIGHTS

- Focus on isospin light isospin=1 hyperons
- We use lowest order Lagrangians and couple channels with vector and pseudoscalar mesons.
- Fix the parameters of the model using experimental data
- Perform a statistical comparison with results from model using NLO terms
- Find evidences for a possible existence of a light Σ (mass around 1400 MeV).

 Λ(1405) is widely accepted a baryon arising from meson-baryon multichannel dynamics [very long list of references: Kaiser, Siegel, Weise, NPA 594, 325 (1995), Oset, Ramos, NPA 635 (1998) 99, Oller, Meißner, PLB 500, 263 (2001), Jido, Oller, Oset, Ramos, Meißner, NPA 725, 181 (2003) Hyodo, Jido, Prog. Part Nucl. Phys. 67, 55 (2012), Hyodo, Weise, PRC98 (2018), C. Wang, L. I. Liu and X. H. Guo, PRD 96, (2017), Z. W. Liu, J. M. M. Hall, D. B. Leinweber, A.W.Thomas and J. J. Wu, PRD 95, (2017).Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset and W.Weise, Nucl. Phys. A 954, 41 (2016). R. Molina and M. Doring, PRD94 (2016)]

 Efforts from lattice community [Hall, Kamleh, Leinweber, Menadue, Owen, Thomas, Young, PRL 114 (2015), Menadue, Kamleh, Leinweber, Mahbub, PRL 108 (2012), Ishii, Doi, Oka, Suganuma, PTP Suppl. 168, 598 (2007), Takahashi and Oka, PTP Suppl. 186, 172 (2010), Hall, Kamleh, Leinweber, Menadue, Owen, Thomas, PRD95 (2017), Gubler, Takahashi and Oka, PRD94 (2016), Briceno, Dudek, Young, Rev. Mod. Phys. 90 (2018)]

Well studied, very relevant topic!

Strange meson-baryon dynamics:

- Important to understand the properties of hyperons
- Possibility of describing $\Lambda(1405)$ as meson-nucleon resonance raises question on the existence of kaonic-nuclear bound states
- Useful for studies of interactions of kaons in nuclear medium
- Does an isovector partner of $\Lambda(1405)$ exists?

Evidences of the existence of a Σ with mass around 1400 MeV:

- A coupled channel study of pseudoscalar-baryon systems, using chiral Lagrangian, based on s-, t-, uchannel diagrams, by fixing the model parameters using data on different $K^-p \rightarrow K^-N$, $\pi\Sigma$, $\pi\Lambda$ processes(Oller, Meißner, PLB 500, 263 (2001)). 1440 – i70 MeV and 1420 – i42 MeV
- Same work was extended later using NLO Lagrangian and by including data on by including data on energy shift and width of the 1s-state in kaonic hydrogen+ data on $K^-p \rightarrow \eta \Lambda$, $\pi^0 \pi^0 \Sigma$ (Guo, Oller, PRC87, 035202 (2013)). Two fits: one corresponding to two Σ s (1376 i33 MeV and 1414 i12) and other none (disfavored).
- Best fit to data on $\gamma + p \rightarrow K^+ + \Sigma^{\pm,0} + \pi^{\mp,0}$ included two Σ s (1413 ± 10) i(26 ± 5) MeV and (1394 ± 20) i(75 ± 20) MeV, (CLAS Collab, PRC 87, 035206 (2013).)
- Theoretical analysis of data on processes: K⁻p → Λπ⁺π⁻, γN → K⁺πΛ, Λp → Λpπ⁰, Λ_c⁺ → ηπ⁺Λ seems to require an isospin resonance with mass around 1380 MeV and width around 60 MeV. (Wu, Dulat, Zou, PRD 80, 017503 (2009), Wu, Dulat, Zou, PRC 81, 045210 (2010), Gao, Wu, Zou, PRC 81, 055203 (2010), Xie, Wu, Zou, PRC 90, 055204 (2014), Xie, Geng, PRD 95, 074024 (2017)).

Evidence not found/not reported:

- A partial-wave analysis (s-/p-wave) of S = -1 low-energy data, including differential cross sections does not find any evidence for Σs around 1400 MeV. But only PB contact interactions taken in to account (Sadasivan, Mai, Döring, PLB789 (2019) 329).
- A study of S=-I coupled systems including constraints from photoproduction date from CLAS does not discuss isovector states (*Mai*, *Meißner*, EPJA 51, 30 (2015)).
- A different analysis of photoproduction data reports finding of a cusp around \overline{KN} threshold (*Roca*, Oset, Phys. Rev. C 88, 055206 (2013).)

- In a previous work, PB-VB coupled channel calculations(Khemchandani, Martinez Torres, Nagahiro and Hosaka, PRD 85, 114020 (2012), we found Σs deep in the complex plane (1427 – i145 MeV, 1438 – i198 MeV).
- But no constraints were made using experimental data.
- Only PB contact interactions taken into account.
- We now include more sources (s-, u-channel)
- Importance of s-, u-channel diagrams also brought forward by Ramos, Feijoo Magas in NPA 954, 58 (2016).
- VB channels do not have large weights in the wave functions but may affect the pole position (some VB thresholds close to PB thresholds, eg $K\Xi$, $\rho\Lambda$).

- Consider all meson-baryon channels with strangeness I composed of pseudoscalar and vector mesons K̄N, KΞ, πΣ, ηΛ, πΛ, ηΣ, K̄*N, K*Ξ, ρΣ, ωΛ, φΛ, ρΛ, ωΣ, φΣ as coupled channels.
- Consider a contact interaction, s-, t-, and u-channel diagrams for VB channels and latter three for PB channels.
 K₁



• PB Lagrangian:

•
$$\mathscr{L}_{PB} = \langle \bar{B}i\gamma^{\mu}\partial_{\mu}B + \bar{B}i\gamma^{\mu}[\Gamma_{\mu}, B] \rangle - M_{B}\langle \bar{B}B \rangle + \frac{1}{2}D'\langle \bar{B}\gamma^{\mu}\gamma_{5}\{u_{\mu}, B\} \rangle + \frac{1}{2}F'\langle \bar{B}\gamma^{\mu}\gamma_{5}[u_{\mu}, B] \rangle$$

$$u_{\mu} = iu^{\dagger}\partial_{\mu}Uu^{\dagger}, \ \Gamma_{\mu} = \frac{1}{2}\left(u^{\dagger}\partial_{\mu}u + u\partial_{\mu}u^{\dagger}\right), \ U = u^{2} = \exp\left(i\frac{P}{f_{P}}\right). \ D'=0.8,$$
$$F' = 0.46$$

$$P = \begin{pmatrix} \pi^{0} + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^{+} & \sqrt{2}K^{+} \\ \sqrt{2}\pi^{-} & -\pi^{0} + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^{0} \\ \sqrt{2}K^{-} & \sqrt{2}\bar{K}^{0} & \frac{-2}{\sqrt{3}}\eta \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^{0} & \Sigma^{+} & p \\ \Sigma^{-} & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^{0} & n \\ \Xi^{-} & \Xi^{0} & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$

• VB Lagrangian:

$$\mathscr{L}_{\mathsf{V}B} = -g \left\{ \langle \bar{B}\gamma_{\mu} \left[V_{8}^{\mu}, B \right] \rangle + \langle \bar{B}\gamma_{\mu}B \rangle \langle V_{8}^{\mu} \rangle + \frac{1}{4M} \left(F \langle \bar{B}\sigma_{\mu\nu} \left[V_{8}^{\mu\nu}, B \right] \rangle + D \langle \bar{B}\sigma_{\mu\nu} \left\{ V_{8}^{\mu\nu}, B \right\} \rangle + \langle \bar{B}\gamma_{\mu}B \rangle \langle V_{0}^{\mu} \rangle + \frac{C_{0}}{4M} \langle \bar{B}\sigma_{\mu\nu}V_{0}^{\mu\nu}B \rangle \rangle \right\}$$

•
$$\mathscr{L}_{V_0BB} = -g\left\{\langle \bar{B}\gamma_{\mu}B\rangle\langle V_0^{\mu}\rangle + \frac{C_0}{4M}\langle \bar{B}\sigma_{\mu\nu}V_0^{\mu\nu}B\rangle\right\}, D=2.4, F=0.82$$

• $V^{\mu\nu} = \partial^{\mu}V^{\nu} - \partial^{\nu}V^{\mu} + ig\left[V^{\mu}, V^{\nu}\right]$
 $V^{\mu} = \frac{1}{2}\begin{pmatrix}\rho^0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*^+}\\\sqrt{2}\rho^- & -\rho^0 + \omega & \sqrt{2}K^{*^0}\\\sqrt{2}K^{*^-} & \sqrt{2}\bar{K}^{*^0} & \sqrt{2}\phi\end{pmatrix}^{\mu}$

• PBVB Lagrangian:

•
$$\mathscr{L}_{\text{PBVB}} = \frac{-ig_{PBVB}}{2f_{v}} \left(F'\langle \bar{B}\gamma_{\mu}\gamma_{5}\left[[P, V^{\mu}], B \right] \rangle + D'\langle \bar{B}\gamma_{\mu}\gamma_{5}\left\{ [P, V^{\mu}], B \right\} \rangle \right)$$

0

 All amplitudes are projected on s-wave and used as an input in the equation T=V+VGT

$$G(\sqrt{s}) = i2M \int \frac{d^4q}{2\pi^4} \frac{1}{(\tilde{P} - q)^2 - M^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon}$$

$$= \frac{2M}{16\pi^2} \left\{ a(\mu) + \ln \frac{M^2}{\mu^2} + \frac{m^2 - M^2 + s}{2s} \ln \frac{m^2}{M^2} + \frac{\tilde{q}}{\sqrt{s}} \left[\ln \left(s - (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) + \ln \left(s + (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) - \ln \left(-s + (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) - \ln \left(s - (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) \right] \right\}$$

$$- \ln \left(-s + (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) - \ln \left(s - (M^2 - m^2) + 2\tilde{q}\sqrt{s} \right) \right] \right\}$$

Must be

regularized, Subtraction constants

Parameters:

- 14 subtraction constants
- decay constants (one for pseudoscalars, one for vectors)
- PBVB coupling

DATA:

- Total cross sections on (175 data points) (Landolt and Börsntein, Numerical data and Functional Relationships in Science and Technology, Group I,Volume 12, Sub-volume a, Total-Cross Sections for reactions of high energy physics) K⁻p → K⁻p, K⁰n, ηΛ, π⁰Λ π⁰Σ⁰, π[±]Σ[∓]
- Kaonic hydrogen (Siddharta collaboration) $\Delta E = 283 \pm 36 \pm 6$ eV and $\Gamma = 549 \pm 89 \pm 22$ eV (*M. Bazzi et al.*, PLB 704, 113 (2011))
- Cross section ratios near threshold

$$\gamma = \frac{\sigma(K^- p \to \pi^+ \Sigma^-)}{\sigma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.12,$$
$$R_c = \frac{\sigma(K^- p \to \text{charged particles})}{\sigma(K^- p \to \text{all})} = 0.664 \pm 0.033,$$
$$R_n = \frac{\sigma(K^- p \to \pi^0 \Lambda)}{\sigma(K^- p \to \text{all neutral states})} = 0.189 \pm 0.015,$$

•
$$\chi^2$$
-fit
 $\chi^2_{\text{d.o.f}} = \frac{\sum\limits_{k=1}^N n_k}{N(\sum\limits_{k=1}^N n_k - n_p)} \sum\limits_{k=1}^N \frac{\chi_k^2}{n_k} \qquad \chi_k^2 = \sum\limits_{i=1}^{n_k} \frac{(y_{k;i}^{\text{th}} - y_{k;i}^{\text{exp}})^2}{\sigma_{k;i}^2}$

N is no. of different data sets, n_k is the no. of data points in the nth data set, n_p is the number of parameters

 $\chi^2 \leqslant \chi_0^2 + \sqrt{2\chi_0^2},$

 $(\chi^2 - n.d.o.f)/(\sqrt{2 n.d.o.f})$ has mean 0, with st deviation 1.

FITS

Parameters	Fit I	Fit II	Parameters	Fit I	Fit II	Parameters	Fit I	Fit II
$a_{ar{K}N}$	-2.00 ± 0.06	-2.12 ± 0.10	$a_{\bar{K}^*N}$	-4.34 ± 0.08	-4.39 ± 0.09	$a_{\omega\Sigma}$	-3.55 ± 1.58	-3.65 ± 1.34
$a_{K\Xi}$	-2.43 ± 0.04	-2.43 ± 0.06	$a_{K^*\Xi}$	-3.86 ± 0.03	-3.33 ± 0.06	$a_{\phi\Sigma}$	-4.67 ± 0.29	-2.51 ± 0.39
$a_{\pi\Sigma}$	-1.09 ± 0.07	-1.18 ± 0.12	$a_{ ho\Sigma}$	1.17 ± 1.29	-2.36 ± 0.07	f_P (MeV)	94.62 ± 1.46	97.24 ± 1.56
$a_{\eta\Lambda}$	-1.25 ± 0.03	-1.27 ± 0.09	$a_{\omega\Lambda}$	-6.50 ± 0.70	-3.86 ± 2.09	$f_v \ ({ m MeV})$	138.12 ± 1.54	113.46 ± 5.21
$a_{\pi\Lambda}$	-0.84 ± 0.26	-1.69 ± 0.31	$a_{\phi\Lambda}$	-6.83 ± 0.60	-5.22 ± 1.13	g_{PBVB}	2.19 ± 0.09	1.81 ± 0.07
$a_{\eta\Sigma}$	-3.62 ± 0.44	-1.97 ± 0.12	$a_{ ho\Lambda}$	-0.77 ± 0.20	-0.49 ± 0.47			





	$\Lambda(14)$	405)	$\Lambda(1670)$	$\Lambda(1800)$
Fit I	$1373^{\pm 5} - i114^{\pm 9}$	$1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
Fit II	$1385^{\pm 5} - i124^{\pm 10}$	$1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	_
ĒN	$0.84^{\pm 0.14} - i1.91^{\pm 0.06}$	$2.44^{\pm 0.05} + i0.69^{\pm 0.08}$	$0.33^{\pm 0.02} - i0.38^{\pm 0.03}$	$0.14^{\pm 0.05} - i 0.12^{\pm 0.07}$
KN	$0.66^{\pm 0.35} - i1.93^{\pm 0.12}$	$2.43^{\pm 0.16} + i 0.63^{\pm 0.23}$	$0.15^{\pm 0.06} - i 0.19^{\pm 0.13}$	
	$-0.51^{\pm 0.05} + i 0.49^{\pm 0.06}$	$0.59^{\pm 0.09} - i 0.19^{\pm 0.04}$	$2.74^{\pm 0.26} + i 0.25^{\pm 0.22}$	$1.26^{\pm 0.60} - i0.39^{\pm 0.28}$
КΞ	$-0.55^{\pm0.13} + i0.27^{\pm0.06}$	$0.72^{\pm 0.14} - i0.14^{\pm 0.08}$	$0.33^{\pm 0.64} + i 0.28^{\pm 0.34}$	
-27	$-2.04^{\pm 0.07} + i 2.29^{\pm 0.08}$	$-0.87^{\pm 0.06} - i1.05^{\pm 0.09}$	$0.27^{\pm 0.02} + i 0.42^{0.06}$	$0.09^{\pm 0.05} - i 0.14^{\pm 0.07}$
$\pi \Sigma$	$-2.05^{\pm 0.11} + i 2.27^{\pm 0.09}$	$-0.90^{\pm 0.08} - i0.96^{\pm 0.15}$	$-0.11^{\pm 0.20} - i 0.13^{\pm 0.35}$	—
~ 1	$-0.71^{\pm 0.07} - i1.24^{\pm 0.04}$	$2.45^{\pm 0.05} + i 0.21^{\pm 0.04}$	$-0.83^{\pm 0.14} + i0.11^{\pm 0.08}$	$-0.50^{\pm 0.23} + i 0.49^{\pm 0.24}$
ηπ	$-0.80^{\pm 0.10} - i1.24^{\pm 0.06}$	$2.34^{\pm 0.13} + i 0.16^{\pm 0.04}$	$-0.19^{\pm 0.10} - i0.20^{\pm 0.06}$	—
$\bar{k}^* M$	$0.86^{\pm 0.08} - i0.04^{\pm 0.10}$	$-0.16^{\pm 0.10} + i 0.26^{\pm 0.03}$	$-0.18^{\pm 0.08} - i0.05^{\pm 0.03}$	$-0.15^{\pm 0.11} + i 0.05^{\pm 0.04}$
ΙΛ ΙΝ	$0.62^{\pm 0.28} - i 0.18^{\pm 0.14}$	$0.04^{\pm 0.36} + i 0.23^{\pm 0.19}$	$0.50^{\pm 0.92} + i 0.01^{\pm 0.10}$	—
$V^* \Xi$	$1.23^{\pm 0.11} - i0.08^{\pm 0.09}$	$-0.36^{\pm 0.12} + i 0.42^{\pm 0.05}$	$-2.05^{\pm 0.25} + i 0.22^{\pm 0.13}$	$1.01^{\pm 0.47} + i 0.22^{\pm 0.18}$
	$1.17^{\pm 0.12} - i 0.40^{\pm 0.12}$	$0.00^{\pm 0.19} + i 0.44^{\pm 0.08}$	$1.04^{\pm 2.99} - i 0.19^{\pm 0.30}$	—
20	$0.16^{\pm 0.11} + i 0.29^{\pm 0.07}$	$-0.24^{\pm 0.09} - i0.01^{\pm 0.02}$	$0.23^{\pm 0.16} - i0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
pД	$0.57^{\pm 0.24} + i0.41^{\pm 0.19}$	$-0.47^{\pm 0.43} + i 0.03^{\pm 0.18}$	$-1.76^{\pm 2.58} + i 0.10^{\pm 0.37}$	—
	$-0.26^{\pm 0.03} + i 0.28^{\pm 0.03}$	$-0.37^{\pm 0.02} - i0.15^{\pm 0.02}$	$0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
$\omega \Lambda$	$-0.23^{\pm 0.10} + i0.33^{\pm 0.06}$	$-0.45^{\pm 0.09} - i0.16^{\pm 0.07}$	$-0.32^{\pm 0.71} + i 0.05^{\pm 0.08}$	—
41	$0.46^{\pm 0.07} - i 0.44^{\pm 0.06}$	$0.62^{\pm 0.05} + i 0.25^{\pm 0.03}$	$-0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
ψn	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13}$	$0.82^{\pm 0.30} + i 0.29^{\pm 0.19}$	$0.60^{\pm 1.19} - i 0.09^{\pm 0.14}$	_

	$\Lambda(14)$	405)	$\Lambda(1670)$	$\Lambda(1800)$
Fit I	$1373^{\pm 5} - i114^{\pm 9}$	$1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
Fit II	$1385^{\pm 5} - i124^{\pm 10}$	$1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	
	$0.84^{\pm 0.14} - i1.91^{\pm 0.06}$	$2.44^{\pm 0.05} + i0.69^{\pm 0.08}$	$0.33^{\pm 0.02} - i0.38^{\pm 0.03}$	$0.14^{\pm 0.05} - i 0.12^{\pm 0.07}$
KN	$0.66^{\pm 0.35} - i1.93^{\pm 0.12}$	$2.43^{\pm 0.16} + i 0.63^{\pm 0.23}$	$0.15^{\pm 0.06} - i 0.19^{\pm 0.13}$	
	$-0.51^{\pm 0.05} + i0.49^{\pm 0.06}$	$0.59^{\pm 0.09} - i 0.19^{\pm 0.04}$	$2.74^{\pm 0.26} + i 0.25^{\pm 0.22}$	$1.26^{\pm 0.60} - i0.39^{\pm 0.28}$
КΞ	$-0.55^{\pm 0.13} + i 0.27^{\pm 0.06}$	$0.72^{\pm 0.14} - i0.14^{\pm 0.08}$	$0.33^{\pm 0.64} + i 0.28^{\pm 0.34}$	
5	$-2.04^{\pm 0.07} + i 2.29^{\pm 0.08}$	$-0.87^{\pm 0.06} - i1.05^{\pm 0.09}$	$0.27^{\pm 0.02} + i 0.42^{0.06}$	$0.09^{\pm 0.05} - i 0.14^{\pm 0.07}$
$\pi\Sigma$	$-2.05^{\pm 0.11} + i 2.27^{\pm 0.09}$	$-0.90^{\pm 0.08} - i0.96^{\pm 0.15}$	$5 - 0.11^{\pm 0.20} - i 0.13^{\pm 0.35}$	_
	$-0.71^{\pm 0.07} - i1.24^{\pm 0.04}$	$2.45^{\pm 0.05} + i 0.21^{\pm 0.04}$	$-0.83^{\pm 0.14} + i 0.11^{\pm 0.08}$	$-0.50^{\pm 0.23} + i 0.49^{\pm 0.24}$
$\eta\Lambda$	$-0.80^{\pm 0.10} - i1.24^{\pm 0.06}$	$2.34^{\pm 0.13} + i 0.16^{\pm 0.04}$	$-0.19^{\pm 0.10} - i0.20^{\pm 0.06}$	—
τ	$0.86^{\pm 0.08} - i 0.04^{\pm 0.10}$	$-0.16^{\pm 0.10} + i 0.26^{\pm 0.03}$	$^{3} -0.18^{\pm 0.08} - i 0.05^{\pm 0.03}$	$-0.15^{\pm 0.11} + i 0.05^{\pm 0.04}$
K IV	$0.62^{\pm 0.28} - i 0.18^{\pm 0.14}$	$0.04^{\pm 0.36} + i 0.23^{\pm 0.19}$	$0.50^{\pm 0.92} + i 0.01^{\pm 0.10}$	_
<i>V</i> *=	$1.23^{\pm 0.11} - i0.08^{\pm 0.09}$	$-0.36^{\pm 0.12} + i 0.42^{\pm 0.03}$	$5 -2.05^{\pm 0.25} + i 0.22^{\pm 0.13}$	$1.01^{\pm 0.47} + i 0.22^{\pm 0.18}$
КΞ	$1.17^{\pm 0.12} - i 0.40^{\pm 0.12}$	$0.00^{\pm 0.19} + i 0.44^{\pm 0.08}$	$1.04^{\pm 2.99} - i 0.19^{\pm 0.30}$	—
×Σ	$0.16^{\pm 0.11} + i 0.29^{\pm 0.07}$	$-0.24^{\pm 0.09} - i0.01^{\pm 0.02}$	$0.23^{\pm 0.16} - i 0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
ρz	$0.57^{\pm 0.24} + i0.41^{\pm 0.19}$	$-0.47^{\pm 0.43} + i 0.03^{\pm 0.18}$	$^{3} -1.76^{\pm 2.58} + i 0.10^{\pm 0.37}$	—
	$-0.26^{\pm 0.03} + i0.28^{\pm 0.03}$	$-0.37^{\pm 0.02} - i 0.15^{\pm 0.02}$	$2 0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
$\omega \Lambda$	$-0.23^{\pm 0.10} + i0.33^{\pm 0.06}$	$-0.45^{\pm 0.09} - i 0.16^{\pm 0.07}$	$7 - 0.32^{\pm 0.71} + i 0.05^{\pm 0.08}$	—
41	$0.46^{\pm 0.07} - i0.44^{\pm 0.06}$	$0.62^{\pm 0.05} + i 0.25^{\pm 0.03}$	$-0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
$\phi \Lambda$	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13}$	$0.82^{\pm 0.30} + i 0.29^{\pm 0.19}$	$0.60^{\pm 1.19} - i0.09^{\pm 0.14}$	<u> </u>

	$\Lambda(14)$	405)	$\Lambda(1670)$	$\Lambda(1800)$
Fit I	$1373^{\pm 5} - i114^{\pm 9}$	$1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
Fit II	$1385^{\pm 5} - i124^{\pm 10}$	$1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	
ĒN	$0.84^{\pm 0.14} - i1.91^{\pm 0.06}$	$2.44^{\pm 0.05} + i0.69^{\pm 0.0}$	⁸ $0.33^{\pm 0.02} - i 0.38^{\pm 0.03}$	$0.14^{\pm 0.05} - i 0.12^{\pm 0.07}$
KIV		- · · ⊥ 0 16 · · · · · · ↓ 0 9	[°] $0.15^{\pm 0.06} - i 0.19^{\pm 0.13}$	
	Comparis	son:	$2.74^{\pm 0.26} + i0.25^{\pm 0.22}$	$1.26^{\pm 0.60} - i0.39^{\pm 0.28}$
CLA	S analysis of ele	ctroproductio	n $0.33^{\pm 0.64} + i 0.28^{\pm 0.34}$	
data:	, poles ~1368 Me	eV. ~1423 Me	V $0.27^{\pm 0.02} + i 0.42^{0.06}$	$0.09^{\pm 0.05} - i 0.14^{\pm 0.07}$
	i Maignar EDI	120(2015)	$-0.11^{\pm 0.20} - i0.13^{\pm 0.35}$	_
• Ivia	1, INTERDICT, EPJ	A31,30(2013)	$-0.83^{\pm 0.14} + i 0.11^{\pm 0.08}$	$-0.50^{\pm 0.23} + i 0.49^{\pm 0.24}$
	$1325^{+15}_{-15} - i$	90_{-18}^{+12} MeV,	$-0.19^{\pm 0.10} - i0.20^{\pm 0.06}$	
	$1429^{+8} - i12$	$2^{+2} MeV$	$-0.18^{\pm 0.08} - i0.05^{\pm 0.03}$	$-0.15^{\pm 0.11} + i 0.05^{\pm 0.04}$
Roca	Oset PRC 88	055206 (2013	$0.50^{\pm 0.92} + i 0.01^{\pm 0.10}$	—
•NOCa	$1, 0.5 \text{ ec}, 1.0 \text{ c}, 1.0 \text$		$-2.05^{\pm 0.25} + i 0.22^{\pm 0.13}$	$1.01^{\pm 0.47} + i 0.22^{\pm 0.18}$
	1352 - i48	MeV,	$1.04^{\pm 2.99} - i0.19^{\pm 0.30}$	—
	1419 – <i>i</i> 29	MeV	$0.23^{\pm 0.16} - i0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
	$0.57^{\pm 0.24} + i 0.41^{\pm 0.19}$	$-0.47^{\pm 0.43} + i 0.03^{\pm 0.3}$	$^{18} -1.76^{\pm 2.58} + i 0.10^{\pm 0.37}$	—
	$-0.26^{\pm 0.03} + i 0.28^{\pm 0.03}$	$-0.37^{\pm 0.02} - i 0.15^{\pm 0.02}$	$02 0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
$\omega \Lambda$	$-0.23^{\pm 0.10} + i 0.33^{\pm 0.06}$	$-0.45^{\pm 0.09} - i 0.16^{\pm 0.09}$	$07 - 0.32^{\pm 0.71} + i 0.05^{\pm 0.08}$	_
1 A	$0.46^{\pm 0.07} - i 0.44^{\pm 0.06}$	$0.62^{\pm 0.05} + i 0.25^{\pm 0.0}$	$^{3} -0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
$\phi \Lambda$	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13}$	$0.82^{\pm 0.30} + i 0.29^{\pm 0.1}$	⁹ $0.60^{\pm 1.19} - i 0.09^{\pm 0.14}$	

		$\Lambda(14)$	405)	$\Lambda(1670)$	$\Lambda(1800)$
	Fit I	$1373^{\pm 5} - i114^{\pm 9}$	$1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
	Fit II	$1385^{\pm 5} - i124^{\pm 10}$	$1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	_
	ĒN	$0.84^{\pm 0.14} - i1.91^{\pm 0.06}$	$2.44^{\pm 0.05} + i0.69^{\pm 0.08}$	$0.33^{\pm 0.02} - i 0.38^{\pm 0.03}$	$0.14^{\pm 0.05} - i 0.12^{\pm 0.07}$
	KIV		o vo⊥0 16 vo oo⊥0 99	$0.15^{\pm 0.06} - i 0.19^{\pm 0.13}$	—
		Comparis	son:	$2.74^{\pm 0.26} + i 0.25^{\pm 0.22}$	$1.26^{\pm 0.60} - i 0.39^{\pm 0.28}$
	CLAS	S analysis of ele	ctroproductio	n $0.33^{\pm 0.64} + i 0.28^{\pm 0.34}$	—
d	ata: c	oles ~1368 Me	eV. ~1423 MeV	$\mathbf{V} 0.27^{\pm 0.02} + i 0.42^{0.06}$	$0.09^{\pm 0.05} - i 0.14^{\pm 0.07}$
	Moi	Maignar EDI	$\Lambda 51 30 (2015)$	$-0.11^{\pm 0.20} - i0.13^{\pm 0.35}$	—
	IVIA	1, IVICIDICI, LFJ	AJ1, J0(2013)	$-0.83^{\pm 0.14} + i0.11^{\pm 0.08}$	$-0.50^{\pm 0.23} + i 0.49^{\pm 0.24}$
		$1325^{+15}_{-15} - i$	90_{-18}^{+12} MeV,	$-0.19^{\pm 0.10} - i0.20^{\pm 0.06}$	
		$1429^{+8} - i12$	2^{+2}_{-2} MeV	$-0.18^{\pm 0.08} - i0.05^{\pm 0.03}$	$-0.15^{\pm 0.11} + i 0.05^{\pm 0.04}$
	Roca	Oset PRC 88	055206(2013)	$0.50^{\pm 0.92} + i 0.01^{\pm 0.10}$	—
	voca	1252		$-2.05^{\pm 0.25} + i 0.22^{\pm 0.13}$	$1.01^{\pm 0.47} + i 0.22^{\pm 0.18}$
		1352 - i48	MeV,	$1.04^{\pm 2.99} - i0.19^{\pm 0.30}$	
		1419 – <i>i</i> 29	MeV	$0.23^{\pm 0.16} - i0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
		$0.57^{\pm 0.24} + i 0.41^{\pm 0.19}$	$-0.47^{\pm 0.43} + i 0.03^{\pm 0.1}$	8 -1.76 ^{±2.58} + <i>i</i> 0.10 ^{±0.37}	—
		$-0.26^{\pm 0.03} + i0.28^{\pm 0.03}$	$-0.37^{\pm 0.02} - i 0.15^{\pm 0.0}$	$2^{2} 0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
	$\omega \Lambda$ -	$-0.23^{\pm 0.10} + i0.33^{\pm 0.06}$	$-0.45^{\pm 0.09} - i 0.16^{\pm 0.0}$	$7 - 0.32^{\pm 0.71} + i 0.05^{\pm 0.08}$	—
	4 1	$0.46^{\pm 0.07} - i 0.44^{\pm 0.06}$	$0.62^{\pm 0.05} + i 0.25^{\pm 0.03}$	$-0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
	$\phi \Lambda$	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13}$	$0.82^{\pm 0.30} + i 0.29^{\pm 0.19}$	$0.60^{\pm 1.19} - i 0.09^{\pm 0.14}$	_

	$\Lambda(1405)$	$\Lambda(1670)$	$\Lambda(1800)$
Fit I	$1373^{\pm 5} - i114^{\pm 9} \qquad 1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
Fit II	$1385^{\pm 5} - i124^{\pm 10} \qquad 1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	
$\bar{K}N$	$0.84^{\pm 0.14} - i 1.91^{\pm 0.06} \qquad 2.44^{\pm 0.05} + i 0.69^{\pm 0.08}$	⁰ Mass, width (p MeV and 25-5	odg): 1670-1680
	S analysis of electroproduction	0	
data;	poles ~1368 MeV, ~1423 MeV	decay rates in	poor agreement:
• Ma	i, Meißner, EPJA51,30 (2015)	channel pdg	Fit I Fitll
	$1325^{+15}_{-15} - i90^{+12}_{-18}$ MeV,	<i>ĒÑ</i> 20-30	% 28% 19%
	$1429^{+8}_{-7} - i12^{+2}_{-2}$ MeV	$-\pi\Sigma$ 25-55	5% 34% 61%
	n, Oset, PRC 88, 055206 (2013)	$-\frac{0}{2.05^{+0.20}} + i 0.22^{+0.10}$	$5\% 25\% 7\% \\ 1.01^{+0.41} + i 0.22^{+0.10}$
	1352 – <i>i</i> 48 MeV,	$1.04^{\pm 2.99} - i 0.19^{\pm 0.30}$	_
	1419 – <i>i</i> 29 MeV	$0.23^{\pm 0.16} - i 0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
	$0.57^{\pm 0.24} + i 0.41^{\pm 0.19} -0.47^{\pm 0.43} + i 0.03^{\pm 0.18}$	$-1.76^{\pm 2.58} + i 0.10^{\pm 0.37}$	_
	$-0.26^{\pm 0.03} + i0.28^{\pm 0.03} - 0.37^{\pm 0.02} - i0.15^{\pm 0.02}$	$0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
$\omega \Lambda$	$-0.23^{\pm 0.10} + i 0.33^{\pm 0.06} - 0.45^{\pm 0.09} - i 0.16^{\pm 0.07}$	$-0.32^{\pm 0.71} + i0.05^{\pm 0.08}$	—
<i>ب</i> ۸	$0.46^{\pm 0.07} - i0.44^{\pm 0.06} 0.62^{\pm 0.05} + i0.25^{\pm 0.03}$	$-0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
ψ1	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13} 0.82^{\pm 0.30} + i 0.29^{\pm 0.19}$	$0.60^{\pm 1.19} - i 0.09^{\pm 0.14}$	

	$\Lambda(14)$	405)	$\Lambda(1670)$	$\Lambda(1800)$
Fit I	$1373^{\pm 5} - i114^{\pm 9}$	$1426^{\pm 1} - i16^{\pm 1}$	$1681^{\pm 1} - i16^{\pm 2}$	$1734^{\pm 7} - i19^{\pm 2}$
Fit II	$1385^{\pm 5} - i124^{\pm 10}$	$1426^{\pm 1} - i15^{\pm 2}$	$1681^{\pm 2} - i7^{\pm 1}$	_
ĒN	$0.84^{\pm 0.14} - i1.91^{\pm 0.06}$	$2.44^{\pm 0.05} + i0.69^{\pm 0.08}$	$0.33^{\pm 0.02} - i0.38^{\pm 0.03}$	$0.14^{\pm 0.05} - i 0.12^{\pm 0.07}$
KN	$0.66^{\pm 0.35} - i1.93^{\pm 0.12}$	$2.43^{\pm 0.16} + i 0.63^{\pm 0.23}$	$0.15^{\pm 0.06} - i 0.19^{\pm 0.13}$	
	$-0.51^{\pm 0.05} + i 0.49^{\pm 0.06}$	$0.59^{\pm 0.09} - i 0.19^{\pm 0.04}$	$2.74^{\pm 0.26} + i 0.25^{\pm 0.22}$	$1.26^{\pm 0.60} - i0.39^{\pm 0.28}$
КΞ	$-0.55^{\pm0.13} + i0.27^{\pm0.06}$	$0.72^{\pm 0.14} - i0.14^{\pm 0.08}$	$0.33^{\pm 0.64} + i 0.28^{\pm 0.34}$	
-27	$-2.04^{\pm 0.07} + i 2.29^{\pm 0.08}$	$-0.87^{\pm 0.06} - i1.05^{\pm 0.09}$	$0.27^{\pm 0.02} + i 0.42^{0.06}$	$0.09^{\pm 0.05} - i 0.14^{\pm 0.07}$
$\pi \Sigma$	$-2.05^{\pm 0.11} + i 2.27^{\pm 0.09}$	$-0.90^{\pm 0.08} - i0.96^{\pm 0.15}$	$-0.11^{\pm 0.20} - i 0.13^{\pm 0.35}$	—
~ 1	$-0.71^{\pm 0.07} - i1.24^{\pm 0.04}$	$2.45^{\pm 0.05} + i 0.21^{\pm 0.04}$	$-0.83^{\pm 0.14} + i0.11^{\pm 0.08}$	$-0.50^{\pm 0.23} + i 0.49^{\pm 0.24}$
ηπ	$-0.80^{\pm 0.10} - i1.24^{\pm 0.06}$	$2.34^{\pm 0.13} + i 0.16^{\pm 0.04}$	$-0.19^{\pm 0.10} - i0.20^{\pm 0.06}$	—
$\bar{k}^* M$	$0.86^{\pm 0.08} - i0.04^{\pm 0.10}$	$-0.16^{\pm 0.10} + i 0.26^{\pm 0.03}$	$-0.18^{\pm 0.08} - i0.05^{\pm 0.03}$	$-0.15^{\pm 0.11} + i 0.05^{\pm 0.04}$
ΙΛ ΙΝ	$0.62^{\pm 0.28} - i 0.18^{\pm 0.14}$	$0.04^{\pm 0.36} + i 0.23^{\pm 0.19}$	$0.50^{\pm 0.92} + i 0.01^{\pm 0.10}$	—
$V^* \Xi$	$1.23^{\pm 0.11} - i0.08^{\pm 0.09}$	$-0.36^{\pm 0.12} + i 0.42^{\pm 0.05}$	$-2.05^{\pm 0.25} + i 0.22^{\pm 0.13}$	$1.01^{\pm 0.47} + i 0.22^{\pm 0.18}$
	$1.17^{\pm 0.12} - i 0.40^{\pm 0.12}$	$0.00^{\pm 0.19} + i 0.44^{\pm 0.08}$	$1.04^{\pm 2.99} - i 0.19^{\pm 0.30}$	—
20	$0.16^{\pm 0.11} + i 0.29^{\pm 0.07}$	$-0.24^{\pm 0.09} - i0.01^{\pm 0.02}$	$0.23^{\pm 0.16} - i0.09^{\pm 0.08}$	$-0.28^{\pm 0.28} - i0.04^{\pm 0.03}$
pД	$0.57^{\pm 0.24} + i0.41^{\pm 0.19}$	$-0.47^{\pm 0.43} + i 0.03^{\pm 0.18}$	$-1.76^{\pm 2.58} + i 0.10^{\pm 0.37}$	—
	$-0.26^{\pm 0.03} + i 0.28^{\pm 0.03}$	$-0.37^{\pm 0.02} - i0.15^{\pm 0.02}$	$0.51^{\pm 0.06} - i 0.09^{\pm 0.03}$	$-0.32^{\pm 0.15} - i0.07^{\pm 0.06}$
$\omega \Lambda$	$-0.23^{\pm 0.10} + i0.33^{\pm 0.06}$	$-0.45^{\pm 0.09} - i0.16^{\pm 0.07}$	$-0.32^{\pm 0.71} + i 0.05^{\pm 0.08}$	—
41	$0.46^{\pm 0.07} - i 0.44^{\pm 0.06}$	$0.62^{\pm 0.05} + i 0.25^{\pm 0.03}$	$-0.66^{\pm 0.10} + i 0.12^{\pm 0.04}$	$0.39^{\pm 0.19} + i 0.11^{\pm 0.07}$
ψn	$0.44^{\pm 0.27} - i 0.58^{\pm 0.13}$	$0.82^{\pm 0.30} + i 0.29^{\pm 0.19}$	$0.60^{\pm 1.19} - i 0.09^{\pm 0.14}$	_

	Σ 's around	1400 MeV	$\Sigma(1620)$ or $\Sigma(1670)$	$\Sigma(1900)$	
Fit I	$1396^{\pm 1} - i5^{\pm 2}$	$1367^{\pm 24} - i57^{\pm 21}$	$1630^{\pm 33} - i104^{\pm 13}$	$1853^{\pm 10} - i150^{\pm 10}$	
Fit II	_	$1399^{\pm 35} - i36^{\pm 9}$	-		
ĒN	$0.18^{\pm 0.03} + i 0.14^{\pm 0.05}$	$0.08^{\pm 0.48} + i0.52^{\pm 0.73}$	$1.47^{\pm 0.08} - i0.017^{\pm 0.07}$	$-0.86^{\pm 0.03} + i 0.79^{\pm 0.02}$	
KN	—	$0.50^{\pm 0.29} + i 0.33^{\pm 0.18}$	—	—	
$K\Xi$	$1.06^{\pm 0.22} + i 1.45^{\pm 0.12}$	$0.62^{\pm 0.47} - i0.42^{\pm 1.00}$	$2.89^{\pm 0.26} - i0.65^{\pm 0.24}$	$0.84^{\pm 0.03} - i0.39^{\pm 0.05}$	
ΛΞ	—	$0.81^{\pm 0.42} + i 0.41^{\pm 0.15}$	—	—	
$\pi \Sigma$	$-0.17^{\pm 0.09} - i020^{\pm 0.03}$	$0.77^{\pm 0.96} - i0.67^{\pm 1.22}$	$0.71^{\pm 0.33} - i1.63^{\pm 0.19}$	$-0.02^{\pm 0.04} + i 0.32^{\pm 0.08}$	
71 21	—	$1.08^{\pm 0.12} + i 0.19^{\pm 0.21}$	-	—	
$\pi \Lambda$	$0.03^{\pm 0.10} + i 0.07^{\pm 0.06}$	$-0.91^{\pm 1.32} + i 0.39^{\pm 0.81}$	$-0.26^{\pm 0.34} - i0.23^{\pm 0.18}$	$0.36^{\pm 0.2} + i1.54^{\pm 0.04}$	
<i>// / 1</i>	—	$-1.40^{\pm 0.18} - i0.07^{\pm 0.10}$	-	-	
$m\Sigma$	$-0.43^{\pm 0.03} - i0.23^{\pm 0.09}$	$0.31^{\pm 0.31} - i0.59^{\pm 1.12}$	$-2.14^{\pm 0.24} - i0.13^{\pm 0.11}$	$0.07^{\pm 0.03} - i0.43^{\pm 0.02}$	
ηΔ	-	$0.27^{\pm 0.10} - i 0.19^{\pm 0.11}$	-	-	
$\bar{k}^* N$	$0.04^{\pm 0.10} + i 0.15^{\pm 0.07}$	$-1.69^{\pm 1.99} + i 0.31^{\pm 0.68}$	$-0.31^{\pm 0.09} - i0.11^{\pm 0.16}$	$0.71^{\pm 0.05} - i0.05^{\pm 0.02}$	
11 11	-	$-3.46^{\pm 0.21} - i0.06^{\pm 0.15}$	-		
$K^* \Xi$	$-0.50^{\pm 0.22} - i0.38^{\pm 0.08}$	$1.40^{\pm 2.11} - i 1.10^{\pm 2.38}$	$-1.80^{\pm 0.47} - i0.37^{\pm 0.14}$	$-0.98^{\pm 0.14} - i0.72^{\pm 0.06}$	
	-	$-0.01^{\pm 0.59} - i0.21^{\pm 0.08}$	-	-	
oΣ	$-0.15^{\pm 0.07} - i0.14^{\pm 0.04}$	$0.76^{\pm 1.02} - i0.58^{\pm 0.85}$	$-0.76^{\pm 0.18} - i0.53^{\pm 0.49}$	$-1.10^{\pm 0.04} - i 0.34^{\pm 0.03}$	
pД		$3.60^{\pm 0.61} - i0.69^{\pm 0.16}$		-	
01	$0.36^{\pm 0.18} + i 0.29^{\pm 0.07}$	$-0.95^{\pm 1.50} + i 0.93^{\pm 1.84}$	$2.44^{\pm 0.50} + i0.94^{\pm 0.27}$	$1.51^{\pm 0.25} + i 0.82^{\pm 0.09}$	
ρn	-	$-1.26^{\pm 0.19} + i 0.09^{\pm 0.07}$			
$\omega \Sigma$	$-0.15^{\pm 0.11} - i0.14^{\pm 0.05}$	$1.03^{\pm 1.35} - i0.55^{\pm 1.10}$	$-0.14^{\pm 0.23} - i0.44^{\pm 0.14}$	$-0.64^{\pm 0.10} - i 0.23^{\pm 0.04}$	
ω2	-	$2.15^{\pm 0.20} - i0.13^{\pm 0.09}$			
$\phi \Sigma$	$0.27^{\pm 0.17} + i 0.24^{\pm 0.08}$	$-1.73^{\pm 2.27} + i 0.90^{\pm 1.82}$	$0.42^{\pm 0.38} + i0.53^{\pm 0.24}$	$1.04^{\pm 0.20} + i 0.39^{\pm 0.07}$	
ΨΔ	-	$-3.23^{\pm 0.39} + i 0.20^{\pm 0.11}$	-	-	

	Σ 's around	1400 MeV	$\Sigma(1620)$ or $\Sigma(1670)$	$\Sigma(1900)$	
Fit I	$1396^{\pm 1} - i5^{\pm 2}$	$1367^{\pm 24} - i57^{\pm 21}$	$1630^{\pm 33} - i104^{\pm 13}$	$1853^{\pm 10} - i150^{\pm 10}$	
Fit II	II - Known ratios for $\Sigma(1620)$				
ĒΝ	$0.18^{\pm 0.03} + i 0.14^{\pm 0.05}$	$\Gamma(\Sigma(1670)) =$	$\bar{K}N$		
$K\Xi$	$1.06^{\pm 0.22} + i 1.45^{\pm 0.12}$	$\frac{\Gamma(\Sigma(1670))}{\Gamma(\Sigma(1670))}$	$\frac{(\pi \pi T)}{\pi \Sigma} < 0.75,$		
$\pi\Sigma$	$-0.17^{\pm 0.09} - i020^{\pm 0.03}$	$0.05 \lesssim \frac{\Gamma(\Sigma(1670))}{\Gamma(\Sigma(1670))}$	$0.05 \lesssim \frac{\Gamma(\Sigma(1670) \to \pi\Lambda)}{\Gamma(\Sigma(1670) \to \pi\Sigma)} \lesssim 0.85$		
$\pi\Lambda$	$- 0.03^{\pm 0.10} + i 0.07^{\pm 0.06} -$	We find 0.5 a	and 0.06		
$\eta\Sigma$	$-0.43^{\pm 0.03} - i0.23^{\pm 0.09}$	Known ratio	s for $\Sigma(1670)$	1 /0	
$ar{K}^*N$	$- 0.04^{\pm 0.10} + i 0.15^{\pm 0.07}$	$0.08 < (\Gamma(\Sigma(1620$	$) \to \bar{K}N)\Gamma(\Sigma(1620) \to \pi$ Γ_{total}	$\left(\frac{\Sigma}{\Sigma}\right)^{1/2} < 0.35$	
$K^*\Xi$	$-0.50^{\pm 0.22} - i 0.38^{\pm 0.08}$	$0.1 < \frac{\left(\Gamma(\Sigma(1620))\right)}{1}$	$() \rightarrow \bar{K}N)\Gamma(\Sigma(1620) \rightarrow \pi)$ Γ_{total}	$\frac{\Lambda))^{1/2}}{2} < 0.15,$	
$ ho\Sigma$	$-0.15^{\pm 0.07} - i 0.14^{\pm 0.04}$	0.08 <	$\frac{\Gamma(\Sigma(1620) \to KN)}{\Gamma_{\text{total}}} < 0.$.35,	
$ ho\Lambda$	$0.36^{\pm 0.18} + i 0.29^{\pm 0.07}$	We find 0.3	7, 0.10, and 0.20	6. In a continue	
$\omega\Sigma$	$-0.15^{\pm 0.11} - i 0.14^{\pm 0.05}$		decay rates in t	ne article.	
$\phi\Sigma$	$- 0.27^{\pm 0.17} + i 0.24^{\pm 0.08} -$	$2.15^{\pm 0.20} - i 0.13^{\pm 0.09}$ $-1.73^{\pm 2.27} + i 0.90^{\pm 1.8}$ $-3.23^{\pm 0.39} + i 0.20^{\pm 0.1}$	$ \begin{array}{c} - \\ 52 \\ 0.42^{\pm 0.38} + i \\ 0.53^{\pm 0.24} \\ - \\ \end{array} $	$- 1.04^{\pm 0.20} + i 0.39^{\pm 0.07} -$	

	Σ 's around	1400 MeV	$\Sigma(1620)$ or $\Sigma(1670)$	$\Sigma(1900)$	
Fit I	$1396^{\pm 1} - i5^{\pm 2}$	$1367^{\pm 24} - i57^{\pm 21}$	$1630^{\pm 33} - i104^{\pm 13}$	$1853^{\pm 10} - i150^{\pm 10}$	
Fit II	_	$1399^{\pm 35} - i36^{\pm 9}$	-		
ĒN	$0.18^{\pm 0.03} + i 0.14^{\pm 0.05}$	$0.08^{\pm 0.48} + i0.52^{\pm 0.73}$	$1.47^{\pm 0.08} - i0.017^{\pm 0.07}$	$-0.86^{\pm 0.03} + i 0.79^{\pm 0.02}$	
KN	—	$0.50^{\pm 0.29} + i 0.33^{\pm 0.18}$	—	—	
$K\Xi$	$1.06^{\pm 0.22} + i 1.45^{\pm 0.12}$	$0.62^{\pm 0.47} - i0.42^{\pm 1.00}$	$2.89^{\pm 0.26} - i0.65^{\pm 0.24}$	$0.84^{\pm 0.03} - i0.39^{\pm 0.05}$	
ΛΞ	—	$0.81^{\pm 0.42} + i 0.41^{\pm 0.15}$	—	—	
$\pi \Sigma$	$-0.17^{\pm 0.09} - i020^{\pm 0.03}$	$0.77^{\pm 0.96} - i0.67^{\pm 1.22}$	$0.71^{\pm 0.33} - i1.63^{\pm 0.19}$	$-0.02^{\pm 0.04} + i 0.32^{\pm 0.08}$	
71 21	—	$1.08^{\pm 0.12} + i 0.19^{\pm 0.21}$	-	—	
$\pi \Lambda$	$0.03^{\pm 0.10} + i 0.07^{\pm 0.06}$	$-0.91^{\pm 1.32} + i 0.39^{\pm 0.81}$	$-0.26^{\pm 0.34} - i0.23^{\pm 0.18}$	$0.36^{\pm 0.2} + i1.54^{\pm 0.04}$	
<i>// / 1</i>	—	$-1.40^{\pm 0.18} - i0.07^{\pm 0.10}$	-	-	
$m\Sigma$	$-0.43^{\pm 0.03} - i0.23^{\pm 0.09}$	$0.31^{\pm 0.31} - i0.59^{\pm 1.12}$	$-2.14^{\pm 0.24} - i0.13^{\pm 0.11}$	$0.07^{\pm 0.03} - i0.43^{\pm 0.02}$	
ηΔ	-	$0.27^{\pm 0.10} - i 0.19^{\pm 0.11}$	-	-	
$\bar{k}^* N$	$0.04^{\pm 0.10} + i 0.15^{\pm 0.07}$	$-1.69^{\pm 1.99} + i 0.31^{\pm 0.68}$	$-0.31^{\pm 0.09} - i0.11^{\pm 0.16}$	$0.71^{\pm 0.05} - i0.05^{\pm 0.02}$	
11 11	-	$-3.46^{\pm 0.21} - i0.06^{\pm 0.15}$	-		
$K^* \Xi$	$-0.50^{\pm 0.22} - i0.38^{\pm 0.08}$	$1.40^{\pm 2.11} - i 1.10^{\pm 2.38}$	$-1.80^{\pm 0.47} - i0.37^{\pm 0.14}$	$-0.98^{\pm 0.14} - i0.72^{\pm 0.06}$	
	-	$-0.01^{\pm 0.59} - i0.21^{\pm 0.08}$	-	-	
oΣ	$-0.15^{\pm 0.07} - i0.14^{\pm 0.04}$	$0.76^{\pm 1.02} - i0.58^{\pm 0.85}$	$-0.76^{\pm 0.18} - i0.53^{\pm 0.49}$	$-1.10^{\pm 0.04} - i 0.34^{\pm 0.03}$	
pД		$3.60^{\pm 0.61} - i0.69^{\pm 0.16}$		-	
01	$0.36^{\pm 0.18} + i 0.29^{\pm 0.07}$	$-0.95^{\pm 1.50} + i 0.93^{\pm 1.84}$	$2.44^{\pm 0.50} + i0.94^{\pm 0.27}$	$1.51^{\pm 0.25} + i 0.82^{\pm 0.09}$	
μn	-	$-1.26^{\pm 0.19} + i 0.09^{\pm 0.07}$			
$\omega \Sigma$	$-0.15^{\pm 0.11} - i0.14^{\pm 0.05}$	$1.03^{\pm 1.35} - i0.55^{\pm 1.10}$	$-0.14^{\pm 0.23} - i0.44^{\pm 0.14}$	$-0.64^{\pm 0.10} - i 0.23^{\pm 0.04}$	
ω2	-	$2.15^{\pm 0.20} - i0.13^{\pm 0.09}$			
$\phi \Sigma$	$0.27^{\pm 0.17} + i 0.24^{\pm 0.08}$	$-1.73^{\pm 2.27} + i 0.90^{\pm 1.82}$	$0.42^{\pm 0.38} + i0.53^{\pm 0.24}$	$1.04^{\pm 0.20} + i 0.39^{\pm 0.07}$	
ΨΔ	-	$-3.23^{\pm 0.39} + i 0.20^{\pm 0.11}$	-	-	

SCATTERING LENGTH

• Scattering length from Siddharta data $a_{K^-p} = (-0.65 \pm 0.10) + i (0.81 \pm 0.15)$

Our values

	Fit I	Fit II
a_{K^-p}	$-0.74_{-0.02}^{+0.01} + i0.69_{-0.01}^{+0.02}$	$-0.74_{-0.02}^{+0.07} + i 0.73_{-0.08}^{+0.03}$
$a^0_{\bar{K}N}$	$-1.58^{+0.03}_{-0.03} + i0.87^{+0.02}_{-0.03}$	$-1.60^{+0.03}_{-0.01} + i 0.89^{+0.04}_{-0.13}$
$a_{\bar{K}N}^1$	$0.09^{+0.02}_{-0.02} + i 0.50^{+0.04}_{-0.02}$	$0.12^{+0.10}_{-0.04} + i 0.55^{+0.02}_{-0.04}$

STATISTICAL COMPARISON

- How to compare the fits in previous works, e.g, Oller, Meißner, PLB 500, 263 (2001) and Guo, Oller, PRC87, 035202 (2013))
- Not nested-models, cannot use F-test
- Other possibilities AIC test Akaike Information criterion

AIC = $\chi_0^2 + \frac{2nk}{k-n-1}$ n is no. of parameters, k is no. of data points

• AIC for our model < AIC for the $\mathcal{O}(p^2)$ fit < AIC for the $\mathcal{O}(p)$ fit.

SUMMARY

- We find evidence for a light Σ , mass around 1400 MeV. The poles on $\Lambda(1405)$ are in agreement with those obtained by analyzing data on electro- and photo-production data.
- We provide resonance-vector-baryon vertex couplings, which can be useful in models requiring such vertices.
- We study resonance arising from VB interactions.
- A statistical comparison with other works based on LO, NLO Lagrangians is made.

• Amplitudes:

•
$$V_{\text{cont}}(i \to j) = -\frac{1}{4f_P^2} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} \mathscr{A}_{ij} \left[\left(2\sqrt{s} - M_i - M_j \right) + \left(2\sqrt{s} + M_i + M_j \right) \times \left(\frac{\overrightarrow{p}_i \cdot \overrightarrow{p}_j + i \, \chi_j^\dagger \ \left(\overrightarrow{p}_j \times \overrightarrow{p}_i \right) \cdot \overrightarrow{\sigma} \, \chi_i}{(M_i + E_i)(M_j + E_j)} \right) \right]$$

•
$$V_s(i \to j) = \frac{1}{2f_P^2} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} \sum_k \frac{\mathscr{B}_{ij}}{s - M_k^2} \left[\left(\sqrt{s} - M_i \right) \left(\sqrt{s} - M_j \right) \left(\sqrt{s} - M_k \right) \left(\frac{\overrightarrow{p}_i \cdot \overrightarrow{p}_j + i \, \chi_j^\dagger \ \left(\overrightarrow{p}_j \times \overrightarrow{p}_i \right) \cdot \overrightarrow{\sigma} \, \chi_i}{(M_i + E_i)(M_j + E_j)} \right) \left(\sqrt{s} + M_j \right) \left(\sqrt{s} + M_k \right) \right]$$

• Amplitudes:

•
$$V_{u}(i \rightarrow j) = -\frac{1}{2f_{P}^{2}}\sqrt{\frac{M_{i} + E_{i}}{2M_{i}}}\sqrt{\frac{M_{j} + E_{j}}{2M_{j}}}\sum_{k}\frac{\mathscr{C}_{ij}^{k}}{u - M_{k}^{2}}\left[u\left(\sqrt{s} + M_{k}\right) + \sqrt{s}\left(M_{j}\left[M_{i} + M_{k}\right]\right)\right]$$
$$+M_{i}M_{k} - M_{j}\left(M_{i} + M_{k}\right)\left(M_{i} + M_{j}\right) - M_{i}^{2}M_{k} + \left(\frac{\overrightarrow{p}_{i} \cdot \overrightarrow{p}_{j} + i\chi_{j}^{\dagger}\left(\overrightarrow{p}_{j} \times \overrightarrow{p}_{i}\right) \cdot \overrightarrow{\sigma}\chi_{i}}{(M_{i} + E_{i})(M_{j} + E_{j})}\right)$$

$$\times \left(u \left(\sqrt{s} - M_k \right) + \sqrt{s} \left(M_j \left[M_i + M_k \right] + M_i M_k \right) + M_j \left(M_i + M_j \right) \left(M_i + M_k \right) + M_i^2 M_k \right) \right]$$

In agreement with: Ramos, Feijoo and Magas, Nucl. Phys. A 954, 58 (2016), Oller, Meißner, PLB 500, 263 (2001), Borasoy, Nissler, Weise, EPJA 25, 79 (2005).

• Amplitudes:

• Contact interaction:

$$\begin{split} V_{\rm CT,VB}^{I} &= -C_{\rm CT,VB}^{I} \frac{g_{1}g_{2}}{2\sqrt{M_{1}M_{2}}} \left\{ -i\vec{\sigma}\cdot\vec{\epsilon}_{2}\times\vec{\epsilon}_{1} \right. \\ &+ \frac{1}{E_{2}+M_{2}} \left(-\epsilon_{1}^{0}\vec{\sigma}\cdot\vec{P}_{2}\vec{\sigma}\cdot\vec{\epsilon}_{2} + \epsilon_{2}^{0}\vec{\sigma}\cdot\vec{P}_{2}\vec{\sigma}\cdot\vec{\epsilon}_{1} \right) \\ &+ \frac{1}{E_{1}+M_{1}} \left(-\epsilon_{2}^{0}\vec{\sigma}\cdot\vec{\epsilon}_{1}\vec{\sigma}\cdot\vec{P}_{1} + \epsilon_{1}^{0}\vec{\sigma}\cdot\vec{\epsilon}_{2}\vec{\sigma}\cdot\vec{P}_{1} \right) \\ &- \frac{1}{2\left(E_{1}+M_{1} \right)\left(E_{2}+M_{2} \right)} \left(\vec{\sigma}\cdot\vec{P}_{2}\vec{\sigma}\cdot\vec{\epsilon}_{1}\vec{\sigma}\cdot\vec{\epsilon}_{2}\vec{\sigma}\cdot\vec{P}_{1} \right. \\ &- \left. \vec{\sigma}\cdot\vec{P}_{2}\vec{\sigma}\cdot\vec{\epsilon}_{2}\vec{\sigma}\cdot\vec{\epsilon}_{1}\vec{\sigma}\cdot\vec{P}_{1} \right) \right\} \sqrt{\frac{M_{1}+E_{1}}{2M_{1}}} \sqrt{\frac{M_{2}+E_{2}}{2M_{2}}} \end{split}$$

• Amplitudes (t-channel):

$$\begin{split} V_{t,\text{VB}}^{I} &= \frac{-m_{Vx}^{2}}{4f_{Vi}f_{Vj}} \frac{1}{t - m_{Vx}^{2}} \left\{ \epsilon_{1} \cdot \epsilon_{2} \left[\left(2\sqrt{s} - M_{1} - M_{2} + (M_{1} - M_{2})\frac{(m_{2}^{2} - m_{1}^{2})}{m_{Vx}^{2}} \right) C_{t1,\text{VB}}^{I} \right. \\ &+ \left(\frac{M_{1} + M_{2}}{2M} \left(2\sqrt{s} - M_{1} - M_{2} \right) - \frac{s - u}{2M} \right) C_{t2,\text{VB}}^{I} + \frac{\vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{(E_{1} + M_{1}) (E_{2} + M_{2})} \left(\left(2\sqrt{s} + M_{1} + M_{2} - (M_{1} - M_{2})\frac{(m_{2}^{2} - m_{1}^{2})}{m_{Vx}^{2}} \right) C_{t1,\text{VB}}^{I} + \left(\frac{M_{1} + M_{2}}{2M} \left(2\sqrt{s} + M_{1} + M_{2} \right) + \frac{s - u}{2M} \right) C_{t2,\text{VB}}^{I} \right) \right] \\ &+ \left(C_{t1,\text{VB}}^{I} + \frac{M_{1} + M_{2}}{2M} C_{t2,\text{VB}}^{I} \right) \left(-2K_{1} \cdot \epsilon_{2} \left[\epsilon_{1}^{0} - \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{E_{1} + M_{1}} - \frac{\vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{\epsilon}_{1}}{E_{2} + M_{2}} + \frac{\epsilon_{1}^{0} \vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{(E_{1} + M_{1}) (E_{2} + M_{2})} \right] \right) \\ &- 2K_{2} \cdot \epsilon_{1} \left[\epsilon_{2}^{0} - \frac{\vec{\sigma} \cdot \vec{\epsilon}_{2} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{E_{1} + M_{1}} - \frac{\vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{e}_{2}}{E_{2} + M_{2}} + \frac{\epsilon_{2}^{0} \vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{(E_{1} + M_{1}) (E_{2} + M_{2})} \right] \right) + \frac{2}{M} \left(K_{1} \cdot \epsilon_{2} P_{1} \cdot \epsilon_{1} + K_{2} \cdot \epsilon_{1} P_{1} \cdot \epsilon_{2} \right) \\ &\times \left(1 - \frac{\vec{\sigma} \cdot \vec{P}_{2} \cdot \vec{\sigma} \cdot \vec{P}_{1}}{(E_{1} + M_{1}) (E_{2} + M_{2})} \right) C_{t2,\text{VB}}^{I} \right\} \sqrt{\frac{M_{1} + E_{1}}{2M_{1}}} \sqrt{\frac{M_{2} + E_{2}}{2M_{2}}}, \end{split}$$

• Amplitudes (s-, u-channel):

$$\begin{split} V_{s,VB}^{I} &= \frac{g_{1}g_{2}}{s - M_{x}^{2}} \left\{ \left(\sqrt{s} + M_{x}\right) \left(\epsilon_{2}^{0} \left[I_{1f}^{s} + \frac{I_{2f}^{s}K_{2}^{0}}{2\sqrt{M_{1}M_{2}}} \right] - \frac{I_{2f}^{s} \left(\vec{\sigma} \cdot \vec{\epsilon}_{2}\vec{\sigma} \cdot \vec{K}_{2}\right)}{2\sqrt{M_{1}M_{2}}} \right) \left(\epsilon_{1}^{0} \left[I_{1i}^{s} + \frac{I_{2i}^{s}K_{1}^{0}}{2\sqrt{M_{1}M_{2}}} \right] - \frac{I_{2i}^{s} \left(\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{1}\right)}{2\sqrt{M_{1}M_{2}}} \right) \right. \\ &+ \left(\sqrt{s} - M_{x}\right) \left(\vec{\sigma} \cdot \vec{\epsilon}_{2} \left[I_{1f}^{s} - \frac{I_{2f}^{s}K_{2}^{0}}{2\sqrt{M_{1}M_{2}}} \right] + \frac{I_{2f}^{s}\epsilon_{2}^{0} \left(\vec{\sigma} \cdot \vec{K}_{2}\right)}{2\sqrt{M_{1}M_{2}}} \right) \left(\left[I_{1i}^{s} - \frac{I_{2i}^{s}K_{1}^{0}}{2\sqrt{M_{1}M_{2}}} \right] \vec{\sigma} \cdot \vec{\epsilon}_{1} + \frac{I_{2i}^{s}\epsilon_{1}^{0} \left(\vec{\sigma} \cdot \vec{K}_{1}\right)}{2\sqrt{M_{1}M_{2}}} \right) \right\}, \end{split}$$

$$V_{u,VB}^{I} &= \frac{g_{1}g_{2}}{u - M_{x}^{2}} \left\{ \epsilon_{1}^{0}\epsilon_{2}^{0} \left(\frac{1}{2} \left[E_{2} - K_{1}^{0} + E_{1} - K_{2}^{0} \right] + M_{x} \right) V_{u1} + \vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{2} \left(\frac{1}{2} \left[E_{2} - K_{1}^{0} + E_{1} - K_{2}^{0} \right] - M_{x} \right) V_{u2} \right. \\ &+ \epsilon_{2}^{0} \vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{2} \, V_{u3} + \epsilon_{2}^{0} \vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1} \, V_{u4} + \epsilon_{1}^{0} \vec{\sigma} \cdot \vec{K}_{2}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u5} + \epsilon_{1}^{0} \vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u6} + \epsilon_{1}^{0}\epsilon_{2}^{0} \left(\frac{\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{2}}{4\sqrt{M_{1}M_{2}}} \, V_{u7} \right. \\ &+ \epsilon_{1}^{0}\epsilon_{2}^{0} \vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1} \, V_{u8} + \epsilon_{1}^{0}\epsilon_{2}^{0} \left(\frac{\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{2}}{2\sqrt{M_{1}M_{2}}} \, V_{u9} + \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{2}\vec{\sigma} \cdot \vec{\epsilon}_{2}}{4\sqrt{M_{1}M_{2}}} \, V_{u10} + \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{2}\vec{\sigma} \cdot \vec{\epsilon}_{2} \\ &+ \epsilon_{2}^{0} \vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{2}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u11} \\ &+ \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{2}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u11} \\ &+ \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u1} \\ &+ \frac{\vec{\sigma} \cdot \vec{\epsilon}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{K}_{1}\vec{\sigma} \cdot \vec{\epsilon}_{2} \, V_{u1} \\ &+ \frac{\vec{\sigma} \cdot \vec{\epsilon} \cdot \vec{\epsilon} \cdot \vec{\epsilon}_{2}}{2\sqrt{M_{1}M_{2}}} \, V_{u11} \\ &+ \frac{\vec{\sigma} \cdot \vec{\epsilon} \cdot \vec{\epsilon} \cdot \vec{\epsilon} \cdot \vec{\epsilon}_{2} \,$$