

On the Study of Dibaryon $d^*(2380)$

---calculations in quark model

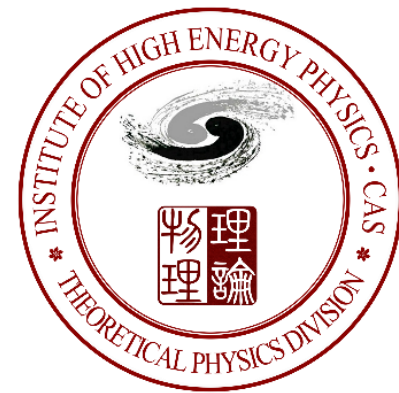
Yubing Dong

Institute of High Energy Physics (IHEP),

Chinese Academy of Sciences

Collaborators: Fei-Huang, Qifang Lyu, Pengnian Shen, Zongye Zhang

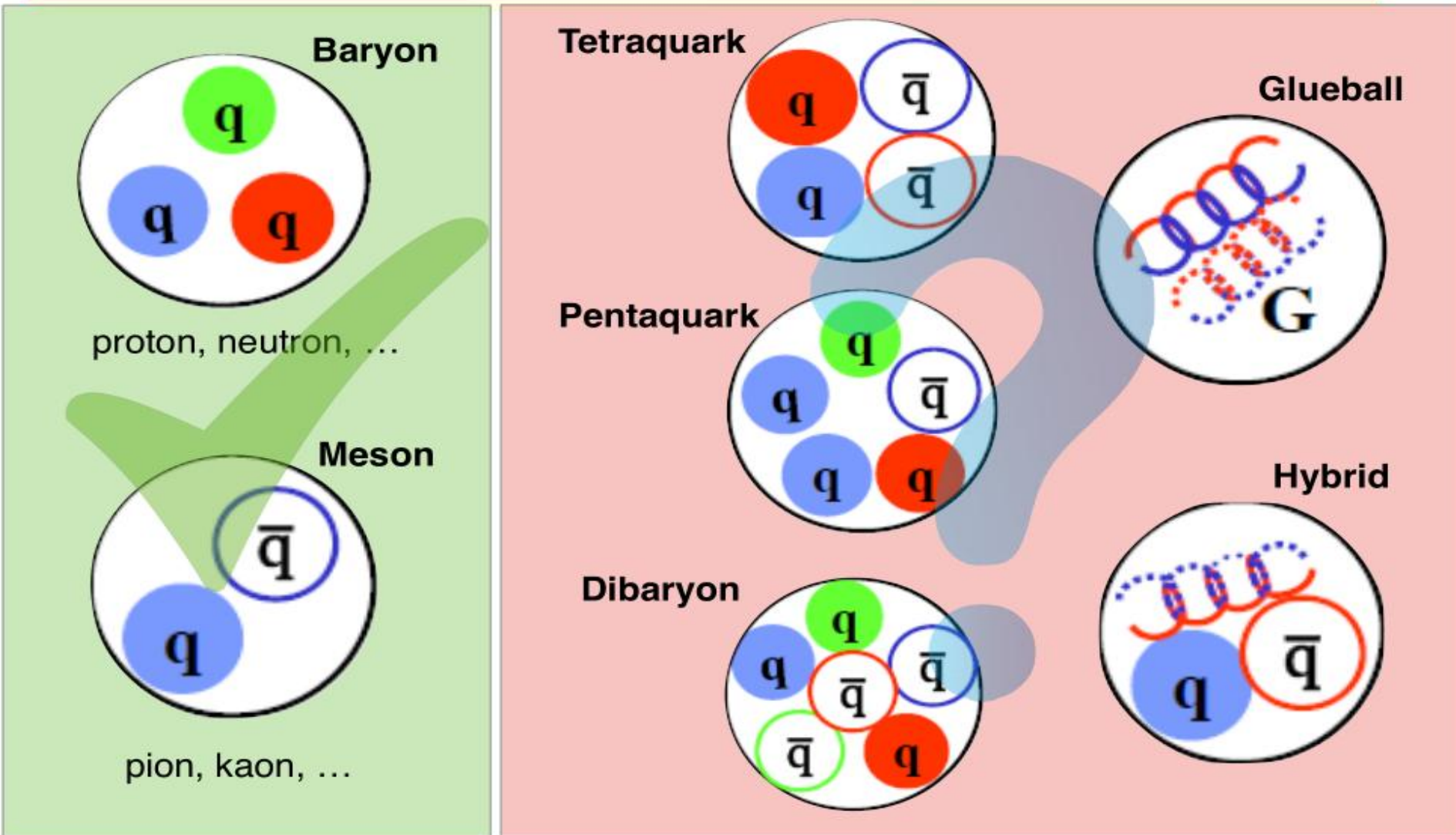
Outline



- 1, Observation of exotics,
dibaryon $d^*(2380)$
- 2, Possible interpretations
- 3, **Ours: Compact 6-quark dominated
structure in chiral constituent quark model
 - (A) Mass and wave function
 - (B) Strong decays
 - (C) Form factors
- 4, Summary and outlook

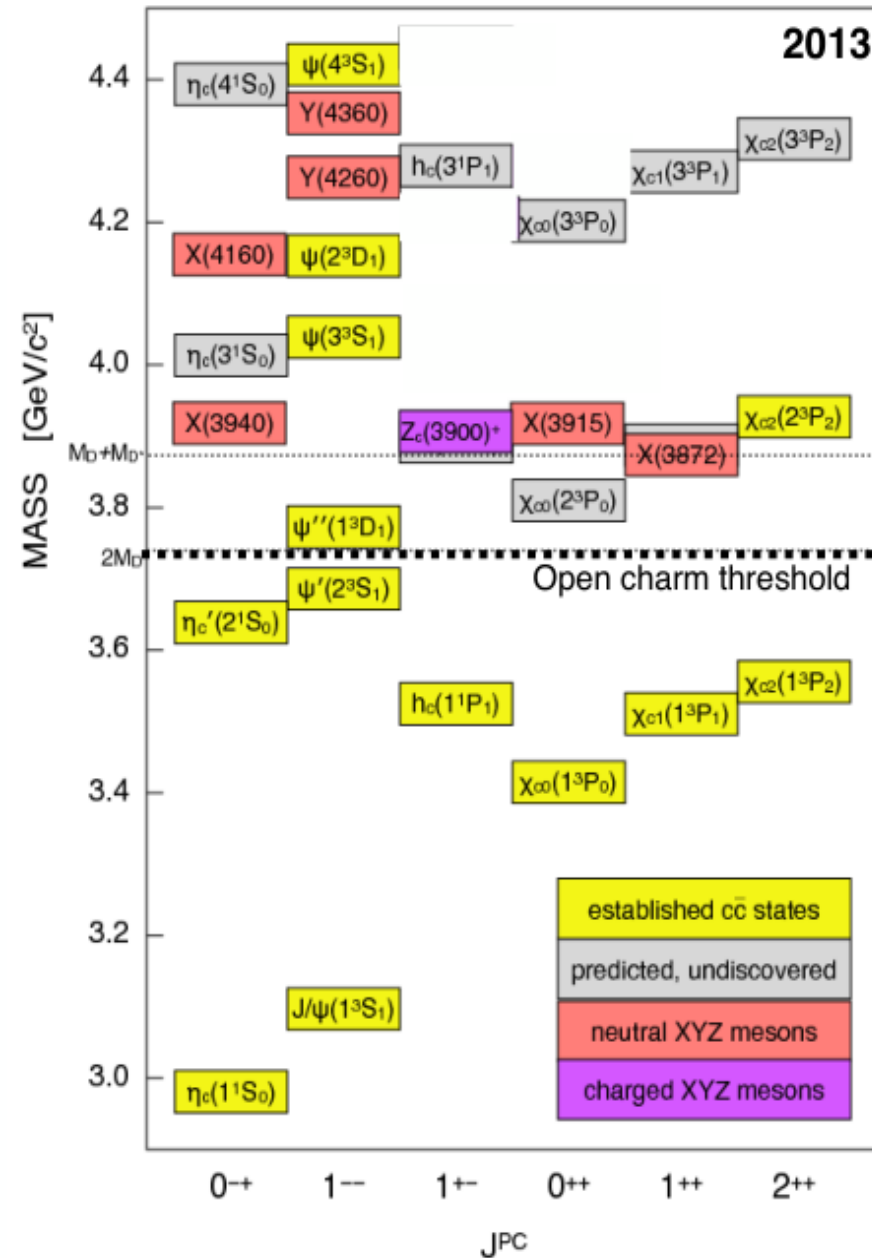
1, Observations of exotics

Ordinary versus "exotic" matter



Charmonium-like particles -

XYZ states

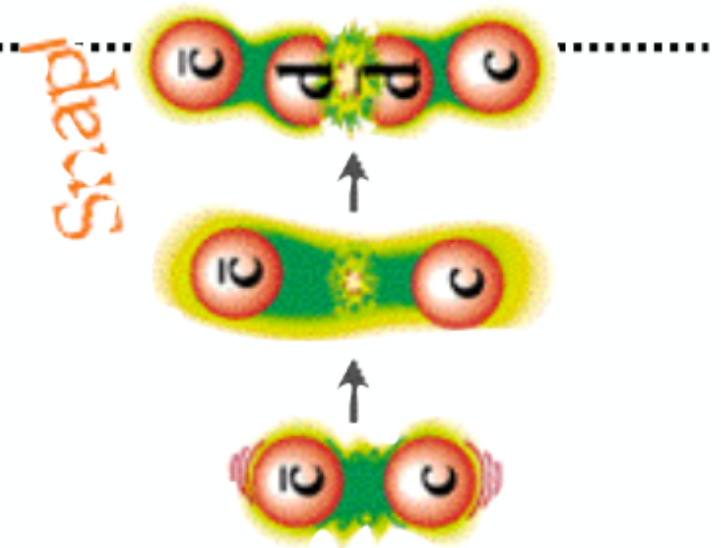


Discovery

Precision

“XYZ” Puzzle

Near threshold,
narrow width

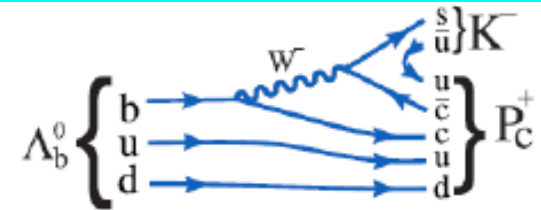


Courtesy of J. Messchendorp

Five-Quark

Pentaquark states $P_c(4380)^+$, & $P_c(4450)^+$: (2015)

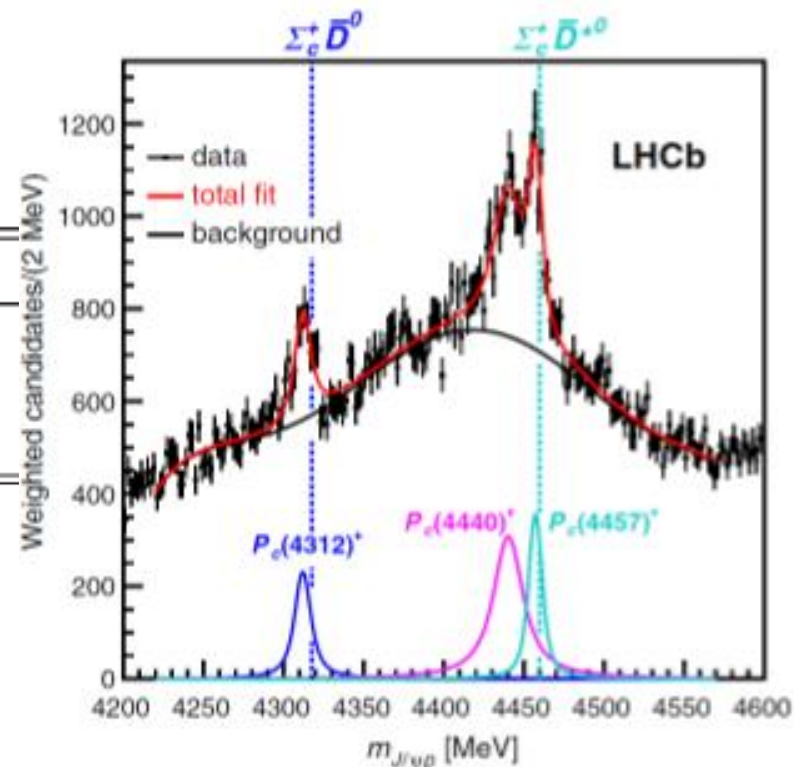
LHCb, Phys.Rev.Lett. 115 (2015) 072001 :
Observation of two N^* from $\Lambda_b^0 \rightarrow J/\psi K^- p$



New observations: three resonances

LHCb, Phys.Rev.Lett. 122 (2019) 222001

State	M [MeV]	Γ [MeV]	(95% C.L.)
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(<27)
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(<49)
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(<20)



Observation: $d^*(2380)$



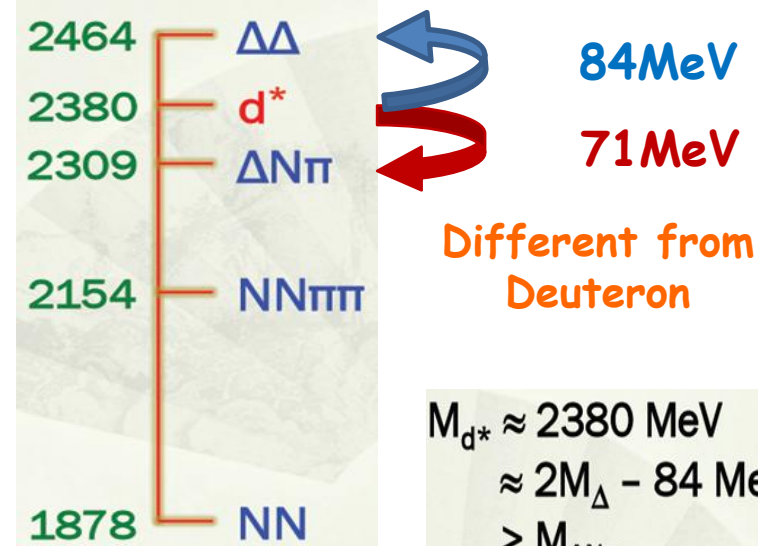
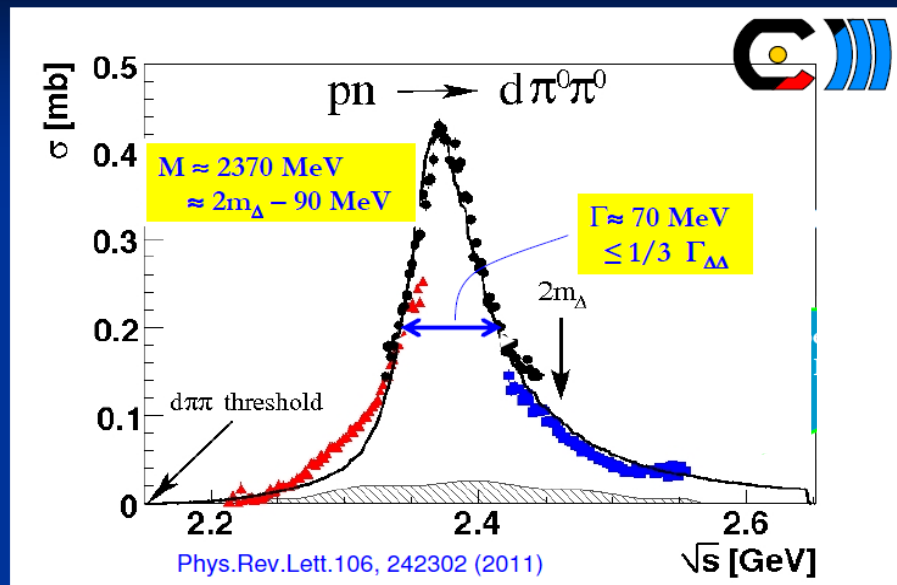
cerncourier.com/cws/article/cern/57836 (2014)

Experiments at the Jülich Cooler Synchrotron (COSY) have found compelling evidence for a new state in the two-baryon system, with a mass of 2380 MeV, width of 80 MeV and quantum numbers $I(J^P) = 0(3^+)$. The structure, containing six valence quarks, constitutes a dibaryon, and could be either an exotic compact particle or a hadronic molecule. The result answers the long-standing question of whether there are more eigenstates in the two-baryon system than just the deuteron ground-state. This fundamental question has been awaiting an answer since at least 1964, when first Freeman Dyson and later Robert Jaffe envisaged the possible existence of non-



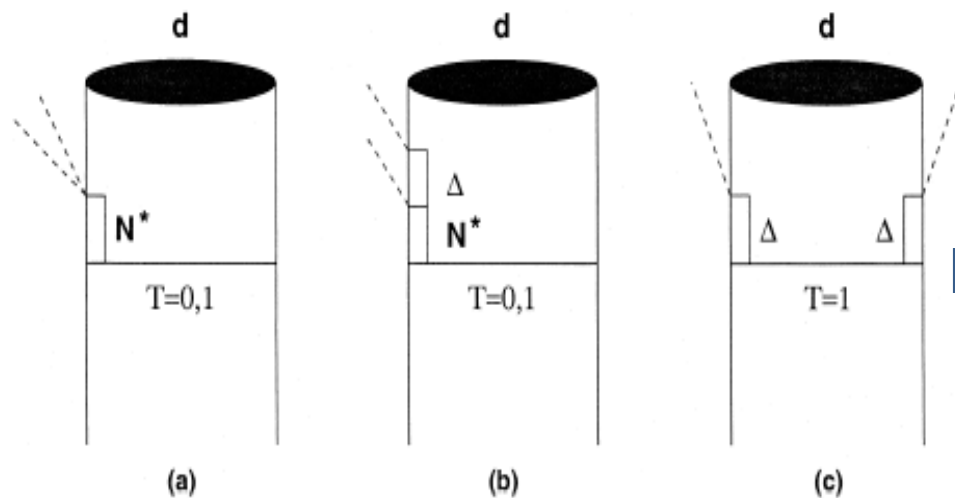
Experiments at the Jülich Cooler Synchrotron (COSY) have found compelling evidence for a new state in the two-baryon system, with a mass of 2380 MeV, width of ~80 MeV and quantum numbers $I(J^P) = 0(3^+)$... Since 2009

The d^* Resonance $I(J^P) = 0(3^+)$



$$\begin{aligned}
 M_{d^*} &\approx 2380 \text{ MeV} \\
 &\approx 2M_\Delta - 84 \text{ MeV} \\
 &> M_{\Delta N\pi} \\
 &> M_{NN\pi\pi} \\
 &> M_{NN}
 \end{aligned}$$

Baryon number=2



Unusual narrow width $2\Gamma_\Delta \approx 230 \text{ MeV}$

$$\begin{aligned}
 \Gamma_{d^*} &\approx 70 \text{ MeV} \\
 &< 1/3 \times 2\Gamma_\Delta
 \end{aligned}$$

Neither NN (Roper),
nor $\Delta\Delta$
Intermediate state

$d^*(2380)$

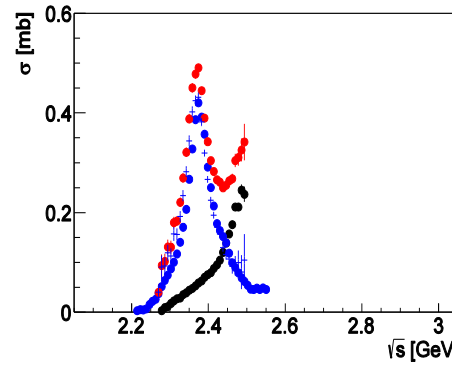
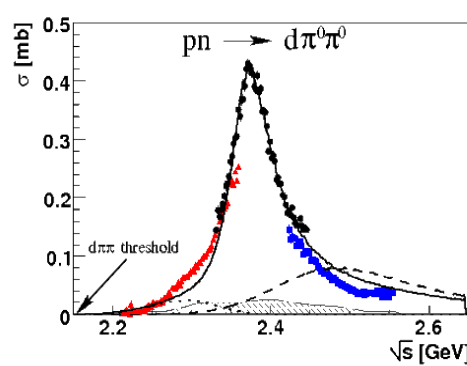
Signals in np procese @ COSY

2 π production processes

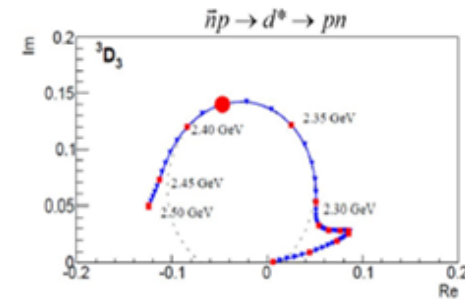
PRL 106 (2011) 242302

PLB 721 (2013) 229

WASA data



np scattering process



Fusion

$d\pi^0\pi^0$

$d\pi^+\pi^-$

$pn \rightarrow d^*(2380)$

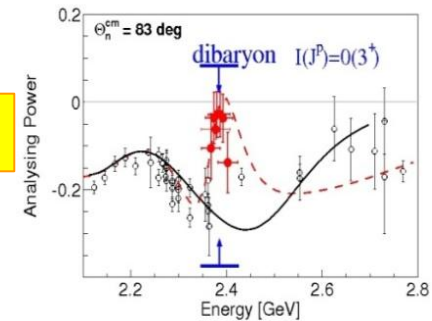
pn

Non-fusion

$pp\pi^-\pi^0$

$pn\pi^0\pi^0$

$pn\pi^+\pi^-$



PRL 112 (2014) 202301

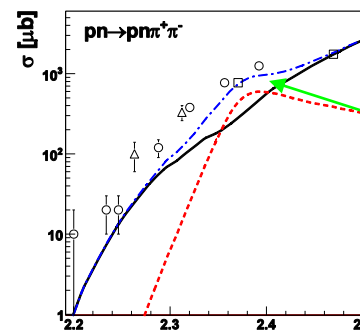
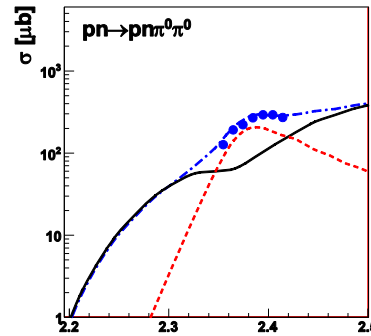
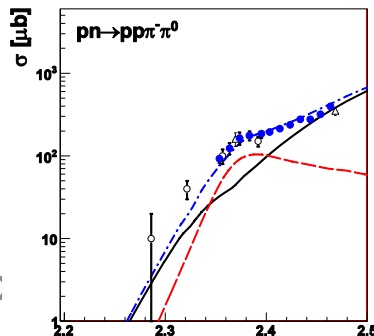
PRC 88 (2013)
055208

PLB 743 (2015) 325

Proc. STORI 2015

8

H
A
D
E
S



8/21/20

Signals in other reactions @ COSY

fusion 2π processes

Measured also in fusion reactions to helium isotopes:



Characters of $d^*(2380)$

- d^* mass locates between $\Delta\Delta$ and $\Delta N\pi$ thresholds

Effect from threshold is expected small

$$M_{\Delta N\pi} = 2310\text{MeV}$$

$$\Gamma_{d^*} \approx 70\text{MeV}$$

$$M_{d^*} \approx 2380\text{MeV}$$

$$M_{\Delta\Delta} = 2464\text{MeV}$$



- d^* narrow width \rightarrow Possible $6q$ structure

Review article: by Heinz Clement,
Progress in Particle and Nuclear Physics,
8/21/2019 93 (2017), 195-142

might be different
from normal hadrons

2, Possible interpretations

▲ Before COSY's observations

$d^*(2380)$

- Consists with COSY's measurement

$I(J^P) = 0(3^+)$

Dyson (64) ----- symmetry analysis

Thomas (83) ----- bag model

Yuan (99) ----- $\Delta\Delta+CC$ quark cluster model

Jaffe(77)

Swart(78)

Oka(80)

Maltman(85)

Goldman(89)

Wang (95).....

▲ After COSY's observations

- Quark model

J.Ping (09/14)-10 coupled channels QM

F.Huang, Y.B.Dong et al. (14-18) -- $\Delta\Delta + CC$ QM

Bashkanov, Brodsky, Clement (13) -- $\Delta\Delta + CC$

A. Compact $6q$
dominated
dibaryon
state

- Hadronic model

Gal (14) --- $\Delta N\pi$

Kukulin (15,16) - $D_{12}\pi$

B. $\Delta N\pi$ (or $D_{12}\pi$)
resonant state

3, Compact 6q dominated $d^*(2380)$ in chiral constituent quark model

Quark model framework


PRC 60 (1999) 045203
CPC 39 (2015) 071001

SU(3) chiral QM + RGM approach

▲ **Interactions:** $V_{ij} = V_{ij}^{\text{conf}} + V_{ij}^{\text{OGE}} + \mathbf{V}_{ij}^{\text{ch}} + \mathbf{V}_{ij}^{\text{chv}}$

q-q potential $\mathbf{V}_{ij}^{\text{ch}} = \sum_a (\mathbf{V}_{ij}^{\text{s(a)}} + \mathbf{V}_{ij}^{\text{ps(a)}})$

Interactive Lagrangian


$$\mathcal{L}_I = -g_{ch} \bar{\Psi} \left(\sum_{a=0}^8 \sigma_a \lambda_a + i \sum_{a=0}^8 \pi_a \lambda_a \gamma_5 \right) \Psi$$

**Model parameters: reproduce experimental data for NN
systems---NN phase shifts,** $\text{BE}_d^{\text{exp}'t} = 2.22 \text{ MeV}$

▲ Trial wave function of d^* : $I(J^P) = 0(3^+)$

$$\Psi_{6q} = \mathcal{A} [\phi_{\Delta}(\xi_1, \xi_2) \phi_{\Delta}(\xi_4, \xi_5) \eta_{\Delta\Delta}(r) + \phi_C(\xi_1, \xi_2) \phi_C(\xi_4, \xi_5) \eta_{CC}(r)]_{S=3, I=0, C=(00)} \cdot$$

6-quark
RGM

$$\Delta : (0s)^3 [3]_{\text{orb}}, S = 3/2, I = 3/2, C = (00),$$

$$C : (0s)^3 [3]_{\text{orb}}, S = 3/2, I = 1/2, C = (11),$$

$\eta_{\Delta\Delta}(r)$ and $\eta_{CC}(r)$
are not orthogonal

▲ Hadronization-----Channel wave function:

Using the projection method to integrate out the internal coordinates inside the clusters (or Hadronization approach)

$$\Psi_{d^*} = |\Delta\Delta\rangle \chi_{\Delta\Delta}(r) + |CC\rangle \chi_{CC}(r)$$

The two components are orthogonal due to the quark exchange effect

$$\chi_{\Delta\Delta}(r) \equiv \langle \phi_{\Delta}(\xi_1, \xi_2) \phi_{\Delta}(\xi_4, \xi_5) | \Psi_{6q} \rangle,$$

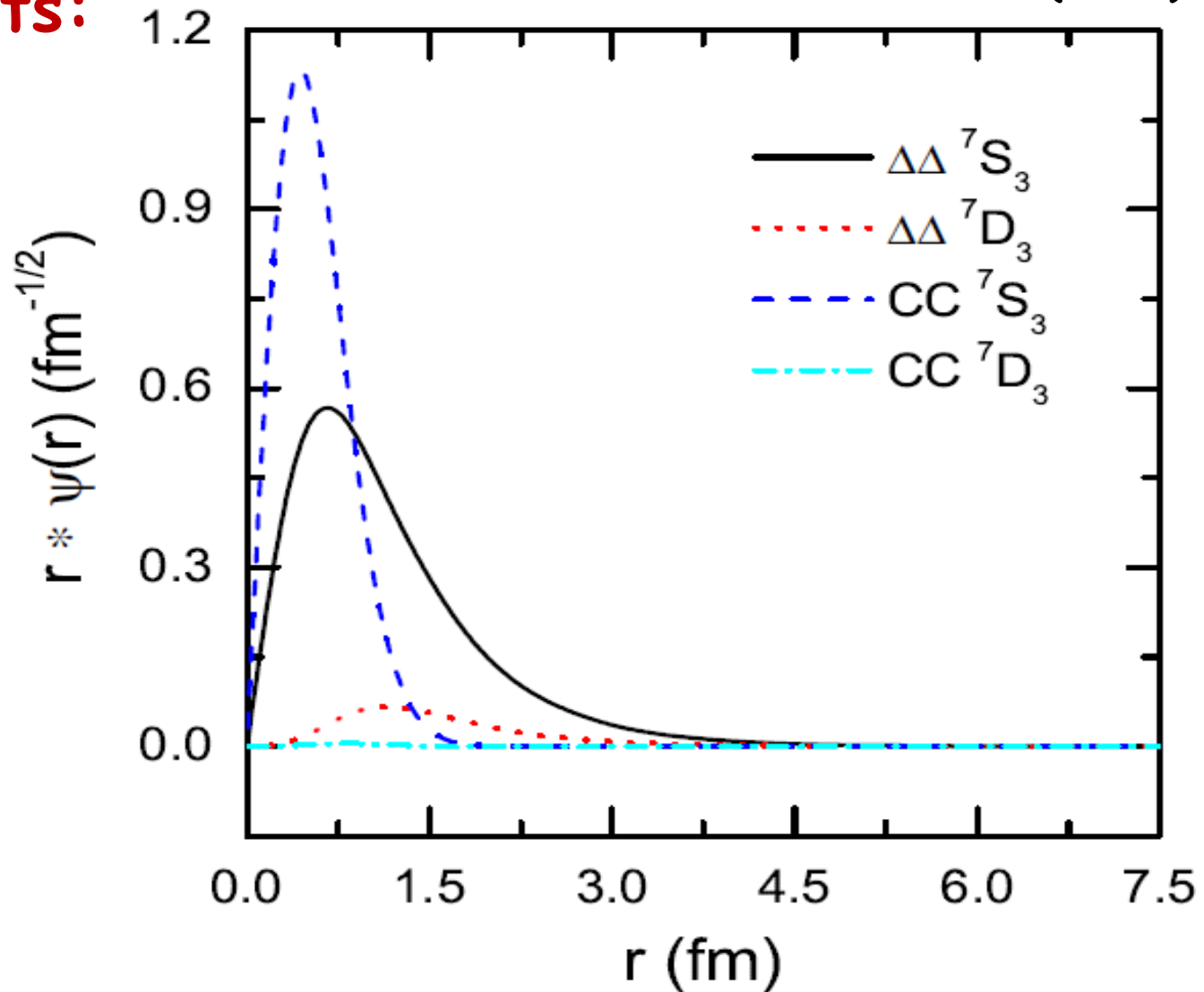
$$\chi_{CC}(r) \equiv \langle \phi_C(\xi_1, \xi_2) \phi_C(\xi_4, \xi_5) | \Psi_{6q} \rangle,$$

(A), Mass and wave function of d^* $I(J^P) = 0(3^+)$

▲ Results:

CPC 39 (2015) 071001

d^* (WFs)



- Binding energy

$$BE_{d^*}^{\text{th}} = 84\text{MeV}$$

$$BE_{d^*}^{\text{exp't}} = 84\text{MeV}$$

		Ext. SU(3) (f/g=0)	
		$\Delta\Delta$ (L=0,2)	$\Delta\Delta$ -CC (L=0,2)
d^* Binding Energy(MeV)		62.3	83.9
Fraction of Wave Function (%)	$\Delta\Delta$ (L=0)	98.01	31.22
	$\Delta\Delta$ (L=2)	1.99	0.45
	CC (L=0)	0	68.33
	CC (L=2)	0	0.00

Reason for the large component of CC (68%)

$$P_{36} = P_{36}^r P_{36}^{sfc}$$

$$I(J^P) = 0(3^+)$$

1). Intrinsic character of d^* ----- $\langle P_{36}^{sfc} \rangle$
quark exchange effect of sfc large (negative: -4/9)

2). Dynamical effect-----

(SI=30) , OGE and vector meson exchange
induced Δ - Δ short range interaction is attractive

Two cluster closer \longrightarrow large CC component

\longrightarrow d^* deep bound and narrow width

d^* might be a 6q dominant state!

(B), Strong decays

PRC91, (2015) 064002

PRC94, (2016) 014003

▲ 2π decay widths

Three-body
decay

$$d^* \rightarrow d\pi^0\pi^0 \quad (d\pi^+\pi^-)$$

$$d^* \rightarrow pp\pi^-\pi^0$$

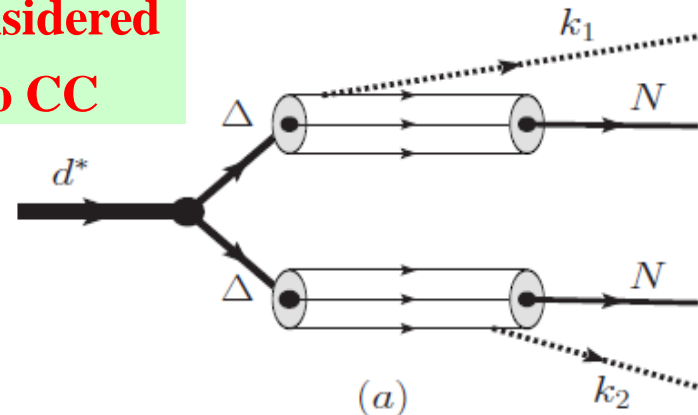
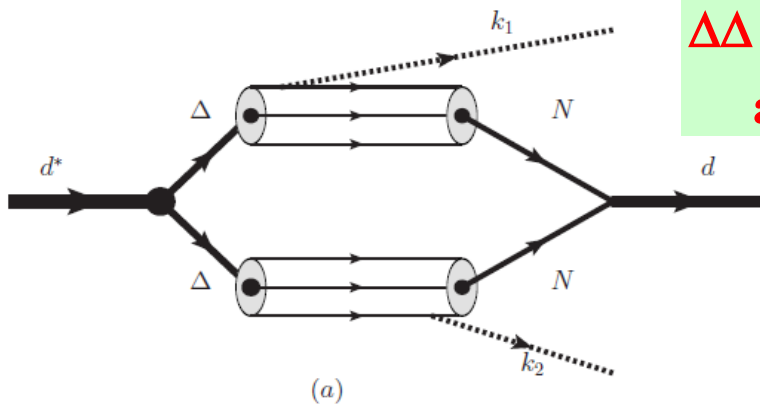
Typical
diagrams

Four-body
decay

$$d^* \rightarrow np\pi^0\pi^0 \quad (np\pi^+\pi^-)$$

$$d^* \rightarrow nn\pi^0\pi^+$$

$\Delta\Delta$ is considered
and no CC



Parameter:

$qq\pi$
Interaction

$$\mathcal{H}_{qq\pi} = g_{qq\pi} \vec{\sigma} \cdot \vec{k}_\pi \tau \cdot \phi \frac{1}{(2\pi)^{3/2} \sqrt{2\omega_\pi}},$$

$$\Delta \rightarrow N\pi$$

8/21/2019

Coupling & form factor

$$\Gamma_{\Delta \rightarrow \pi N} = \frac{4}{3\pi} k_\pi^3 (g_{qq\pi} I_o)^2 \frac{\omega_N}{M_\Delta},$$

	Theor.(MeV)	Expt.(MeV)
$d^* \rightarrow d\pi^+\pi^-$	16.8	16.7
$d^* \rightarrow d\pi^0\pi^0$	9.2	10.2
$d^* \rightarrow pn\pi^+\pi^-$	20.6	21.8
$d^* \rightarrow pn\pi^0\pi^0$	9.6	8.7
$d^* \rightarrow pp\pi^0\pi^-$	3.5	4.4
$d^* \rightarrow nn\pi^0\pi^+$	3.5	4.4
$d^* \rightarrow pn$	8.7	8.7
Total	71.9	74.9

* Too large width for ($\Delta\Delta$) component only

$M_{d^*}(\text{MeV})$	(100%) $\Delta\Delta$ 2374	Expt 2375
Decay channel	$\Gamma(\text{MeV})$	$\Gamma(\text{MeV})$
$d^* \rightarrow d\pi^0\pi^0$	17.0	10.2
$d^* \rightarrow d\pi^+\pi^-$	30.8	16.7
Total	132.8	74.9

Discussions:

* FSI is about 26~30%

* Isospin breaking factor

$$\frac{\Gamma(d^* \rightarrow d\pi^+\pi^-)}{\Gamma(d^* \rightarrow d\pi^0\pi^0)} \sim 1.8 \quad (1.6, \quad 2.0)$$

$$\frac{\Gamma(d^* \rightarrow pn\pi^+\pi^-)}{\Gamma(d^* \rightarrow pn\pi^0\pi^0)} \sim 2.2 \quad (2.5, \quad 2.5)$$

↑
Ideal!

* All partial and total widths agree with data

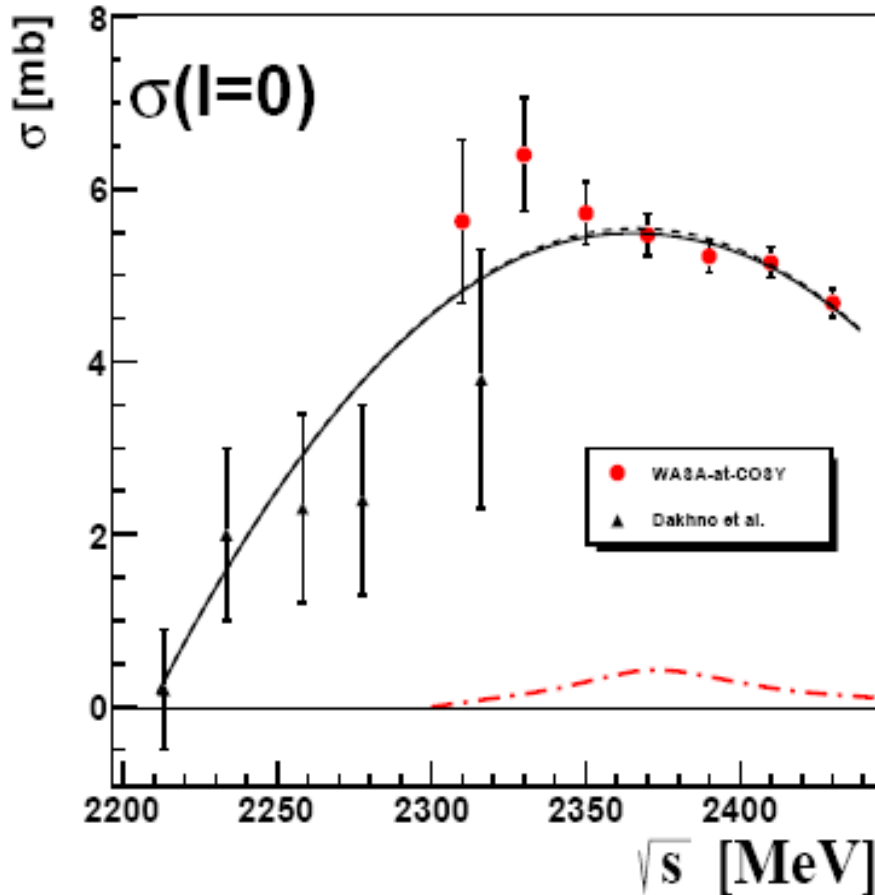
$$\Gamma^{exp't} = 70 \sim 75 \text{ MeV}$$

$$\Gamma^{th} \approx 72 \text{ MeV}$$

The narrow width is due to large CC component

▲ Single- π decay

$$\sigma_{NN \rightarrow NN\pi}(I=0) = 3(2\sigma_{np \rightarrow pp\pi^-} - \sigma_{pp \rightarrow pp\pi^0})$$



● Experimental status

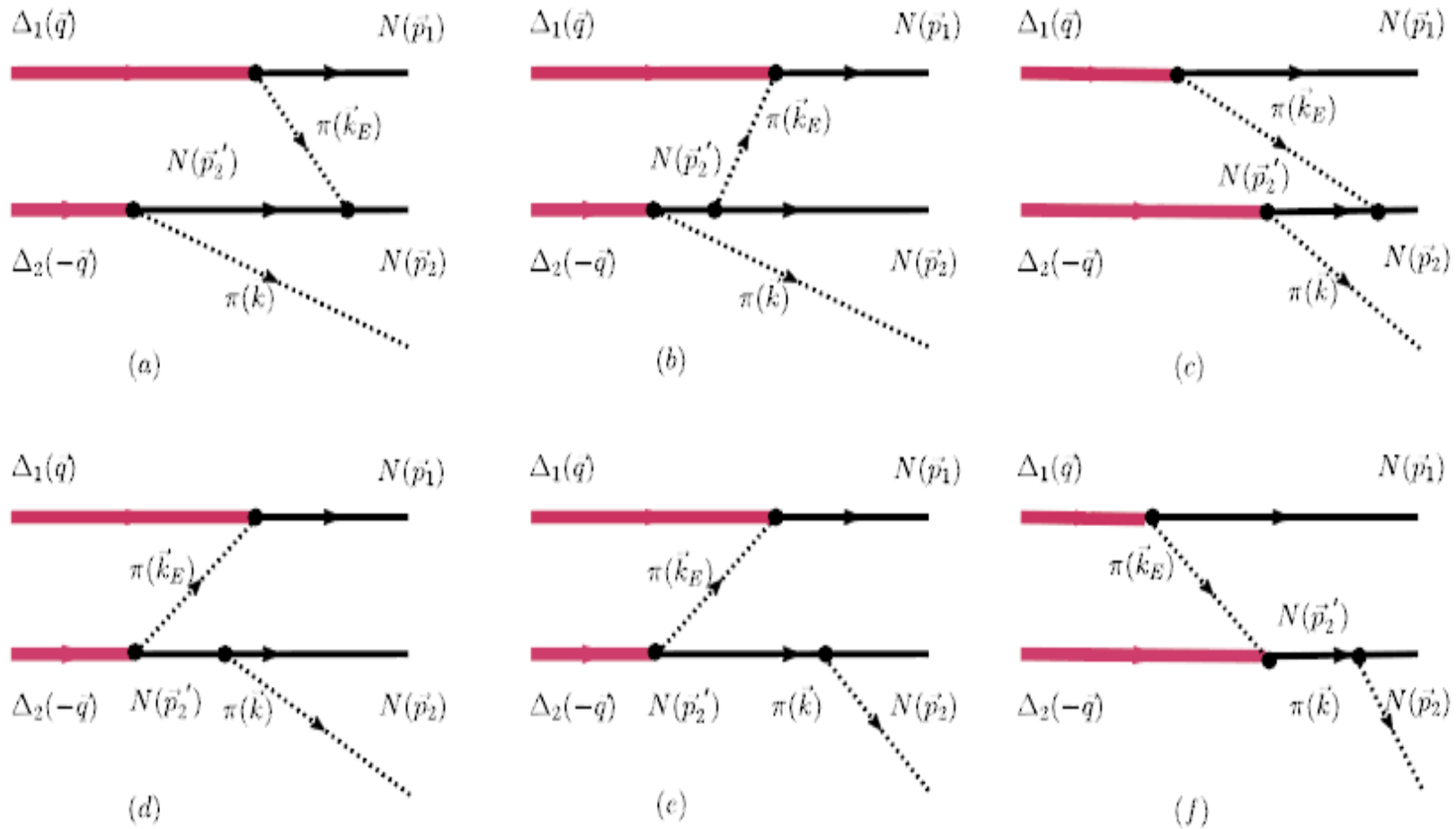
The WASA-@-COSY
Collaborations,
arXiv:1702.07212v1 [nucl-ex]
PLB774 (2017), 599-607

Dash-dotted line illustrates a
10% d^* resonance contribution

Upper limit of branching
ratio for $d^*(2380) \rightarrow NN\pi$
is 9%.

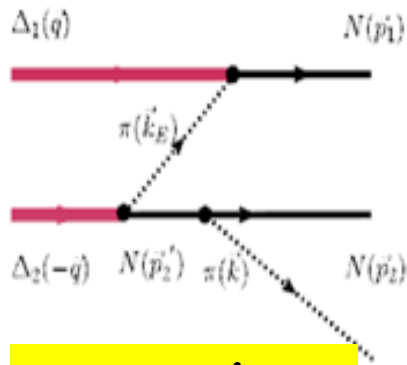
This channel might serve as a test!

► compact 6q dominated case:



Typical diagrams: one
pion emitted from
cluster II

Fig. 1. Six possible ways to emit pion only from the $\Delta\Delta$ component of d^* in the $d^* \rightarrow NN\pi$ decay process. The outgoing pion with momenta \vec{k} is emitted from Δ_2 . The other six sub-diagrams with pion emitted from Δ_1 are similar, and then are not shown here for reducing the size of the figure.



$$\Psi_{d^*} = |\Delta\Delta\rangle \chi_{\Delta\Delta}(r) + |CC\rangle \chi_{CC}(r)$$

$$\Delta : (0s)^3 [3]_{\text{orb}}, S = 3/2, I = 3/2, C = (00),$$

$$C : (0s)^3 [3]_{\text{orb}}, S = 3/2, I = 1/2, C = (11),$$

Sample

$$L = L_{\pi NN} + L_{\Delta N \pi}$$

Intermediate states: (N, N*, Δ, Δ*)

Low-lying resonances need to be considered

From quark model

$$\frac{g_{\pi\Delta\Delta}^2}{4\pi} = \frac{1}{25} \frac{M_{\Delta}^2}{M_N^2} \frac{g_{\pi NN}^2}{4\pi}, \quad g_{\pi\Delta\Delta} \text{ small}$$

1, $C \rightarrow \Delta$, interaction should be color and isospin-dependent

2, $CC(SI=3,0) \rightarrow NN^*(1400)$, D-wave of OGE is required

The suppressions enable to ignore the contribution from the CC component in d^*

Our prediction, 1% is compatible with the Exp't upper-limits

(C), Form factors

Form factors: 2S+1 relative to size arXiv:1704.01253

Nucleon (1/2): $\langle N(p') | J_N^\mu | N(p) \rangle = \bar{U}_N(p') \left[F_1(Q^2) \gamma^\mu + i \frac{\sigma^{\mu\nu} q_\nu}{2M_N} F_2(Q^2) \right] U(p),$

$$G_E(Q^2) = F_1(Q^2) - \eta F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$

Breit frame

$$\langle N(\vec{q}/2) | J_N^0 | N(-\vec{q}/2) \rangle = (1 + \eta)^{-1/2} \chi_{s'}^+ \chi_s G_E(Q^2)$$

$$\langle N(\vec{q}/2) | \vec{J}_N | N(-\vec{q}/2) \rangle = (1 + \eta)^{-1/2} \chi_{s'}^+ \frac{\vec{\sigma} \times \vec{q}}{2M_N} \chi_s G_M(Q^2).$$

Deuteron (1): $J_{jk}^\mu(p', p) = \epsilon_j'^{* \alpha}(p') S_{\alpha\beta}^\mu \epsilon_k^\beta(p)$

$$S_{\alpha\beta}^\mu = - \left[G_1(Q^2) g_{\alpha\beta} - G_3(Q^2) \frac{Q_\alpha Q_\beta}{2m_D^2} \right] P^\mu - G_2(Q^2) (Q_\alpha g_\beta^\mu - Q_\beta g_\alpha^\mu),$$

$$G_C(Q^2) = G_1(Q^2) + \frac{2}{3} \eta_D G_2(Q^2), \quad G_M(Q^2) = G_2(Q^2),$$

$$G_Q(Q^2) = G_1(Q^2) - G_2(Q^2) + (1 + \eta_D) G_3(Q^2),$$

Breit frame

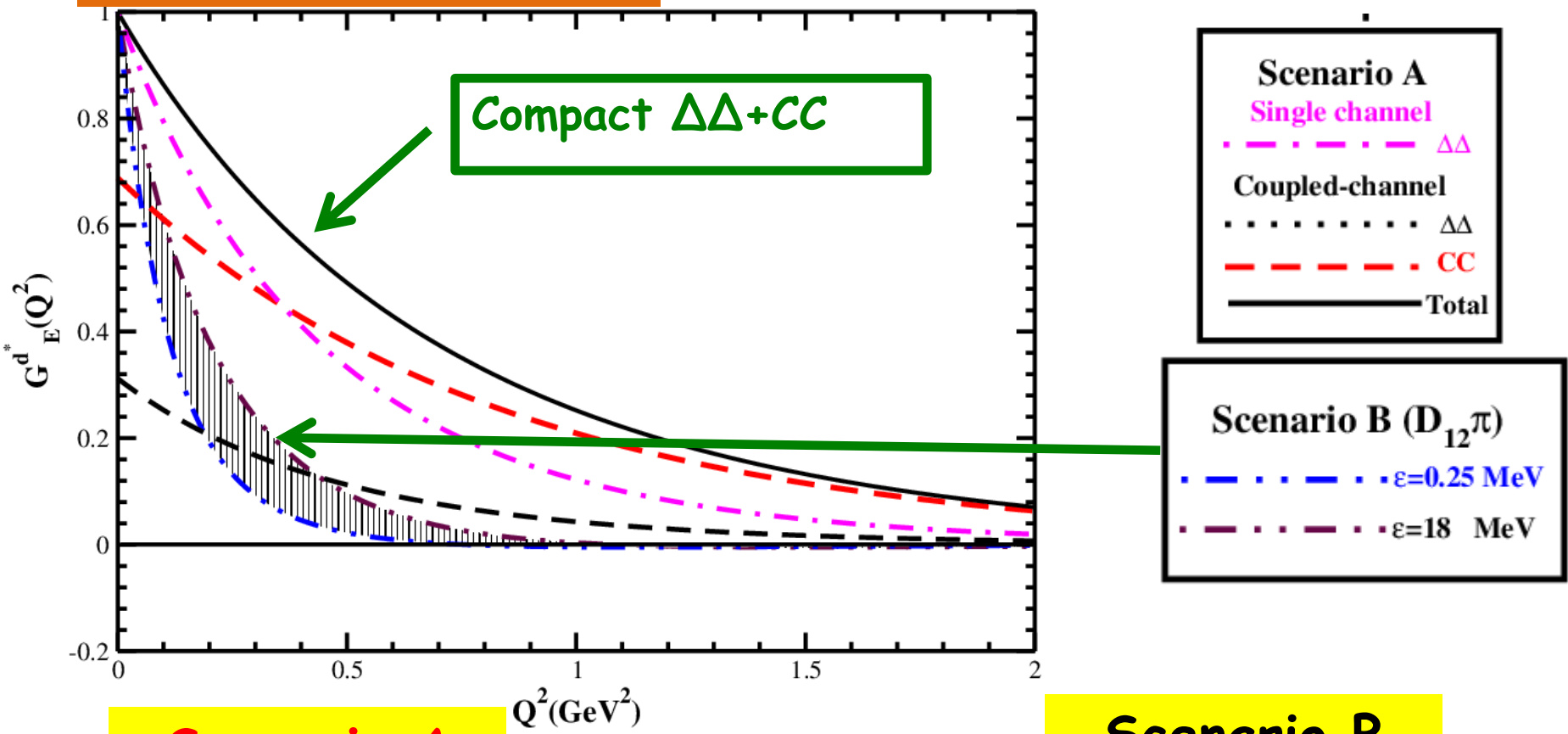
$$G_C(Q^2) \longrightarrow \frac{1}{3} \sum_\lambda \langle p', \lambda | J^0 | p, \tilde{\lambda} \rangle.$$

Form factors of $d^*(2380)$, spin=3 system

$2S+1=7$ form factors

PRD96 094001 (2017)

Charge Distributions (related to the size of system)



Scenario A

Scenario B

$d^*(2380)$		
Cases	A1	A2
<i>rms</i> (fm)	1.09	0.72

rms

D_{12}		
Cases	B1	B2
<i>rms</i> (fm)	2.64	1.87

Magnetic Moment

Naïve quark model
Nucleon

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \rightarrow -\frac{2.79}{1.91_{EXPT.}}$$

$d^*(2380)$

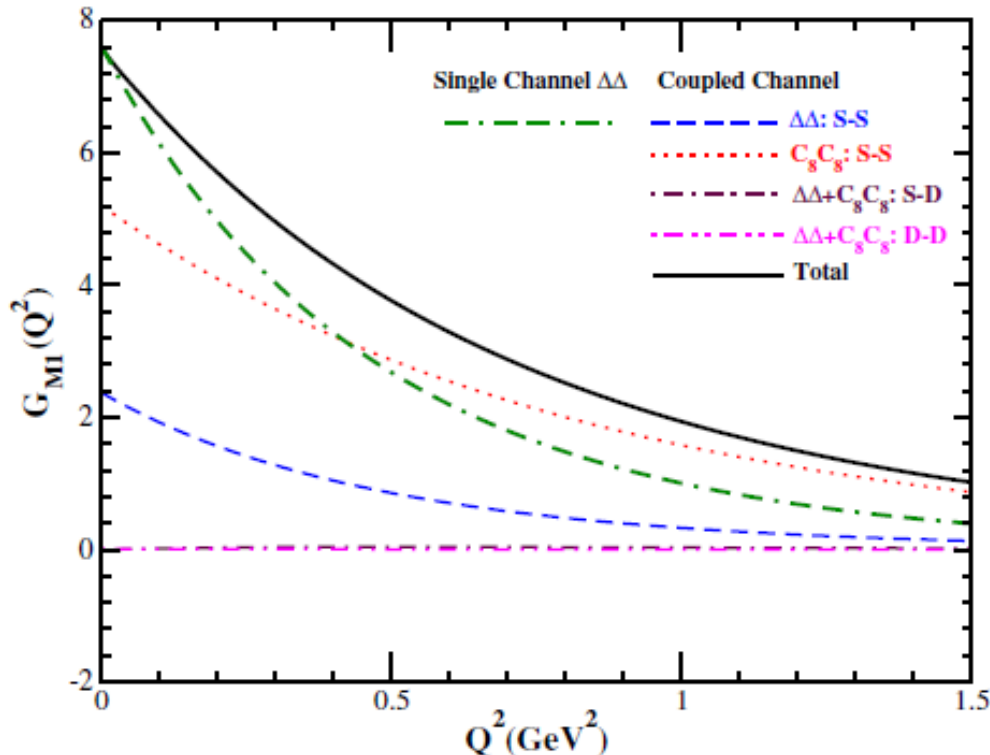
$\Delta\Delta + CC$

$$\mu_{d^*} = \frac{M_{d^*}}{m_q} \approx 7.6$$

$d^*(2380)$

$D_{12}\pi$

$$\mu_{d^*} = \frac{2M_{d^*}}{3m_q} \approx 5.1$$



$$G_{M1}^{NCQM}(0) = [\mathcal{P}_{\Delta\Delta}^S + \mathcal{P}_{C_8C_8}^S] \frac{3M_{d^*}}{M_N} \sim \frac{3M_{d^*}}{M_N} = 7.62,$$

$$R_{C_8C_8}^{\Delta\Delta} = \frac{G_{M1}^{\Delta\Delta}}{G_{M1}^{C_8C_8}} = \frac{2.37}{5.20} = 0.4558,$$

$$-\frac{d}{dQ^2} G_{M1}^{\Delta\Delta}(Q^2) \Big|_{Q^2 \rightarrow 0} = 5.014 \text{ GeV}^{-2} = 0.195 \text{ fm}^2,$$

$$-\frac{d}{dQ^2} G_{M1}^{C_8C_8}(Q^2) \Big|_{Q^2 \rightarrow 0} = 6.139 \text{ GeV}^{-2} = 0.239 \text{ fm}^2,$$

The magnetic dipole form factor $M1$ of d^* .

4. Summary and outlook

d^* : Hexaquark dominated state :
(CC: component $\sim 66-68\%$ in $\Delta\Delta+CC$)

Compact 6q dominated

Dong et al., PLB769 (2017) 223

$\Delta N\pi$ (or $D_{12}\pi$) system

A.Gal, PLB769 (2017) 436

Good \leftarrow **Mass** Double-pion strong decays \rightarrow Good

$d^*(2380)$ single- π decay

Exp't BR $\leq 9\%$

$d^*(2380) \rightarrow NN\pi$

our predicted BR of 1% is compatible with the exp't upper limit of 9%

the resultant BR for $\Delta N\pi$ (or $D_{12}\pi$) is about 18%.

in the mixing case

$$\alpha\Gamma_{<} + (1 - \alpha)\Gamma_{>} = \Gamma_{NN\pi\pi}^{d^*}$$

$$\Gamma_{>} = 100 \text{ MeV} \quad \Gamma_{NN\pi\pi}^{d^*} = 60 \text{ MeV}$$

$$\text{if } \Gamma_{<} = 44 \text{ MeV} \quad \alpha = \frac{5}{7}$$

BR can be 9%

Suggest other experimental searches

- $\gamma + d$ Process (Mainz, Jlab.)
- $\Upsilon \rightarrow \bar{d}^* + X$ Process (Belle)
[BR($\Upsilon \rightarrow \bar{d} + X$) $\sim 2.86 \times 10^{-5}$]
- $e^+ + e^- \rightarrow \bar{d}^* + p + n$ Processes
(Belle?)

If the d^* is further confirmed by experiments, Our interpretation looks reasonable. Thus, it might be a state with 6q structure dominant and moreover, the more information about the short range interaction is expected.

Thanks !

BACKUP SLICES

Before the discovery of d^*

- A pioneer discussion from symmetry: J.Dyson, PRL 13, 815 (1964)

Two baryon systems
SU(6) classification :

Anti-symmetric representations:
Non-strange states

$(I, J) = (3,0)(2,1)(1,0)(1,2)(0,1)(0,3)$ 6 states

Casimir operator reduced
a mass formula

$$M = A + B' (T(T+1)) + B'' (J(J+1))$$

If $B' = B'' = B$, the obtained deuteron mass
1876MeV, and then, obtain A,

Choose $B = 50\text{MeV}$, Then, $M_{d^*} = 2376\text{MeV}$

A. Compact 6q dominated exotic state

(a) In 1999, proposed d^* with $\Delta\Delta+CC$ structure

X.Q.Yuan, Z.Y.Zhang, Y.W.Yu, P.N.Shen, PRC 60 (1999) 045203

- ▶ d^* binding energy: 40-80 MeV
- ▶ CC enhances binding energy by 20 MeV

(b) In 2013, proposed narrow d^* width due to Harvey formula

$$|\Psi_{d^*}\rangle = \sqrt{\frac{1}{5}}|\Delta\Delta\rangle + \sqrt{\frac{4}{5}}|6Q\rangle$$

Bashkanov, Brodsky, H.Clement, Phys.Lett.B727 (2013) 438

(c) In 2014, gave CC fraction of 68% in $d^*(\Delta\Delta+CC)$

F.Huang, Z.Y.Zhang, P.N.Shen, W.L.Wang, CPC 39 (2015) 071001

Decay widths

Three-body decay

$$\Gamma_{d^* \rightarrow d\pi^0\pi^0} = \frac{1}{2!} \int d^3k_1 d^3k_2 d^3p_d (2\pi) \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{p}_d) \\ \times \delta(\omega_{k_1} + \omega_{k_2} + E_{p_d} - M_{d^*}) |\overline{\mathcal{M}}_{if}^{\pi^0\pi^0}|^2$$

$$\mathcal{M}_{if}^{\pi^0\pi^0} = \frac{1}{\sqrt{3}} \sum F_1 F_2 k_{1,\mu} k_{2,\nu} I_S^0 I_I^0 C_{1\nu,1\mu}^{jm_j} C_{3m_{d^*},jm_j}^{1m_d} \\ \times \int d^3q \left[\frac{\chi_d^*(\vec{q} - \frac{1}{2}\vec{k}_{12})}{E_\Delta(q) - E_N(q - k_1) - \omega_1} \right. \\ + \frac{\chi_d^*(\vec{q} + \frac{1}{2}\vec{k}_{12})}{E_\Delta(q) - E_N(q - k_2) - \omega_2} \\ + \frac{\chi_d^*(\vec{q} + \frac{1}{2}\vec{k}_{12})}{E_\Delta(-q) - E_N(-q - k_1) - \omega_1} \\ \left. + \frac{\chi_d^*(\vec{q} - \frac{1}{2}\vec{k}_{12})}{E_\Delta(-q) - E_N(-q - k_2) - \omega_2} \right] \chi_{d^*}(\vec{q})$$

Four-body decay

$$\Gamma_{d^* \rightarrow pn\pi^0\pi^0} = \frac{1}{2!2!} \int d^3k_1 d^3k_2 d^3p_1 (2\pi) \delta(\Delta E) \\ \times |\overline{\mathcal{M}}(k_1, k_2; p_1)|^2$$

$$\mathcal{M}(k_1, k_2; p_1) = \mathcal{M}^{\text{bare}}(k_1, k_2; p_1) \times \mathcal{I} \quad \longleftrightarrow \text{FSI}$$

$$\mathcal{I} = \mathcal{J}^{-1}(k) = C(k^2) \frac{\sin \delta e^{i\delta}}{k}$$

$$\mathcal{M}^a(k_1, k_2; p_1) = \int d^3p_2 d^3q [\mathcal{H} \mathcal{S}_f \mathcal{H}] \Psi_{d^*}(q) \\ \times \delta^3(\vec{p}_1 + \vec{k}_1 - \vec{q}) \delta(\vec{p}_2 + \vec{k}_2 + \vec{q}) \\ = \int d^3p_2 \delta^3(\vec{p}_1 + \vec{p}_2 + \vec{k}_1 + \vec{k}_2) [\mathcal{H} \mathcal{S}_f \mathcal{H}] \\ \times \Psi_{d^*}(-\vec{p}_2 - \vec{k}_2)$$

$$d^* \rightarrow np\pi^0\pi^0 \quad (np\pi^+\pi^-)$$