The vector-vector (Hidden-Gauge) approach and its recent relativistic extensions

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August 17, 2019

Overview

1. **The Hidden-Gauge formalism (HGF)**

2. **Beyond the static ρ exchange with on-shell factorization**

3. **Improved calculation (ρ-ex. meson not on-shell)**

4. **The N/D approach**

The Hidden-Gauge formalism (HGF)

The VV interaction in HGF

Starting from a nonlinear sigma model based on $G/H = SU(2)_L \otimes SU(2)_R/SU(2)_V$: Bando,Kugo,Yamawaki

$$L = (f_\pi^2/4)\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \ , \qquad U(x) = \exp[2i\pi(x)/f_\pi] \tag{1}$$

and introduce new variables ξ_L, ξ_R and the field V_μ:

$$U(x) \equiv \xi_L^\dagger(x)\xi_R(x) \ , V_\mu = (1/2i)(\partial_\mu \xi_L \cdot \xi_L^\dagger + \partial_\mu \xi_R \cdot \xi_R^\dagger) \tag{2}$$

Any linear combination $L = L_A + aL_V$ of the invariants:

$$L_V = -\frac{f_\pi^2}{4} Tr(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger)^2 \quad L_A = -\frac{f_\pi^2}{4} Tr(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger)^2$$

is equivalent to the original one, Eq. (1). A kinetic term is added, $-(1/4g^2)(V_{\mu\nu})^2$, and choosing $a = 2$ it is obtained

- 1)$m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$ (KSFR relation)
- 2)ρ dominance of the electromagnetic form factor of pions ($gV_\mu(\pi \times \partial^\mu \pi)$)

And, fixing the gauge $\xi_L^\dagger = \xi_R \equiv \xi$ the Lagrangian becomes in the Weinberg's Lagrangian (nonlinear realization of the chiral symmetry)

The VV interaction in HGF

Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \quad (3)$$

$$\mathcal{L}^{(2)} = \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle; \qquad \mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}M_V^2 \langle [V_\mu - \frac{i}{g}\Gamma_\mu]^2 \rangle$$

$$D_\mu U = \partial_\mu U - ieQA_\mu U + ieUQA_\mu; \quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]; \quad U = e^{i\sqrt{2}P/f}$$

Upon expansion of $[V_\mu - \frac{i}{g}\Gamma_\mu]^2$

$$\begin{aligned}
\mathcal{L}_{V\gamma} &= -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle \\
\mathcal{L}_{V\gamma PP} &= eg A_\mu \langle V^\mu (QP^2 + P^2 Q - 2PQP) \rangle \\
\mathcal{L}_{VPP} &= -ig \langle V^\mu [P, \partial_\mu P] \rangle \\
\mathcal{L}_{\gamma PP} &= ie A_\mu \langle Q [P, \partial_\mu P] \rangle \\
\widetilde{\mathcal{L}}_{PPPP} &= -\frac{1}{8f^2} \langle [P, \partial_\mu P]^2 \rangle. \qquad (5)
\end{aligned}$$

The VV interaction in HGF

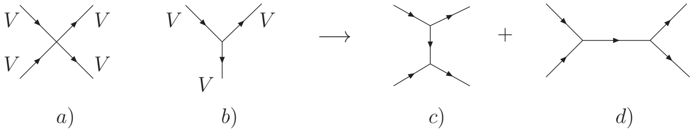
$$\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ieQA_\mu)u + u(\partial_\mu - ieQA_\mu)u^\dagger] \quad u^2 = U$$

$$\frac{F_V}{M_V} = \frac{1}{\sqrt{2}g}, \quad \frac{G_V}{M_V} = \frac{1}{2\sqrt{2}g}, \quad F_V = \sqrt{2}f, \quad G_V = \frac{f}{\sqrt{2}}, \quad g = \frac{M_V}{2f}$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu$$

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle \rightarrow \mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle$$

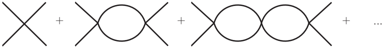
$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle$$



The VV interaction in HGF

Spin projectors

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu$$
$$\mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu - \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu)$$
$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu + \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu) - \frac{1}{3}\epsilon_\alpha\epsilon^\alpha\epsilon_\beta\epsilon^\beta\}$$



$$T = [I - VG]^{-1}V$$

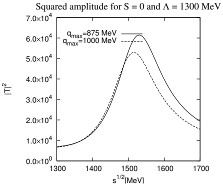
Approach

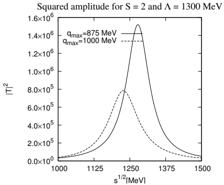
$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (|\vec{k}|,0,0,k^0)/m$$

$$k^\mu = (k^0,0,0,|\vec{k}|)$$
$$\vec{k}/m \simeq 0,\ k_j^\mu\epsilon_\mu^{(l)} \simeq 0$$

$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (0,0,0,1)$$

	Theory		Experiment		
(I, J)	M[MeV]	Γ[MeV]	Name	M[MeV]	Γ[MeV]
(0, 0)	1532	212	$f_0(1370)$	1200 to 1500	200 to 500
(0, 2)	1275	100	$f_2(1270)$	1275.1 ± 1.2	$185.1^{+2.9}_{-2.4}$





Convolution + 2π, 4π box diagrams

[1] R. Molina, D. Nicmorus, and E. Oset, PRD 78 (2008). [2] L. Geng and E. Oset, PRD79 (2009)

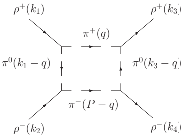
$$\tilde{G}(s) = \frac{1}{N^2}\int_{(m_\rho-2\Gamma_\rho)^2}^{(m_\rho+2\Gamma_\rho)^2} d\tilde{m}_1^2 (-\frac{1}{\pi})\mathcal{I}m\frac{1}{\tilde{m}_1^2 - m_\rho^2 + i\Gamma\tilde{m}_1}$$

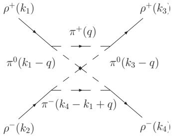
$$\times\int_{(m_\rho-2\Gamma_\rho)^2}^{(m_\rho+2\Gamma_\rho)^2} d\tilde{m}_2^2 (-\frac{1}{\pi})\mathcal{I}m\frac{1}{\tilde{m}_2^2 - m_\rho^2 + i\Gamma\tilde{m}_2}$$

$$\times G(s, \tilde{m}_1^2, \tilde{m}_2^2);$$

$$G = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2}\frac{\omega_1+\omega_2}{\omega_1\omega_2[(P^0)^2 - (\omega_1+\omega_2)^2 + i\epsilon]},$$

$$N = \int_{(m_\rho-2\Gamma_\rho)^2}^{(m_\rho+2\Gamma_\rho)^2} d\tilde{m}_1^2 (-\frac{1}{\pi})\mathcal{I}m\frac{1}{\tilde{m}_1^2 - m_\rho^2 + i\Gamma\tilde{m}_1}$$



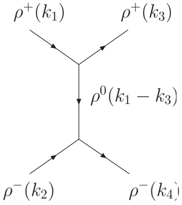


Beyond the static ρ exchange with on-shell factorization

On-shell factorization

[3] D. Gülmez, U. G. Meißner, and J. A. Oller, Eur. Phys. J. C **77**(2017)

$$D(\rho) = \frac{1}{q^2 - M_\rho^2 + i\epsilon} = \frac{1}{-2\vec{k}^{\,2}(1-\cos\theta) - M_\rho^2 + i\epsilon}$$



On-shell momenta

Assume:

$q^0 = 0$; $\quad q = (k_1 - k_3)$; $\quad k_i^2 = m_\rho^2$;

$k^2 = (\frac{E}{2})^2 - m_\rho^2$; ($k^2 < 0$ if $E < 2m_\rho$)

s-wave projection

$$D_\rho(s\text{−wave}) = -\frac{1}{4k^2}\log\left(\frac{4k^2 + M_\rho^2}{M_\rho^2} + i\epsilon\right)$$

If $4k^2 + M_\rho^2 \equiv s - 4M_\rho^2 + M_\rho^2 = 0$, $s = 3M_\rho^2$, $D_\rho(s\text{−wave}) = \infty$. For $s < 3M_\rho^2$, $\mathrm{Im}D_\rho(s - \text{wave}) \neq 0$.

I	J	Contact	Exchange	Total(thr.) $[I^G(J^{PC})]$
0	0	$8g^2$	$-8g^2\left(\frac{3s}{4M_\rho^2}-1\right)$	$-8g^2[0^+(0^{++})]$
0	2	$-4g^2$	$-8g^2\left(\frac{3s}{4M_\rho^2}-1\right)$	$-20g^2[0^+(2^{++})]$

Table 1: Potential V for the scalar and tensor channels with $I=0$.

$$V(s)=V_c+V_{\text{ex}}D_\rho(s-\text{wave})(-M_\rho^2) \tag{6}$$

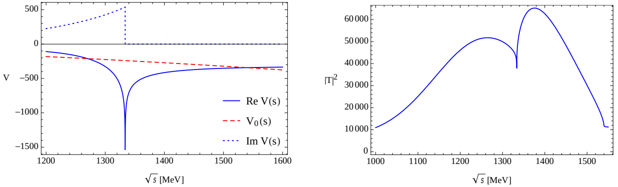


Table 2: Left: Dashed line: $V_0=V_c+V_{\text{ex}}$ from Ref. [1]. Solid line: Re $V(s)$ of Eq. (6). Dotted line: Im $V(s)$ of Eq. (6). Right: $|T|^2$ with $V(s)$ from Eq. (6).

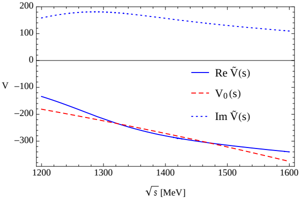
No singularity is found for $J=2$!

Convolution of the D_ρ function

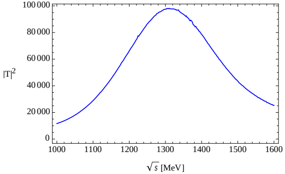
$$\boxed{\tilde{V}(s) = V_c + V_{\rm ex}\tilde{D}_\rho(s-\text{wave})(-M_\rho^2); \quad T = [1-\tilde{V}\tilde{G}]^{-1}\tilde{V};} \tag{7}$$

$$\hat{D}_\rho = \frac{1}{N}\int_{(M_\rho-2\Gamma_\rho)^2}^{(M_\rho+2\Gamma_\rho)^2} d\tilde{m}_\rho^2 \left(-\frac{1}{\pi}\right) \mathrm{Im}\,\frac{1}{\tilde{m}_\rho^2 - M_\rho^2 + i\Gamma\tilde{m}_\rho} \times \left[-\frac{1}{4p^2}\log\left(\frac{4p^2+\tilde{m}_\rho^2}{\tilde{m}_\rho^2} + i\epsilon\right)\right] \tag{8}$$

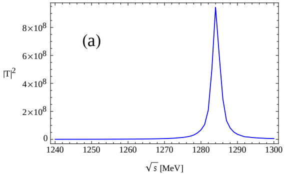
$$\Gamma(\tilde{m}) = \Gamma_\rho\left(\frac{\tilde{m}^2-4m_\pi^2}{M_\rho^2-4m_\pi^2}\right)^{3/2}\theta(\tilde{m}-2m_\pi)$$



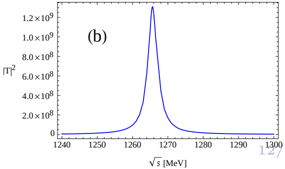
$\tilde{V}$ from Eq. (7)



$\mathrm{Im}\tilde{V} = 0$

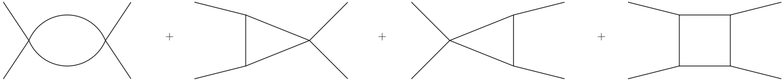


V from Ref. [1]



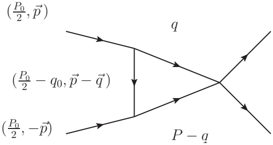
Improved calculation (ρ-ex. meson not on-shell)

One-loop calculation



$$t = \int_{|\vec{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(q)^2} \frac{1}{2\omega(\vec{p}-\vec{q})} \frac{1}{P^0 - 2\omega(q) + i\epsilon}$$

$$\times \frac{1}{\frac{P^0}{2} - \omega(q) - \omega(\vec{p}-\vec{q}) + i\epsilon}$$



Two cuts: (No singularities, $\mathrm{Im}t = 0$ for $P^0 < 2M_\rho$)

1) $P^0 - 2\omega(q) + i\epsilon = 0$

2) $\frac{P^0}{2} - \omega(q) - \omega(\vec{p}-\vec{q}) + i\epsilon = 0$

Since $\omega(q) \geq M_\rho$ for $P^0 < 2M_\rho$, 2) never vanishes.

On-shell fact.: $\frac{1}{2\omega(\vec{p}-\vec{q})} \frac{1}{\frac{P^0}{2} - \omega(q) - \omega(\vec{p}-\vec{q})} \to \frac{-1}{2M_\rho^2}$, exact at thr. with $\vec{q} = 0$.

One-loop calculation

Define $G_{\rho,\text{eff}}$: $G_{\rho,\text{eff}}(s)G(s) = t(s)$, and

$$\tilde{V}_{\text{ex}} = V_{\text{ex}}(-M_\rho^2)G_{\rho,\text{eff}}. \tag{9}$$

$(\tilde{V}_{\text{ex}} + V_c)^2\, G$ gives rise to the diagrams (a), (b), and (c). Approximation for (d) as $\tilde{V}_{\text{ex}}^2 G$.
(d) is exactly evaluated in PRD79, Geng and Oset(2009).
$2.5 - 4.5\%$ difference in total one-loop contribution.

$$\boxed{T = [1 - V_{\text{eff}}G]^{-1}V_{\text{eff}}; \quad V_{\text{eff}} = \tilde{V}_{\text{ex}} + V_c} \tag{10}$$

One-loop calculation

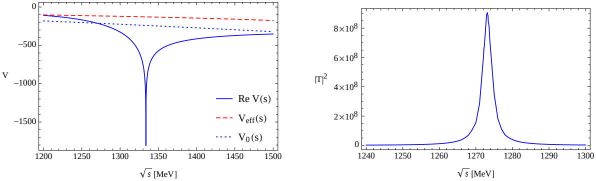


Table 3: Left: Comparison of $V_{\rm eff}$, Re $V(s)$ and the potential from Ref. [1]. Right: $|T|^2$ evaluated with $V_{\rm eff}$ and $\tilde{G}$. The value of $q_{\rm max}$ is 1500 MeV.

Stable parameters of $f_2(1270)$

$$\langle p|\psi\rangle = g\frac{\theta(q_{\rm max}-p)}{E-\omega_1(p)-\omega_2(p)}$$

$p^2\langle p|\psi\rangle^2$. $<p>\simeq 500$ MeV

$$g_T^2 = M_R\Gamma_R\sqrt{|T|^2_{\rm max}}$$

$g_T = 10.7\,\text{GeV}$, $q_{\rm max} = 1.5$ GeV.
($g_T = 11.7\,\text{GeV}$ in Ref. [1])
Both: $\sqrt{s_0} = 1273$ MeV, $\Gamma = 3$ MeV.

If $p = 50$ MeV, $\sqrt{s_0} = 1254$ MeV, $\Gamma = 2$ MeV and $g_T = 10.0$ GeV.
If $p = 800$ MeV, $\sqrt{s_0} = 1300$ MeV, $\Gamma = 5$ MeV and $g_T = 11.0$ GeV.

The N/D approach

[4] M. L. Du, D. Gülmez, F. K. Guo, U. G. Meißner and Q. Wang, Eur. Phys. J. C **78** (2018)

$$\text{Scattering amplitude :} \quad T = N(s)D^{-1}(s) \tag{11}$$

$$N(s) = \sum_{m=0}^{n-1} \overline{a}'_m s^m + \frac{(s-s_0)^n}{\pi} \int_{-\infty}^{s_{\text{left}}} ds' \frac{\text{Im}T(s')D(s')}{(s'-s_0)^n (s'-s)} \,,$$

$$D(s) = \sum_{m=0}^{n-1} \overline{a}_m s^m + \frac{(s-s_0)^n}{\pi} \int_{s_{\text{th}}}^{\infty} ds' \frac{\rho(s')N(s')}{(s'-s)(s'-s_0)^n} \,.$$

Perturbative approach [4]:

$\boxed{N(s) = V(s)}$ Contains the singularity

$$D_2(s) = \gamma_0 + \gamma_1(s-s_0) + \frac{1}{2}\gamma_2(s-s_0)^2 + \frac{(s-s_0)\, s^2}{\pi} \int_{s_0}^{\infty} ds' \frac{\rho(s')V(s')}{(s'-s_0-i\,\epsilon)(s'-s-i\,\epsilon)s'^2} \,;$$

$$s_0 = s_{\text{th}} \,; \qquad \rho(s) = \frac{\sigma(s)}{16\pi\, s} \,; \qquad \sigma(s) = 2\,p\sqrt{s} = \sqrt{(s-s_0)\,s} \,, \tag{12}$$

- γ_i, $i=1,2$: Matching $D_2(s)$ to $1-V\,G$ at threshold:

$$P_2(s) \equiv \gamma_0 + \gamma_1(s-s_0) + \frac{1}{2}\gamma_2(s-s_0)^2 \tag{13}$$

vs.

$$\omega_2(s) = 1 - V(s)G(s) - \frac{(s-s_0)s^2}{\pi}\int_{s_0}^{\infty} ds' \frac{\rho(s')V(s')}{(s'-s_0-i\,\epsilon)(s'-s-i\,\epsilon)s'^2} \tag{14}$$

$$\boxed{\gamma_0 = \omega_2(s_0); \qquad \gamma_1 = \omega_2'(s_0); \qquad \gamma_2 = \omega_2''(s_0)} \tag{15}$$

D(s) at $\mathcal{O}((s-s_0)^3)$

Extra subtraction at $s = 0$:

$$D_3(s) = \gamma_0 + \gamma_1(s-s_0) + \frac{1}{2}\gamma_2(s-s_0)^2 + \frac{1}{3!}\gamma_3(s-s_0)^3 + \frac{(s-s_0)s^3}{\pi}\int_{s_0}^{\infty} ds' \frac{\rho(s')V(s')}{(s'-s_0-i\,\epsilon)(s'-s-i\,\epsilon)s'^3} \,. \tag{16}$$

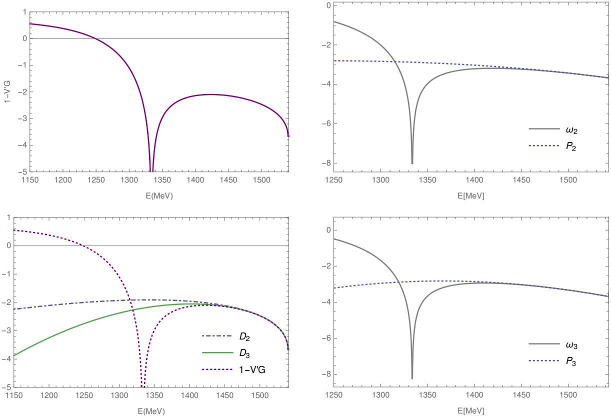
$$P_3(s) \equiv \gamma_0 + \gamma_1(s-s_0) + \frac{1}{2}\gamma_2(s-s_0)^2 + \frac{1}{3!}\gamma_3(s-s_0)^3 \tag{17}$$

vs.

$$\omega_3(s) = 1 - V(s)G(s) - \frac{(s-s_0)s^3}{\pi}\int_{s_0}^{\infty} ds' \frac{\rho(s')V(s')}{(s'-s_0-i\,\epsilon)(s'-s-i\,\epsilon)s'^3} \tag{18}$$

$$\boxed{\gamma_0 = \omega_3(s_0); \qquad \gamma_1 = \omega_3'(s_0); \qquad \gamma_2 = \omega_3''(s_0); \qquad \gamma_3 = \omega_3'''(s_0)} \tag{19}$$

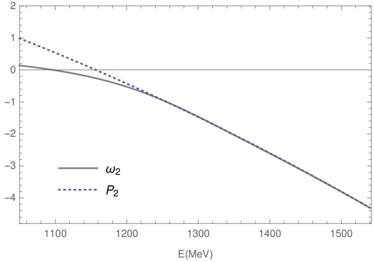
$$D_\rho(p) = \frac{1}{p^2 - m_\rho^2 + i\,\epsilon} \xrightarrow{s-wave} -\frac{1}{4\,p^2}\mathrm{Log}\left(\frac{4\,p^2 + m_\rho^2}{m_\rho^2} + i\,\epsilon\right) \equiv D_\rho^{(s.w.)}$$

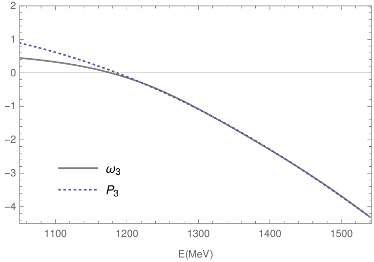


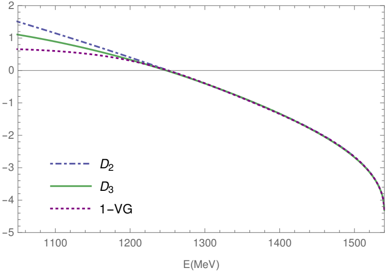
$$V'_{ex} = V_{ex}(-m_\rho^2)D_\rho^{(s.w.)}$$
$$V' = V_c + V'_{\mathrm{ex}}$$

(Real parts)

Result with convoluted potential







$$\hat{D}_\rho = \frac{1}{N}\int_{(M_\rho-2\Gamma_\rho)^2}^{(M_\rho+2\Gamma_\rho)^2} d\tilde{m}_\rho^2 \left(-\frac{1}{\pi}\right) \mathrm{Im}\,\frac{1}{\tilde{m}_\rho^2 - M_\rho^2 + i\Gamma\tilde{m}_\rho}$$

$$\times\left[-\frac{1}{4p^2}\log\left(\frac{4p^2+\tilde{m}_\rho^2}{\tilde{m}_\rho^2}+i\epsilon\right)\right] \tag{20}$$

Parameters:	γ_0	$\gamma_1 \times 10^6$ (MeV^{-2})	$\gamma_2 \times 10^{12}$ (MeV^{-4})	$\gamma_3 \times 10^{18}$ (MeV^{-6})
D_2	-3.7	-2.0	-2.4	-
D_3	-3.7	-3.0	-3.9	7.7
D_2	-4.3	-4.1	0.04	-
D_3	-4.3	-5.1	-0.35	2.8

Table 4: Value of the parameters $\gamma's$ (no convolution: upper two lines; convolution: lower two lines.)

L. S. Geng, R. Molina and E. Oset, "On the chiral covariant approach to $\rho\rho$ scattering," Chin. Phys. C **41**, 124101 (2017)

R. Molina, L. S. Geng, and E. Oset, "Comments on the dispersion relation method to the vector-vector interaction,"(2018)

Conclusions

- Since the potential in the vector-vector interaction is more attractive for $J=2$ than $J=0$, the presence of an state more bound for $J=2$ is unavoidable.
- The state found in HGF has stable properties and similar to the $f_2(1270)$.
- In fact, the radiative decay agrees very well with experiment. Crystal Ball $\Gamma(f_2(1270)\to\gamma\gamma)=2.71^{+0.26}_{-0.23}$ KeV. Our result: 2.6 KeV. Nagahiro, Sekihara, Oset, Hirenzaki, Molina, PRD79, 114023(2009)