The role of charged exotic states in $e^+e^- \to \psi'(J/\psi)\ \pi^+\pi^-$

Daniel Molnar

THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Collaborators:
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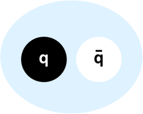
Phys. Lett. B 797 (2019) 134851

1 Exotic States

Quark Model

- **Mesons:**



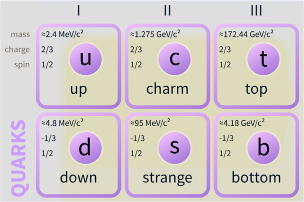
$$P = (-1)^{L+1} \quad C = (-1)^{L+S}$$

L	S	J^{PC}
0	0	0^{-+}
	1	1^{--}
1	0	1^{+-}
	1	$0^{++}\ 1^{++}\ 2^{++}$
2	0	2^{-+}
	1	$1^{--}\ 2^{--}\ 3^{--}$

Forbidden Quantum Numbers:

$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \cdots$

Quarks



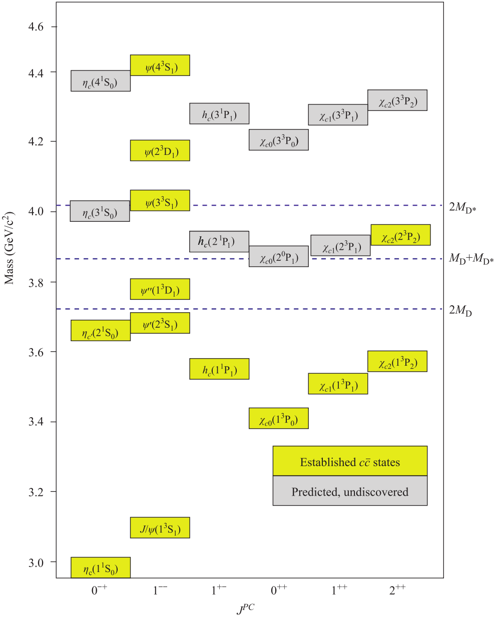
- Heavy Quarks:

$$\frac{d^2}{dr^2}u(r) + 2\mu\left[E - V(r) - \frac{l(l+1)}{2\mu r^2}\right]u(r) = 0$$

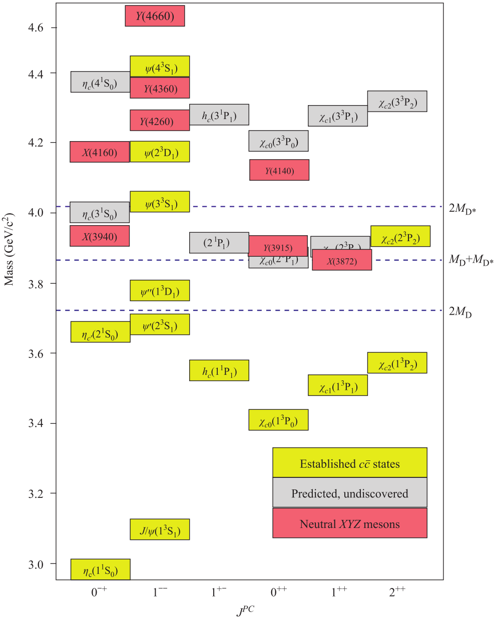
- Phenomenological Potential:

$$V(r) \simeq -\frac{\kappa}{r} + br + \text{spin-terms}$$

S. L. Olsen *Front.Phys.* (2015)



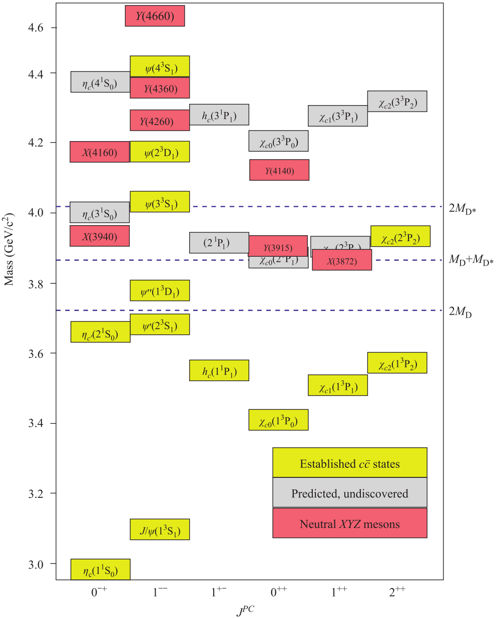
S. L. Olsen *Front.Phys. (2015)*



Alternative explanations:

- Hybrids and Glueballs
- Tetraquark
- Molecular state
- Hadrocharmonium
- Kinematics effects

S. L. Olsen *Front.Phys.* *(2015)*



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No Unique Structure

No new physics! All interpretations are based on QCD.

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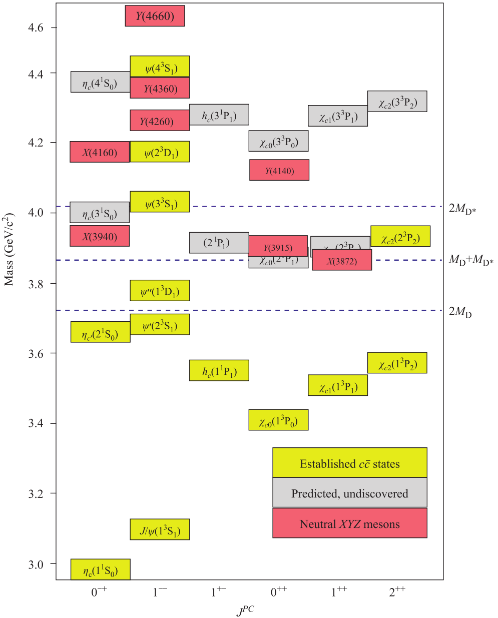

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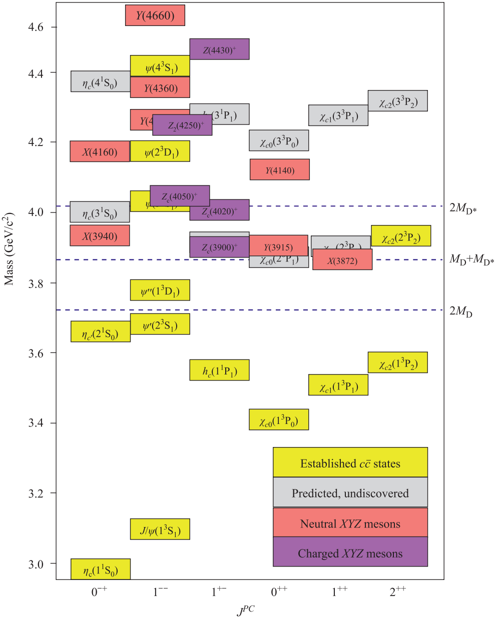
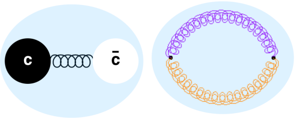
Charged Exotic Mesons

- Confirmed in 2013 by Belle and BESIII
- $c\bar{c} + q_i\bar{q}_j \ (i \neq j)$

S. L. Olsen *Front.Phys.* (2015)

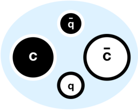


Hybrids, Glueballs



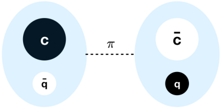
↓ Charged exotic states
↓ Search for the missing J^{PC} (GlueX, Compass and Panda)

Tetraquark



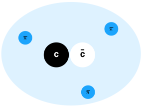
↓ Too many states predicted

Molecular State



↓ Not enough to explain radiative decays
↓ Production at high-p_T

Hadrocharmonium



↓ Decays predominantly into D-mesons

If the force $\begin{cases} \downarrow, & \text{not state} \\ \uparrow, & \text{D-mesons} \end{cases}$

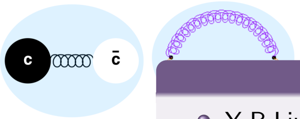
Non-Resonance Interpretation

Threshold effects: Kinematics effects due to the opening of a channel producing a cusp effect.

Triangle singularities: when particles can go simultaneously on shell in a triangle loop.

Both should be a small contributions in the final observable.

Hybrids, Glueballs

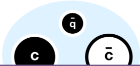


↓Charged exot

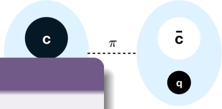
↓Search for the

(GlueX, Compa

Tetraquark



Molecular State



to explain

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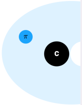
at high-p_T

Hadroch



↓Decays predo

D-mesons

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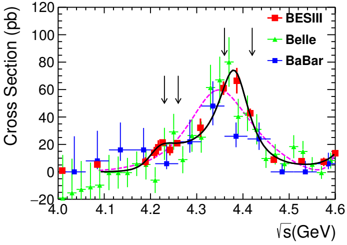
Recent Review Papers

- Y-R Liu *et al.*, Progress in Particle and Nuclear Physics 04, 003 (2019)
- S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90, 15003 (2018)
- R.F. Lebed, R.E. Mitchell, E. S. Swanson, Progress in Particle and Nuclear Physics 93, 143 (2017)
- F-K,Guo *et al.*, Rev. Mod. Phys. 90 014004 (2018)
- H-X. Chen *et al.* Physics Reports 639, 1-121 (2016)
- M.R. Shepherd, J.J. Dudek, R.E. Mitchell, Nature 534 (2016) no.7608, 487-493

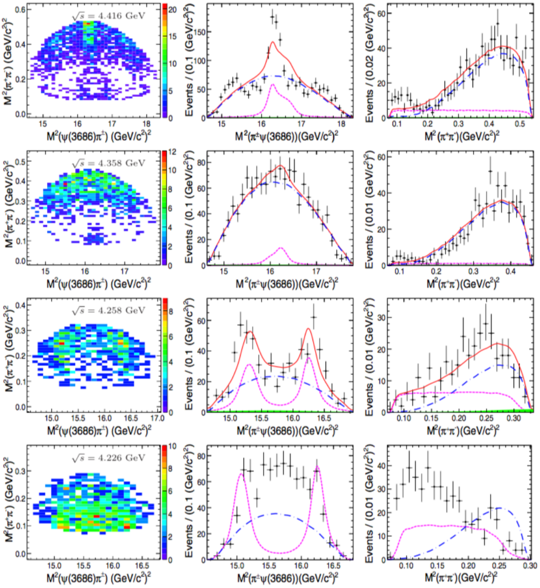
$e^+e^- \to \psi(2S)\,\pi^+\pi^-$

- **Y(4220) + Y(4390)**



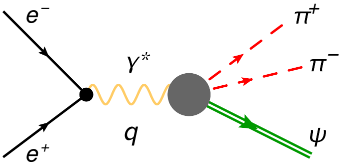
$Z_c(3900) + Z_c(4030)$

- Two charged exotic states!
- No consistent description
- Below $K\bar{K}$ threshold



BESIII PRD (2017)

2 Formalism



$$s = (p_{\pi^+} + p_{\pi^-})^2$$
$$t = (p_\psi + p_{\pi^-})^2$$
$$u = (p_\psi + p_{\pi^+})^2$$

Double Differential Cross Section

$$\frac{\partial^2\sigma}{\partial s \partial t} = \frac{1}{3}\frac{e^2}{(2\pi)^3}\frac{1}{2^5}\frac{(q^2+2m_e^2)}{\sqrt{q^2(q^2-4m_e^2)}}\frac{1}{(q^2)^3}\sum_{\lambda_1\lambda_2}|\mathcal{H}_{\lambda_1\lambda_2}|^2$$

$$\langle \pi\pi\psi(\lambda_2)|\,\mathcal{T}\,|\gamma^*(\lambda_1)\rangle = (2\pi)^4\delta(q - p_\psi - p_{\pi^+} - p_{\pi^-})\,\mathcal{H}_{\lambda_1\lambda_2}$$

Independent Helicity Amplitudes

P-symmetry: $\mathcal{H}_{++}$, $\mathcal{H}_{+-}$, $\mathcal{H}_{+0}$, $\mathcal{H}_{0+}$ and $\mathcal{H}_{00}$

- CP-symmetry: $J_{\pi\pi} \to$ even
- Bose-symmetry: $I_{\pi\pi} = 0\ ,\ 2$
- $I_\psi = 0\ ;\ I_\gamma = 0\ ,\ 1 \implies \boxed{I_{\pi\pi} = 0}$

Isobar Decomposition

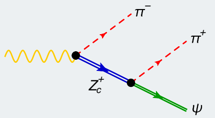
$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = \sum_{J\text{ even}}^{\infty} (2J+1) h^{(J)}_{\lambda_1\lambda_2}(s) d^{(J)}_{\Lambda,0}(\theta)$$

Khuri & Treiman Phys.Rev. (1960)

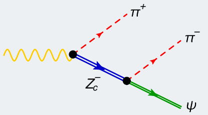
$$\approx \sum_{J\text{ even}}^{J_{max}} (2J+1) \left\{ h^{(J),t}_{\lambda_1\lambda_2}(t)\, d^{(J)}_{\Lambda,0}(\theta_t) + h^{(J),u}_{\lambda_1\lambda_2}(u)\, d^{(J)}_{\Lambda,0}(\theta_u) + h^{(J),s}_{\lambda_1\lambda_2}(s)\, d^{(J)}_{\Lambda,0}(\theta_s) \right\}$$

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) \overset{(J_{max}=0)}{=} h^{(0),t}_{\lambda_1\lambda_2}(t) + h^{(0),u}_{\lambda_1\lambda_2}(u) + h^{(0),s}_{\lambda_1\lambda_2}(s)$$

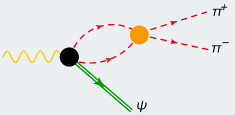
t-channel



u-channel



s-channel



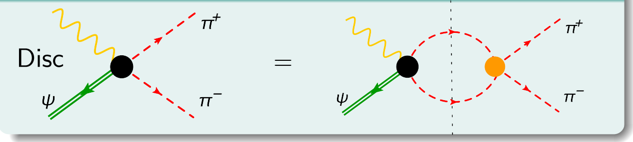
$$h^{(0)}_{\lambda_1\lambda_2}(s) = \frac{1}{2}\int_{-1}^{+1} dz\, \mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = \underbrace{h^{(0),s}_{\lambda_1\lambda_2}(s)}_{\text{only r.h.c.}} + h^{(0),L}_{\lambda_1\lambda_2}(s)$$

$$h^{(0),L}_{\lambda_1\lambda_2}(s) \equiv \frac{1}{2}\int_{-1}^{+1} dz\, \left(h^{(0),t}_{\lambda_1\lambda_2}(t) + h^{(0),u}_{\lambda_1\lambda_2}(u)\right)$$

$J_{\pi\pi} = 0$

$I_{\pi\pi} = 0$

Unitarity in the s-channel



$$\text{Disc}\, h_{\lambda_1\lambda_2}(s) \equiv \frac{1}{2\,i}\left(h_{\lambda_1\lambda_2}(s+i\epsilon) - h_{\lambda_1\lambda_2}(s-i\epsilon)\right) = t^*_{\pi\pi}(s)\ \rho(s)\ h_{\lambda_1\lambda_2}(s)\,\theta(s > 4m_\pi^2)$$

- The pions interaction amplitude can be written in terms of the phase shift:

$$t^*_{\pi\pi}(s) = \frac{e^{-i\delta_{\pi\pi}(s)}\sin\delta_{\pi\pi}(s)}{\rho(s)}$$

- Since $h_{\lambda_1\lambda_2}(s) = |h_{\lambda_1\lambda_2}(s)|e^{i\phi(s)} \implies \phi(s) = \delta_{\pi\pi}(s)$, Watson Theorem
- Therefore, we can use the $\delta_{\pi\pi}(s)$ through the Omnès Function:

$$\Omega(s) = exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_{\pi\pi}(s')}{s'-s}\right]$$

Omnès, Nuovo Cim. (1958)
Muskhelishvili, (1953)

Dispersion Relation

- Assuming no kinematic constrains, we look for a solution in terms of the Omnès function:

$$h^s_{\lambda_1\lambda_2}(s) = \Omega(s)\ G_{\lambda_1\lambda_2}(s)$$

- The unitarity relation for the Omnès function is

$$\mathrm{Disc}\,\Omega(s) = t^*_{\pi\pi}(s)\ \rho(s)\ \Omega(s)\,\theta(s > 4m_\pi^2)$$

- Since $\mathrm{Disc}\,h_{\lambda_1\lambda_2}(s) = \mathrm{Disc}\,h^s_{\lambda_1\lambda_2}(s)$, one can write down a DR for $G_{\lambda_1\lambda_2}$:

$$G_{\lambda_1\lambda_2} = -\int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\,(\Omega)^{-1}(s')\,h^L_{\lambda_1\lambda_2}(s')}{s'-s}$$

Helicity Amplitude with Rescattering

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = h^t_{\lambda_1\lambda_2}(t) + h^u_{\lambda_1\lambda_2}(u) + \Omega(s)\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\mathrm{Disc}\,(\Omega)^{-1}(s')\,h^L_{\lambda_1\lambda_2}(s')}{s'-s}\right\}$$

- 2 subtraction constants to reduce the sensitive to high energy.

Left-Hand Cuts

Invariant Amplitudes

- The helicity amplitude $\mathcal{H}^{\mu\nu}$ can be written in the general form as

$$\mathcal{H}^{\mu\nu} = \sum_{i=1}^{5} F_i L_i^{\mu\nu}$$

where F_i are the invariant amplitudes and $L_i^{\mu\nu}$ is a complete set of Lorenz structures.

- For the S-wave:

$$h_{++}^{(0)}(s) = \frac{s - q^2 - m_\psi^2}{2} f_1(s) - q^2\, m_\psi^2 f_4(s)$$

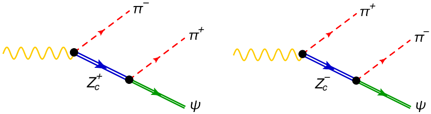
$$h_{00}^{(0)}(s) = -q m_\psi \left(f_1(s) - \frac{s - q^2 - m_\psi^2}{2}\, f_4(s) \right)$$

with f_i the partial wave expansion of F_i.

- The helicity amplitudes are correlated

$$h_{++}^{(0),L}(s) \pm h_{00}^{(0),L}(s) \sim \mathcal{O}(s - (q \pm m_\psi)^2)$$

Z_c Exchange Mechanism



$$\mathcal{H}_{\lambda_1\lambda_2}^{Z_c} = (V_{Z_\pm\psi\pi})^{\beta\nu}\, S_{\nu\mu}(Q_{z_\pm})\, (V_{\gamma^*\pi Z_\pm})^{\mu\alpha} \times \epsilon_\alpha(p_{\gamma^*}, \lambda_1)\, \epsilon_\beta^*(p_\psi, \lambda_2)$$

- We observe that

$$\mathcal{H}_{+-}^{Z_c} = \mathcal{H}_{+0}^{Z_c} = \mathcal{H}_{0+}^{Z_c} \approx 0$$

$$\sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}^{Z_c}|^2 = 2\,|\mathcal{H}_{++}^{Z_c}|^2 + |\mathcal{H}_{00}^{Z_c}|^2 \approx 3\,|\mathcal{H}_{++}^{Z_c}|^2$$

$\therefore$ the effects of the kinematical constraints can be ignored!

- pole contribution: $t = m_z^2$ and $u = m_z^2$ in the numerators.

Anomalous Threshold

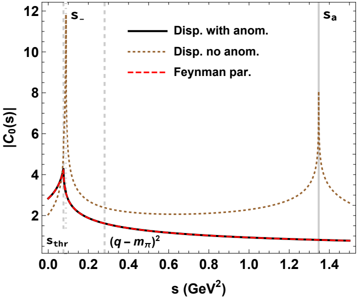
- Depending on the kinematics new nonphysical singularities might appear $(q^2 > 2m_\pi^2 + 2m_z^2 - m_\psi^2)$.
- The anomalous piece that emerges because the anomalous branch point moves onto the first Riemann sheet distorting the integration contour. Effectively, that can be written as

$$\int_{-1}^{1} \frac{dz}{t - m_z^2} = \int_{-1}^{1} \frac{dz}{u - m_z^2} = -\frac{2}{k(s)} log\left(\frac{X(s)+1}{X(s)-1}\right) - i\frac{4\pi}{k(s)}\theta(s_- < s < s_a)$$

- $s_a = 2m_\pi^2 + m_\psi^2 + q^2 - 2m_z^2$ is the position where the argument of the logarithm changes sign.
- Analytical continuation: $q \to q + i\epsilon$

Cross-Check

- Comparison of the DR with the scalar triangle loop calculated via traditional method.



S. Mandelstam, PRL (1960); W. Lucha *et al*, PRD (2007); M. Hoferichter *et al*, Mod. Phys. Conf. Ser. (2014)

Inverse Amplitude Method

Modified-IAM for (J=0):

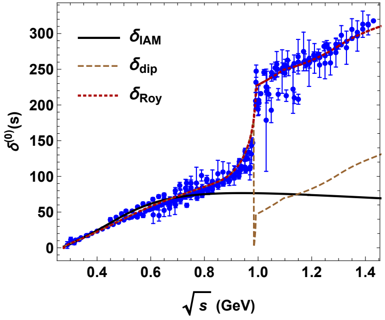
$$t_{\pi\pi}(s) = \frac{|t_{LO}(s)|^2}{t_{LO}(s) - t_{NLO}(s) + A^{mIAM}(s)}$$

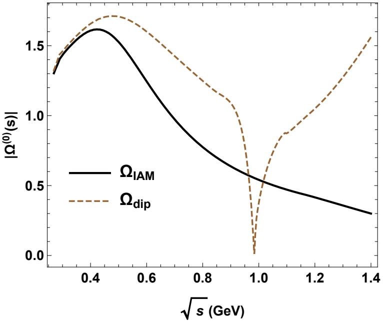
- Correct positions of Adler zeros;
- Consistent description of $f_0(500)$.

GomezNicola et al. PRD (2008)

Omnès Function

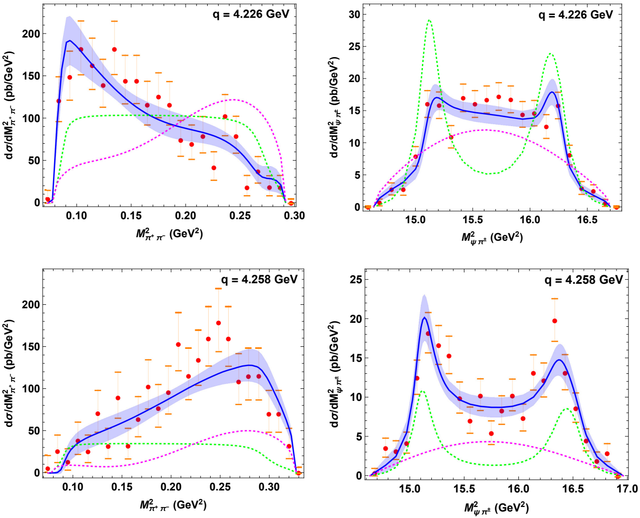
$$\Omega(s) = exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_{\pi\pi}(s')}{s'-s}\right]$$





3 $\psi(2S)\ \pi^+\pi^-$

$Z_c(3900)$



$\chi^2_{red} = 1.16$

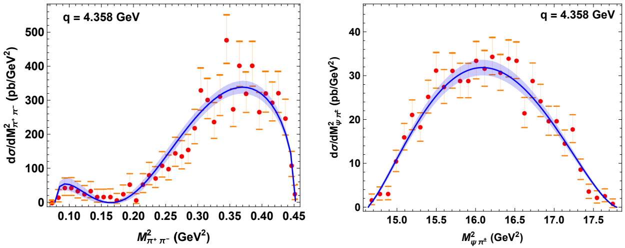
– Combined Fit
-- Only Z_c
-- Only $\pi\pi$-FSI

- $a(q^2)$ and $b(q^2)$ are complex numbers
- Global normalization: $N(q^2, C_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

$\chi^2_{red} = 1.01$

Data is normalized using the total cross section

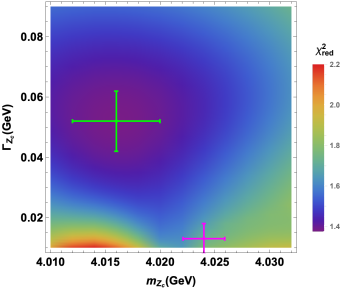
No Z_c Intermediate State



$$\chi^2_{red} = 0.83$$

- No intermediate state is required;
- Two real subtraction constants and the $\pi\pi$ Omnès function describe well the data;
- The left-hand cuts are dominated by the contact interation or possible D-meson loops (absorbed in the subtraction constants).

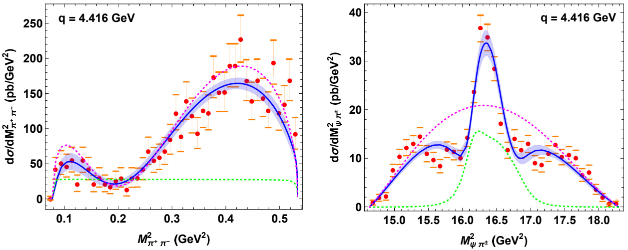
$Z_c(4016)$ or $Z_c(4020)$?



m_{Z_c} (GeV)	Γ_{Z_c} (MeV)
4.016(4)	52(10)
4.024(2)	13(5)

$Z_c(4020)$ observed by BESIII in

- $e^+e^- \to D^*\bar{D}^*\pi$
- $e^+e^- \to h_c\pi\pi$

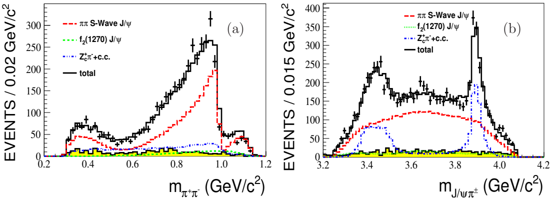


$\chi^2_{red} = 1.38$

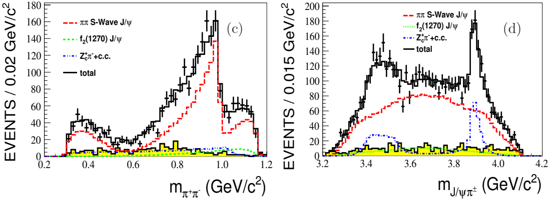
– Combined Fit
-- Only Z_c
-- Only $\pi\pi$-FSI

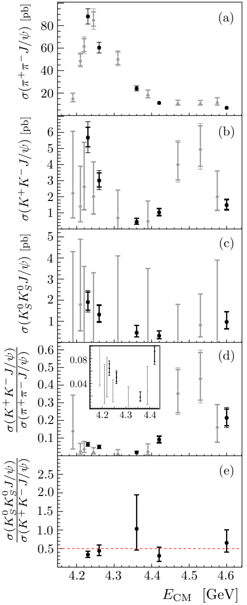
4 Perspectives

$e^+e^- \to J/\psi\ \pi^+\pi^-$



BESIII PRL (2017)





BESIII PRD (2018)

- Above $K\bar{K}$ threshold $\to$ couple channel rescattering;
- No evidence of the strange partner of $Zc(3900)$;
- $\sigma(J/\psi K^+K^-)$ is suppressed compared to $\sigma(J/\psi\ \pi^+\pi^-)$.

Strange Partner of $Z_c(3900)$?

$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix}=\begin{bmatrix}h_{Z_c}(t,u)\\ h_{Z_c^s}(t,u)\end{bmatrix}+\underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}}\\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}}\end{bmatrix}}_{\bar{\Omega}}\left\{\begin{bmatrix}a+b\,s\\ c+d\,s\end{bmatrix}-\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s'-s}\begin{bmatrix}h^L_{Z_c}(s')\\ h^L_{Z_c^s}(s')\end{bmatrix}\right\}$$

Strange Partner of $Z_c(3900)$?

$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix} = \begin{bmatrix} h_{Z_c}(t,u)\\ \cancel{h_{Z_c}}(t,u)\end{bmatrix} + \underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar K}\\ \Omega_{K\bar K,\pi\pi} & \Omega_{K\bar K,K\bar K}\end{bmatrix}}_{\bar\Omega} \left\{ \begin{bmatrix} a+b\,s\\ c+d\,s\end{bmatrix} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\bar\Omega)^{-1}(s')}{s'-s}\begin{bmatrix}h^L_{Z_c}(s')\\ \cancel{h^L_{Z_c}}(s')\end{bmatrix}\right\}$$

Naive Insights:

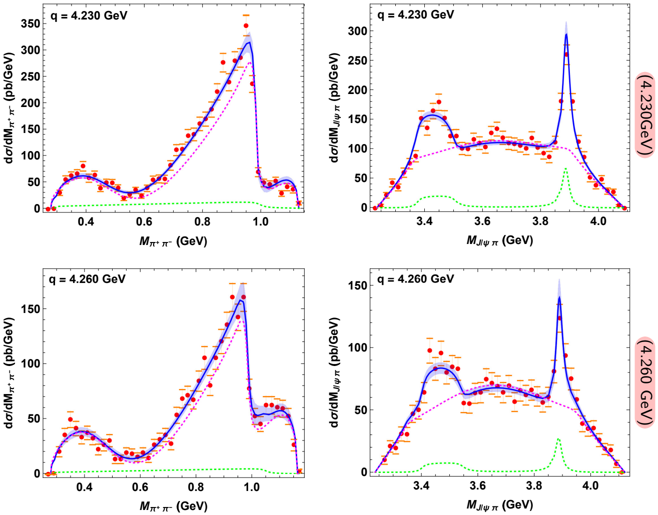
- $(m_s \approx 95\ \mathrm{MeV}) >> (m_u \approx 2.2\ \mathrm{MeV})$ and $(m_d \approx 4.7\ \mathrm{MeV})$

$$\Longrightarrow\ \mathbf{m_{Z_c^{(s)}} > m_{Z_c}}$$

- For $q = 4.26(4.23)$ GeV, $\gamma^*(q) \to K\ Z_c^{(s)}$ and $Z_c^{(s)} \to J/\psi\ K$ would constrain the $Z_c^{(s)}$ mass:

$$\mathbf{3.59 < m_{Z_c^{(s)}} < 3.77(3.74)\ GeV}$$

$e^+e^- \to J/\psi\ \pi^+\pi^-$ (Preliminary)



$\chi^2_{red} = 2.17$

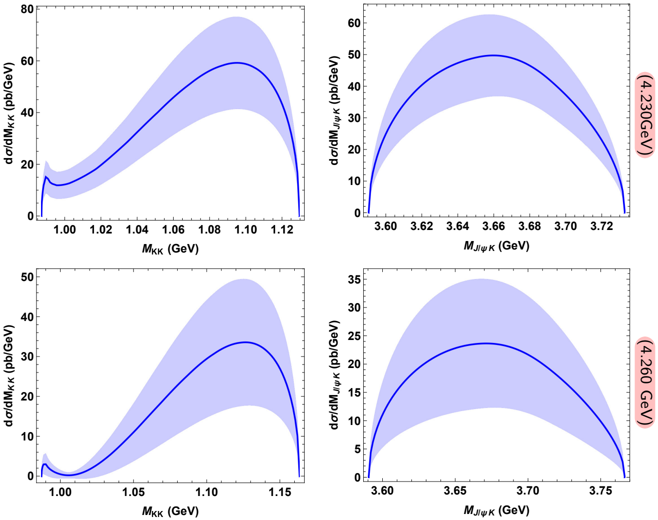
Extra Constrain:

$e^+e^- \to J/\psi\ K^+K^-$

- $a(q^2)$, $b(q^2)$, $c(q^2)$ and $d(q^2)$ are complex numbers
- Global normalization: $N(q^2, C_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

$\chi^2_{red} = 1.03$

Predictions for $e^+e^- \to J/\psi\ K^+K^-$ (Preliminary)



$$\sigma^{\text{pred}}_{4.23} = 5.29 \pm 1.47 \text{ pb}$$
$$\sigma^{\text{BES}}_{4.23} = 5.27 \pm 0.75 \text{ pb}$$

$$\sigma^{\text{pred}}_{4.26} = 3.08 \pm 1.54 \text{ pb}$$
$$\sigma^{\text{BES}}_{4.26} = 3.08 \pm 0.44 \text{ pb}$$

5 Summary

Summary

$e^+e^- \to \psi(2S)\,\pi^+\,\pi^-$

- We perform an amplitude analysis of the reaction $e^+e^- \to \psi(2S)\,\pi^+\pi^-$ at different e^+e^--CM energies q;
- The $\pi\pi$-FSI is treated using the dispersion theory and we studied quantitatively the contribution of the charged exotic mesons as intermediate states;
- The exotic state $Zc(3900)$ plays an important role to explain the invariant mass distribution at both $q = 4.226$ and $q = 4.258$ GeV;
- To explain the enhancement in the experimental data at $q = 4.416$ GeV a heavier charged state is needed instead, with $m_Z = 4.016(4)$ GeV and $\Gamma_Z = 52(10)$ MeV. However, this state could be the already known $Z_c(4020)$;
- For $q = 4.358$ GeV no intermediate Z_c state is necessary for left-hand cuts in order to describe both $\psi\pi$ and $\pi\pi$ line shapes. It points to another left-hand contribution which we absorbed in the subtraction constants:
- The $\pi\pi$-FSI is the main mechanism to describe the $\pi\pi$-line shape for all the energies.

Thank you for listening!

THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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Appendix

Inverse Amplitude Method

- partial wave elastic unitarity

$$\mathrm{Im}[t_{\pi\pi}(s)] = \rho_{\pi\pi}(s)\,|t_{\pi\pi}(s)|^2 \implies \mathrm{Im}\left[\frac{1}{t_{\pi\pi}(s)}\right] = -\rho_{\pi\pi}(s)$$

- The ChPT amplitude only satisfy the unitarity condition perturbatively:

$$\mathrm{Im}[t_{\mathrm{LO}}] = 0\,; \qquad \mathrm{Im}[t_{\mathrm{NLO}}] = \rho_{\pi\pi}(s)\,|t_{\mathrm{LO}}(s)|^2 \qquad \text{with } t_{\pi\pi} = t_{\mathrm{LO}} + t_{\mathrm{NLO}} + \cdots$$

- One can write down a dispersion relation for the ChPT amplitudes as

$$t_{\mathrm{LO}}(s) = \sum_{l=0}^{k} a_l s^l\,; \qquad t_{\mathrm{NLO}}(s) = \sum_{l=0}^{k} b_l s^l + \frac{s^k}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^k}\,\frac{\mathrm{Im}\,[t_{\mathrm{NLO}}]}{s'-s-i\epsilon} + I_{LC}\,[t_{\mathrm{NLO}}]$$

- Same analytic structure for t and t^{-1}: $G(s) \equiv t_{\mathrm{LO}}^2/t_{\pi\pi} \implies \mathrm{Im}[G(s)] = -\mathrm{Im}[t_{\mathrm{NLO}}]$
- Thus $G(s) \simeq t_{\mathrm{LO}}(s) - t_{\mathrm{NLO}}(s)$, considering that $I_{LC}\,[G(s)] = -I_{LC}\,[t_{\mathrm{NLO}}]$ and ignoring pole contributions:

$$\boxed{t_{\pi\pi}(s) \simeq \frac{|t_{\mathrm{LO}}(s)|^2}{t_{\mathrm{LO}}(s) - t_{\mathrm{NLO}}(s)}}$$

Truong, PRL (1988)
Dobado & Peláez, PRD (1993)

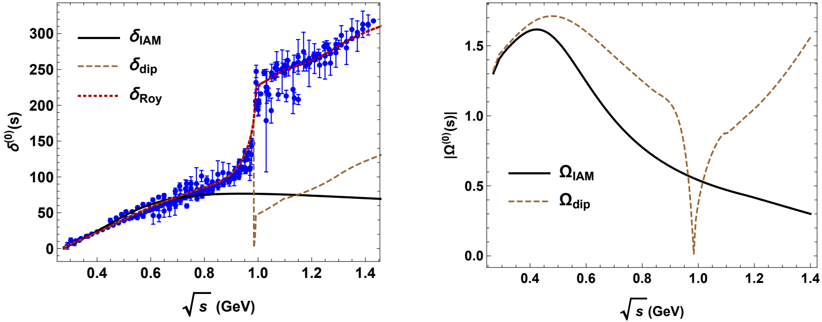
Inverse Amplitude Method

- Spurious poles emerges below threshold for the scalar waves (J=0), thus to reproduce correctly the Adlers zeros the IAM must be modified as

$$t_{\pi\pi}(s) = \frac{|t_{\rm LO}(s)|^2}{t_{\rm LO}(s) - t_{\rm NLO}(s) + A^{\rm mIAM}(s)}$$

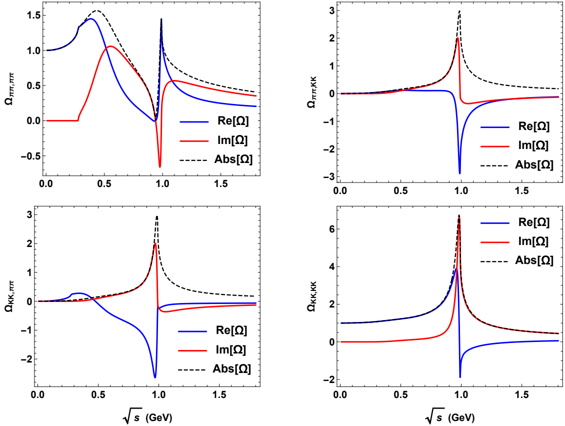
GomezNicola et al. PRD (2008)

with the adler zero $s_A = s_{LO} + s_{NLO} + \mathcal{O}(p^6)$ and $t_{\rm LO}(s_{\rm LO} + s_{\rm NLO}) + t_{\rm NLO}(s_{\rm LO} + s_{\rm NLO}) = 0$



Couple Channel

$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix}=\begin{bmatrix}h_{Z_c}(t,u)\\ h_{Z_c^s}(t,u)\end{bmatrix}+\underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}}\\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}}\end{bmatrix}}_{\bar{\Omega}}\left\{\begin{bmatrix}a+b\,s\\ c+d\,s\end{bmatrix}-\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s'-s}\begin{bmatrix}h^L_{Z_c}(s')\\ h^L_{Z_c^s}(s')\end{bmatrix}\right\}$$



Danilkin & Vanderhaeghen, PLB (2019)