Partial Wave Analysis with the PAWIAN software package

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Outline

- Introduction to PAWIAN
- Description of resonances
- Analyticity, unitarity and unstable particles
- Application example
- Summary and Outlook

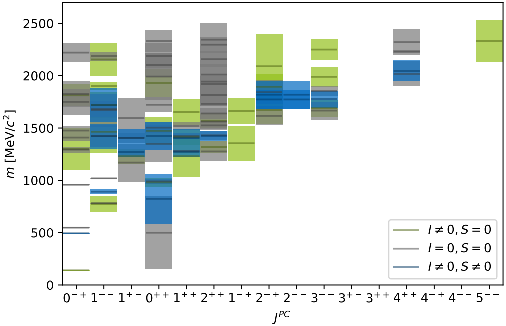
Challenges in Meson Spectroscopy

- Many broad and overlapping states discovered
- Assignment to $q\overline{q}$ multiplets still ambiguous

→ Fundamental to gain deeper understanding of strong interaction

→ Optimally: Combine different **production processes** and **decay modes**

[M.Kümmel, data from PDG2018]



Modern high statistics experiments provide huge and clean data samples
Adequate tools using sophisticated methods are needed to properly analyze this data

What is PAWIAN?

PArtial Wave Interactive ANalysis Software

- PWA software package under development in Bochum to ...

  [B. Kopf et. al., Hyperfine Interact. 229 (2014) no.1-3, 69-74]

  - provide a generic (amplitudes, dynamics, ...) easy-to-use software package (GPLv3)
  - support different production processes ($\overline{p}p$, e^+e^-, $\gamma\gamma$, $\pi\pi$-scattering, CEP, ...)

$\Rightarrow$ Several analyses have already been performed

$\Rightarrow$ **PAWIAN** features full configuration of hypotheses and other input settings via plain-text configuration files

$\Rightarrow$ Event based maximum likelihood fits (Minimizer: MINUIT2, ...)

$\Rightarrow$ Event generator, histogramming, analysis tools, ...

$\Rightarrow$ **Coupled channel analyses** across different production processes

$\Rightarrow$ Parallelization: Server-client (network) mode

Workflow with PAWIAN

Data Selection
(kin. Fits, cuts, ...)

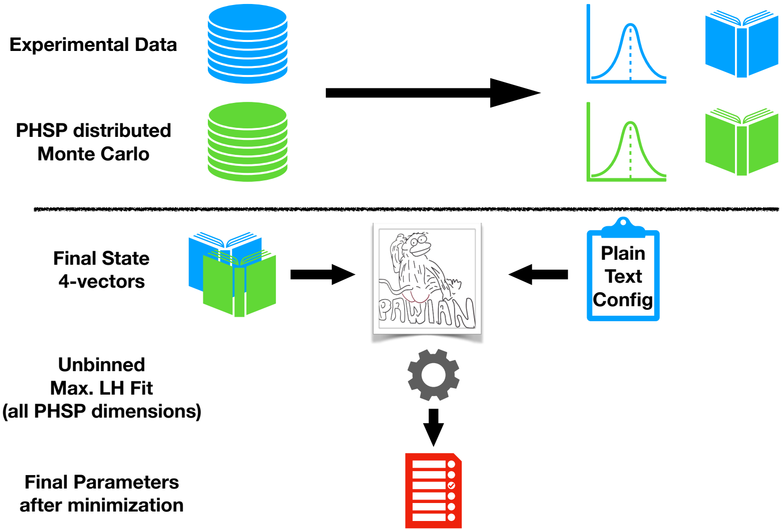
Selected Data

Final State 4-vectors

Experimental Data

PHSP distributed Monte Carlo

Workflow with PAWIAN



Data Selection
(kin. Fits, cuts, ...)

Selected
Data

Final State
4-vectors

Experimental Data

**PHSP distributed
Monte Carlo**

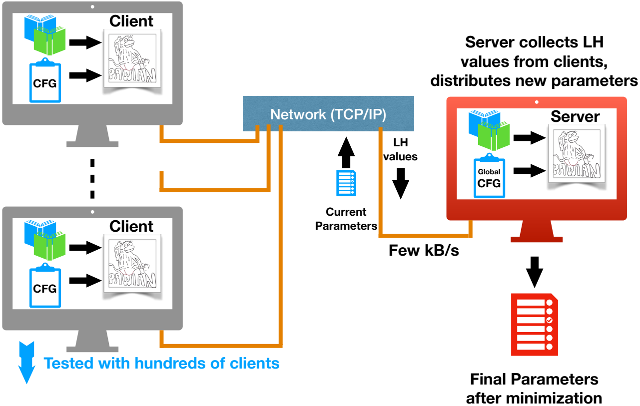
**Final State
4-vectors**

**Plain
Text
Config**

**Unbinned
Max. LH Fit
(all PHSP dimensions)**

**Final Parameters
after minimization**

Workflow with PAWIAN



Client

CFG

Client

CFG

Tested with hundreds of clients

Network (TCP/IP)

Current Parameters

LH values

Few kB/s

Server collects LH values from clients, distributes new parameters

Server

Global CFG

Final Parameters after minimization

- Only server communicates with minimizer, current parameters saved periodically (restarting of fit easily possible any time!)
- Parallelization on event level → No communication between clients necessary

$$-\ln \mathcal{L}_k \approx -\sum_{i=1}^{n_{data}} Q_i \cdot \ln w(\vec{\tau}_i, \vec{\alpha}) + \left(\sum_{i=1}^{n_{data}} Q_i\right) \cdot \ln\left(\frac{\sum_{j=1}^{n_{MC}} w(\vec{\tau}_j, \vec{\alpha})}{n_{MC}}\right) + \dots$$

- Amplitudes are cached → only parts need to be re-evealuated in every minimization step
- Very small network / I/O load → Multiple server processes per machine feasible

Example: The Configuration File

Plain Text Config

- → When cleanly selected data and PHSP distributed MC are available, setup of PWA fit quick and easy for many cases
- → No limitation on number of final state particles or intermediate resonances
- ▶ *K*-Matrix: Full configuration (number of poles, channels, order of background terms, ...) via individual CFG files
- ▶ Some relevant lines in the config file (many more options available):

```
datFile = /path/to/reconstructed_data.dat
mcFile = /path/to/reconstructed_PHSP-MC.dat
```

Plain-text files w/ final state four vectors (Data and PHSP MC)

```
finalStateParticle = K+
finalStateParticle = K-
finalStateParticle = pion0
```

Name and order of final state particles

```
productionFormalism = Heli
```

Spin-formalism: Helicity, Canonical, Rarita-Schwinger (partly),...

```
production = f2(1270) pion0 BlattWBarrier
production = aMatrix0 pion0 BlattWBarrier
```

Production channels (w/ or w/o barrier factors)

```
decay = Cano f2(1270) To K+ K-
decay = Cano aMatrix0 To K+ K-
```

Decay channels

```
addDynamics = f2(1270) BreitWignerBlattWRel
addDynamics = aMatrix0 KMatrix ./a0_pieta_2channel_2poles.cfg K+ K-
```

Dynamics for individual resonances (optional)
Choice of Breit-Wigner, *K*-Matrix

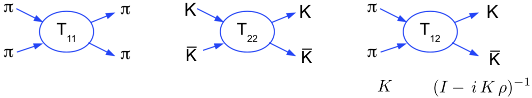
```
serverPort = 24611
serverAddress = pc14
noOfClients = 250
```

Configuration options for server/client network mode

Description of Resonances

- Breit-Wigner parameterization most commonly used. OK, if resonance is isolated, appearing only in one channel, far from thresholds
- K-matrix parameterization with consideration of analyticity:
  - S-matrix describes two-body scattering process via $S = I + 2i\sqrt{\rho}T\sqrt{\rho}$
  - Where T can be written as $T = (I - iK\rho)^{-1}K$
  - Elements of the K-matrix: $K_{ij} = \sum_\alpha \frac{g_{\alpha i} g_{\alpha j}}{m_\alpha^2 - s} + \sum_k c_{kij} s^k$

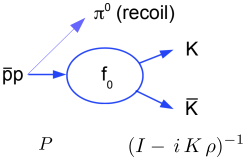
→ Example: 2 channels ($\pi\pi$ and $K\overline{K}$)



  - Extension of formalism to production of resonances using P-vector approach: [I.Aitchison, Nucl.Phys.A 189 (1972) 417]
  - Dynamical function becomes: $F = (I - iK\rho)^{-1}P$ with

$$P_i = \sum_\alpha \frac{\beta_\alpha g_{\alpha i}}{m_\alpha^2 - s} + \sum_k c_{ki} s^k$$

Example:
$\overline{p}p \to f_0\pi^0 \to (K\overline{K})\pi^0$



Analyticity

- For coupled channels: Resonances can be below threshold for specific channels!

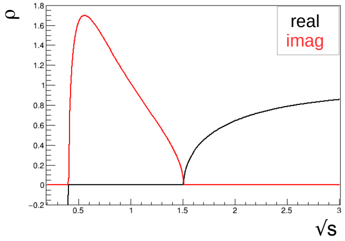
→ Treatment of phase-space factors becomes important

- Standard phase-space factors: $\rho = \sqrt{\left[1-\left(\frac{m_a+m_b}{m}\right)^2\right]\cdot\left[1-\left(\frac{m_a-m_b}{m}\right)^2\right]}$

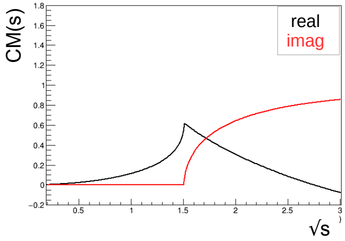
→ Violate contraints from analyticity due to unphysical behavior below threshold (pole at $m = 0$ and unphysical cuts for $m_a \neq m_b$)

- Replace standard phase-space factors ρ with function to respect constraints from analyticity
  (Chew-Mandelstam function [Basdevant, Berger PRD19 (1979) 239])
- $T = (1 - iK\rho)^{-1}K$ replaced by $T = (1 - K\,CM(s))^{-1}K$

standard ρ for $\eta\eta'$



CM function for $\eta\eta'$



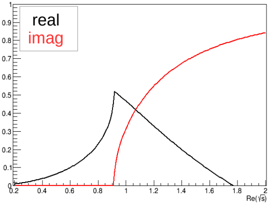
Unstable Particle Scattering

- Chew-Mandelstam function so far only valid for “zero-width approximation” case

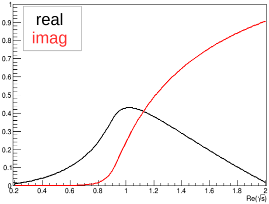
→ Extension to particles with non-negligible widths (like ω, ρ, K^*, …)
[Basdevant, Berger PRD19 (1979) 239]

- Correct analytical properties, satisfying quasi-two-body unitarity
- Chew-Mandelstam function for unstable particles used → resonance branch cuts located on 2^{nd} Riemann sheet of three-particle complex energy plane

CM for $\rho\pi$ (stable)



CM for $\rho\pi$ (unstable)



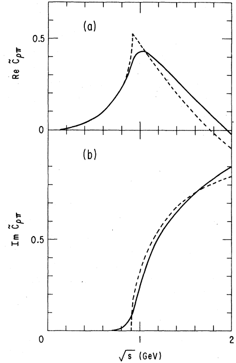
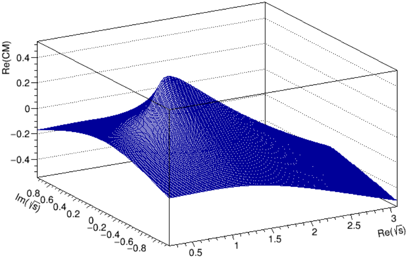
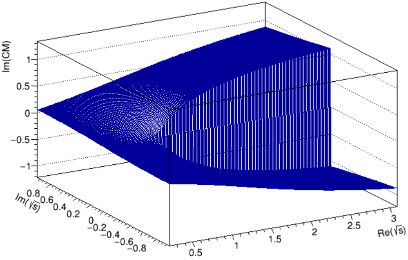


FIG. 4. The function $\tilde{C}_{\rho\pi}(s)$ (solid line) is compared with the stable particle Chew-Mandelstam function (dashed line): (a) real parts, (b) imaginary parts.

Extraction of Pole Parameters

[M.Kuhlmann, RUB]





- Downside: This *CM*-function involves integrals that have to be calculated numerically

→ Calculations very time consuming (not for online use)

→ Solution:
Using pre-calculated complex-valued lookup-tables in PAWIAN (existing for the cases: $K^*\pi, \omega\pi, \rho\pi, \ldots$)

→ Resonance parameters are extracted from the genuine poles of the T-matrix in the complex energy plane

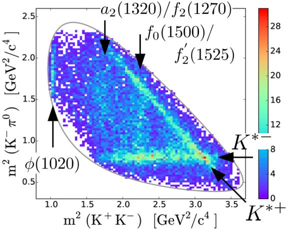
Coupled Channel Analysis using CB/LEAR Data Preliminary

- Simultaneous fit of $\overline{p}p \to K^+K^-\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ channels

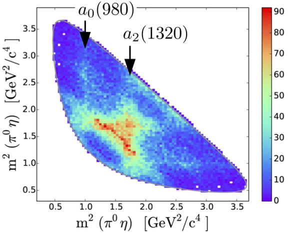
→ Advantages: Common amplitudes, common description of the dynamics (K-matrix), less fit parameters

- Clean data samples were prepared at $p_{\overline{p}} = 0.9\,\mathrm{GeV}/c$
- Kinematic fits, event-based background subtraction

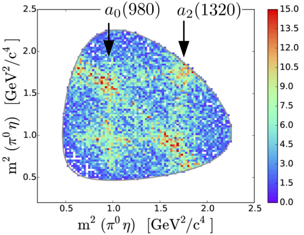
$K^+K^-\pi^0$ ($\sim$ 17.5k events)



$\pi^0\pi^0\eta$ ($\sim$ 90.4k events)



$\pi^0\eta\eta$ ($\sim$ 10.5k events)



- Additional constraints by coupling to **scattering data** (phase & elasticity):
  - $I = 0$ $S-$ and $D-$wave $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$ $(\eta\eta')$,
    $I = 1$ P-wave $\pi\pi \to \pi\pi$
    PRD 83 (2011) 074004, Nucl.Phys.B 64 (1973) 134-162, Nucl.Phys.B 100 (1975) 205
    Nucl.Phys.B 269 (1986) 485, Nuov.Cim.A 80 (1984) 363

Fit Result

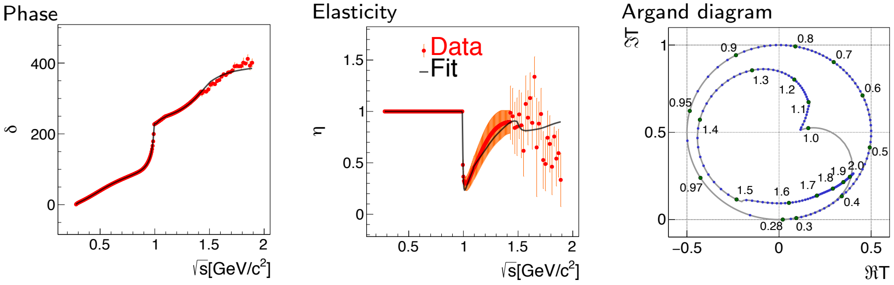
Preliminary

- f_0 K-Matrix: 5 poles, 5 channels
- f_2 K-Matrix: 4 poles, 4 channels
- ρ K-Matrix: 2 poles, 3 channels
- a_0 and a_2 K-Matrix: 2 poles, 2 channels (each)

→ **All pole positions and coupling strengths are free parameters!**

- $(K\pi)_S$-wave K-Matrix: 1 pole, 2 channel [Phys.Lett.B653 (2007) 1-11]
- Breit-Wigner description for isolated resonances ($\phi(1020)$, $K^{*\pm}(892)$, $\pi_1(1400)$)
- Overall: Good description of data for $\bar{p}p$ channels and also for scattering data:

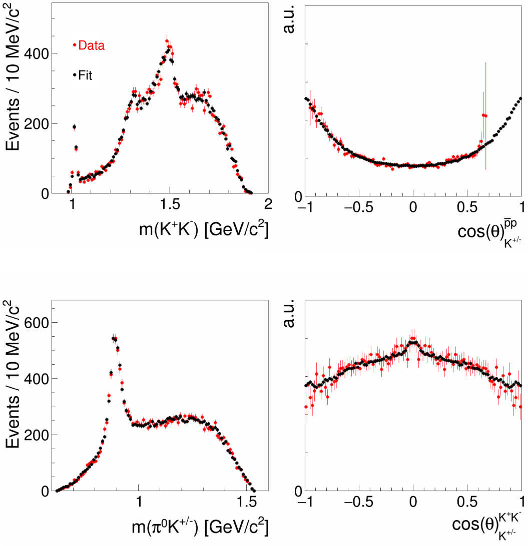
$\pi\pi \rightarrow \pi\pi$, $I = 0$ S-wave:


Phase

Elasticity

Argand diagram

$\bar{p}p \rightarrow K^+K^-\pi^0$ Preliminary



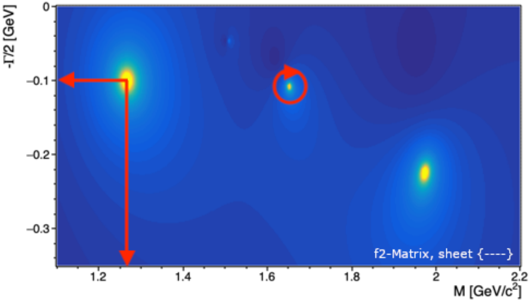
(angular distributions efficiency corrected)

Fit quality: $p = 0.848$

[Aslan,Zech
NIM A 537 (2005) 626-636]

	contribution (in %)
$f_0\ \pi^0$	$10.7 \pm 0.4 \pm 1.5$
$f_2\ \pi^0$	$19.3 \pm 0.8 \pm 6.6$
$\rho\ \pi^0$	$6.2 \pm 0.6 \pm 3.2$
$a_0\ \pi^0$	$0.8 \pm 0.0 \pm 0.5$
$a_2\ \pi^0$	$1.0 \pm 0.0 \pm 0.8$
$K^*(892)^\pm\ K^\mp$	$45.7 \pm 1.6 \pm 10.5$
$(K\pi)^\pm_S\ K^\mp$	$14.4 \pm 0.8 \pm 4.3$
$\phi(1020)\ \pi^0$	$2.7 \pm 0.3 \pm 0.5$
Σ	$100.9 \pm 2.0 \pm 5.8$

Extracted Resonance Parameters (Example) Preliminary



- All resonance parameters are encoded in the K-matrix
- → Masses and widths from position of poles in complex plane of the T-matrix (Riemann sheet closest to the physical sheet)
- → ≈ 50 resonance properties extracted from this coupled channel fit

- Partial widths / branching fractions extracted using the residues of the poles:

$$Res^{\alpha}_{k\to k} = \frac{1}{2\pi i} \oint_{C_{z_\alpha}} \sqrt{\rho_k} \cdot T_{k\to k}(z) \cdot \sqrt{\rho_k}\, dz$$

name	pole mass [MeV/c^2]	pole width Γ [MeV]	$\Gamma_{\pi\pi}/\Gamma[\%]$	Γ_{KK}/Γ [%]	$\Gamma_{\eta\eta}/\Gamma[\%]$
$f_2(1270)$	$1266.3 \pm 0.2 \pm 3.4$	$200.1 \pm 0.5 \pm 14.2$	$88.2 \pm 0.1 \pm 4.5$	$6.8 \pm 0.5 \pm 4.5$	$0.0 \pm 0.5 \pm 0.1$
$f_2'(1525)$	$1506.8 \pm 1.1 \pm 4.6$	$87.6 \pm 1.2 \pm 9.2$	$4.1 \pm 1.8 \pm 1.2$	$54.0 \pm 3.7 \pm 9.1$	$5.6 \pm 4.5 \pm 0.6$

Summary

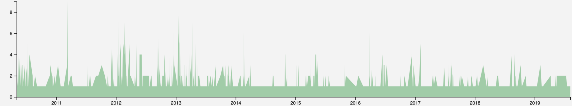
- PAWIAN software package is in a good shape

- Various analyses (single and coupled channel) underway using
  - e^+e^- data from BESIII (J/ψ, $\psi(3686)$, $\psi(3770)$, ...)
  - $\overline{p}p$ annihilation data (CrystalBarrel/LEAR, $0.9\,\mathrm{GeV}/c \leq p_{\overline{p}} \leq 1.94\,\mathrm{GeV}/c$)

  → Momentum overlap with PANDA → valuable studies to prepare for challenges ahead
  - simulated data for feasibility studies for PANDA
- Sophisticated techniques to consider analyticity/unitarity
- Growing user base and active community

→ Discussion forum: `https://pawiantalk.ep1.rub.de`

→ Git repository: `https://jollyj.ep1.rub.de/EP1/Pawian`



⇒ Future: Coupling of e^+e^- and $\overline{p}p$ data ⇒ Stay tuned for more results!

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