Partonic structure of neutral pseudoscalars via two photon transition form factors

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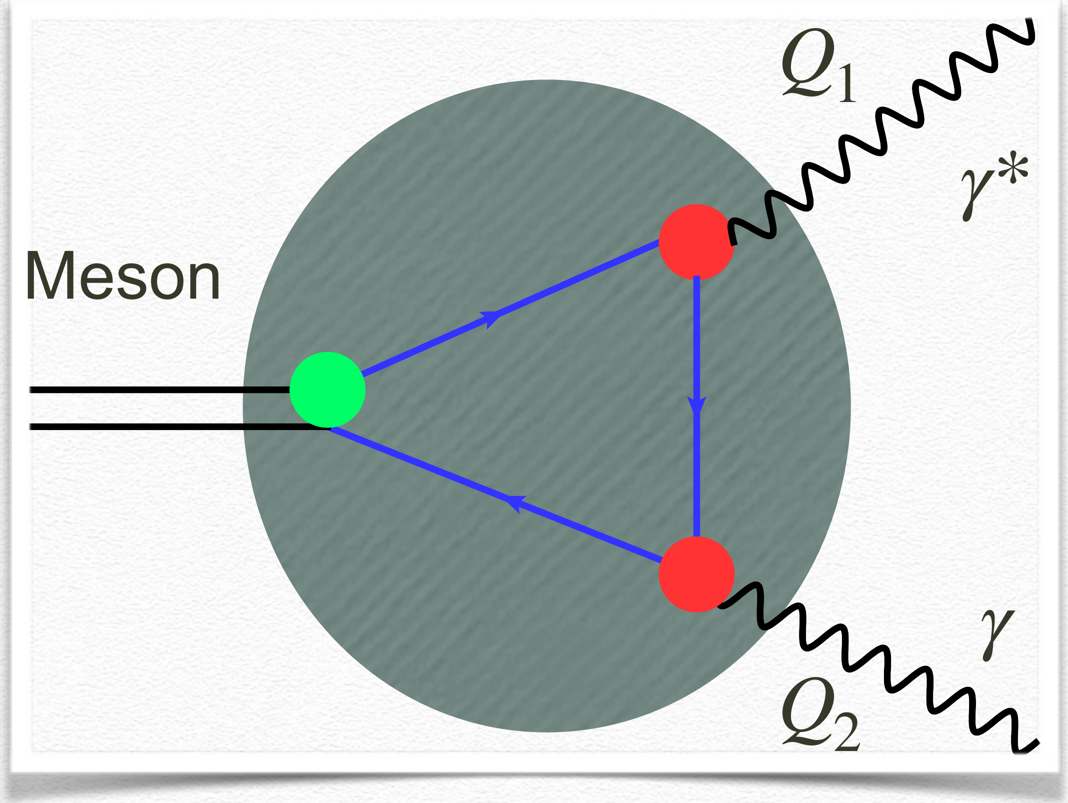
The 18th International Conference on Hadron Spectroscopy and Structure
(HADRON2019)
Guilin, China, Aug. 17-21, 2019.

Two photon transition form factors

- Understanding the structure of hadrons in terms of QCD's quarks and gluons is one of the central goals of modern hadron physics.
- Form factors: the closest thing to a snapshot, the size, shape and makeup of hadrons.



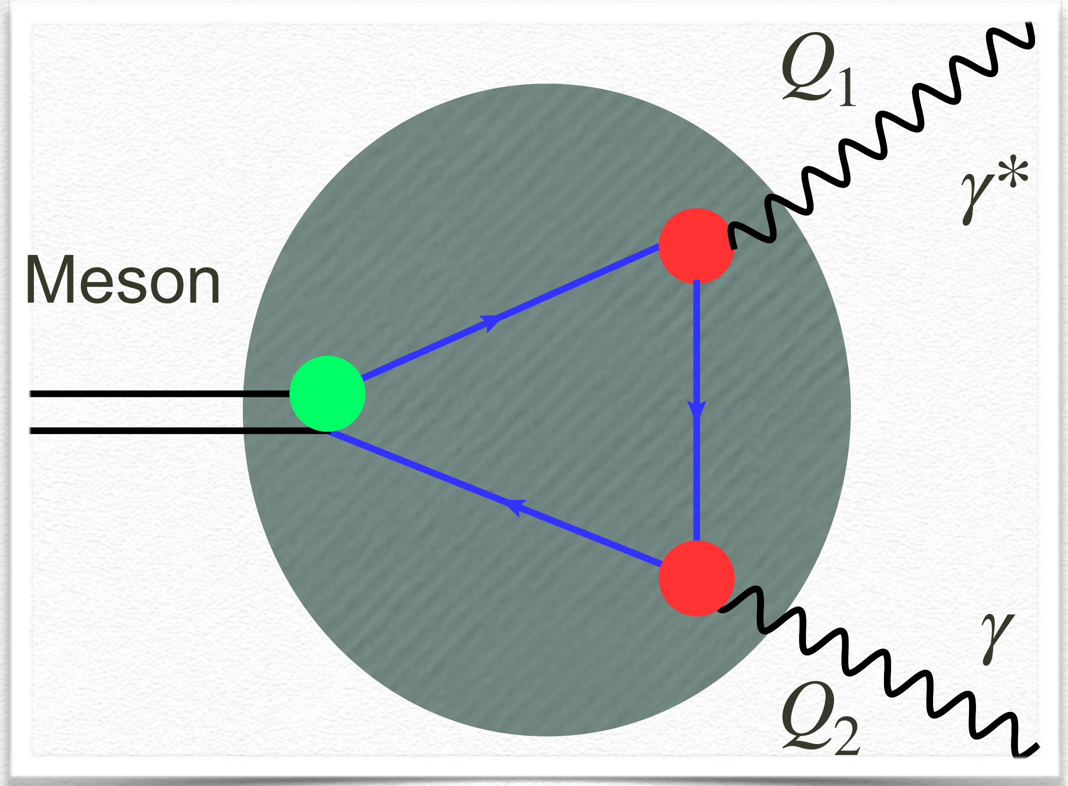
- Two photon transition form factors
- Transfer photon momentum Q_1, Q_2
  - $Q_1^2 \approx Q_2^2 \approx 0$, two photon decay width
  - $Q_1^2 \neq 0\,, Q_2^2 \approx 0$, single-virtual transition form factor (TFF)
  - $Q_1^2 \neq 0\,, Q_2^2 \neq 0$, double-virtual TFF

Single-virtual transition form factors: $Q_1^2 = Q^2 \neq 0\,, Q_2^2 \approx 0$

- Strong-QCD prediction: $Q^2 F_M^q(Q^2) \overset{Q^2 > Q_0^2}{\approx} 4\pi^2 f_M^q e_q^2 \tilde{w}_M^q(Q^2)$

PhysRevD.22.2157

- TFF: $F_M^q(Q^2)$, $q\bar{q}$ component of M, power-law scaling.



- $Q_0 > \Lambda_{QCD}$, e_q, electric charge
- f_M^q, $q\bar{q}-$component leptonic decay constant
- $\tilde{w}_M^q(Q^2) = \int_0^1 dx \frac{1}{x} \varphi_M^q(x, Q^2)$
- $\varphi_M^q(x, Q^2)$ dressed-valence q-parton contribution to the meson's distribution amplitude (DA)

Transition form factors (TFF) & Distribution amplitude (DA)

- Strong-QCD prediction: $Q^2 F_M^q(Q^2) \overset{Q^2>Q_0^2}{\approx} 4\pi^2 f_M^q e_q^2 \tilde{w}_M^q(Q^2)$

In the limit $\tau^2 := \Lambda_{QCD}^2/Q^2 \simeq 0$

- Asymptotic TFF
  $Q^2 F_M^q(Q^2) \overset{\tau\simeq 0}{\approx} 12\pi^2 f_M^q e_q^2$

$\Leftarrow$

- Asymptotic DA:
  $\varphi_M(x, Q^2) \overset{\tau\simeq 0}{\approx} \varphi_\infty = 6x(1-x)$

Phys. Lett. B 87, 359 (1979)

Phys. Lett. B 94, 245 (1980)

Models

Large-Q^2: $$F_M^q(Q^2) = \frac{A}{1 + Q^2/(12\pi^2 f_M^q e_q^2)}$$

Low-Q^2: $$F_M^q(Q^2) = \frac{B}{1 + Q^2/\Lambda_M^2}$$

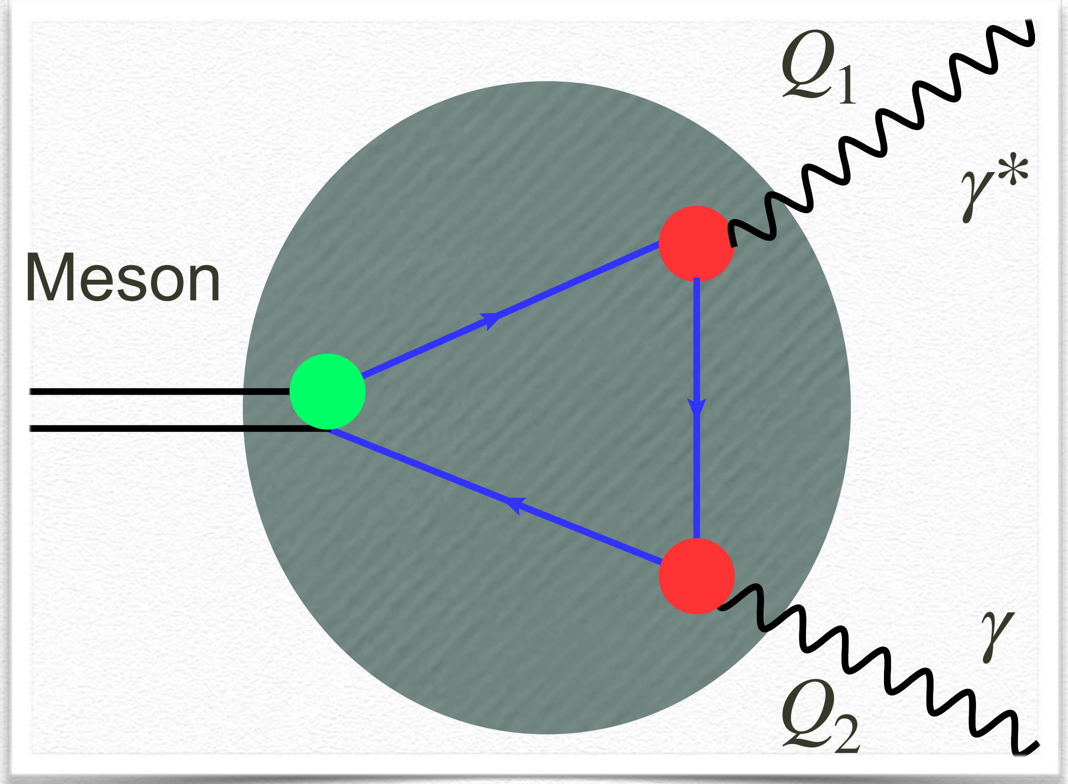
Single-virtual transition form factors with DSEs: formulae I

- Impulse approximation

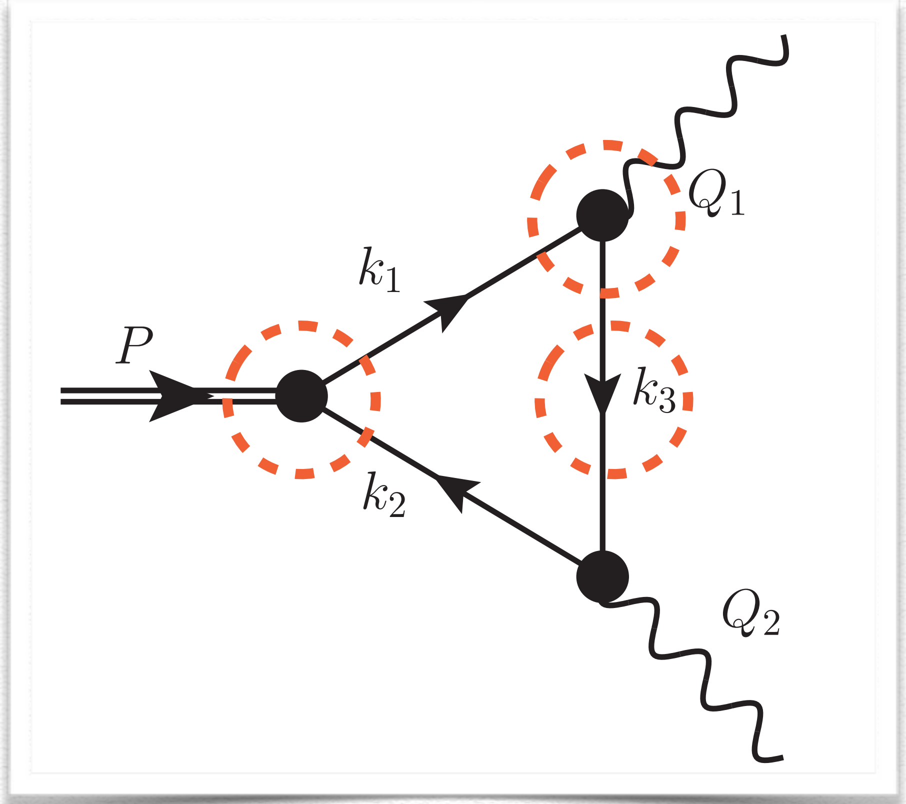
$$\Lambda^{q}_{\alpha\beta}(Q_1^2, Q_2^2) = e^2 N_c \int_{dk}^{\Lambda} \mathrm{tr}\left[S(k_1)\Gamma^{q\bar{q}}_{M}(k;P)S(k_2)\, i\Gamma_{\beta}(k+Q_1/2;Q_2)S(k_3)i\Gamma_{\alpha}(k-Q_2/2;Q_1)\right]$$

arXiv:nucl-th/0201017

- TFF and vertex $\Lambda^{\gamma^*\gamma^*\pi^0}_{\mu\nu} = \dfrac{2i\alpha_{em}g_{\gamma\gamma\pi}}{\pi\tilde{f}_{\pi}}\epsilon_{\mu\nu\rho\sigma}Q_{1\rho}Q_{2\sigma}F_{\gamma^*\gamma^*\pi^0}(Q_1^2, Q_2^2)$



$\Rightarrow$



Single-virtual transition form factors with DSEs: formulae II

- Impulse approximation

$$\Lambda^{q}_{\alpha\beta}(Q_1^2, Q_2^2) = e^2 N_c \int_{dk}^{\Lambda} \mathrm{tr}\left[S(k_1) \Gamma_M^{q\bar{q}}(k;P) S(k_2)\, i\Gamma_\beta(k+Q_1/2;Q_2) S(k_3)\, i\Gamma_\alpha(k-Q_2/2;Q_1) \right]$$

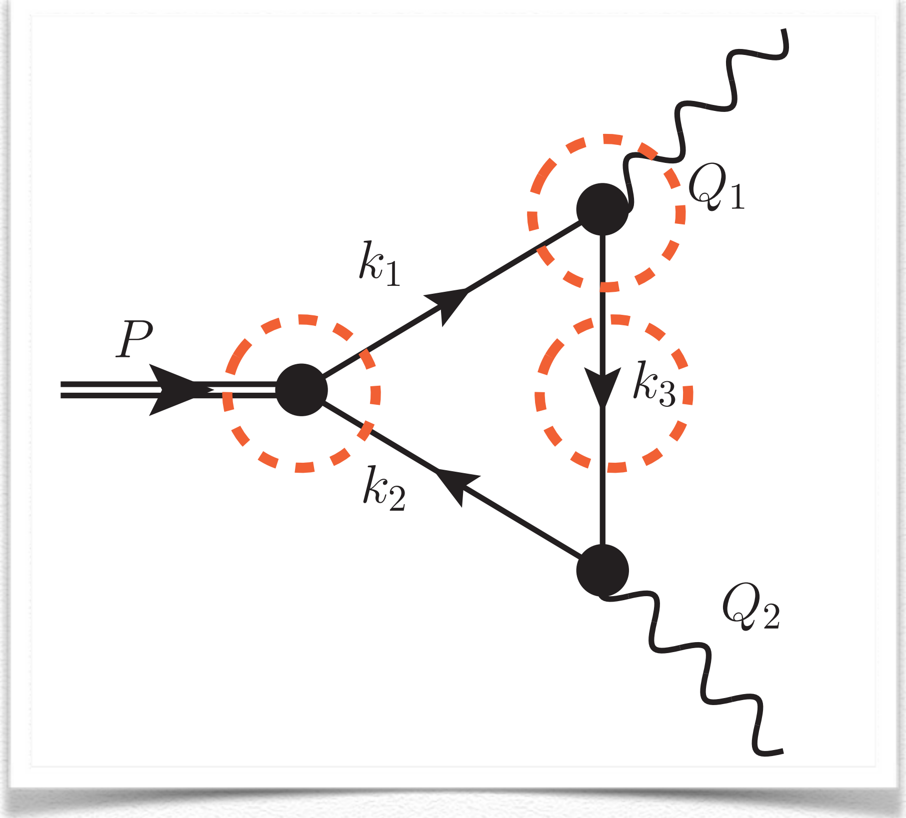
- $S(k_i)$: dressed quark propagator

$$k_1 = k + P/2,\ k_2 = k - P/2$$

$$k_3 = k + (Q_1 - Q_2)/2$$

- $\Gamma_M^{q\bar{q}}$: meson Bethe-Salpeter amplitude $\Leftarrow$

- $\Gamma_{\alpha/\beta}$: quark-photon vertex

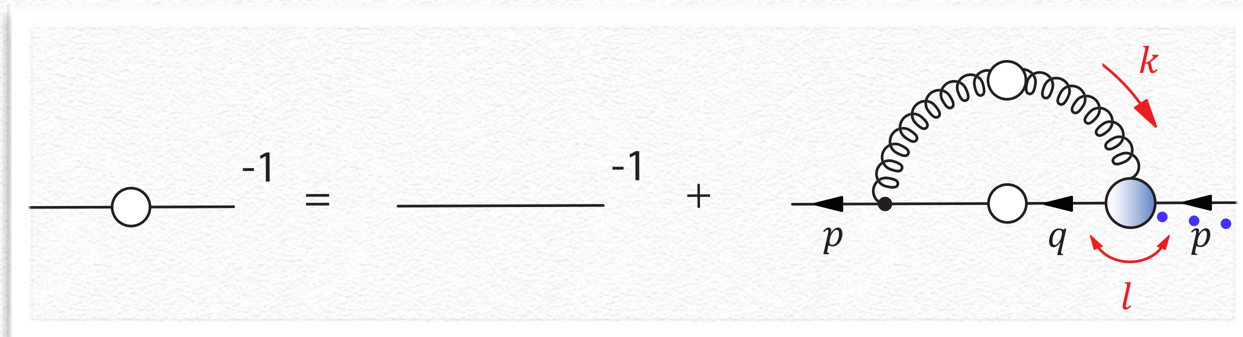


$$\Lambda^{\pi^0\to\gamma\gamma^*}_{\alpha\beta}(Q_1;Q_2) = \frac{2}{\sqrt{2}}\left((\hat{Q}^u)^2 - (\hat{Q}^d)^2\right)\Lambda^u_{\alpha\beta}(Q_1;Q_2)$$

DSEs: Equations for mesons

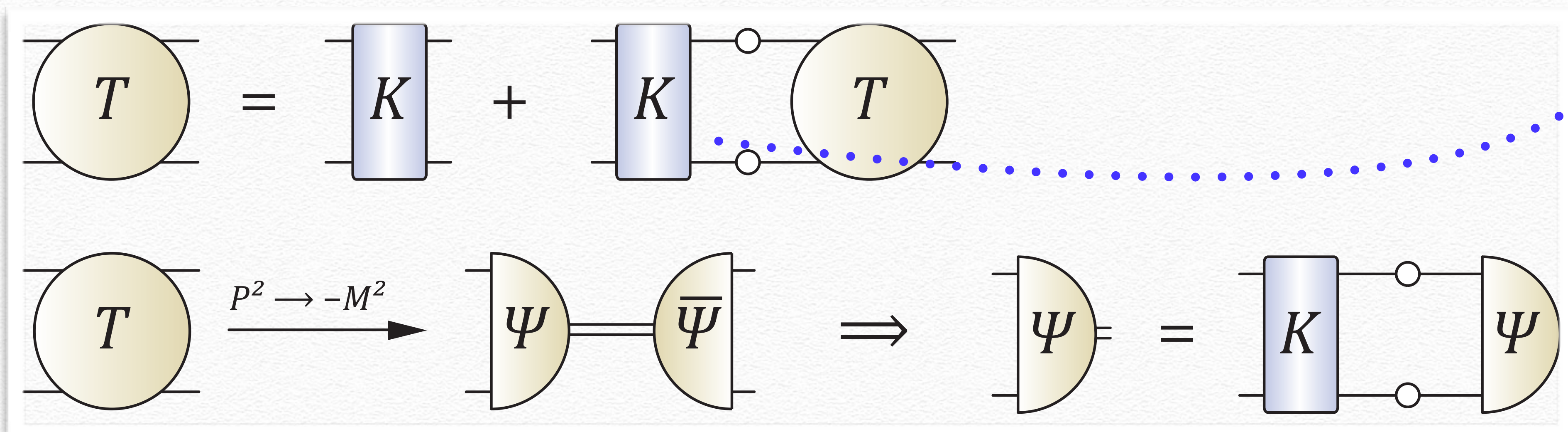
- $S(k_i)$: dressed quark propagator

quark gluon vertex



- $\Gamma_M^{q\bar{q}}$: meson Bethe-Salpeter amplitude

scattering kernel



- $\Gamma_{\alpha/\beta}$: quark-photon vertex, inhomogeneous vector Bethe-Salpeter equation

picture from G. Eichmann, arXiv:0909.0703

DSEs: Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \mathscr{F}^a + i\gamma_5 \mathscr{F}^a S^{-1}(k_-) - 2i\mathscr{M}^{ab}\Gamma^b_5(k;P) - A^a(k;P)$$

- An identity between correlation functions, symmetry conserved.
- Non-Abelian anomaly: $A^a(k;P)$.
- Neutral Pseudoscalars: quark gluon vertex $\Leftrightarrow$ scattering kernel
  - π^0, η_c, η_b, anomaly $A^a(k;P) = 0$. Rainbow-Ladder approximation.
  - η, η', anomaly $A^a(k;P) \neq 0$. Phenomenally introduce $A^a(k;P)$.

DSEs: Axial-vector Ward-Takahashi identity. π^0, η_c, η_b

- Anomaly $A^a(k;P) = 0$

$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \mathscr{F}^a + i\gamma_5 \mathscr{F}^a S^{-1}(k_-) - 2i\mathscr{M}^{ab}\Gamma^b_5(k;P)$$

- DSEs

$$\Gamma_{5\mu}(p,P) = \gamma_5\gamma_\mu + \int_k K(p,k,P) S^a(k_+)\Gamma_{5\mu}(k,P)S^b(k_-)$$

$$S^{-1}(p) = i\gamma\cdot p + m + \Sigma(p)$$

$$\Gamma_5(p,P) = \gamma_5 + \int_k K(p,k,P) S^a(k_+)\Gamma_5(k,P)S^b(k_-)$$

- Self energy & scattering kernel

$$\Sigma\left(p_+\right) i\gamma_5 + i\gamma_5\Sigma\left(p_-\right) = -\int_k K(p,k,P)\left(S^a(k_+)i\gamma_5 + i\gamma_5 S^a(k_-)\right)$$

- Rainbow
- Ladder

$$\Sigma(p) = \int_{dk} g^2 D_{\mu\nu}(p-k)\gamma_\mu S(k)\gamma_\nu \Leftrightarrow K_{RL}(p,k,P) = -g^2 D_{\mu\nu}(p-k)\frac{\lambda^a}{2}\frac{\lambda^a}{2}\gamma_\mu\gamma_\nu$$

quark gluon vertex $\Leftrightarrow$ scattering kernel

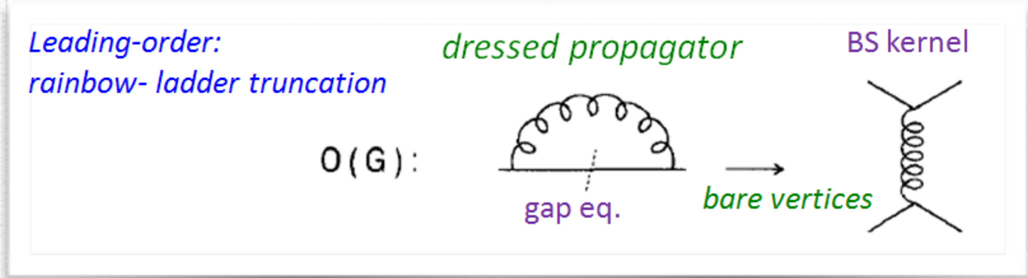
DSEs: Axial-vector Ward-Takahashi identity. η, η'

- Anomaly $A^a(k;P) \neq 0$

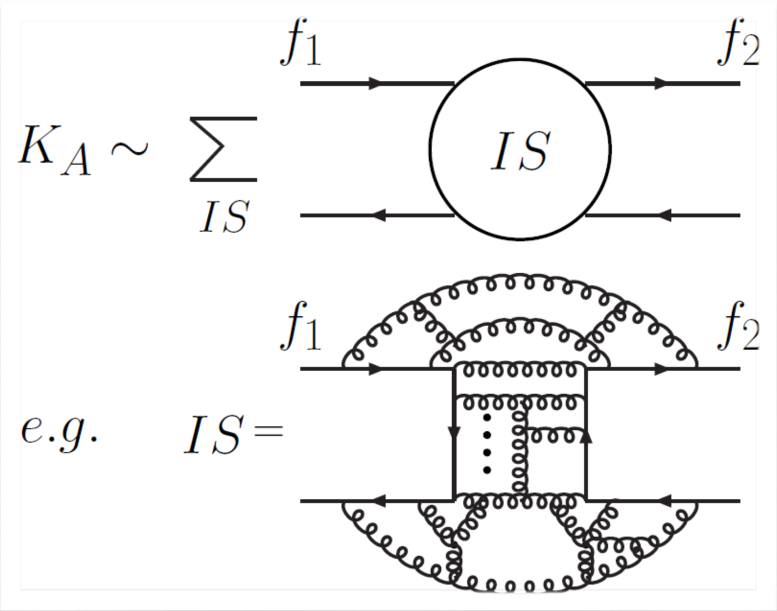
$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \mathscr{F}^a + i\gamma_5 \mathscr{F}^a S^{-1}(k_-) - 2i\mathscr{M}^{ab}\Gamma^b_5(k;P) - A^a(k;P)$$

- No simple relation between self energy and scattering kernel

- "hairpin" structure: intermediate state (IS) involve infinitely many lines



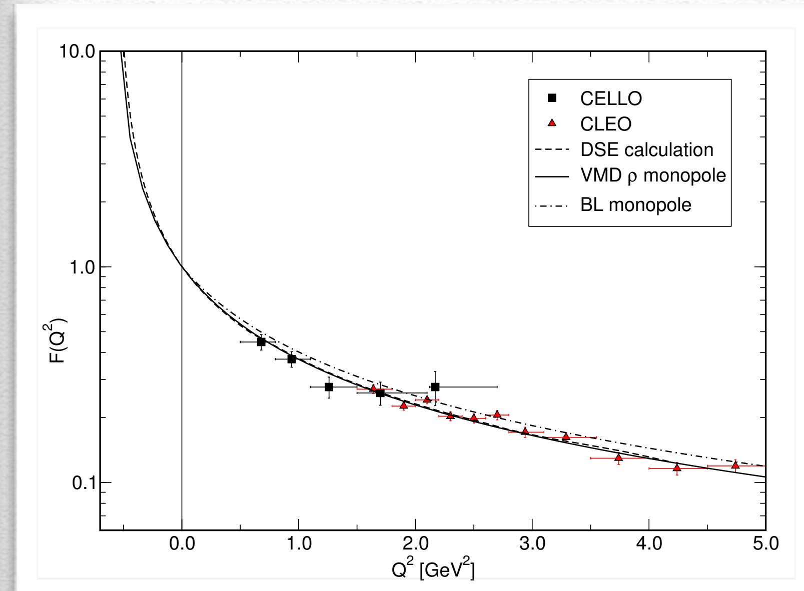
$$[K_A(k,q,P)]^{\alpha\beta}_{\rho\sigma}$$
$$= \xi(s)\cos^2(\theta)_\xi[zi\gamma_5]_{\alpha\beta}[zi\gamma_5]_{\rho\sigma}$$
$$+\frac{1}{\chi^2}\xi(s)\sin^2(\theta)_\xi[zi\gamma_5\gamma\cdot P]_{\alpha\beta}[zi\gamma_5\gamma\cdot P]_{\rho\sigma}$$

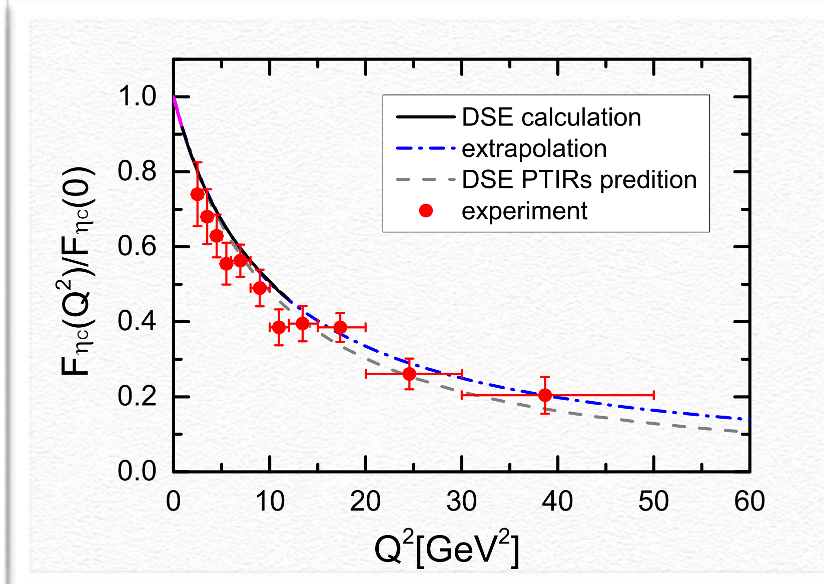


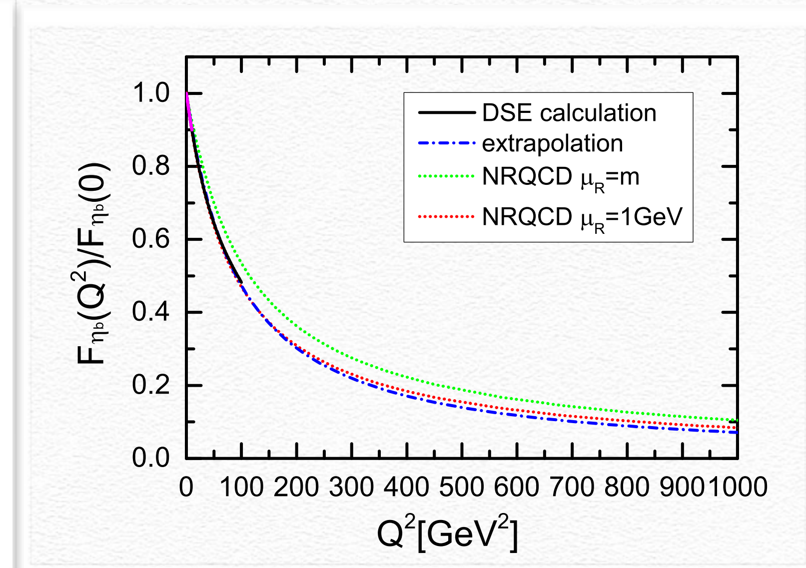
- $\xi(s) = 0 \rightarrow$ anomaly kernel: $K_A = 0$

arXiv:0708.1118

Single-virtual TFFs with DSEs: direct calculation







arXiv:nucl-th/0201017

arXiv:1611.05960

- Singularity of quark propagator

- F_{π^0} upper-limit $5GeV^2$ ↭ BELLE $Q^2 \in [4,40]GeV^2$
- F_{η_c} upper-limit $12GeV^2$ ↭ BABAR $Q^2 \in [2,50]GeV^2$
- F_{η_b} upper-limit $100GeV^2$ ↭ No experiment data

- Need extrapolation

Single-virtual TFFs with DSEs: extrapolation methods

- Method based on Distribution amplitude (DA)
- Off-shell method
  - Introducing a virtuality eigenvalue $\lambda(v)$ into the Bethe-Salpeter equations
  - Considering the v-dependence of the pointwise behaviour of the Bethe-Salpeter amplitude

arXiv:1702.06100

Single-virtual TFFs with DSEs: extrapolation methods

- Method based on Distribution amplitude (DA)

$$\langle 0|\psi(-z)\gamma_5\gamma.n\psi(z)|\pi(P)\rangle = f_\pi n\cdot P\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi(x)\,,$$
$$= tr_{CD}Z_2\int_{dq}^{\Lambda} e^{-iz\cdot q - iz\cdot(q-P)}\gamma_5\gamma.n\chi(q;P)\,.$$

- Projecting Bethe-Salpeter wave function onto the light front

$$f_\pi\phi(x) = tr_{CD}Z_2\int_{dq}^{\Lambda}\delta(n\cdot q_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi(q;P)$$

  - n, light-like four-vector, $n^2 = 0$. f_π, decay constant.
  - $\chi(q;P)$ Bethe-Salpeter wave function

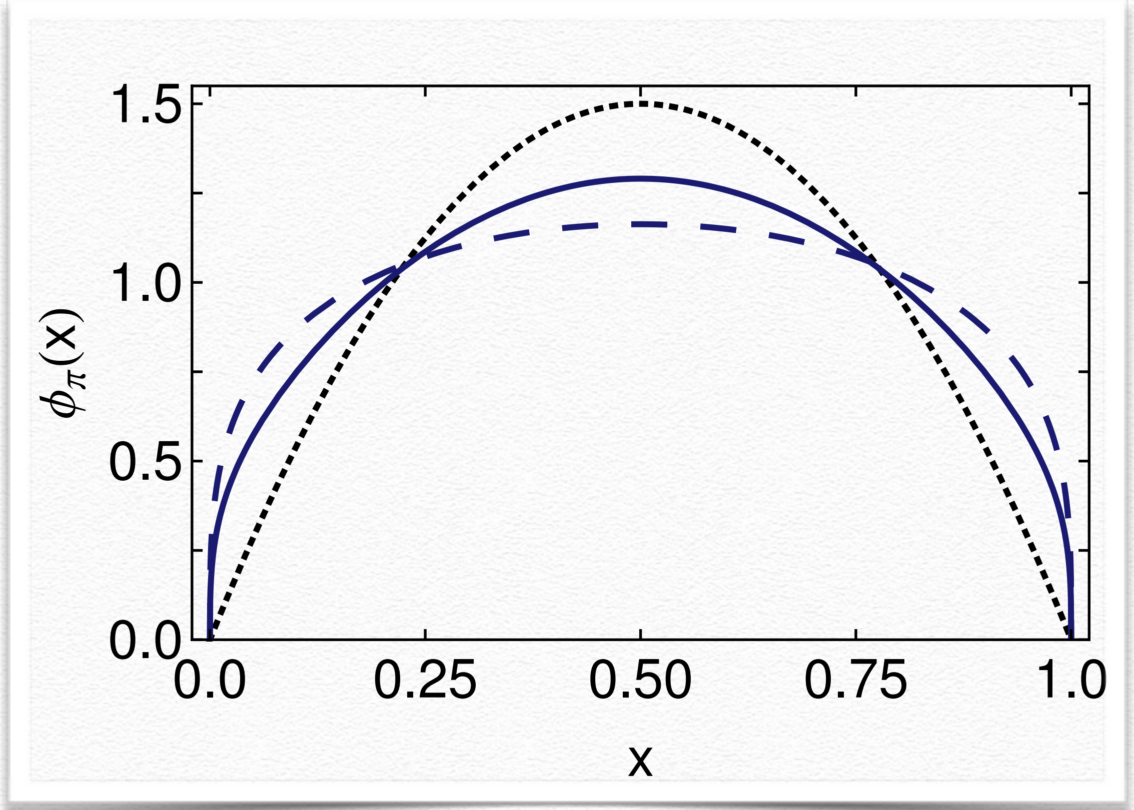
$$\chi(q,P) = S(q_+)\Gamma(q,P)S(q_-)$$

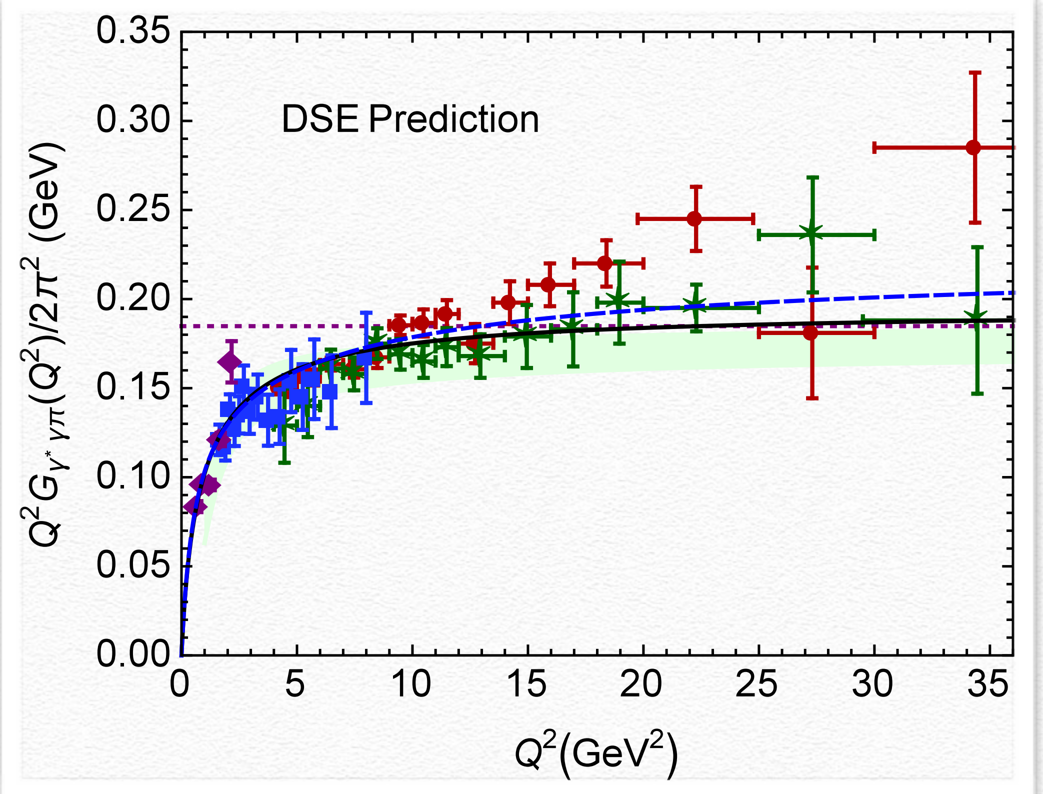
Single-virtual TFFs with DSEs: extrapolation methods

- Method based on Distribution amplitude (DA)
  - Perturbation theory integral representations (PTIRs):
  - Represent the Quark propagator with conjugate poles
  - Represent Bethe-Sapeter wave function with Nakanishi representation
  - Combine denominators of the integral by Feynman parameters

Prog. Theor. Phys. Suppl. 43, 1 (1969)

Single-virtual TFFs with DSEs: π^0





DA arXiv:1301.0324

TFF arXiv:1510.02799

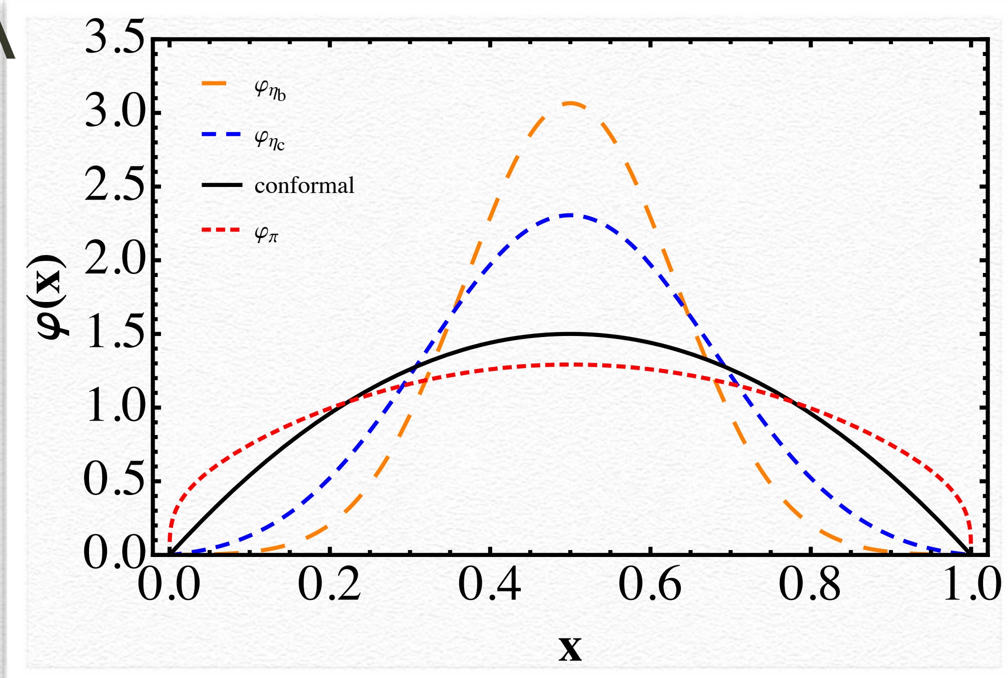
- Solid line: broad, concave function at hadronic scale;
- Dotted line: asymptotic prediction: $\phi_{asy}(x) = 6x(1-x)$

- Consistent with all non-BaBar data
- Approach pQCD limit $2f_\pi$ from above

$$Q^2 F_\pi(Q^2) \overset{Q^2 > Q_0^2}{=} 4\pi^2 f_\pi \frac{1}{3} \int_0^1 dx \frac{\phi_\pi(x;Q^2)}{x}$$

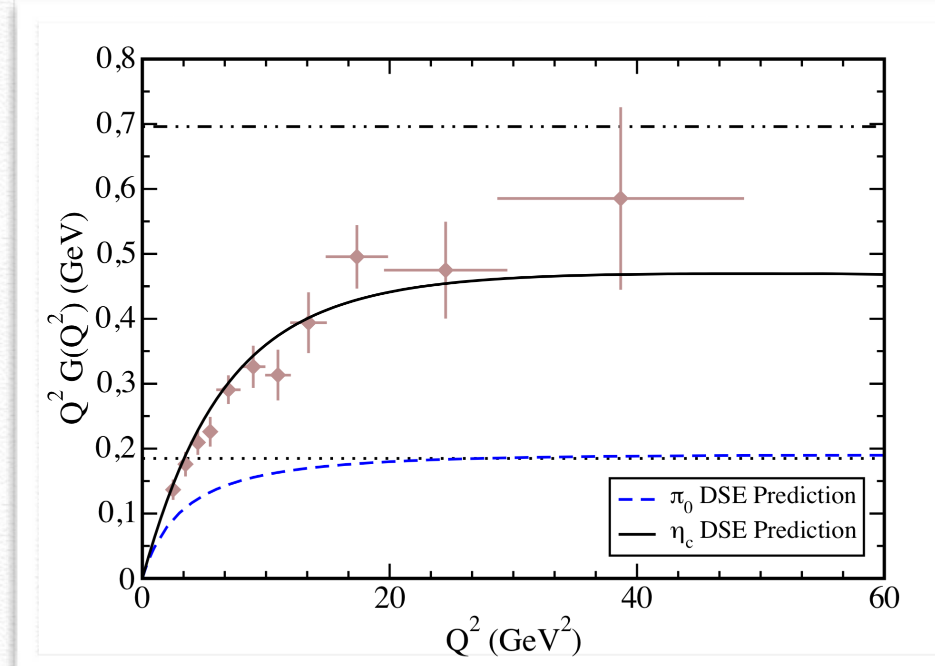
Single-virtual TFFs with DSEs: η_c & η_b

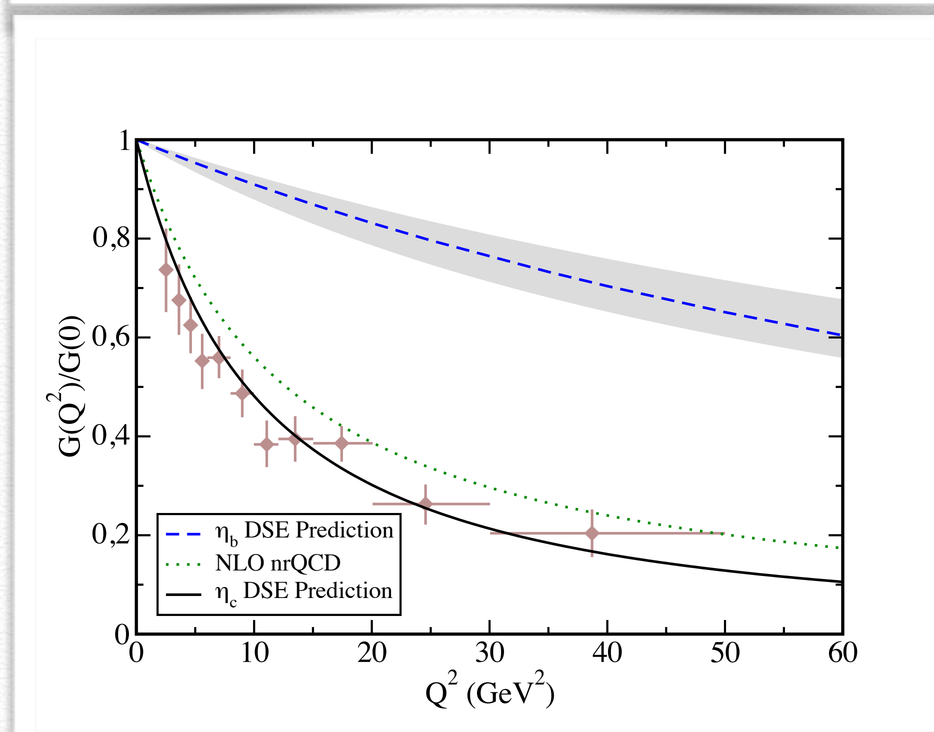
❖ DA



arXiv:1511.04943

❖ TFF





- DA: piecewise convex-concave-convex function

$$\Lambda_{QCD}/m_q(\zeta) \to 0, \phi(x) \to \delta(x - 1/2)$$

- TFF: consistent with BaBar data

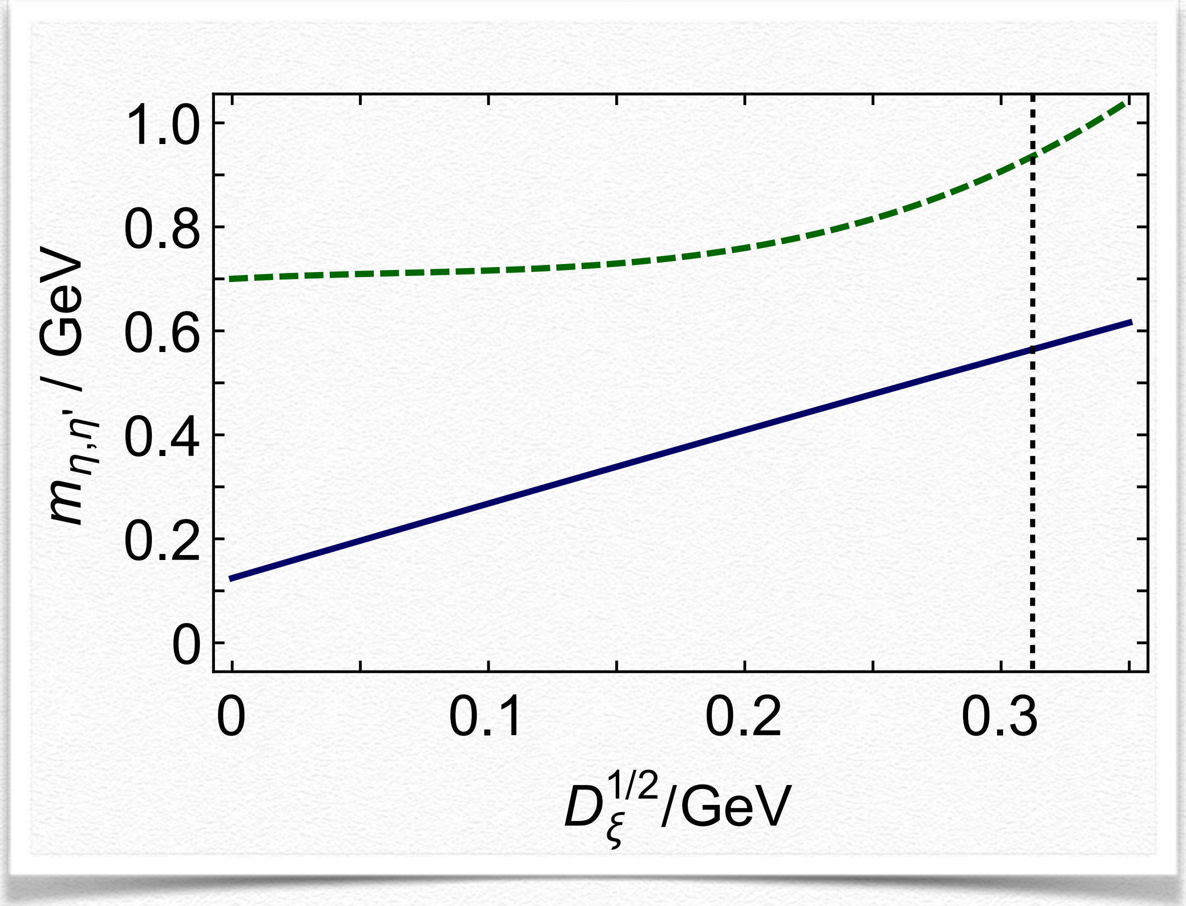
- Approach pQCD limit $\frac{8}{3} f_{\eta_c}$ from below

$$\lim_{Q^2\to\infty} Q^2 F_{\eta_c}(Q^2) = 4\pi^2 \int_0^1 dx \frac{\frac{4}{9} f_{\eta_c} \phi_{\eta_c}(x)}{1-x}$$

arXiv:1610.06575

Single-virtual TFFs with DSEs: η & η'

- Meson Mass



- Ideal mixing: no anomaly
  $m_\eta = m_\pi, m_{\eta'} = m_{s\bar{s}} = 0.7GeV$

- Meson mass grow with the anomaly strength, reach the empirical values
  $m_\eta = 0.56GeV, m_{\eta'} = 0.96GeV$

Single-virtual TFFs with DSEs: η & η'

- Decay constants: pseudovector projection of the Bethe-Salpeter wave function

$$f^{l,s}_{\eta,\eta'} P_\mu = Z_2 tr \int_{dk}^{\Lambda} \gamma_5 \gamma_\mu \chi^{l,s}_{\eta,\eta'}(k;P)$$

	f^l_η	f^s_η	$f^l_{\eta'}$	$f^s_{\eta'}$
herein-direct	0.072	-0.092	0.070	0.104
herein-fit	0.074	-0.094	0.068	0.101
phen.[1] [2]	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

- Flavor mixing angle

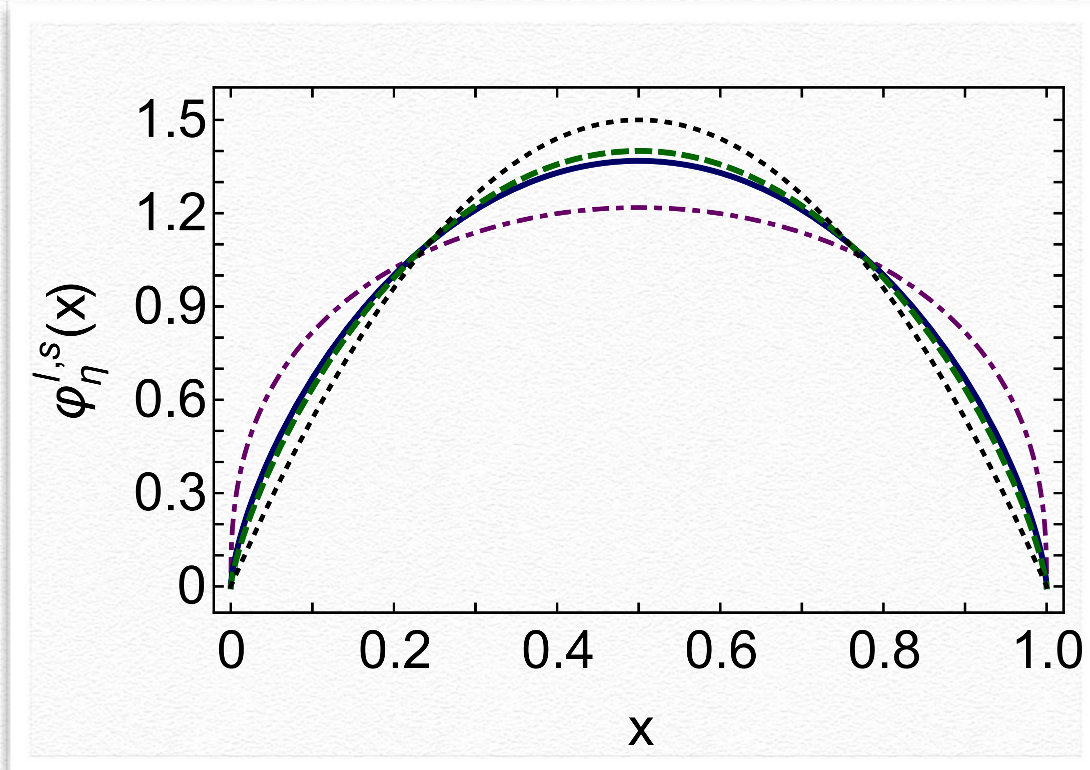
$$\begin{pmatrix} f^l_\eta & f^s_\eta \\ f^l_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f^l \cos\theta & -f^s \sin\theta \\ f^l \sin\theta & f^s \cos\theta \end{pmatrix}$$

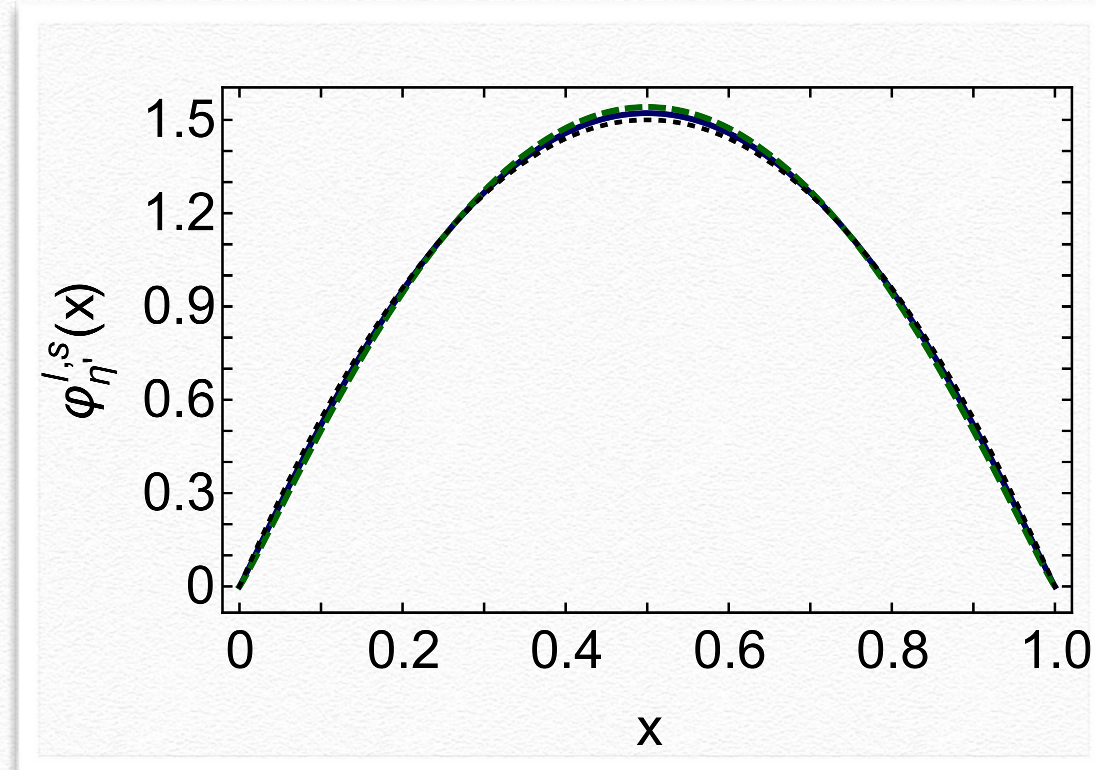
$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f^8 \cos\theta_8 & -f^0 \sin\theta_0 \\ f^8 \sin\theta_8 & f^0 \cos\theta_0 \end{pmatrix}$$

- $\theta = 42.8^\circ$
  $f^l = 0.101 GeV = 1.08 f_\pi$
  $f^s = 0.138 GeV = 1.49 f_\pi$

- $\theta_8 = -21^\circ, \theta_0 = -2.8^\circ$
  $f^8 = 1.34 f_\pi$
  $f^0 = 1.26 f_\pi$

Single-virtual TFFs with DSEs: η & η'





- DAs: ϕ_η

  $\phi^{asy} <_N \phi_\eta <_N \phi_\pi$

- DAs: $\phi_{\eta'}$

  $\phi_{\eta'} \sim \phi^{asy}$

- Light- and s-quark component DAs have similar profiles

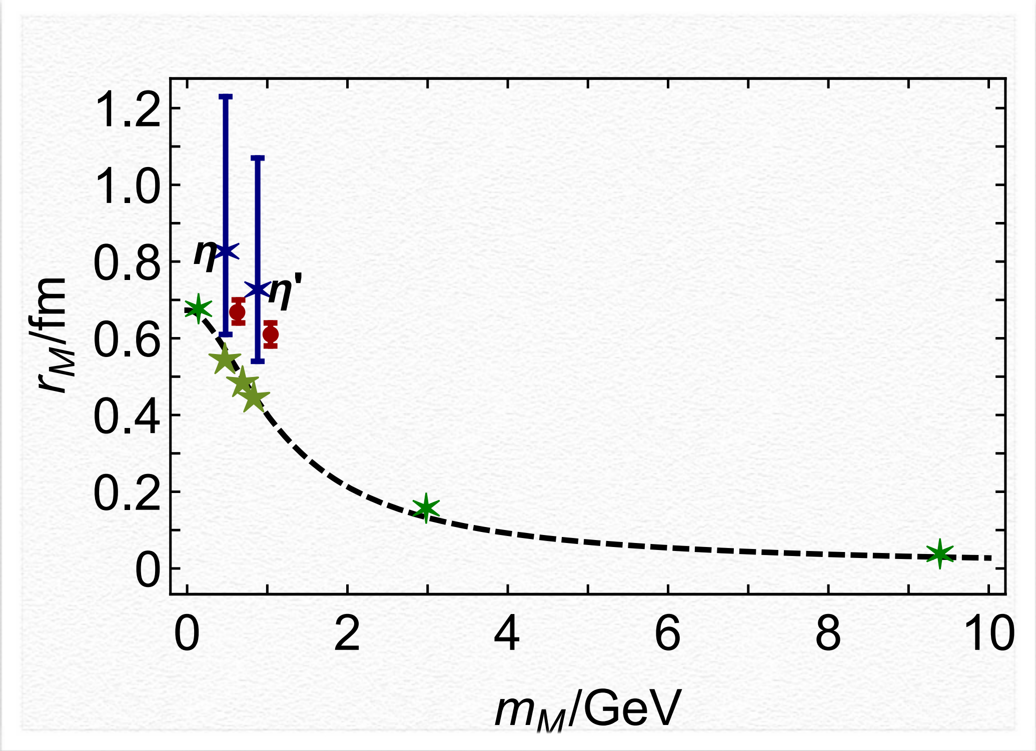
Single-virtual TFFs with DSEs: η & η' Low Q^2

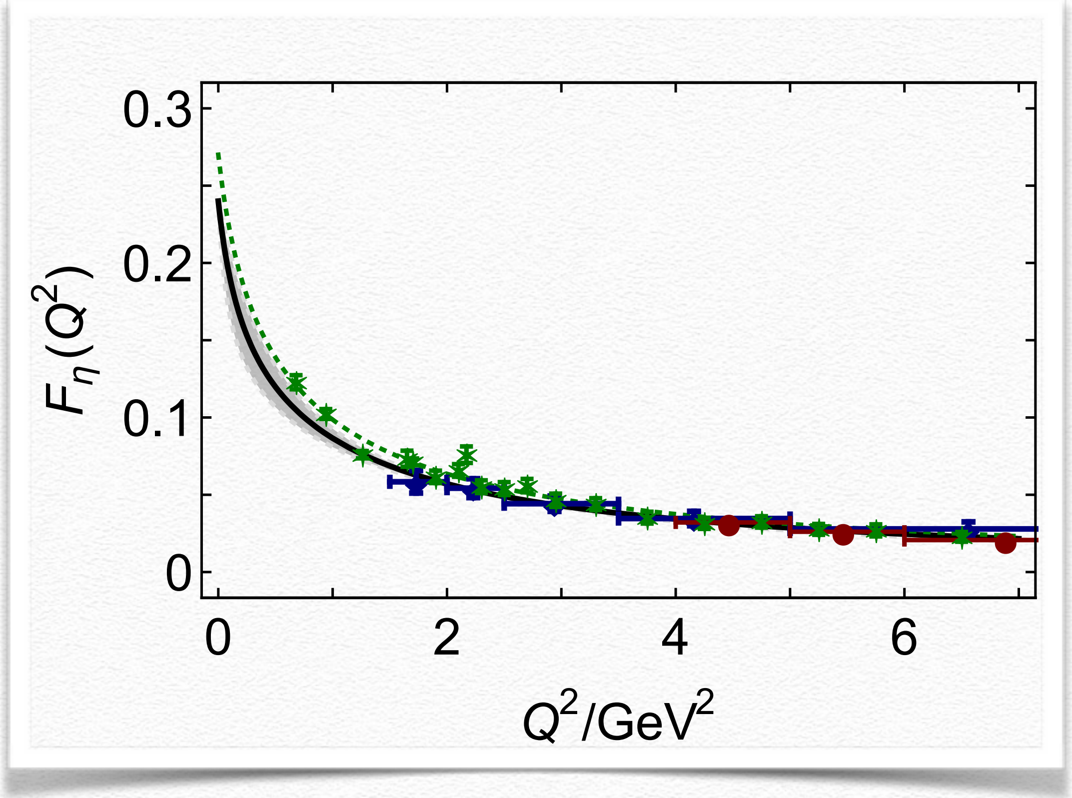
- TFFs consistent with Babar data
- Radii:

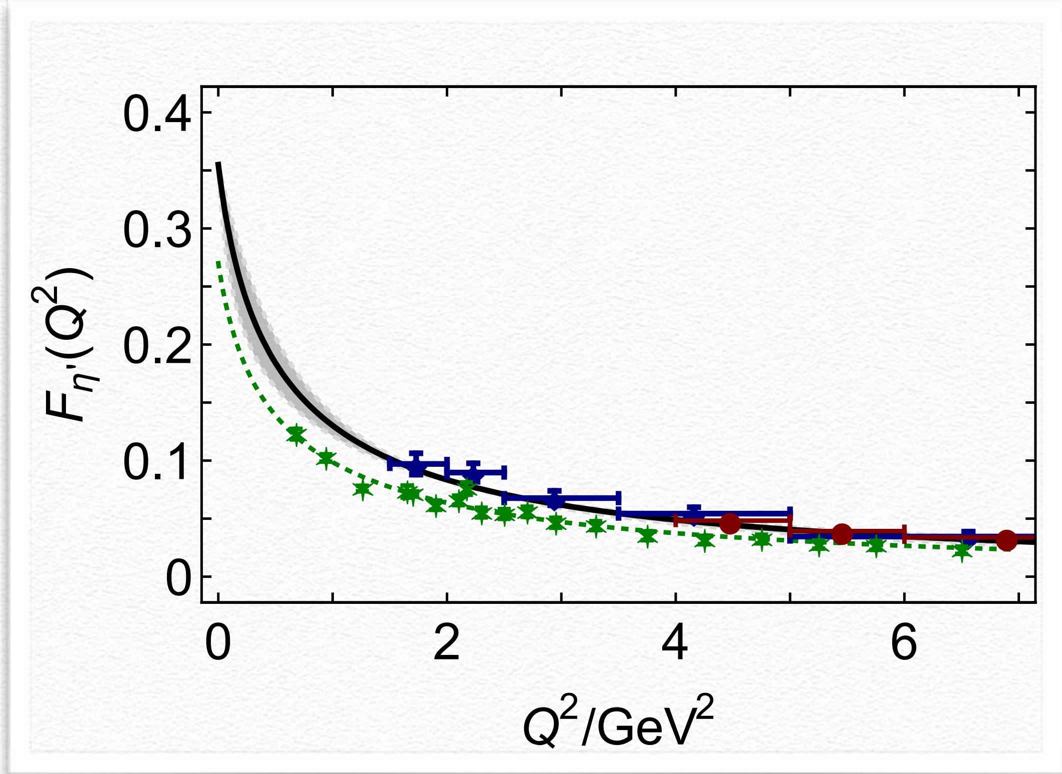
$$r_\eta = 0.83^{+0.40}_{-0.22} fm\,,\; r_{\eta'} = 0.73^{+0.34}_{-0.19} fm$$

$$r^2_{\eta,\eta'} := \frac{-6}{F_{\eta,\eta'}(0)} \frac{d}{dQ^2} F_{\eta,\eta'}(Q^2)\,|_{Q^2=0}$$

- $\pi^0, \eta, \eta', \eta_c, \eta_b$ & $m_{0^-}/GeV = 0.47, 0.69, 0.83$
- Non-Abelian anomaly: η, η' larger than a fit by 24% and 48%

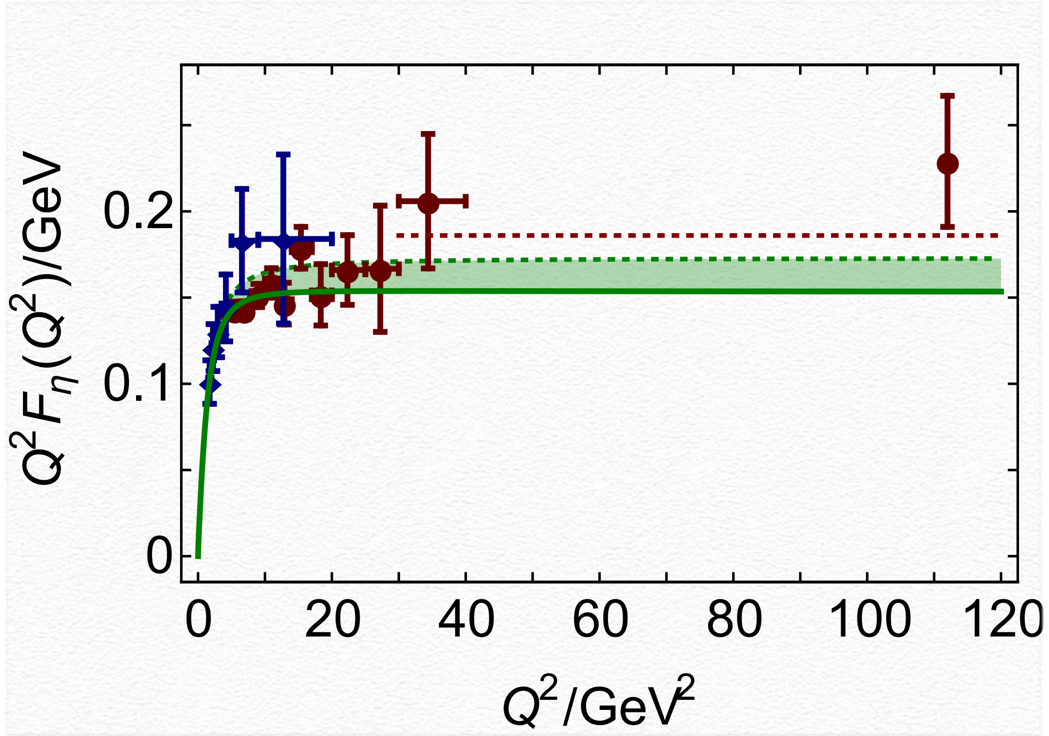


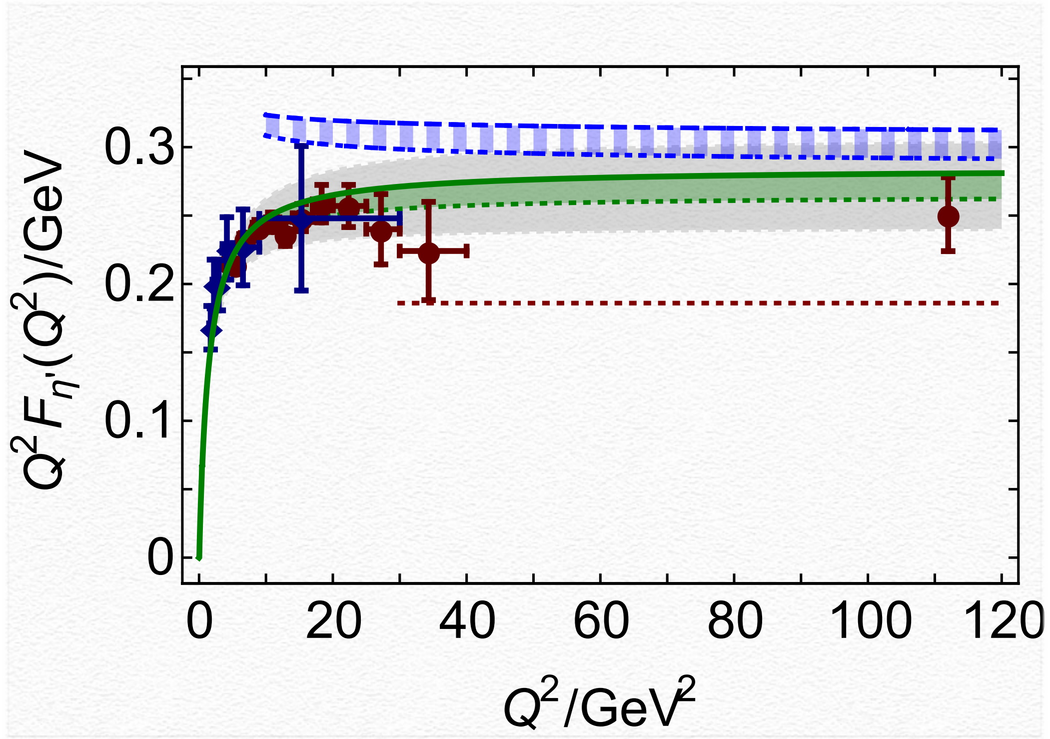




arXiv:1810.12313

Single-virtual TFFs with DSEs: η & η' Large Q^2





- $Q^2F_\eta(Q^2 \to \infty) = 0.15GeV$
- $Q^2F_{\eta'}(Q^2 \to \infty) = 0.30GeV$

- Omission of $\varphi^0_{\eta,\eta'} - \varphi^g_{\eta,\eta'}$ mixing $\lim_{Q^2\to\infty} Q^2F_{\eta,\eta'}(Q^2) = 6\left[\frac{5}{9}f^l_{\eta,\eta'}(Q^2) + \frac{\sqrt{2}}{9}f^s_{\eta,\eta'}(Q^2)\right]$

η & η' topological charge

$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \mathscr{F}^a + i\gamma_5 \mathscr{F}^a S^{-1}(k_-) - 2i\mathscr{M}^{ab}\Gamma^b_5(k;P) - A^a(k;P)$$

- Axial-vector Ward-Takahashi identity

$$\Downarrow$$

$$m^2_{\eta,\eta'} f^a_{\eta,\eta'} = \delta^{a0} n_{\eta,\eta'} + 2\mathscr{M}^{ab}\rho^b_{\eta,\eta'}$$

- General mass formula

- Topological charge: $n_{\eta,\eta'}$

$$n_{\eta,\eta'} = \sqrt{\frac{3}{2}}\nu_{\eta,\eta'}, \nu_{\eta,\eta'} = \langle 0 | \mathscr{Q}(x) = i\frac{\alpha_s}{4\pi} tr_C[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma(x)}] | \eta, \eta' \rangle$$

- Pseudoscalar projection of the Bethe-Salpeter wave function: $\rho^b_{\eta,\eta'}$

ρ^8_η	ρ^0_η	$\rho^8_{\eta'}$	$\rho^0_{\eta'}$
0.50^2	0.033^2	-0.37^2	0.57^2

$$\Rightarrow \quad \nu_\eta = (0.29 GeV)^3, \quad \nu_{\eta'} = (0.37 GeV)^3 .$$

- Topological charge content of the η' is 2.1 times that of η

Summary

- Two photon single-virtual transition form factors of neutral pesudoscalars using DSEs, constrain Bethe-Salpeter kernel by axial-vector Ward-Takahashi identity.
- $\gamma\gamma^* \to \pi^0$: consistent with all non-BaBar data; approach pQCD limit $2f_\pi$ from above; DA: broad concave function.
- $\gamma\gamma^* \to \eta_c, \eta_b$: consistent with BaBar data; approach pQCD limit $\frac{8}{3}f_{\eta_c}$ from below; DA: narrow piecewise convex-concave-convex function; $\phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy}$
- $\gamma\gamma^* \to \eta, \eta'$: consistent with BaBar data; DA: $\phi^{asy} <_N \phi_\eta <_N \phi_\pi, \phi_{\eta'} \sim \phi^{asy}$; non-abelian anomaly: η, η' larger than a non-mixing pseudoscalar radii fit by 24% and 48%; $Q^2F_\eta(Q^2 \to \infty) = 0.15GeV, Q^2F_{\eta'}(Q^2 \to \infty) = 0.30GeV$; Topological charge content of the η' is 2.1 times that of η.

Outlook

- Double virtual transition form factors, $Q_1^2 \neq 0\,, Q_2^2 \neq 0$, BaBar, (2018)
- Time like form factor: $Q^2 < 0$.

Thank you!