Quark-mass dependence of light meson masses and decay constants

Xiao-Yu Guo*, Matthias F.M. Lutz

[PRD98,056005; NPA988,36; NPA988,48]

GSI Helmholtzzentrum, Darmstadt

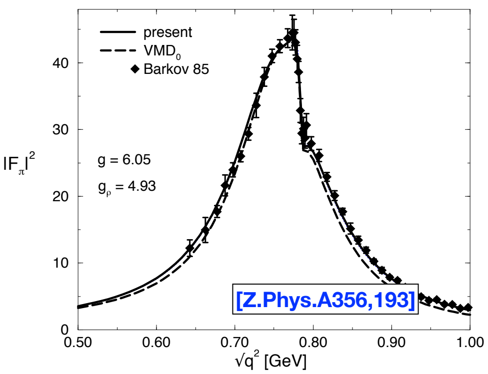
Hadron 2019, Guilin

August 20, 2019

Outline

Introduction

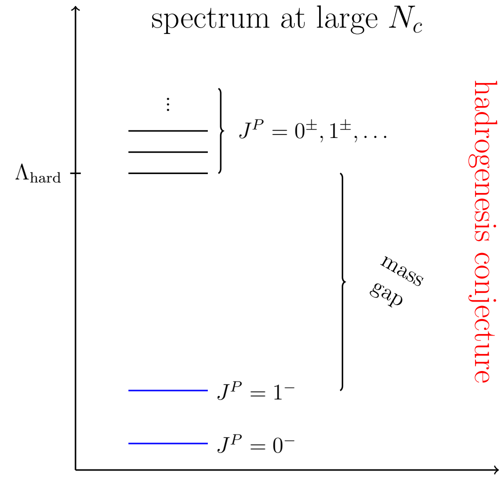
- strong interaction at low momentum transfer
  $\rightarrow$ perturbative QCD fails
- effective degrees of freedom?
- chiral perturbation theory (χPT)
  - $\rightarrow$ Goldstone bosons π, K, η as dynamical degrees of freedom
  - $\rightarrow$ power counting
  - are more massive states relevant?



- vector-meson dominance (VMD)
  - $\rightarrow$ considering ρ as a dynamical degree of freedom
  - $\rightarrow$ instrumental to describe hadron properties
- how to systematically incorporate such fields?
- how to power count?

Introduction

- an effective field theory approach
  - consider Goldstone-bosons π, K, η and vector mesons ρ, ω, K^*, ϕ as effective degrees of freedom
  - construct a chiral SU(3) Lagrangian
- how to power count?
  - conjecture a scale separation in the large N_c limit of chiral QCD
  - $m_{0^-} < 1\text{GeV}$, $m_{1^-} \lesssim 1\text{GeV}$, other meson masses $\sim (2-3)\text{GeV}$
  - naturalness implications for the low-energy constants (LECs)

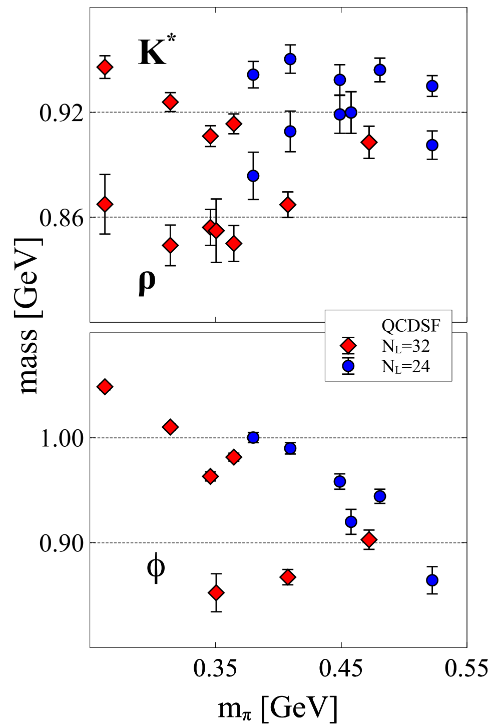


[Terschlüsen,Leupold,Lutz:EPJA48,190]

- masses and momenta of π, K, η and ρ, ω, K^*, ϕ are soft

- dynamical vector mesons at the one-loop level
  - dimensional power counting
  - an order-by-order renormalizability condition [Terschlüsen,Leupold:PRD94,014021]
  - specific correlations for LECs follow [PRD98,056005]

- is our approach compatible with QCD lattice results?
  - large data set for meson masses and decay constants available at different quark masses
  - we consider results from QCDSF, PACS-CS, HPQCD, CLS, HSC, ETMC



[Phys.Rev.D84,054509]

Effective Lagrangian with one-loop corrections

- leading order two-point interactions

$$\mathcal{L}_2^{(2)} = -f^2\left(\mathrm{tr}\, U_\mu U^\mu + \frac{1}{2}\mathrm{tr}\chi_+\right) - \frac{1}{4}\mathrm{tr}\left(\partial^\mu \Phi_{\mu\alpha}\partial_\nu \Phi^{\nu\alpha}\right) + \frac{1}{8}M^2\mathrm{tr}\left(\Phi_{\mu\nu}\Phi^{\mu\nu}\right)$$

  - the Goldstone-boson fields Φ enter with

$$U_\mu = \frac{1}{2}e^{-i\frac{\Phi}{2f}}\left(\partial_\mu e^{i\frac{\Phi}{f}}\right)e^{-i\frac{\Phi}{2f}} + \ldots$$

  - the symmetry breaking terms $\chi_\pm$ are proportional to the quark-mass matrix with

$$\chi_\pm \sim \mathrm{Diag}(m, m, m_s)$$

  - the vector meson fields are in the tensor-field representation with

$$\Phi_{\mu\nu} = \begin{pmatrix} \rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}\rho^+_{\mu\nu} & \sqrt{2}K^{*+}_{\mu\nu} \\ \sqrt{2}\rho^-_{\mu\nu} & -\rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}K^{*0}_{\mu\nu} \\ \sqrt{2}K^{*-}_{\mu\nu} & \sqrt{2}\bar{K}^{*0}_{\mu\nu} & \sqrt{2}\phi_{\mu\nu} \end{pmatrix}$$

- leading order three-point interaction terms

$$\mathcal{L}_2^{(3)} = \frac{i}{2} f\, h_1 \,\mathrm{tr}\big(U_\mu\, \Phi^{\mu\nu}\, U_\nu\big) + \frac{i}{8} h_2\, \varepsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\Big\{\big[\Phi_{\mu\nu},\, (D^\tau \Phi_{\tau\alpha})\big]_+\, U_\beta\Big\}$$
$$- \frac{i}{4}\, \frac{M^2}{f}\, h_3 \,\mathrm{tr}\left\{\Phi_{\mu\tau}\, \Phi^{\mu\nu}\, \Phi^\tau{}_\nu\right\}$$

  - we count

$$M^2 \sim m_{0^-}^2 \sim Q^2$$

  - the order-by-order renormalizability condition requests (to avoid $\sim 1/M^n$ contributions in the induced loops)

$$\mathcal{L}_4^{(3)} = M^2 \frac{i}{8}\, h_4\, \varepsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\Big\{\big[(D_\alpha \Phi_{\mu\nu}),\, \Phi_{\tau\beta}\big]_+\, U^\tau\Big\}$$
$$+ M^2 \frac{i}{4}\, h_5\, \varepsilon^{\mu\nu\alpha\beta}\, \mathrm{tr}\left\{\Phi_{\mu\nu}\, \chi_-\, \Phi_{\alpha\beta}\right\} + \dots$$

- estimates from tree-level studies [Leupold,Lutz:NuPA813,96; Terschlüsen,et al:.EPJA48,190]
  - hadronic and radiative decays of vector mesons $\to h_1, h_2$
  - magnetic-dipole and electric-quadrupole moments of vector mesons $\to h_3$

- leading order four-point interaction terms

$$
\begin{aligned}
\mathcal{L}_2^{(4)} = & \frac{1}{8}\,\mathrm{tr}\left\{ g_1 \left[\Phi_{\mu\nu}\,, U_\alpha\right]_+ \left[U^\alpha, \Phi^{\mu\nu}\right]_+ + g_2 \left[\Phi_{\mu\nu}\,, U_\alpha\right]_- \left[U^\alpha, \Phi^{\mu\nu}\right]_- \right\} \\
& + \frac{1}{8}\,\mathrm{tr}\left\{ g_3 \left[\, U_\mu\,, U^\nu\right]_+ \left[\Phi_{\nu\tau}\,, \Phi^{\mu\tau}\right]_+ + g_5 \left[\Phi^{\mu\tau}, U_\mu\right]_- \left[\Phi_{\nu\tau}\,, U^\nu\right]_- \right\} \\
& + \frac{1}{8}\frac{M^2}{f^2}\mathrm{tr}\left\{ g_6 \left[\Phi_{\mu\nu}, \Phi_{\alpha\beta}\right]_+ \left[\Phi^{\alpha\beta}, \Phi^{\mu\nu}\right]_+ + g_7 \left[\Phi_{\mu\nu}, \Phi_{\alpha\beta}\right]_- \left[\Phi^{\alpha\beta}, \Phi^{\mu\nu}\right]_- \right\} \\
& + \frac{1}{8}\frac{M^2}{f^2}\mathrm{tr}\left\{ g_8 \left[\Phi^{\mu\nu}, \Phi_{\mu\beta}\right]_+ \left[\Phi^{\alpha\nu}, \Phi^{\alpha\beta}\right]_+ + g_9 \left[\Phi^{\mu\nu}, \Phi_{\mu\beta}\right]_- \left[\Phi^{\alpha\nu}, \Phi^{\alpha\beta}\right]_- \right\}
\end{aligned}
$$

  - the order-by-order renormalizability condition requests (the 1-loop level μ-dependence cancelled by counter terms)

$$
4g_1 + g_3 = \frac{1}{2} h_2^2, \qquad g_5 = g_3 + 4g_2\,.
$$

- next-to-leading order interactions involving vector-mesons

$$
\begin{aligned}
\mathcal{L}_4^{(V)} = & \frac{e_2}{8} M^4 \,\mathrm{tr}\{\Phi_{\mu\nu}\}\,\mathrm{tr}\{\Phi^{\mu\nu}\} + \frac{b_1}{8} M^2 \,\mathrm{tr}\left\{\Phi^{\mu\nu}\,\Phi_{\mu\nu}\,\chi_+\right\} \\
& + \frac{b_2}{8} M^2 \,\mathrm{tr}\{\Phi_{\mu\nu}\,\Phi^{\mu\nu}\}\,\mathrm{tr}\{\chi_+\} + \frac{b_3}{8} M^2 \,\mathrm{tr}\{\Phi_{\mu\nu}\}\,\mathrm{tr}\{\Phi^{\mu\nu}\,\chi_+\} \\
& + \frac{c_1}{8} \,\mathrm{tr}\,\{\Phi_{\mu\nu}\,\chi_+\,\Phi^{\mu\nu}\,\chi_+\} + \frac{c_2}{8}\,\mathrm{tr}\,\{\Phi_{\mu\nu}\,\Phi^{\mu\nu}\,\chi_+^2\} \\
& + \frac{c_3}{8} \,\mathrm{tr}\,\{\Phi_{\mu\nu}\,\Phi^{\mu\nu}\}\,\mathrm{tr}\,\{\chi_+^2\} + \frac{c_4}{8}\,\mathrm{tr}\,\{\Phi_{\mu\nu}\,\Phi^{\mu\nu}\,\chi_+\}\,\mathrm{tr}\,\{\chi_+\} \\
& + \frac{c_5}{8} \,\mathrm{tr}\,\{\Phi^{\mu\nu}\,\chi_+\}\,\mathrm{tr}\,\{\Phi_{\mu\nu}\chi_+\} + \frac{c_6}{8}\,\mathrm{tr}\,\{\Phi^{\mu\nu}\}\,\mathrm{tr}\,\{\Phi_{\mu\nu}\,\chi_+^2\}\,,
\end{aligned}
$$

  - the order-by-order renormalizability condition requests the b_1 term to scale with M^2
  - c_{3-6} terms are large N_c suppressed but required in the reproduction of lattice data

- next-to-leading order interactions without vector-mesons

$$\mathcal{L}_4^{(P)} = -8\,L_4\,\mathrm{tr}\{U_\mu\,U^\mu\}\,\mathrm{tr}\{\chi_+\} - 8\,L_5\,\mathrm{tr}\{U_\mu\,U^\mu\,\chi_+\} + 4\,L_6\,\mathrm{tr}\{\chi_+\}\,\mathrm{tr}\{\chi_+\} + 4\,L_7\,\mathrm{tr}\{\chi_-\}\,\mathrm{tr}\{\chi_-\} + 2\,L_8\,\mathrm{tr}\{\chi_+\chi_+ + \chi_-\chi_-\} + \ldots,$$

  - terms were constructed by Gasser-Leutwyler [Ann.Phys.158,142]
  - integrate out vector-meson fields at the one-loop level given our subtraction scheme we find

$$L_4 \to L_4\,, \qquad L_5 \to L_5\,, \qquad L_6 \to L_6\,.$$

$$L_7 \to L_7 - \frac{7\,h_1^2}{49152\,\pi^2} - \frac{h_2^2}{1536\,\pi^2} \simeq L_7 - 0.31 \times 10^{-3}\,,$$

$$L_8 \to L_8 + \frac{7\,h_1^2}{16384\,\pi^2} + \frac{h_2^2}{512\,\pi^2} \simeq L_8 + 0.92 \times 10^{-3}.$$

  - renormalization terms are dominated by the h_2 coupling constant

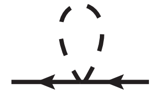
$h_1 \simeq 1.96$ from $\rho \to \pi\pi$ decay

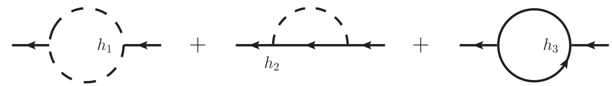
$h_2 \simeq 1.95$ from $\omega \to \rho\pi$ decay

Chiral extrapolations of meson masses and decay constants

- contributions from loop diagrams
  - vector meson masses

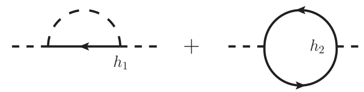
$$M_V^2 = \Pi_V^{\text{tree}} + \underbrace{\Pi_V^{\text{tadpole}} + \Pi_V^{\text{bubble}}/Z_V}_{\text{depend on } (M_V^2, m_P^2)},$$





  - pseudoscalar meson masses

$$m_P^2 = \underbrace{\Pi_P^{\chi-\text{PT}}}_{(\color{red}{m_q})} + \underbrace{\Pi_P^{\text{bubble}}/Z_P}_{(M_V^2, m_P^2)},$$



$$f_P = \underbrace{f_P^{\chi-\text{PT}}}_{(\color{red}{m_q})} + \underbrace{(\sqrt{Z_P} - 1) f - \frac{f}{\sqrt{Z_P}} \frac{\Pi_P^{\text{bubble}}}{m_P^2}}_{(M_V^2, m_P^2)}$$

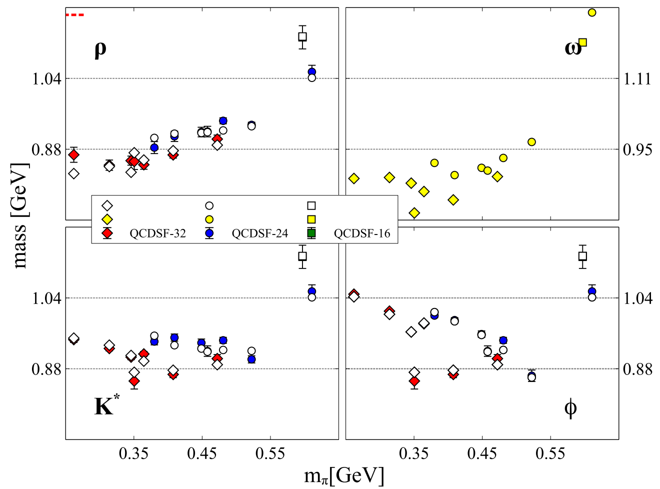
with $Z_P = 1 + \frac{\partial}{\partial\, m_P^2} \Pi_P^{\text{bubble}}(m_P^2)$

- adjust the LECs to QCD lattice data at different quark masses

A global fit to QCD lattice data

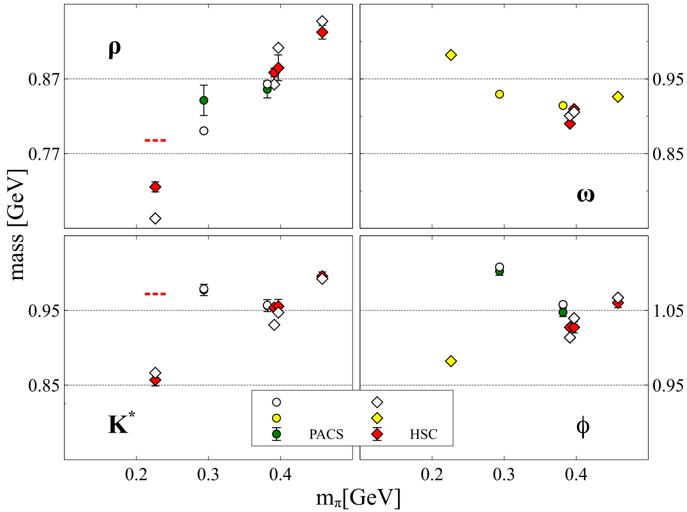
- fit to QCD lattice data of meson masses and decay constants [arXiv:1810.07376, 1810.07078]
  - use on-shell masses in the loop contributions
  - finite-volume corrections are taken into account
  - we consider results from 6 lattice groups
  - estimate the systematic error by the condition $\chi^2/N \sim 1$

- masses from QCDSF-UKQCD [Phys.Rev.D84,054509]
  - consider finite-box energy levels from quark-antiquark interpolators
  - levels are well reproduced
  - predictions for ω-meson masses on the ensembles



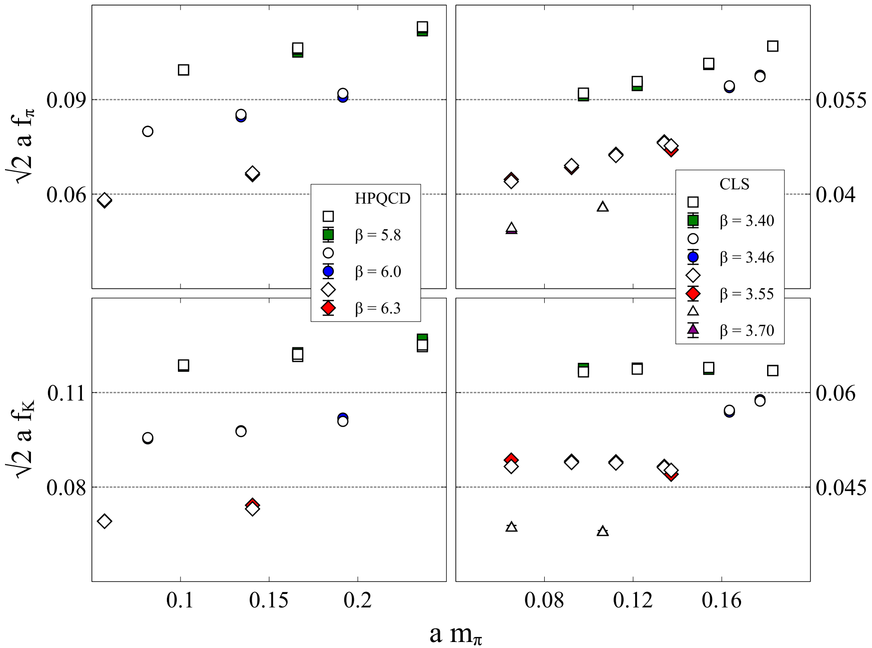
A global fit to QCD lattice data

- masses from PACS-CS [Phys.Rev.D79,034503]
  - energy levels from quark-antiquark interpolators
  - levels are well reproduced
  - predictions for ω-meson masses on the ensembles
- masses from HSC [Phys.Rev.D79,034503]
  - levels from quark-antiquark interpolators well reproduced
  - level from $\pi\,K$ interpolators well reproduced [Nucl.Phys.B932,29]
  - tension in level from $\pi\pi$ interpolators, not well separated from the two-body $\pi\pi$ state (dashed line) [Nucl.Phys.B910,842]



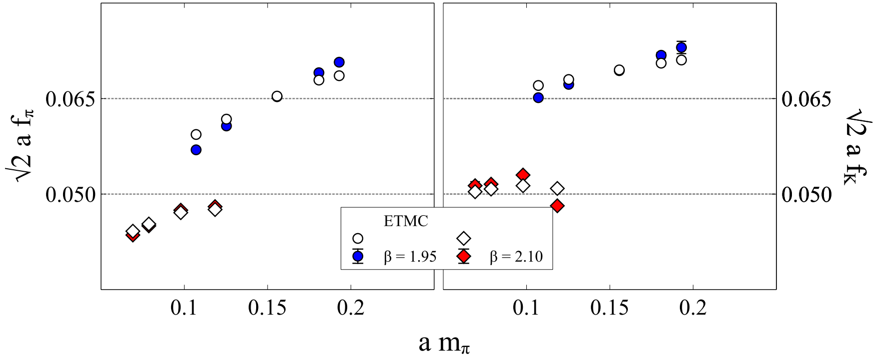
A global fit to QCD lattice data

- decay constants from HPQCD and CLS [Phys.Rev.D88,074504; Phys.Rev.D95,074504]



A global fit to QCD lattice data

- decay constants from ETMC [Phys.Rev.D97,054508]
  - f_K depends on wave-function renormalization factor Z in the twisted-mass set up
  - tuning $Z = 0.6884, 0.7428$ at $\beta = 1.95, 2.10$, within the ranges from two approaches determined by lattice

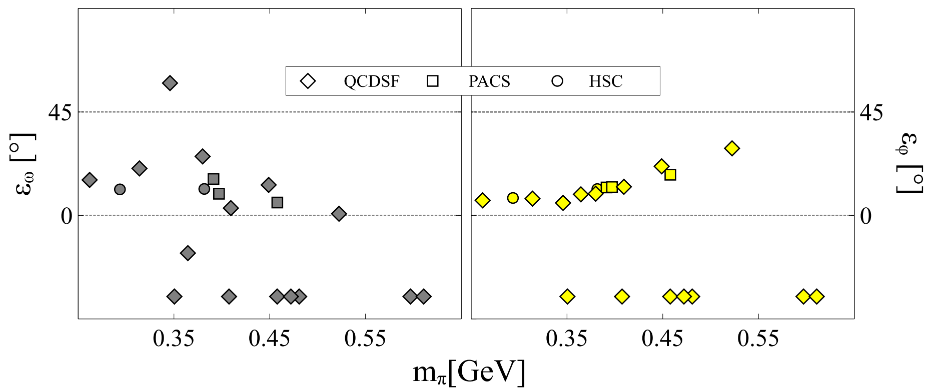


Some predictions for QCD lattice simulations

- $\omega - \phi$ mixing angle ϵ is mass-dependent

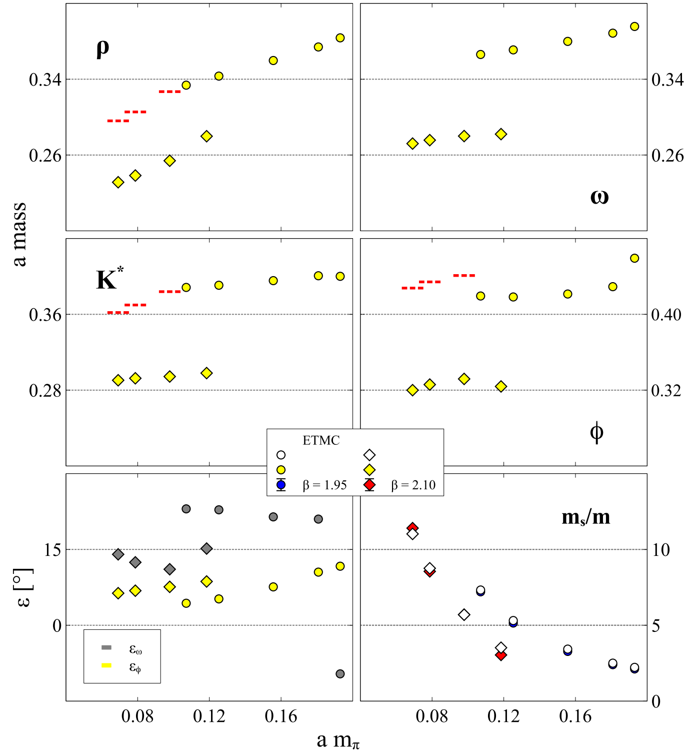
$$\omega = \omega' \cos\,\epsilon + \phi' \sin\,\epsilon \qquad\qquad \phi = \phi' \cos\,\epsilon - \omega' \sin\,\epsilon$$

  - of physical relevance are $\epsilon_\omega = \epsilon$ at M_ω and $\epsilon_\phi = \epsilon$ at M_ϕ
  - $|\epsilon_\phi| \simeq 3.32^\circ$ from $\phi \to \pi^0\gamma$ [Klingl, et,al: Z.Phys.A356,193]
  - from our global fit to lattice data $\epsilon_\omega \simeq 21^\circ$ at physical quark masses
- predict a striking quark-mass dependence of the mixing angles



Some predictions for QCD lattice simulations

- predictions for ETMC ensembles
  [Phys.Rev.D95,074504]
  - the lowest two-body energy levels (dashed lines) are well separated from our predicted energy levels
  - the quark masses m_s and m are determined from the lattice values of m_π and m_K
  - our predictions for the ratio m_s/m match well with the results from lattice



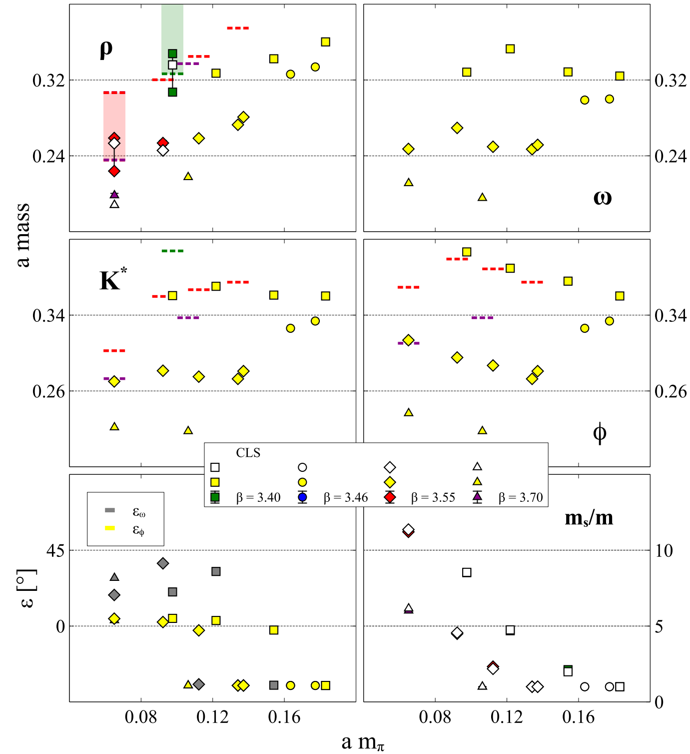
Some predictions for QCD lattice simulations

- predictions for CLS ensembles

  [Phys.Rev.D95,074504]

  - the lowest two-body energy levels (dashed lines) are not always well separated from our predicted energy levels
  - the quark masses m_s and m are determined from the lattice values of m_π and m_K
  - our predictions for the ratio m_s/m match well with the results from lattice
  - no data yet from CLS but levels from $\pi\pi$ interpolators

    [Nucl.Phys.B939,145]

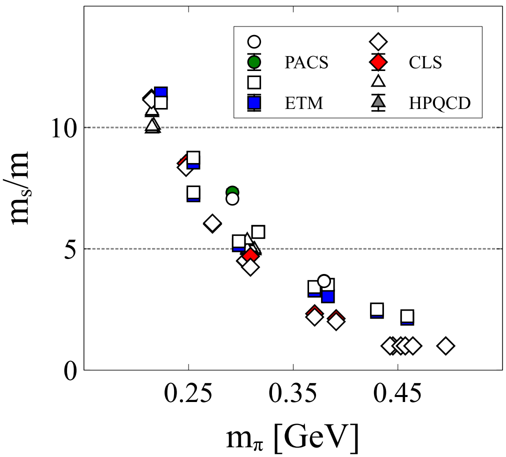


Some predictions for QCD lattice simulations

- our prediction for the quark mass ratio

  $$m_s/m = 26.13 - 26.92$$

  - compares well with lattice result $m_s/m = 26.66(32)$ [ETMC: Nucl.Phys.B887,19]
  - our ratios are compatible with lattice results on all considered ensembles



- LECs of Gasser and Leutwyler

	Ours: w/ VM	2-loop χPT
f [MeV]	67.51 - 73.57	64 - 71
$10^3 L_4$	-0.04 - 0.13	0.3 - 0.76
$10^3 L_5$	-0.01 - 0.04	0.50 - 1.01
$10^3 L_6$	-0.07 - 0.04	0.14 - 0.49
$10^3 L_7$	-0.14 - -0.11	-0.34 - -0.19
$10^3 L_8$	0.65 - 0.75	0.17 - 0.47

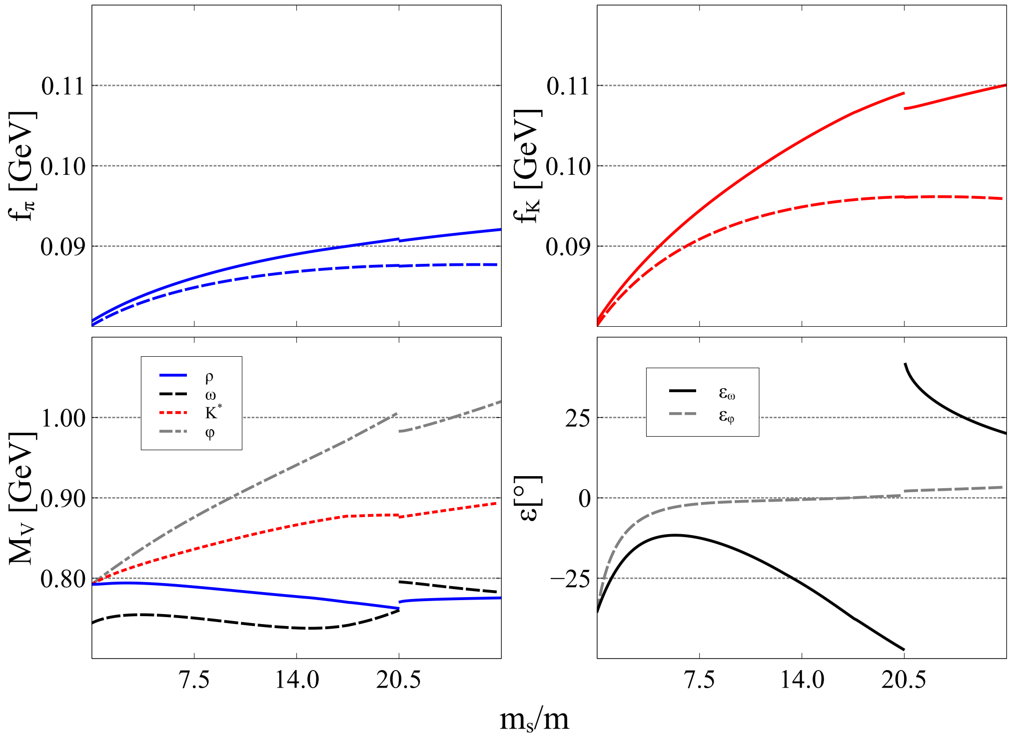
- the χPT results are at 2-loop level

  [Bijnens,Ecker:Ann.Rev.Nucl.Part.Sci.64,149]
- the impact of vector meson loops seem most dominant in L_5

Summary

- dynamical vector-meson degrees of freedom at the one-loop level
  - the vector-meson fields were incorporated in the chiral SU(3) Lagrangian based on the hadrogenesis conjecture
  - order-by-order renormalizability was obtained
  - the one-loop corrections for the meson masses and decay-constants were derived in a finite box
- a global fit to lattice QCD results
  - our results are compatible with lattice data sets from PACS, QCDSF, HSC, HPQCD, CLS and ETMC
  - predictions for vector-meson masses and the $\omega - \phi$ mixing angles on various lattice ensembles
  - results for Gasser-Leutwyler LECs $L_4 - L_8$ (vector-meson degrees of freedom are important)

Thank you for your attention!


0.11
0.10
0.09
f_π [GeV]
f_K [GeV]
1.00
0.90
0.80
M_V [GeV]
ρ
ω
K*
φ
25
0
−25
ε[°]
ε_ω
ε_φ
7.5
14.0
20.5
m_s/m