Configuration mixing of positive parity excited baryons in the large N_c limit

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[1] C.T. Willemyns, N.N. Scoccola, Phys. Rev. D **98**, 034019 (2018)

[2] C.T. Willemyns, C. Schat, Phys. Rev. D **95**, 094007 (2017)

Outline

1. Motivation
2. Baryons in the large N_c limit
3. Towers with configuration mixing
4. Summary

Understanding baryon structure directly from QCD is a basic problem of hadronic physics

- Chiral Perturbation Theory, Nambu-Jona-Lasinio model, etc...

- Lattice QCD
- Large N_c QCD

G. 't Hooft proposed a QCD generalized to N_c colors

QCD	3 colors and an SU(3) gauge group
⇓	
Large N_c QCD	N_c colors and an SU(N_c) gauge group

Large N_c QCD admits a consistent perturbative expansion in terms of $1/N_c$

[3] K. G. Wilson, Phys. Rev. D **10**, 2445 (1974)

[4] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974)

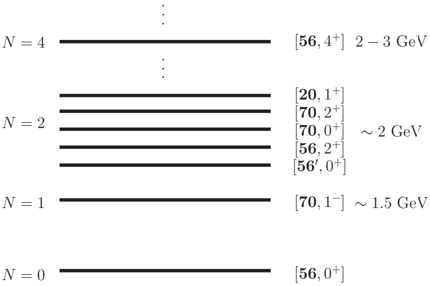
Baryon classification scheme

Baryon classification scheme $\longrightarrow$ Quark Model (QM)

In the QM

Baryons belong to the $SU(6) \times O(3)$ irreducible representations.

They organize in bands of the harmonic oscillator.



Experimental data and Lattice QCD calculations seem to "agree" with QM classification scheme

Spin-flavor symmetry for baryons in large N_c QCD

Gervais, Sakita and Dashen, Manohar found that in the large N_c limit a spin-flavor "contracted" $SU(2N_f)_c$ arises for **ground state** baryons.

$SU(2N_f)$	$SU(2N_f)_c$
$[S^i, T^a] = 0$, $[S^i, S^j] = i\epsilon^{ijk} S^k$, $\quad [T^a, T^b] = i f^{abc} T^c$, $[S^i, G^{ja}] = i\epsilon^{ijk} G^{ka}$, $\quad [T^a, G^{ib}] = i f^{abc} G^{ic}$, $[G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2N_f}\delta^{ab}\epsilon^{ijk} S^k + \frac{i}{2}\epsilon^{ijk} d^{abc} G^{kc}$	$[S^i, T^a] = 0$, $[S^i, S^j] = i\epsilon^{ijk} S^k$, $\quad [T^a, T^b] = i f^{abc} T^c$, $[S^i, X_0^{ja}] = i\epsilon^{ijk} X_0^{ka}$, $\quad [T^a, X_0^{ib}] = i f^{abc} X_0^{ic}$ $[X_0^{ia}, X_0^{jb}] = 0$

$$X_0^{ia} = \lim_{N_c \to \infty} \frac{G^{ia}}{N_c} .$$

$$SU(2N_f)\ (\mathrm{QM}) \xrightarrow[N_c \to \infty]{} SU(2N_f)_c$$

[5] J. L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984), Phys. Rev. D **30**, 1795 (1984).

[6] R. F. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993)

Baryons in the large N_c limit

QCD baryon $\longrightarrow$ Large N_c baryon

Ground state baryons are symmetric states in spin-flavor

$$\text{GS baryon} : \quad \underbrace{N_c \text{ quarks}}_{\text{Symmetric in spin-flavor}}$$

We couple a quark and a core to get the symmetries of the excited states

$$\text{Excited baryon} : \underbrace{N_c - 1 \text{ quarks core}}_{\text{Symmetric in spin-flavor}} + \underbrace{\text{excited quark}}_{\ell}$$

Large N_c baryons

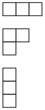
"QM states" with N_c colors

For $N_c = 3$:

56 $\rightarrow$ Symmetric

70 $\rightarrow$ Mixed-symmetric

20 $\rightarrow$ Antisymmetric

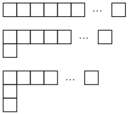


For large N_c:

"**56**" $\rightarrow$ Symmetric

"**70**" $\rightarrow$ Mixed-symmetric

"**20**" $\rightarrow$ Antisymmetric



$\mathbf{8} = (\mathbf{1}, \mathbf{1})$

$$\begin{array}{ccccc} & \mathbf{1} & & \mathbf{1} & \\ \mathbf{1} & & \mathbf{2} & & \mathbf{1} \\ & \mathbf{1} & & \mathbf{1} & \end{array}$$

$"\mathbf{8}" = \left(1, \frac{N_c-1}{2}\right)$

$$\begin{array}{ccccccccccc} & & & & \mathbf{1} & & \mathbf{1} & & & & \\ & & & \mathbf{1} & & \mathbf{2} & & \mathbf{1} & & & \\ & & \mathbf{1} & & \mathbf{2} & & \mathbf{2} & & \mathbf{1} & & \\ & \mathbf{1} & & \mathbf{2} & & \mathbf{2} & & \mathbf{2} & & \mathbf{1} & \\ \mathbf{1} & & \mathbf{2} & & \mathbf{2} & & \mathbf{2} & & \mathbf{2} & & \mathbf{1} \\ & \iddots & & & & & & & & \ddots & \end{array}$$

Operator expansion for excited states

$$O = \sum_{i=1} c_i O_i + \mathcal{O}(1/N_c)$$

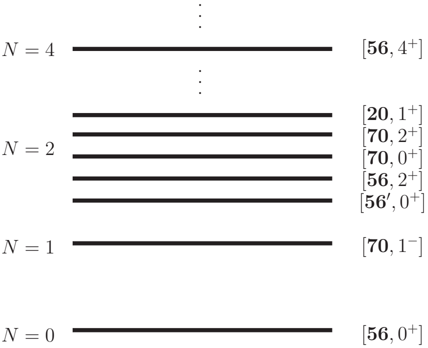
The buildings blocks of the operator expansions are the group generators:

- SU(6) generators: $S,\ T,\ G \rightarrow s,\ t,\ g, \quad S_c,\ T_c,\ G_c$
- coupled to an angular momentum operator ℓ.

Using reduction rules one can find an operator basis for excited states.

$$H = \sum_{i=1}^{5} c_i^{\mathbf{T},\mathbf{T}'} O_i + \mathcal{O}(1/N_c)$$

States studied in the $1/N_c$ expansion



$\mathcal{N} = 2$

Carlson Carone PLB **484**.260 (2000)

Goity Schat Scoccola PLB **564**.83 (2003)

Matagne Stancu PLB **631**.7 (2005)

Matagne Stancu PRD **74**.034014 (2006)

$\mathcal{N} = 1$

Pirjol Schat PRD **67**.096009 (2003)

Goity Schat Scoccola PRD **66**.114014 (2002)

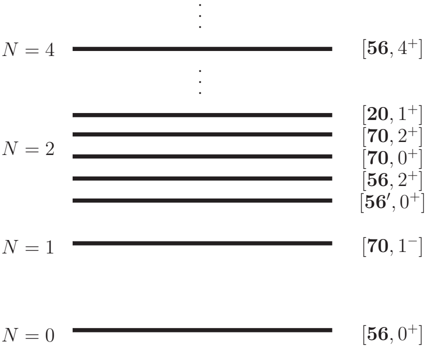
Schat Goity Scoccola PRL **88**.102002 (2002)

Carlson Carone Goity Lebed PRD **59**.114008 (1999)

$\mathcal{N} = 0$

Witten NPB **160**.57 (1979)

States studied in the $1/N_c$ expansion



- Since the QM symmetries are not QCD symmetries, these states can mix
- In large N_c QCD too

> Configuration mixing effects are not N_c suppressed

- A complete study of the band $N = 2$ was necessary

[7] J. Goity, Yad. Fiz. **68**.655 (2005).

Towers of large N_c states

Large N_c QCD $\rightarrow$ towers **K**

For $N_f = 2$, K is determined by the spin-flavor symmetry

$$\mathbf{K} = \mathbf{L} \quad \text{for S}$$
$$\mathbf{K} = \mathbf{L} + \mathbf{1} \quad \text{for MS, A}$$

For $[\text{“}\mathbf{70}\text{”}, 2^+] \rightarrow$ three towers $K = 1, 2, 3$

For $N_f = 3$ this relation is not trivial

- Only states with same K number mix

[8] D. Pirjol, T. M. Yan, PRD **57**.1449 (1998)

[9] D. Pirjol, T. M. Yan, PRD **57**.5434 (1998)

Main results:

S and MS states fall into only nine towers

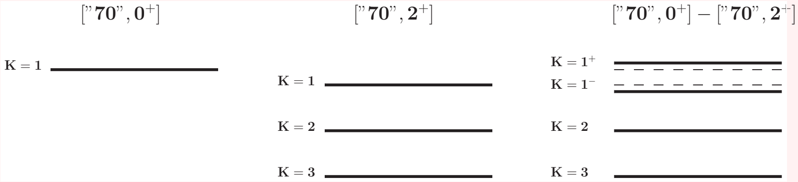
$$\begin{aligned}
m_0 &= \bar{c}_1^{S_0} N_c, \\
m_{1^\pm} &= \bar{m}_1 \pm \delta_1, & \text{with } \delta \sim \mathcal{O}(1) \\
m_{2^\pm} &= \bar{m}_2 \pm \delta_2, \\
m_3 &= \bar{c}_1^{MS_2} N_c + c_2^{MS_2} - \frac{2}{7} c_3^{MS_2} \\
m_{\frac{1}{2}} &= \bar{c}_1^{MS_0} N_c - 3 c_5^{MS_0} \\
m_{\frac{3}{2}} &= \bar{c}_1^{MS_2} N_c - \frac{3}{2} \bar{c}_2^{MS_2} + 3 c_4^{MS_2} - 3 c_5^{MS_2} \\
m_{\frac{5}{2}} &= \bar{c}_1^{MS_2} N_c + \bar{c}_2^{MS_2} - 2 c_4^{MS_2} - 3 c_5^{MS_2}
\end{aligned}$$

Antisymmetric states of the $\mathcal{N} = 2$ band fall into 6 towers

- Configuration mixing effects are $\mathcal{O}(1)$

Configuration mixing effects

- Configuration mixing happens between [“**70**”, 0^+] – [“**70**”, 2^+] and [“**56**”, 2^+] – [“**70**”, 2^+].



$$\delta_1 = \sqrt{\left(\frac{1}{2}\left(\bar{c}_1^{MS_0} - \bar{c}_1^{MS_2}\right) N_c + \frac{3}{4}\bar{c}_2^{MS_2} + \frac{1}{2} c_3^{MS_2}\right)^2 + 2\left(c_3^{MS_0, MS_2}\right)^2}$$

- Configuration mixing of the [“**20**”, 1^+] is an $\mathcal{O}(1/N_c)$ effect.

Summary

The $1/N_c$ expansion of QCD for baryons gives an analytical insight into the low energy regime

- Study baryons in the large N_c is complicated (many states to consider, group theory)
- We know how to build baryons states and operators with N_c colors. The core+quark approach in large N_c is suitable to study S, MS, and also A states.
- We considered configuration mixing and found that all the states of the $N = 2$ band organize into $9 + 6$ towers.
- Configuration mixing happens between $[``\mathbf{70}", 0^+] - [``\mathbf{70}", 2^+]$ and $[``\mathbf{56}", 2^+] - [``\mathbf{70}", 2^+]$, other mixings are subleading.

Thank you for your attention

References

[1] C.T. Willemyns, N.N. Scoccola, Phys. Rev. D **98**, 034019 (2018)
[2] C.T. Willemyns, C. Schat, Phys. Rev. D **95**, 094007 (2017)
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[9] D. Pirjol, T. M. Yan, PRD **57**.5434 (1998)