Triangle singularities in $J/\psi \to \eta\pi^0\phi$

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BESIII observation of $a_0(980)^0 - f_0(980)$ mixing

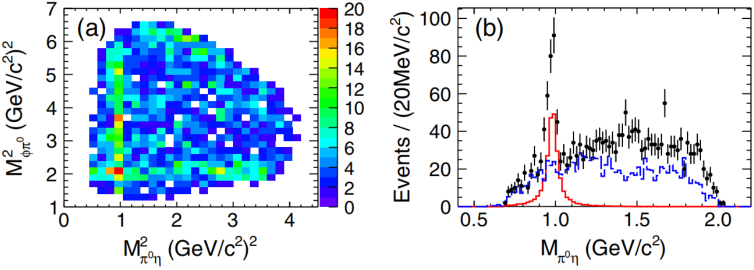


Figure: Dalitz plot for $J/\psi \to \eta\pi^0\phi$ (a) and mass projections on $m_{\pi^0\eta}$ (b) with the η reconstructed from two photons in Ref. [Ablikim et al., 2018].

- one band around 1 GeV on $\pi^0\eta$ distribution:

  $a_0(980)^0 - f_0(980)$ mixing

- another band around 1.4 GeV on $\pi^0\phi$ distribution:

  a resonance or a kinematic effect

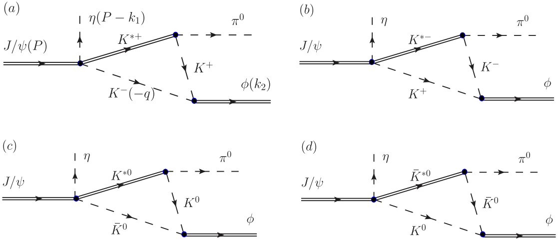
BESIII observation of $a_0(980)^0 - f_0(980)$ mixing

Resonance

maybe related to isovector state C(1480) Ref. [Bityukov et al., 1987] or isoscalar $h_1(1380)$.

Kinematic effect

triangle diagrams in the J/ψ decay into $\eta\pi^0\phi$ involving $K^*\bar{K}K$ intermediate states that can lead to a peak around 1.4 GeV on $\pi^0\phi$ distribution without a resonance.



Effective Lagrangian

To obtain the decay amplitude, we use the following effective Lagrangian:

$$\mathcal{L}_{JVPP} = \quad g_1 J_\mu \langle V^\mu PP \rangle \qquad \mathcal{L}_{VPP} = \quad -ig_2 \langle V^\mu [P, \partial_\mu P] \rangle$$

where g_1, g_2 are coupling constants, J_μ is the field operator of J/ψ.

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} V_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*+} \\ \sqrt{2}\rho^- & -\rho^0 + \omega & \sqrt{2}K^{*0} \\ \sqrt{2}K^{*-} & \sqrt{2}\bar{K}^{*0} & \sqrt{2}\phi \end{pmatrix}_\mu$$

Numerical calculation setting

- we set coupling constants $g_1 = 1$ and $g_2 = 4.5$.
- we use dimensional regularization with the $\overline{\text{MS}}$ subtraction scheme to regularize the UV divergence and set the scale $\mu = 1.2$ GeV.

General discussion on triangle singularity(TS)

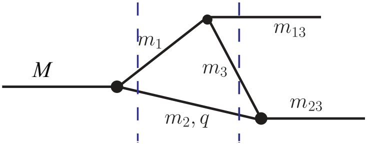


Figure: Triangle diagram with the intermediate particles with masses $m_{1,2,3}$. M, m_{13} and m_{23} are the invariant masses of external particles.

TS at the physical boundary when(Ref. [Bayar et al., 2016]):

$$q_{\text{on}+} = q_{a-} \quad \text{with} \quad q_{\text{on}+} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}, \quad q_{a-} = \gamma\left(\beta\, E_2^* - p_2^*\right)$$

The region of m_{23} for having a triangle singularity in the invariant mass distribution of M at the physical boundary is

$$m_{23} \in \left[m_2 + m_3, \sqrt{\left[(m_1 + m_2)\left(m_3^2 + m_1 m_2\right) - m_2 m_{13}^2\right]/m_1}\right]$$

Final state distribution for $J/\psi \to \eta\pi^0\phi$

	Pole position on $\pi^0\phi$ distribution(MeV)	
Type	Charged	Neutral
Threshold	1385.3	1393.6
TS	1385.7	1395.6

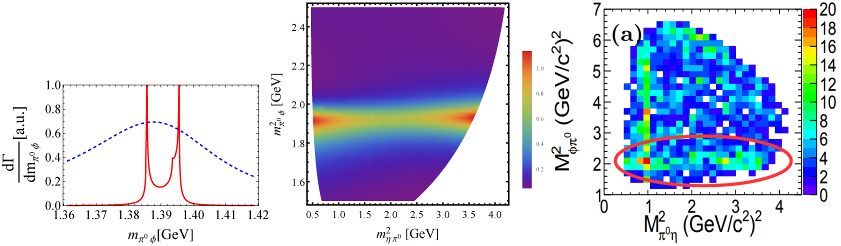
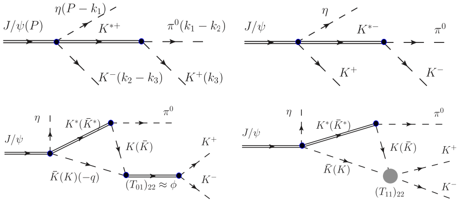


Figure: The $\pi^0\phi$ distribution(left) and Dalitz plot(middle)from our calculation in arbitrary units (a.u.). Dalitz plot(right) from BESIII.

Final state distribution for $J/\psi \to \eta\pi^0 K\bar{K}$

Motivation

- the ϕ resonance is reconstructed from $K\bar{K}$ with $m_{K\bar{K}} \in [m_\phi - 10\text{MeV}, m_\phi + 10\text{MeV}]$.
- the effect of TS is very sensitive to kinematics, here only happens when the $m_{K\bar{K}} \in [987.4, 1025.9]$ MeV and $[995.2, 1033.7]$ MeV for the charged and neutral intermediate states, respectively.



Final state distribution for $J/\psi \to \eta\pi^0 K\bar{K}$

$\Gamma_{K^*} = 0$ MeV

$\Gamma_{K^*} = 50$ MeV

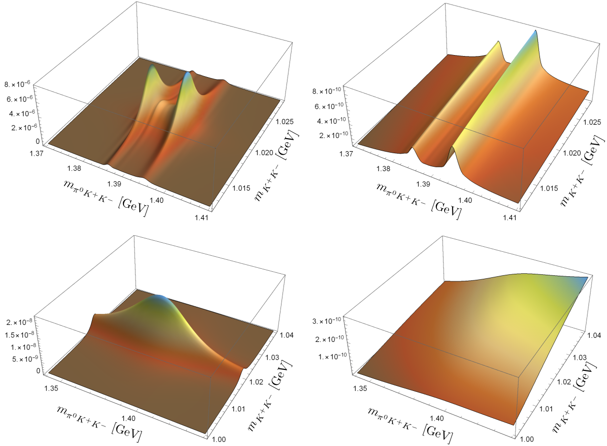


Figure: Differential width for the $J/\psi \to \eta\pi^0 K\bar{K}$ in arbitrary units. The left (right) two plots show results considering triangle diagrams with $I = 0 (I = 1)$ $K\bar{K}$ FSI. The K^* width is considered in the lower-line plots.

Final state distribution for $J/\psi \to \eta\pi^0 K\bar{K}$

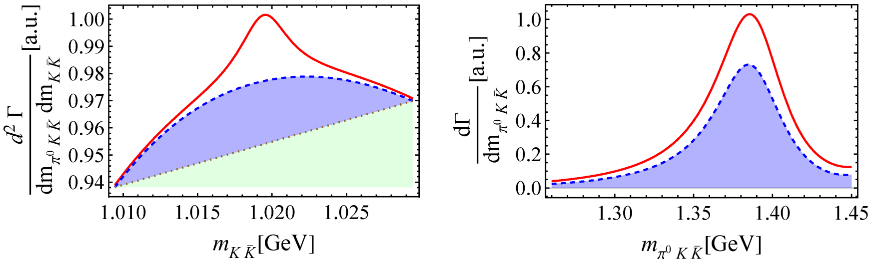


Figure: Left panel: the final state K^+K^- distribution in a narrow ϕ window $m_{K\bar{K}} \in [1010, 1030]$ MeV with $m_{\pi^0 K\bar{K}}$ fixed at 1.39 GeV. Right panel: the $\pi^0 K^+ K^-$ invariant mass distribution with $m_{K\bar{K}}$ integrated in the ϕ window, where we have subtracted the sideband contribution. The K^* width has been taken into account in these plots.

Summary

- triangle diagrams with $K^* K \bar{K}$ intermediate particles for $J/\psi \to \eta\pi^0\phi$ can give a band in the Dalitz plot which is located at the $m_{\pi^0\phi}$ around 1.4 GeV by virtue of TS.
- TS effect is most evident with the $m_{K^+K^-}$ is in the ϕ resonance mass region, but its effect would become weaker and eventually invisible if $m_{K^+K^-}$ is sufficiently away from the ϕ mass region.

Proposal

we suggest experimentalists to take more data for the $J/\psi \to \eta\pi^0\phi$ to make the Dalitz plot more clear, and to check whether the structure persists in other K^+K^- invariant mass regions.

THANK YOU FOR YOUR ATTENTION !

Back up

Amplitude for $J/\psi \to \eta\pi^0\phi$

Amplitudes for each vertex:

$$-it_{J/\psi,K^{*+}K^-\eta} = i\frac{1}{\sqrt{6}}g_1\epsilon^\mu_{J/\psi}\epsilon^{*\nu}_{K^{*+}}g_{\mu\nu} \qquad -it_{J/\psi,K^{*0}\bar{K}^0\eta} = i\frac{1}{\sqrt{6}}g_1\epsilon^\mu_{J/\psi}\epsilon^{*\nu}_{K^{*0}}g_{\mu\nu}$$

$$-it_{J/\psi,K^{*-}K^+\eta} = i\frac{1}{\sqrt{6}}g_1\epsilon^\mu_{J/\psi}\epsilon^{*\nu}_{K^{*-}}g_{\mu\nu} \qquad -it_{J/\psi,\bar{K}^{*0}K^0\eta} = i\frac{1}{\sqrt{6}}g_1\epsilon^\mu_{J/\psi}\epsilon^{*\nu}_{\bar{K}^{*0}}g_{\mu\nu},$$

$$-it_{K^{*+},K^+\pi^0} = -i\frac{\sqrt{2}}{2}g_2\epsilon^\mu_{K^{*+}}(p_{K^+} - p_{\pi^0})_\mu \quad -it_{K^{*-},K^-\pi^0} = -i\frac{\sqrt{2}}{2}g_2\epsilon^\mu_{K^{*-}}(p_{\pi^0} - p_{K^-})_\mu$$

$$-it_{K^{*0},K^0\pi^0} = -i\frac{\sqrt{2}}{2}g_2\epsilon^\mu_{K^{*0}}(p_{\pi^0} - p_{K^0})_\mu \qquad -it_{\bar{K}^{*0},\bar{K}^0\pi^0} = -i\frac{\sqrt{2}}{2}g_2\epsilon^\mu_{\bar{K}^{*0}}(p_{\bar{K}^0} - p_{\pi^0})_\mu$$

$$-it_{\phi,K^+K^-} = -ig_2\epsilon^\mu_\phi(p_{K^+} - p_{K^-})_\mu \qquad -it_{\phi,K^0\bar{K}^0} = -ig_2\epsilon^\mu_\phi(p_{K^0} - p_{\bar{K}^0})_\mu$$

Amplitude for triangle diagrams:

$$\mathcal{M}_{J/\psi\to\eta\pi^0\phi} = 2(\mathcal{M}_C - \mathcal{M}_N) \qquad \mathcal{M}_{id} = -i\frac{g_1 g_2^2}{2\sqrt{3}}\epsilon^\mu_{J/\psi}\epsilon^\nu_\phi\mathcal{M}^{id}_{\mu\nu}$$

$$\mathcal{M}^{id}_{\mu\nu} = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{\left[-g_{\mu\lambda} + (q+k_1)_\mu(q+k_1)_\lambda/m^2_{K^*_{id}}\right](q+2k_2-k_1)^\lambda(2q+k_2)_\nu}{(q^2 - m^2_{K_{id}} + i\epsilon)[(q+k_1)^2 - m^2_{K^*_{id}} + i\epsilon][(q+k_2)^2 - m^2_{K_{id}} + i\epsilon]}$$

Amplitude for $J/\psi \to \eta\pi^0 K\bar{K}$

Amplitude for P-wave $K\bar{K}$ FSI vertex:

$$-it_{(I,1)} = -i\tilde{t}_{(I,1)}(p_K^{\text{in}} - p_{\bar{K}}^{\text{in}})^\mu (p_K^{\text{out}} - p_{\bar{K}}^{\text{out}})_\mu$$

$$-i\tilde{t}_{(0,1)} = i\frac{g_2^2}{m_{K\bar{K}}^2 - m_\phi^2 + im_\phi\Gamma_\phi} \qquad -i\tilde{t}_{(1,1)} = -i\frac{3(T_{11})_{22}}{4q_{cm}^{\text{in}} q_{cm}^{\text{out}}}$$

One can find the specific form of the T-matrix in Ref. [Oller et al., 1999]. Transform the particle basis from the isospin space to the charged space:

$$|K\bar{K}\rangle_{I=0} = -\frac{1}{\sqrt{2}}|K^+K^-\rangle - \frac{1}{\sqrt{2}}|K^0\bar{K}^0\rangle$$

$$|K\bar{K}\rangle_{I=1,I_3=0} = -\frac{1}{\sqrt{2}}|K^+K^-\rangle + \frac{1}{\sqrt{2}}|K^0\bar{K}^0\rangle$$

Amplitude for $J/\psi \to \eta\pi^0 K\bar{K}$

Amplitudes for this process:

$$-i\mathcal{M}_{\text{tree}} = -i\frac{1}{2\sqrt{3}}g_1 g_2 \epsilon^{\mu}_{J/\psi}\left[\frac{-(p_{\pi^0}-p_K)_\mu + (p_{\pi^0}+p_K)_\mu (m^2_{\pi^0}-m^2_K)/m^2_{K^*}}{m^2_{\pi^0 K\bar{K}} - 2m_{\pi^0 K\bar{K}}E_{\bar{K}} + m^2_K - m^2_{K^*} + i\epsilon}\right.$$
$$\left. + \frac{-(p_{\pi^0}-p_{\bar{K}})_\mu + (p_{\pi^0}+p_{\bar{K}})_\mu (m^2_{\pi^0}-m^2_K)/m^2_{K^*}}{m^2_{\pi^0 K\bar{K}} - 2m_{\pi^0 K\bar{K}}E_{K} + m^2_K - m^2_{K^*} + i\epsilon}\right]$$

$$-i\mathcal{M}^{I=0}_{\text{loop}} = -\frac{1}{2\sqrt{3}}g_1 g_2 \epsilon^{\mu}_{J/\psi}(\mathcal{M}^C_{\mu\nu} - \mathcal{M}^N_{\mu\nu})\tilde{t}_{(0,1)}(p_{\bar{K}} - p_K)^\nu$$

$$-i\mathcal{M}^{I=1}_{\text{loop}} = -\frac{1}{2\sqrt{3}}g_1 g_2 \epsilon^{\mu}_{J/\psi}(\mathcal{M}^C_{\mu\nu} + \mathcal{M}^N_{\mu\nu})\tilde{t}_{(1,1)}(p_{\bar{K}} - p_K)^\nu$$

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_{\text{tree}} + \mathcal{M}^{I=0}_{\text{loop}} + \mathcal{M}^{I=1}_{\text{loop}}$$

1 to 4-body differential decay width:

$$\frac{d^2\Gamma_{J/\psi\to\eta\pi^0 K\bar{K}}}{dm_{\pi^0 K\bar{K}}\,dm_{K\bar{K}}} = \frac{|\mathbf{p}_\eta||\mathbf{p}'_{\pi^0}||\mathbf{p}''_K|}{(2\pi)^8 2^5 m^2_{J/\psi}}\frac{1}{3}\int d\Omega_\eta d\Omega'_{\pi^0} d\Omega''_K \sum_{\text{spin}} |\mathcal{M}_{\text{tot}}|^2$$

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Observation of $a_0^0(980)$-$f_0(980)$ Mixing.
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