



Identification of a visible cusp structure in

$$\Lambda_c \rightarrow p K \pi^+$$

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Mainly based on [arXiv:1906.07942](https://arxiv.org/abs/1906.07942)

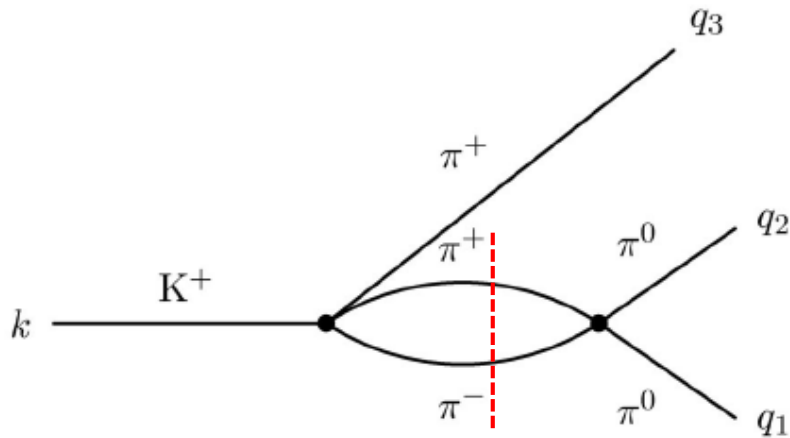
Hadron 2019, Guilin, August 20, 2019

Outline

- **Brief review of Cusp phenomena**
- **Narrow resonance-like structure “ $X(1663)$ ” in $\Lambda_c \rightarrow p K^- \pi^+$**
- **Threshold cusp enhanced by the nearby triangle singularity**
- **Summary**

Cusp effect

Induced by the charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$



Two-body cut

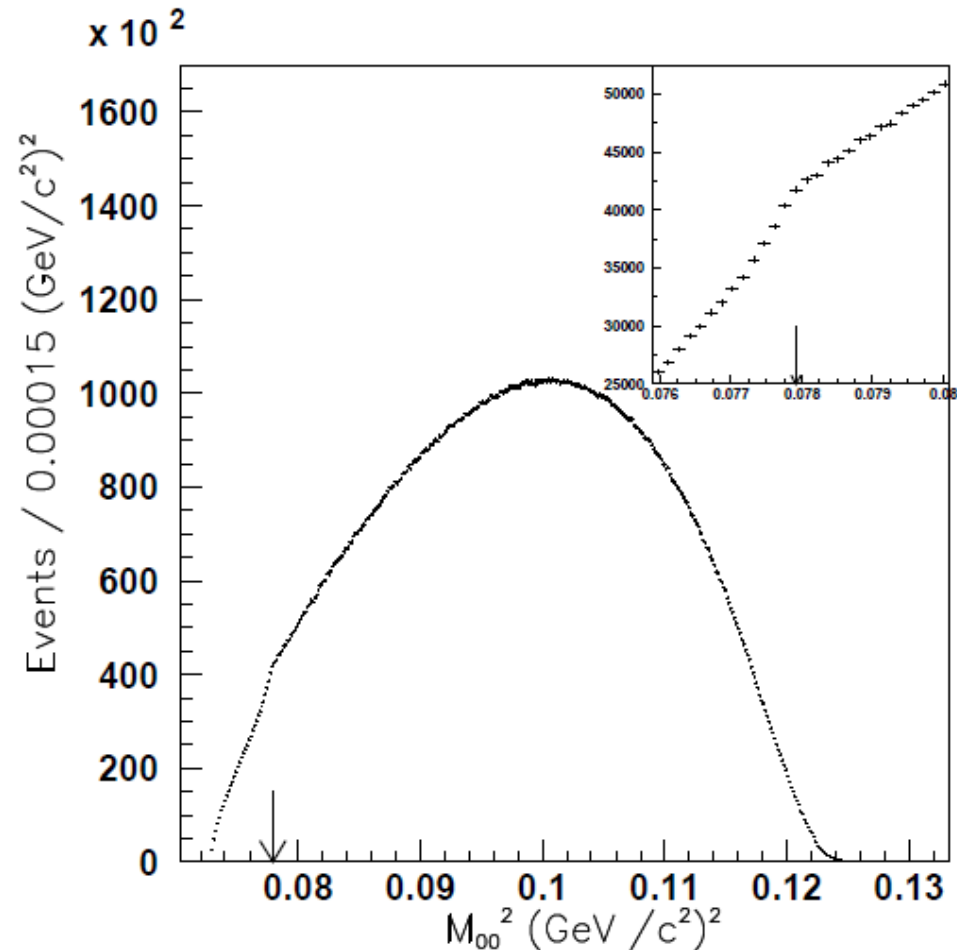
Budini & Fonda, PRL6,419(1961);
Cabibbo, PRL93,121801(2004);

Branching ratio

$$K^+ \rightarrow \pi^+\pi^-\pi^+ \quad ((5.59 \pm 0.04)\%)$$

much larger than

$$K^+ \rightarrow \pi^0\pi^0\pi^+ \quad ((1.761 \pm 0.022)\%)$$



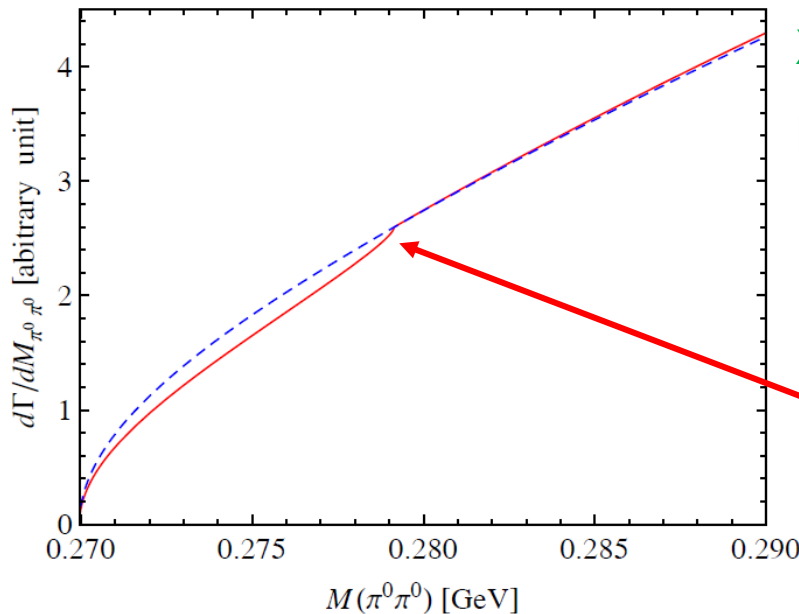
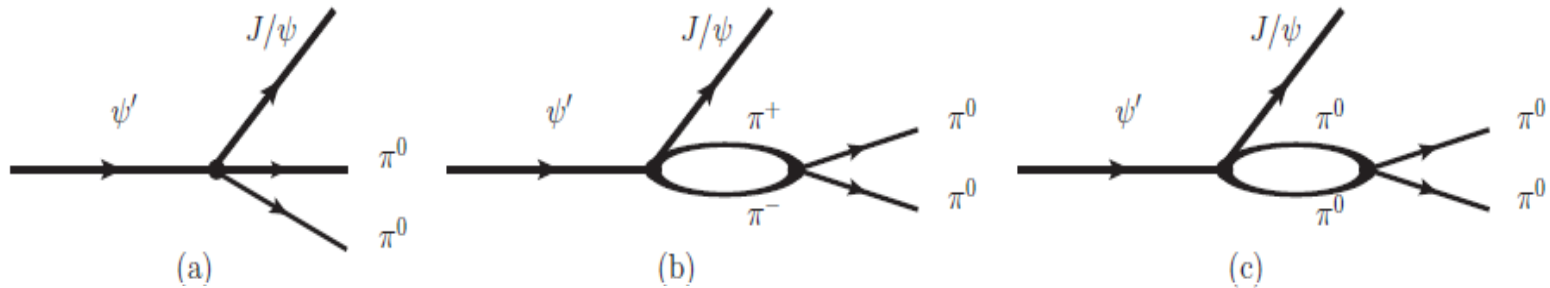
NA48/2, PLB633,173 (2006)
 6×10^7 events

Cusp effect

Could be used to extract the $\pi\pi$ scattering length

$K \rightarrow 3\pi$, one of the most accurate experiments

Heavy quarkonium dipion transitions: $\psi' \rightarrow J/\psi\pi^0\pi^0$,
 $Y(3S) \rightarrow Y(2S)\pi^0\pi^0$ (better)



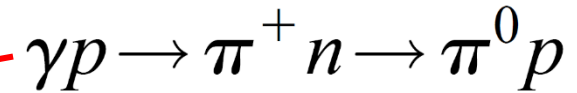
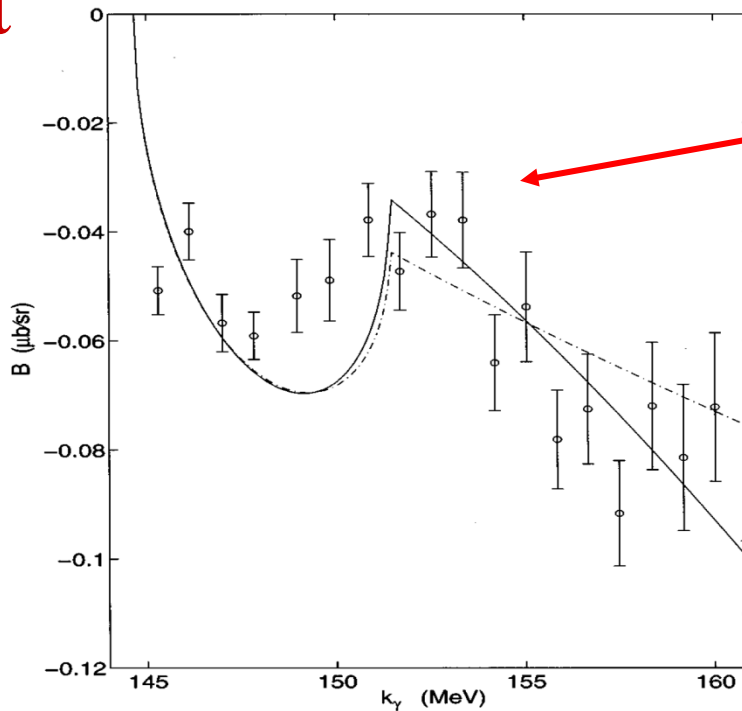
X.H Liu, F.K. Guo, E. Epelbaum,
EPJC73,2284(2013)

cusp

Cusp effect

Early study:

E.P. Wigner, “*On the Behavior of Cross Sections Near Thresholds*”, PR73, 1002 (1948)



A.M. Bernstein et al, “*Observation of a unitary cusp in the threshold $\gamma p \rightarrow \pi^0 p$ reaction*”, PRC55, 1509(1997)

Threshold of final states <
Threshold of intermediate
states

D. V. Bugg, “*Reinterpreting several narrow ‘resonances’ as threshold cusps*”, PLB598, 8 (2004):

The threshold $ppbar$ peak in BES data for $J/\psi \rightarrow \gamma ppbar$ may be fitted as a cusp;

a cusp at the ΣN threshold $K^- d \rightarrow \pi^- (\Lambda p)$

...

Difficult to observe: high statistics; perfect energy resolution; clear background

Cusp effect

- Possible correlation with some XYZ states: $Z_b(10610/10650)$, $Z_c(3900)$, $Z_c(4020)$

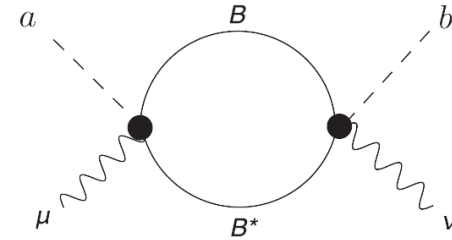
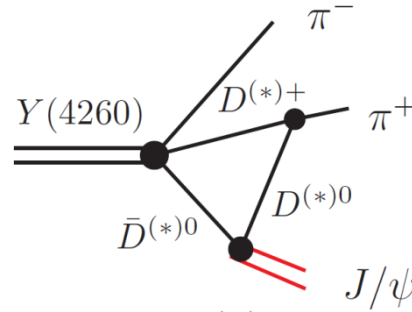
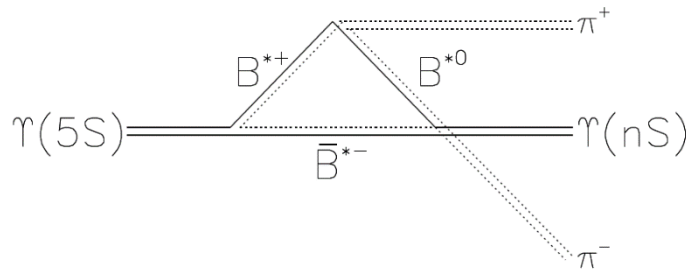
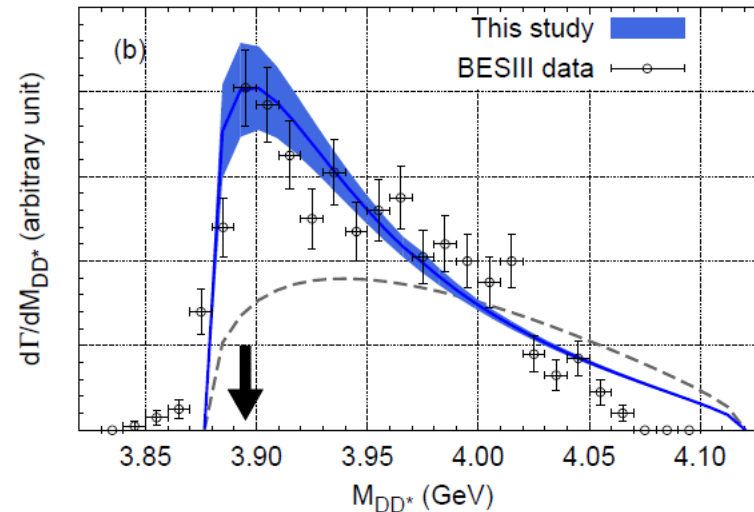
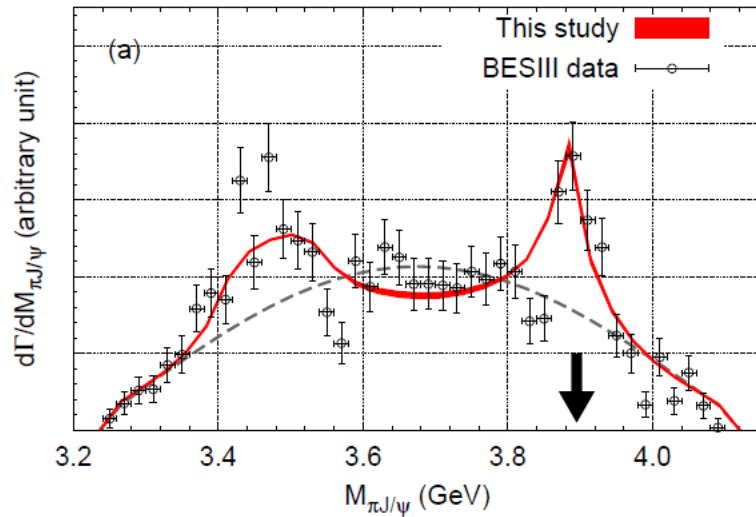


FIG. 1. Coupled channels in $Y\pi$ scattering.

D.V. Bugg,
EPL96, 11002(2011)

D.Y. Chen, X. Liu,
PRD88, 11002(2013)

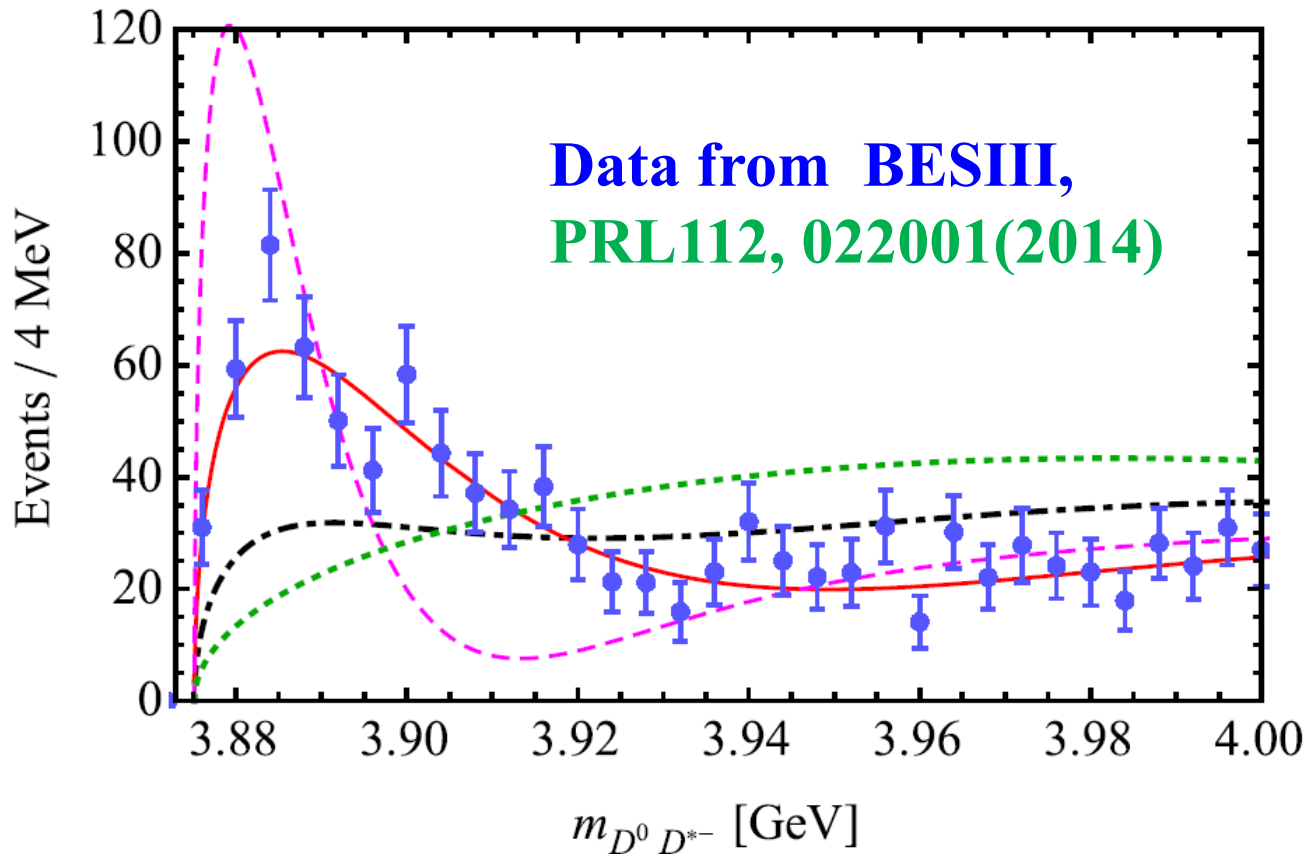
E. Swanson,
PRD91, 034009(2015)



Y. Ikeda [HAL QCD Collaboration], arXiv:1706.07300 **Strong $J/\psi\pi$ - DD^* coupling**

Cusp effect

- A sharp peak **cannot** be resulted by a pure threshold cusp in the elastic channel [Guo, Hanhart, Wang, Zhao, PRD91, 051504(2015)]: $Z_c(3900)$ was also observed in the DD^* invariant mass distributions

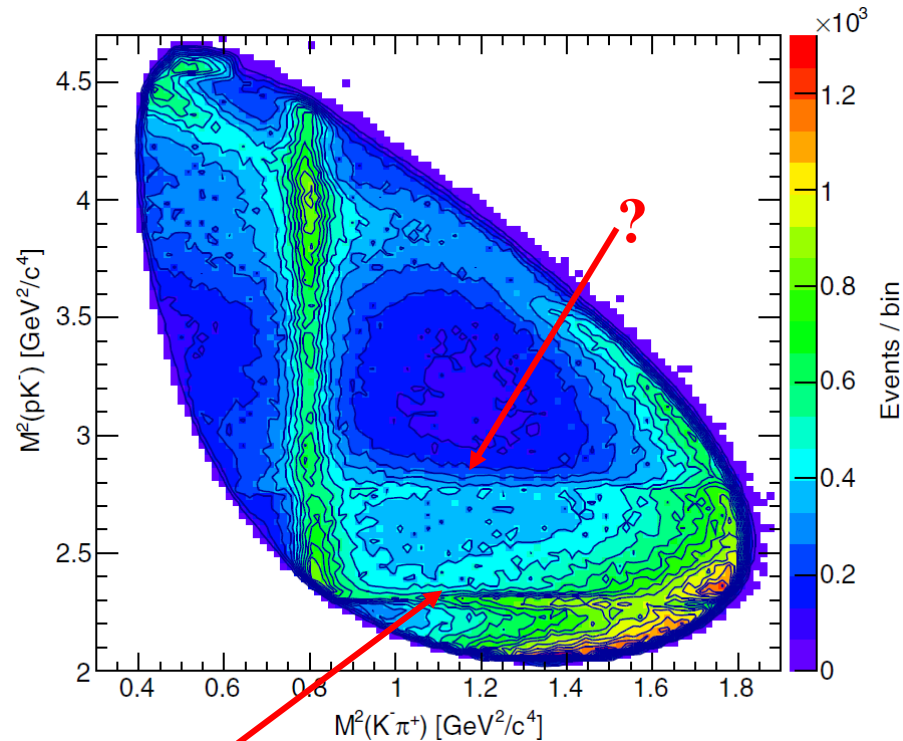


Observation of “X(1663)”

Dalitz plot for $\Lambda_c \rightarrow p K^- \pi^+$

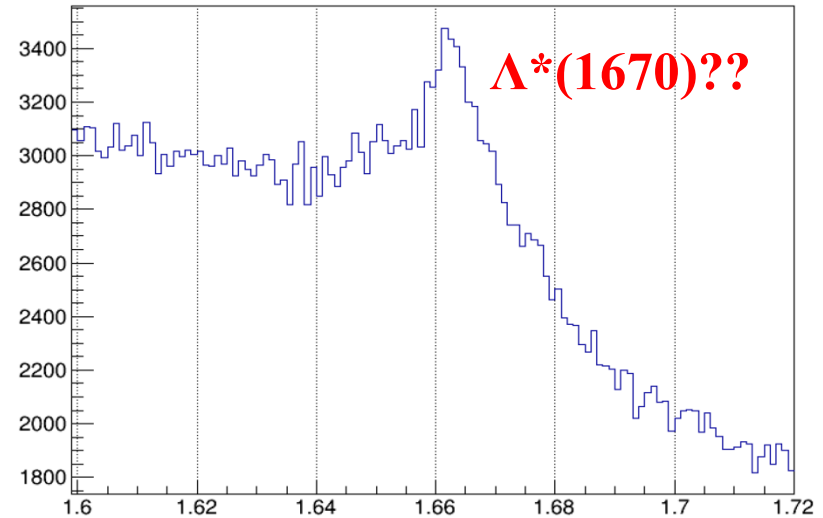
Belle, PRL117,011801(2016)

1.452×10^6 events



$\Lambda(1520)$

■ 1D projection -- $M(pK^-)$



✓ Bin width: 1 MeV

✓ $M \approx 1663$ MeV

✓ $\Gamma \approx 10$ MeV

✓ $\Lambda\eta$ threshold: 1663.545 MeV

From C.P. Shen's talk, no published result concerning “X(1663)”

Observation of “X(1663)”

Hyperons around 1663 MeV [PDG]

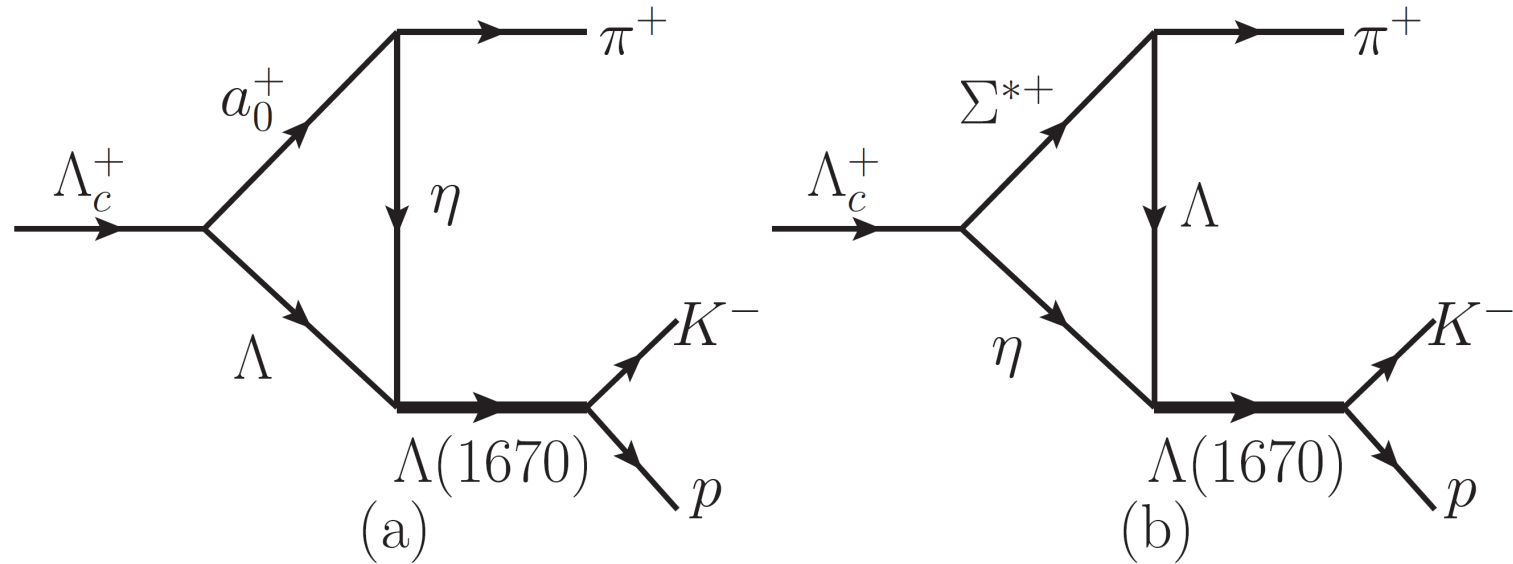
[MeV]	Mass	Width	J ^P
X(1663)	1663	~10	?
$\Lambda^*(1670)$	1660 to 1680 ≈ 1670	25 to 50 ≈ 35	1/2-
$\Lambda^*(1690)$	1685 to 1695 ≈ 1690	50 to 70 ≈ 60	3/2-
$\Sigma^*(1660)$	1630 to 1690 ≈ 1660	40 to 200 ≈ 100	1/2+
$\Sigma^*(1670)$	1665 to 1685 ≈ 1670	40 to 80 ≈ 60	3/2-

No established hyperons correspond to this “X(1663)”

Two groups claim there is a narrow Λ^* with J=3/2:

- **Liu & Xie [PRC85, 038201; PRC86,055202]**
J^P=3/2-(D03), M=1668.5 ± 0.5 MeV, $\Gamma=1.5 \pm 0.5$ MeV
- **Kamano *et al.* [PRC90, 065204; PRC92, 025205]**
J^P=3/2+(P03), M=1671+2-8 MeV, $\Gamma=10+22-4$ MeV

Contributions from rescattering processes



✓ Cabibbo-favored process

✓ Strong couplings

✓ Exp. value: $Br(\Lambda_c \rightarrow \Lambda \eta \pi^+) \sim (2.2 \pm 0.5)\%$

$Br(\Lambda_c \rightarrow \Sigma(1385) \eta \rightarrow \Lambda \eta \pi^+) \sim (1.06 \pm 0.32)\%$

Experimental data concerning $\Lambda_c \rightarrow \Lambda \eta \pi^+$

CLEO, PRL74,3534 (1995)

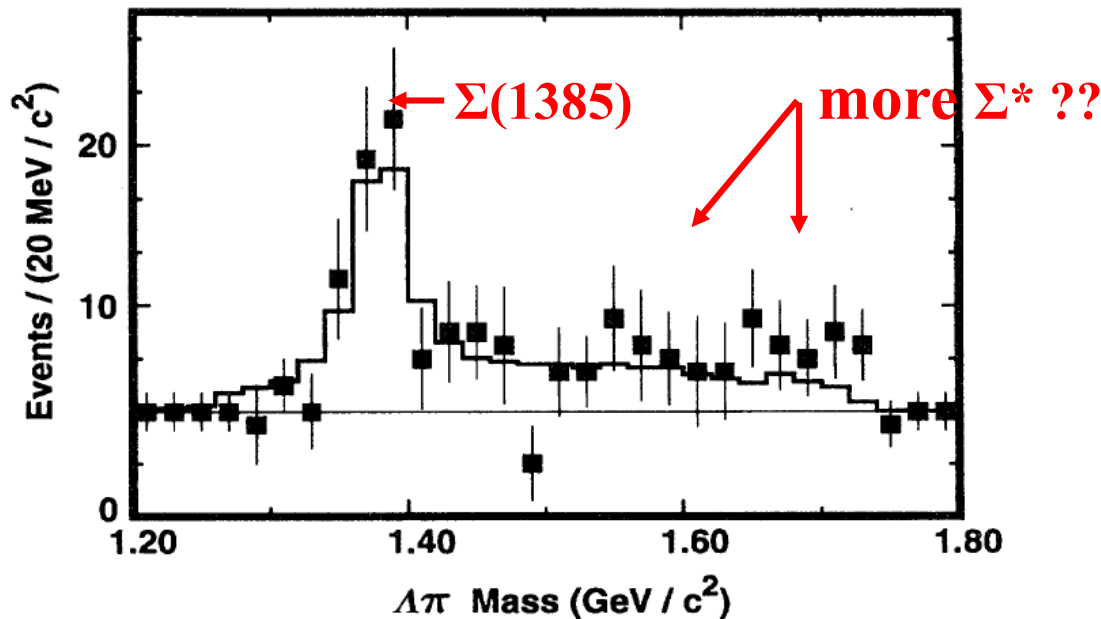


FIG. 3. The $\Lambda \pi^+$ invariant mass distribution for $\Lambda_c^+ \rightarrow \Lambda \eta \pi^+$ candidates showing the $\Sigma^{*+}(1385)$ resonance. The points are data and the histogram is a fit to a Breit-Wigner signal and a phase space background, both of which include the effects of detector acceptance and resolution.

$a_0 \rightarrow \eta \pi^+$. We searched for the Λa_0 decay. However, because the a_0 width is quite large and not well measured (50–300 MeV/ c^2) [5], we could not constrain the Λa_0 decay component.

Kinematic region of triangle singularity (TS)

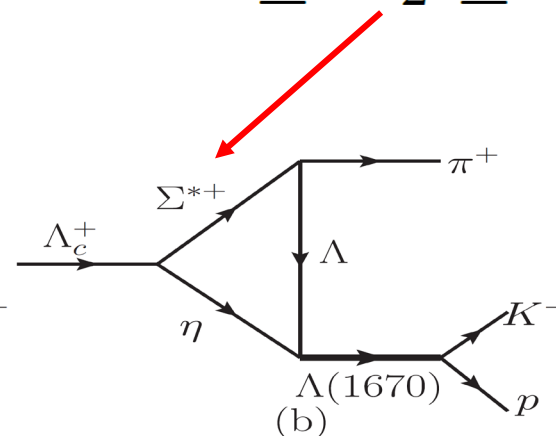
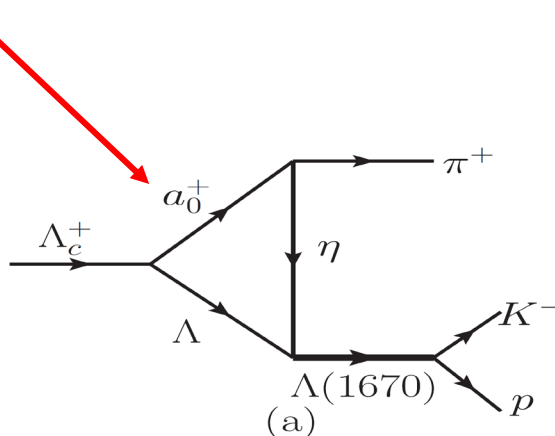
$$\frac{m_1 m_{\pi^+}^2 + m_3 M_{\Lambda_c^+}^2}{m_1 + m_3} - m_1 m_3 \leq m_2^2 \leq (M_{\Lambda_c^+} - m_1)^2$$

$$(m_1 + m_3)^2 \leq s^- \leq (m_1 + m_3)^2 + \frac{m_1 [(m_2 - m_3)^2 - m_{\pi^+}^2]}{m_2}$$

Liu, Oka, Zhao, PLB753,297 (2016)

$$1.06 \leq m_2 \leq 1.17 \text{ GeV}$$

$$1.70 \leq m_2 \leq 1.74 \text{ GeV}$$



TS location

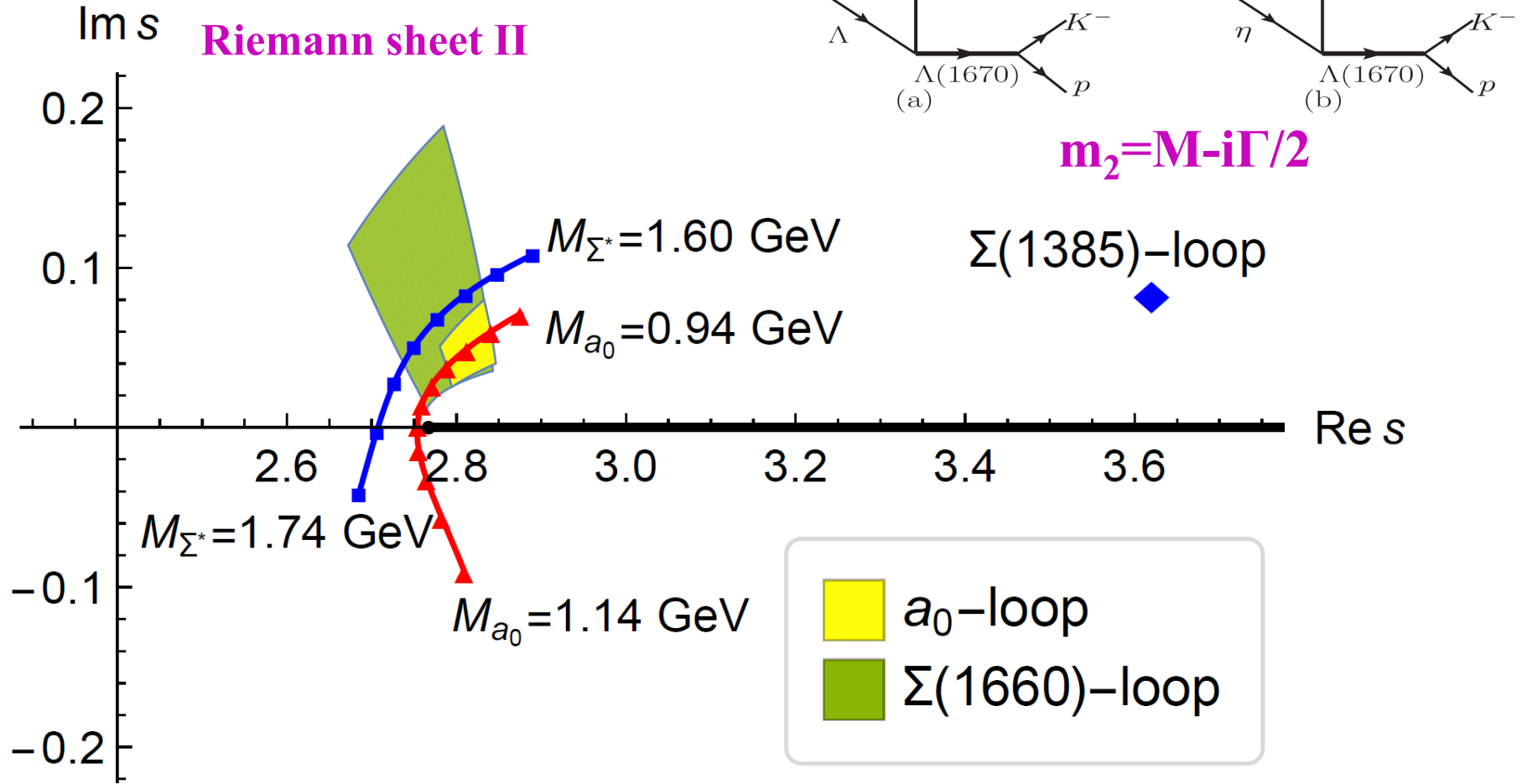


FIG. 2: The TS location of $\mathcal{T}(s, m_2^2)$ in the complex s -plane. The thick line on the real axis represents the unitary cut starting from s_{th} . The trajectory marked with triangle (box) is obtained by varying M_{a_0} (M_{Σ^*}) and fixing $\Gamma_{a_0} = 75 \text{ MeV}$ ($\Gamma_{\Sigma^*} = 100 \text{ MeV}$).

Rescattering Amplitude

$$\mathcal{T} = \frac{1}{s - M_{\Lambda(1670)}^2 + iM_{\Lambda(1670)}\Gamma_{\Lambda(1670)}} \times \int \frac{d^4q_1}{(2\pi)^4} \frac{\mathcal{A}}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)},$$

with $\mathcal{A} = \mathcal{M}(\Lambda_c^+ \rightarrow \Lambda a_0^+) \mathcal{M}(a_0^+ \rightarrow \eta \pi^+) \mathcal{M}(\eta \Lambda \rightarrow \Lambda(1670)) \mathcal{M}(\Lambda(1670) \rightarrow K^- p)$ and $\mathcal{A} = \mathcal{M}(\Lambda_c^+ \rightarrow \eta \Sigma^{*+}) \mathcal{M}(\Sigma^{*+} \rightarrow \Lambda \pi^+) \mathcal{M}(\eta \Lambda \rightarrow \Lambda(1670)) \mathcal{M}(\Lambda(1670) \rightarrow K^- p)$ for a_0 -loop and Σ^* -loop

$$\mathcal{M}(\Lambda_c^+ \rightarrow \Lambda a_0^+ / \eta \Sigma^{*+}) = g_A \bar{u}_f u_i + i g_B \bar{u}_f \gamma_5 u_i$$

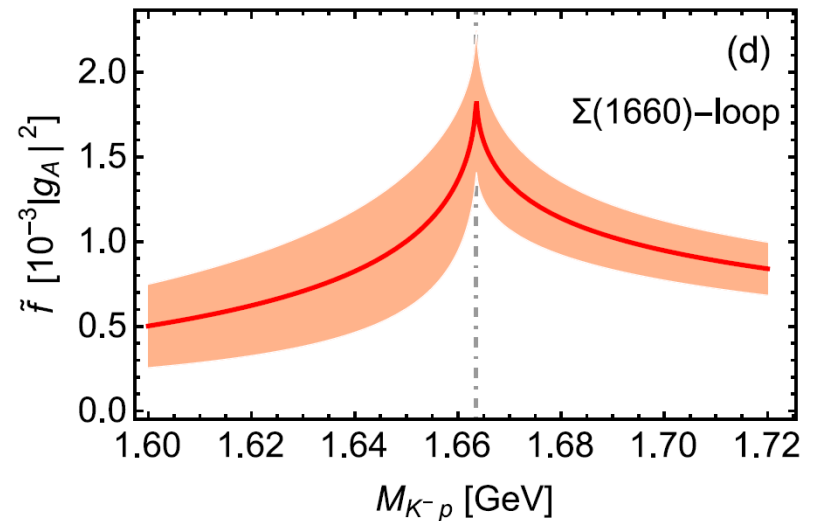
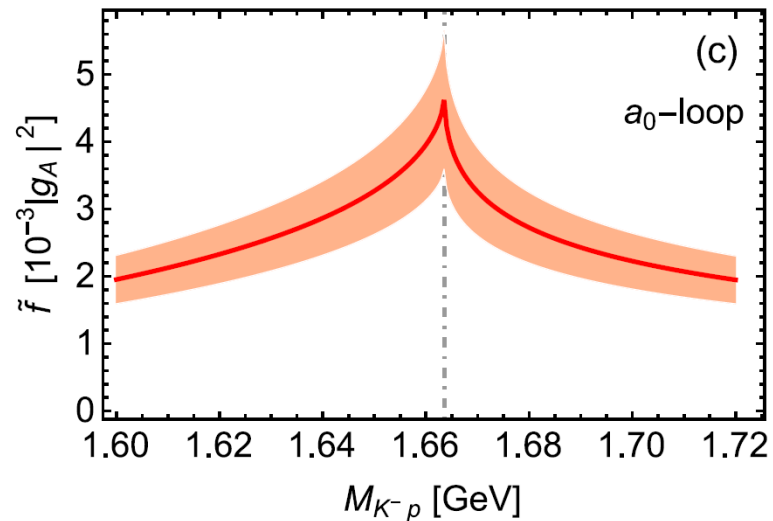
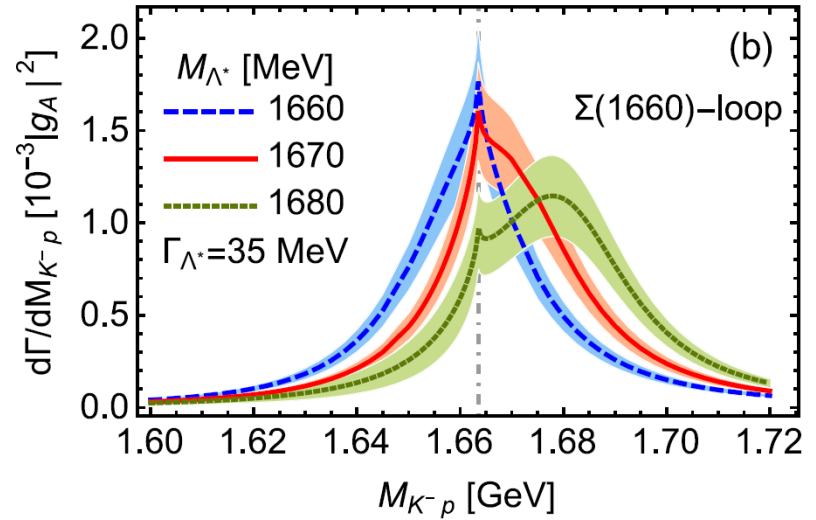
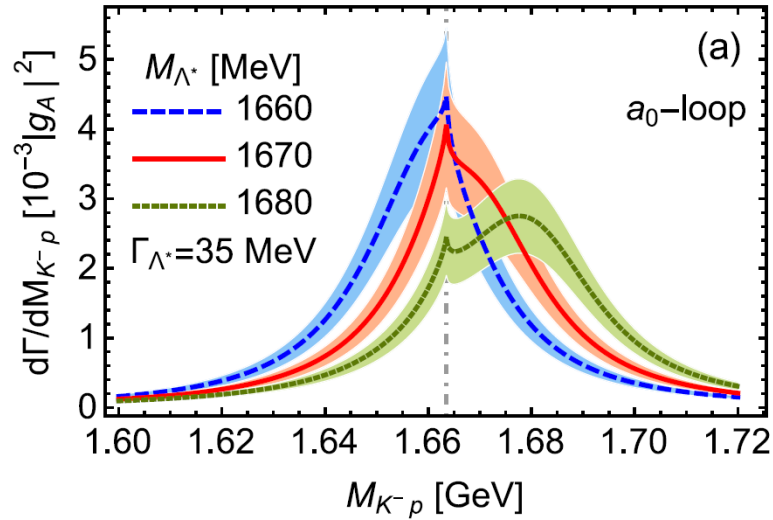
Estimation:

$$R \equiv |g_B|/|g_A| \sim \mathbf{1}$$

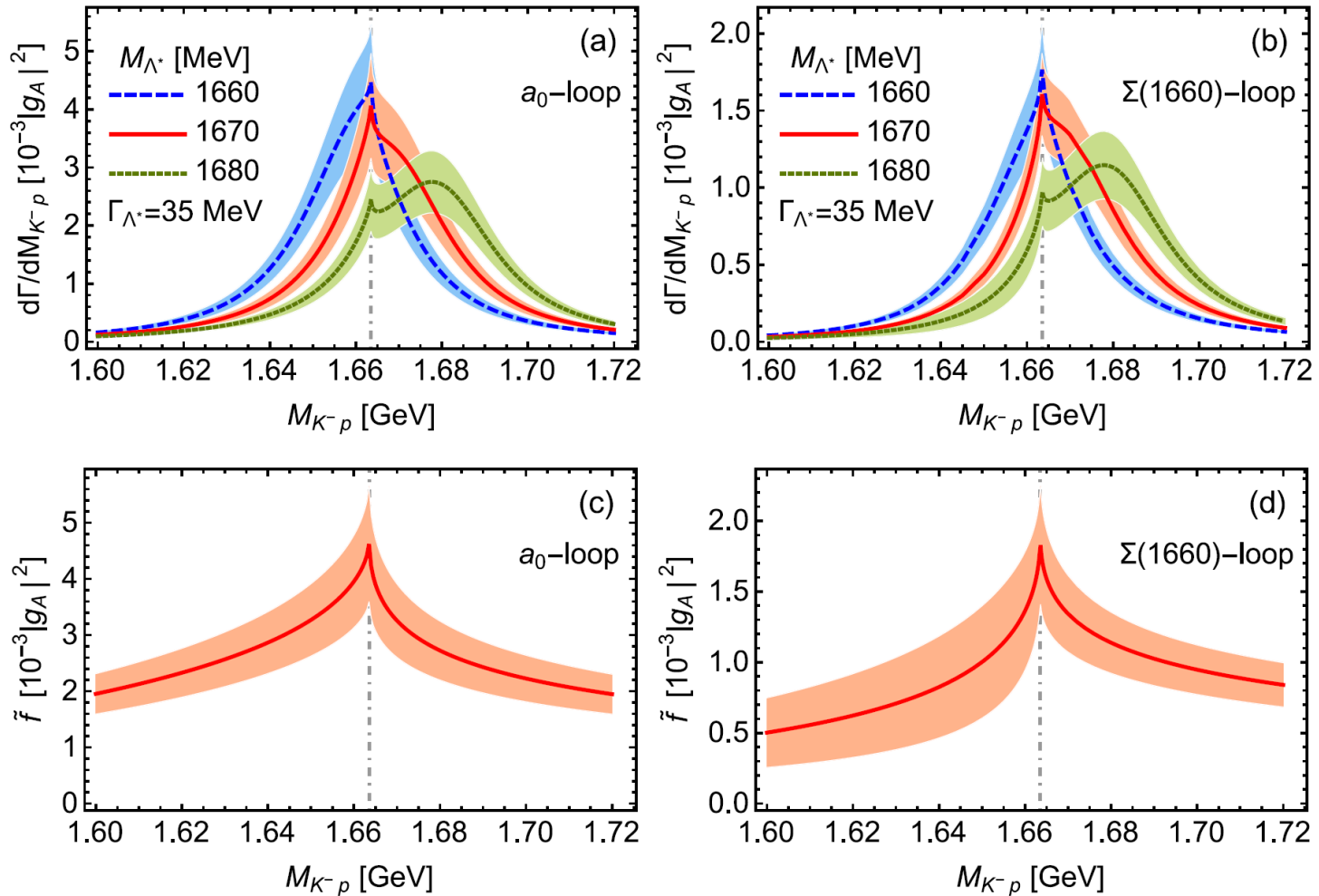
$$|g_A|_{\max}^2 \approx 0.32 \text{ GeV}^{-1} / \tau_{\Lambda_c} \text{ for } \Lambda a_0^+ \text{ channel}$$

$$|g_A|_{\max}^2 \approx 3.14 \text{ GeV}^{-1} / \tau_{\Lambda_c} \text{ for } \Sigma(1660)\eta \text{ channel}$$

Invariant Mass Distributions



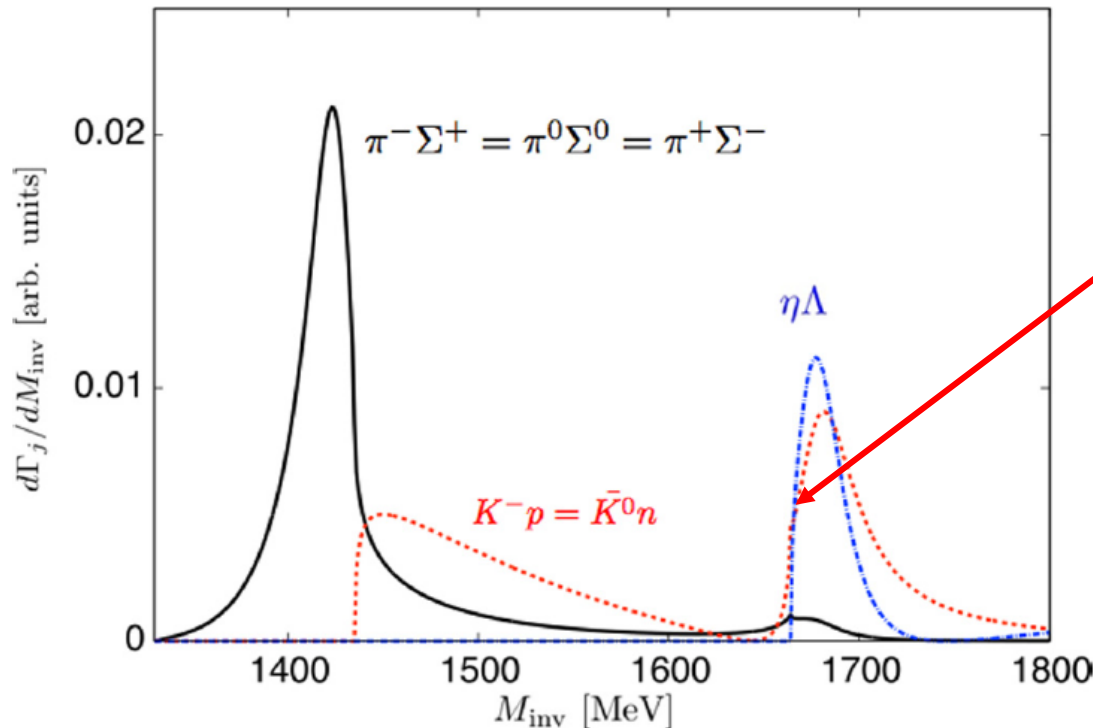
Invariant Mass Distributions



$$\tilde{f}(M_{K-p}) = \left| \frac{s - M_{\Lambda^*}^2 + iM_{\Lambda^*}\Gamma_{\Lambda^*}}{M_{\Lambda^*}\Gamma_{\Lambda^*}} \right|^2 \times \frac{d\Gamma}{dM_{K-p}}$$

Invariant Mass Distributions

The narrow resonance-like structure “X(1663)” is not simply resulted from the threshold cusp and $\Lambda(1670)$ interference



Tiny cusp at $\Lambda\eta$ threshold

FIG. 6. (Color online) Invariant mass distribution of the decay $\Lambda_c^+ \rightarrow \pi^+ MB$. The solid, dotted, and dash-dotted lines represent the $\bar{K}N = \{K^-p, \bar{K}^0n\}$, $\pi\Sigma = \{\pi^0\Sigma^0, \pi^-\Sigma^+, \pi^+\Sigma^-\}$, and $\eta\Lambda$ channels, respectively. The meson-baryon amplitude is taken from Ref. [19], where the $\Lambda(1520)$ contribution in d wave is not included.

K. Miyahara, T. Hyodo, E. Oset, PRC92,055204 (2015)

Argand plot

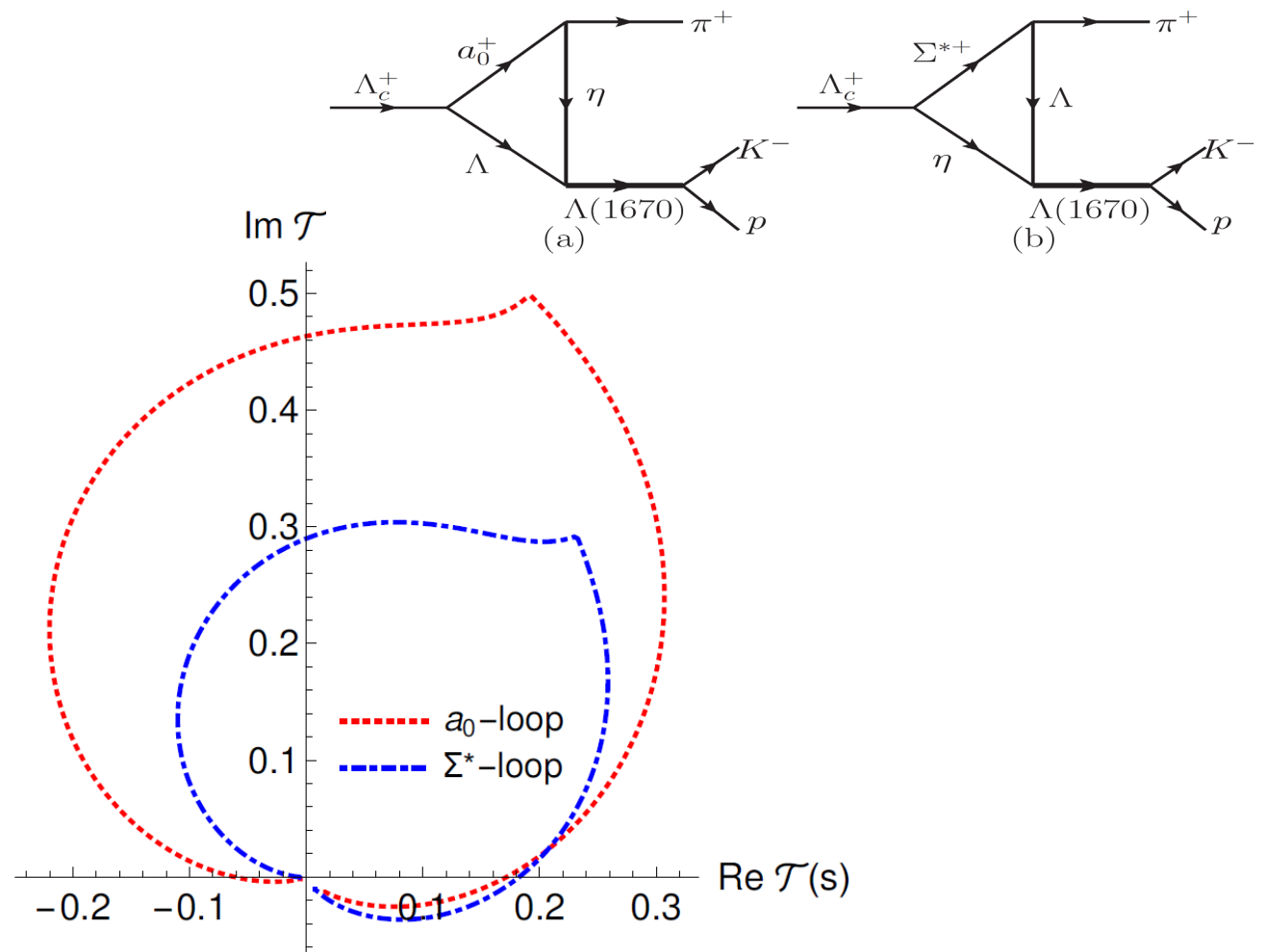


FIG. 4: Argand plot of the rescattering amplitude, with s increasing from $(M_{K^-} + m_p)^2$ to $(M_{\Lambda_c^+} - m_{\pi^+})^2$ counterclockwise. The mass and width of $\Lambda^*(1670)$ are taken to be PDG averaged values.

Summary

- The narrow resonance-like structure observed by Belle in Kp invariant mass spectrum can be identified as a cusp phenomenon resulted from the rescatterings:
 - The cusp is enhanced and narrowed by the nearby unphysical TSs.
 - No established narrow hyperons in the vicinity of $\Lambda\eta$ threshold.
 - Huge data sample, very narrow bin width ~ 1 MeV
- Direct prediction:
 - $\Lambda\eta$ also strongly couples to the $\Sigma\pi$ channel around 1670 MeV, one can expect that the similar cusp could also be observed in $\Lambda_c \rightarrow \Sigma\pi\pi$ decays

Thanks!