Quark Wigner distributions Using Light-front Wave Functions

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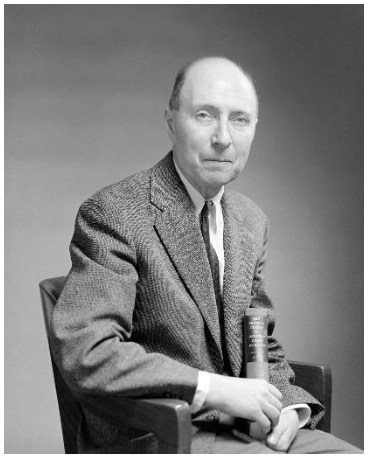
Wigner function



Eugene Wigner

JUNE 1, 1932 *PHYSICAL REVIEW* *VOLUME 40*

On the Quantum Correction For Thermodynamic Equilibrium

By E. WIGNER
Department of Physics, Princeton University
(Received March 14, 1932)

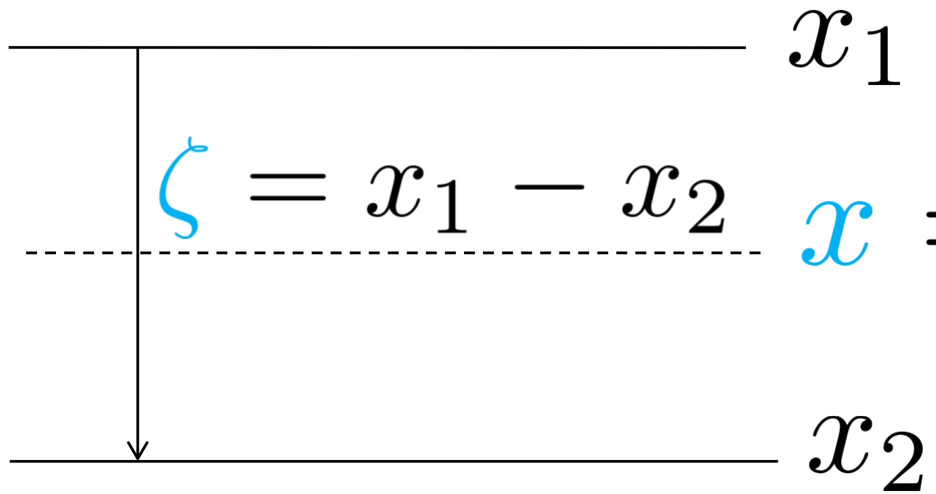
$$\psi(x_1 \ldots x_n) \xrightarrow[\text{distribution}]{\text{Probability}} P\left(x_1 \ldots x_n; p_1 \ldots p_n\right)$$

$$P = \left(\frac{1}{h\pi}\right)^n \int dy_1 \ldots dy_n \;\; \psi(x_1 + y_1 \ldots x_n + y_n)^* \psi(x_1 - y_1 \ldots x_n - y_n) e^{2i(p_1 y_1 + \ldots + p_n y_n)/h}$$

may take negative values

Intuitive picture of Wigner function

Quantum jump for a particle from x_1 to x_2



$$x = \frac{x_1 + x_2}{2}$$

$$\langle x_2 \mid \hat{\rho} \mid x_1 \rangle$$

$\hat{\rho} \rightarrow$ density operator describing the state of this particle

Intuitive picture of Wigner function

The momentum p of the particle is related to the jump from x_1 to $x_2 \Rightarrow \zeta$

$$W(x,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} d\zeta \;\; e^{-\frac{i}{\hbar}p\zeta} \left\langle x + \frac{\zeta}{2} \middle| \hat{\rho} \middle| x - \frac{\zeta}{2} \right\rangle$$

Wigner function is a FT of the density operator ρ in position space expressed in terms of the two variables x and p

Intuitive picture of Wigner function

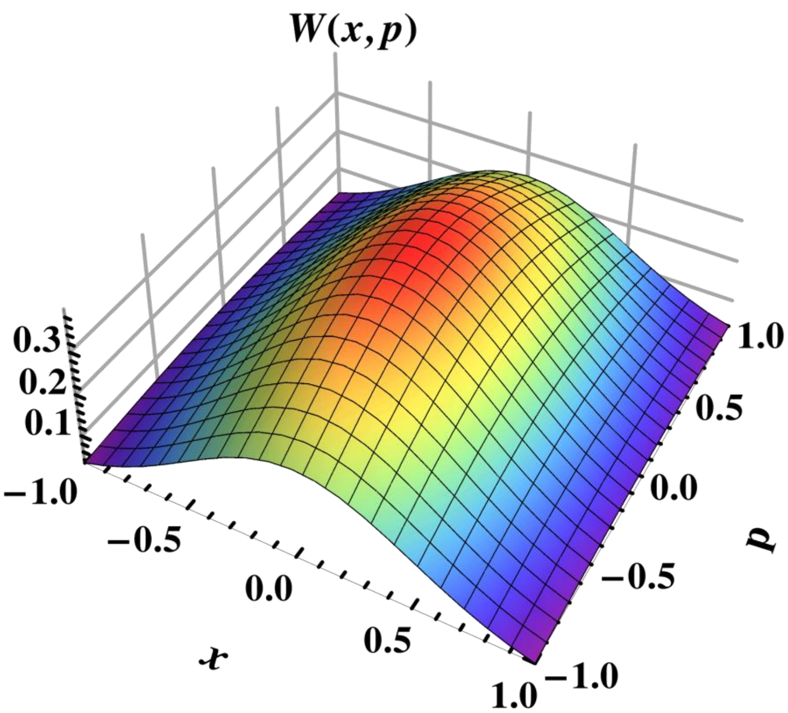
The momentum p of the particle is related to the jump from x_1 to $x_2 \Rightarrow \zeta$

$$W(x,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} d\zeta \;\; e^{-\frac{i}{\hbar} p\zeta} \;\; \Psi^*\left(x - \frac{\zeta}{2}\right) \; \Psi\left(x + \frac{\zeta}{2}\right)$$

Wigner function is a FT of the density operator ρ in position space expressed in terms of the two variables x and p

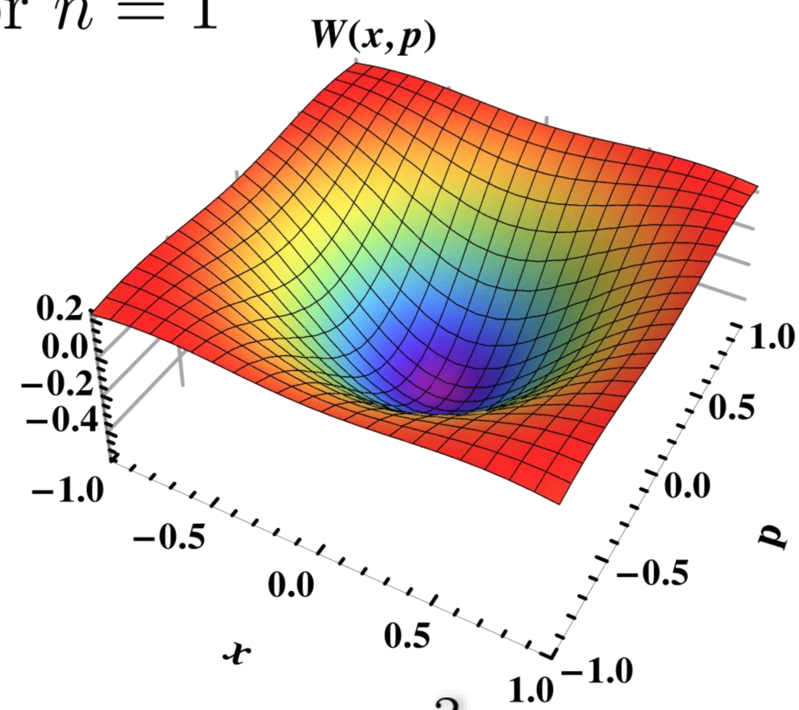
Examples - Wigner function

Gaussian function



$$\psi(x) = e^{-x^2}$$

Quantum Harmonic Oscillator for $n = 1$



$$\psi_1(x) = e^{-x^2} H_1(x)$$

Hermite polynomial

How to define Wigner distribution for a quantum field

Wigner distribution definition

$$\rho^{\Gamma}_{\lambda,\lambda'}(b_\perp, k_\perp, x) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} W^{\Gamma}_{\lambda,\lambda'}(\Delta_\perp, k_\perp, x)$$

The most general two-parton correlation function

Dirac structure

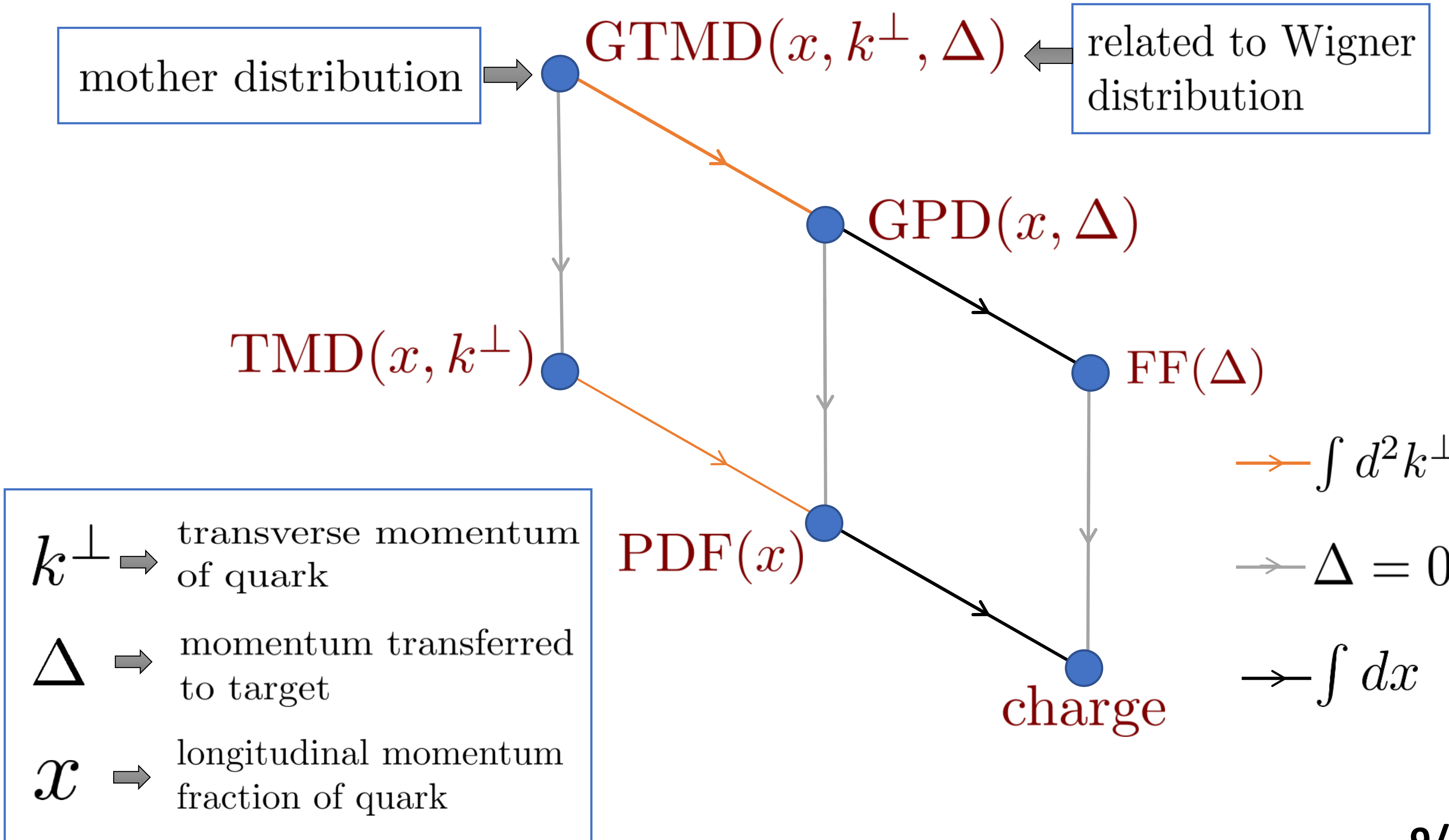
$$W^{\Gamma}_{\lambda\lambda'}(\Delta_\perp, k_\perp, x) = \int \frac{dz^- d^2 z^\perp}{2(2\pi)^3} e^{ik.z} \left\langle p', \lambda' \left| \bar{\psi}\left(\frac{-z}{2}\right) \Gamma \, \mathcal{G} \psi\left(\frac{z}{2}\right) \right| p, \lambda \right\rangle \Bigg|_{z^+=0}$$

initial/final state helicity

gauge link

[C. Lorce and B. Pasquini, PRD 84 (2011) 014015]

Mother distribution function



Different Polarization

$$\rho^{\Gamma}_{\lambda,\lambda'}(b_\perp, k_\perp, x) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} W^{\Gamma}_{\lambda,\lambda'}(\Delta_\perp, k_\perp, x)$$

$$\Gamma = \{\gamma^+, \gamma^+\gamma^5, \sigma^{+j}\gamma^5\}$$

U L T

16 possible leading twist quark Wigner distributions

$$\rho^{\gamma^+}_{UU} = \rho^{\gamma^+}_{\uparrow,\uparrow} + \rho^{\gamma^+}_{\downarrow,\downarrow}$$

$$\rho^{\gamma^+}_{LU} = \rho^{\gamma^+}_{\uparrow,\uparrow} - \rho^{\gamma^+}_{\downarrow,\downarrow}$$

We calculate the Wigner distributions in the dressed quark model

Dressed quark model

$$\left\langle p', \Lambda' \middle| \hat{\mathcal{O}} \middle| p, \Lambda \right\rangle$$

target is a dressed quark state

which can be expanded in Fock space

$$\left| p, \Lambda \right\rangle = b^\dagger(p)|0\rangle + \sum_{\sigma_1 \sigma_2} \int [dp_1] \int [dp_2] \delta^3(p - p_1 - p_2) \Psi^{\Lambda}_{\sigma_1 \sigma_2}(p_1, p_2) b^\dagger(p_1) a^\dagger(p_2)|0\rangle$$

Light-front wave function which contains all the parton information

Obtained by solving the light-front bound state eq.

[A. Harindranath *et al*, PRD 59 (1999) 094013]

Overlap representation

$$W^{\Gamma}_{\Lambda'\Lambda}(x,k^{\perp},\Delta)=\int\frac{dz^{-}d^{2}z_{\perp}}{2(2\pi)^{3}}e^{ik.z}\left\langle p',\Lambda'\left|\bar{\psi}\left(\frac{-z}{2}\right)\Gamma\,\mathcal{G}\psi\left(\frac{z}{2}\right)\right|p,\Lambda\right\rangle$$

$$W^{\gamma^{+}}_{\Lambda,\Lambda'}(x,k^{\perp},\Delta)=\sum_{\sigma_1,\sigma_2,\lambda_1}\Psi^{*\Lambda'}_{\lambda_1\sigma_2}(x,q'^{\perp})\chi^{\dagger}_{\lambda_1}\chi_{\sigma_1}\Psi^{\Lambda}_{\sigma_1\sigma_2}(x,q^{\perp})$$

$$\Psi^{\sigma}_{\sigma_1\sigma_2}(x,q^{\perp})=\frac{1}{\left[m^{2}-\frac{m^{2}+(q^{\perp})^{2}}{x}-\frac{(q^{\perp})^{2}}{1-x}\right]}\frac{g}{\sqrt{2(2\pi)^{3}}}T^{a}\chi^{\dagger}_{\sigma_1}\frac{1}{\sqrt{1-x}}$$
$$\left[-2\frac{q^{\perp}}{1-x}-\frac{(\sigma^{\perp}.q^{\perp})\sigma^{\perp}}{x}+\frac{im\sigma^{\perp}(1-x)}{x}\right]\chi_{\sigma}(\epsilon^{\perp}_{\sigma_2})^{*}$$

Final analytic results

$$\rho^{\gamma^+}_{UU}(b_\perp, k_\perp) = \int d^2\Delta_\perp \int dx \, \frac{\cos(\Delta_\perp \cdot b_\perp)}{D(q_\perp)D(q'_\perp)} \left[I_1 + \frac{4m^2(1-x)}{x^2} \right]$$

$$\rho^{\gamma^+}_{LU}(b_\perp, k_\perp) = \int d^2\Delta_\perp \int dx \, \frac{\sin(\Delta_\perp \cdot b_\perp)}{D(q_\perp)D(q'_\perp)} \left[4(k_x\Delta_y - k_y\Delta_x) \frac{(1+x)}{x^2(1-x)} \right]$$

Evaluated numerically

$$I_1 = 4\left((k_\perp)^2 - \frac{\Delta_\perp^2(1-x)^2}{4}\right) \frac{(1+x^2)}{x^2(1-x)^3}$$

$$D(k_\perp) = \left(m^2 - \frac{m^2 + (k_\perp)^2}{x} - \frac{(k_\perp)^2}{1-x}\right)$$

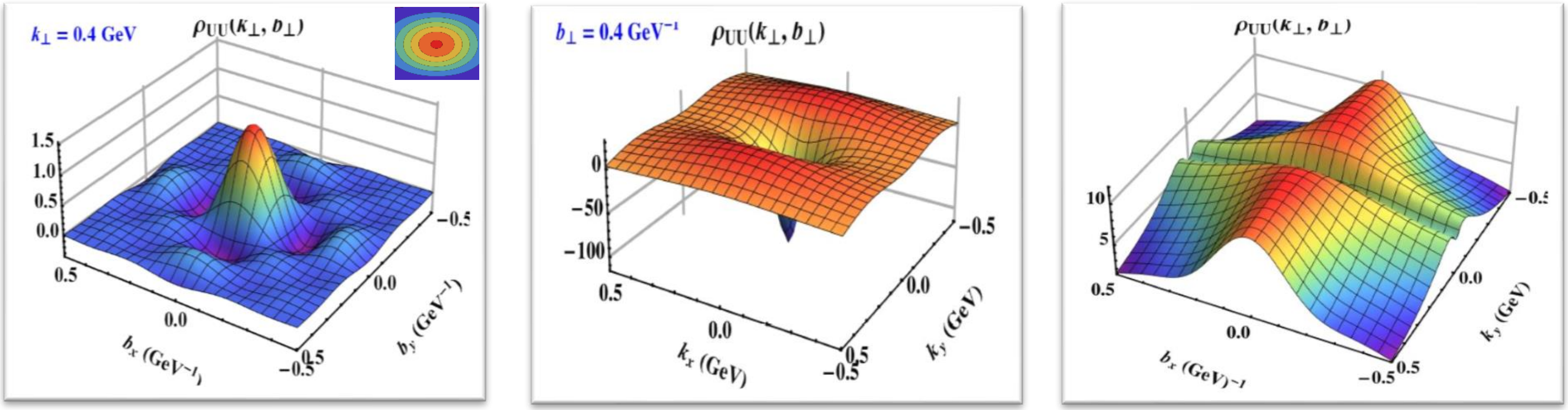
where A_x, A_y are x, y component of $A_\perp$

[S.N, A. Mukherjee & J.More PRD 95 (2017), 074039]

Results - Unpolarized quark

$b - space$ $k - space$ $m - space$

UU



[S.N, A. Mukherjee & J.More PRD 95 (2017), 074039]

- positive peak centered around $b_x = b_y = 0$ as seen in [1] & [2].
- sharp negative peak centered around $k_x = k_y = 0$ unlike [1] & [2].
- probabilistic interpretation in the b_x - k_y plane.

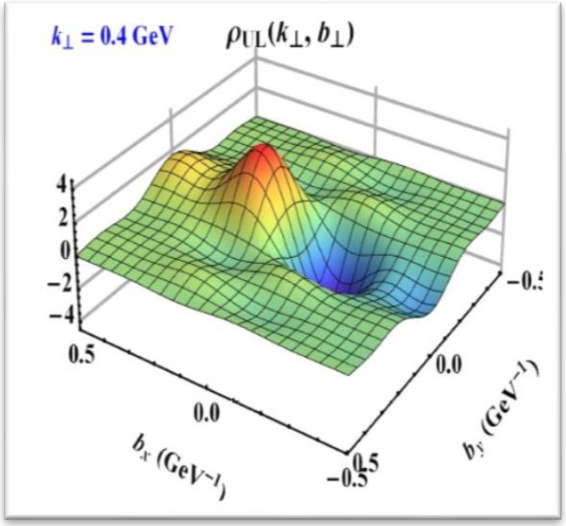
[1] T. Liu and B. Q. Ma, Phys. Rev. D 91, 034019 (2015).

[2] C. Lorcé and B. Pasquini, Phys. Rev. D 93, 034040 (2016).

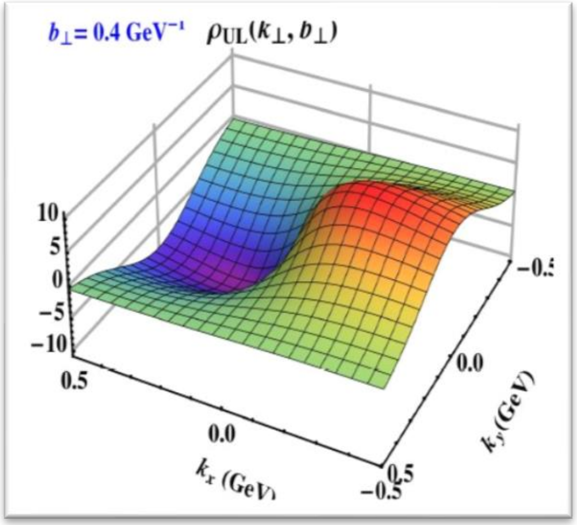
Results - Longitudinally Polarized Quark

UL

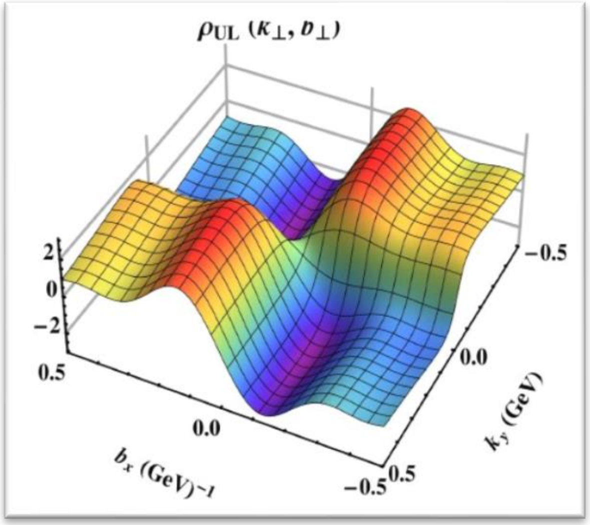
$b - space$



$k - space$



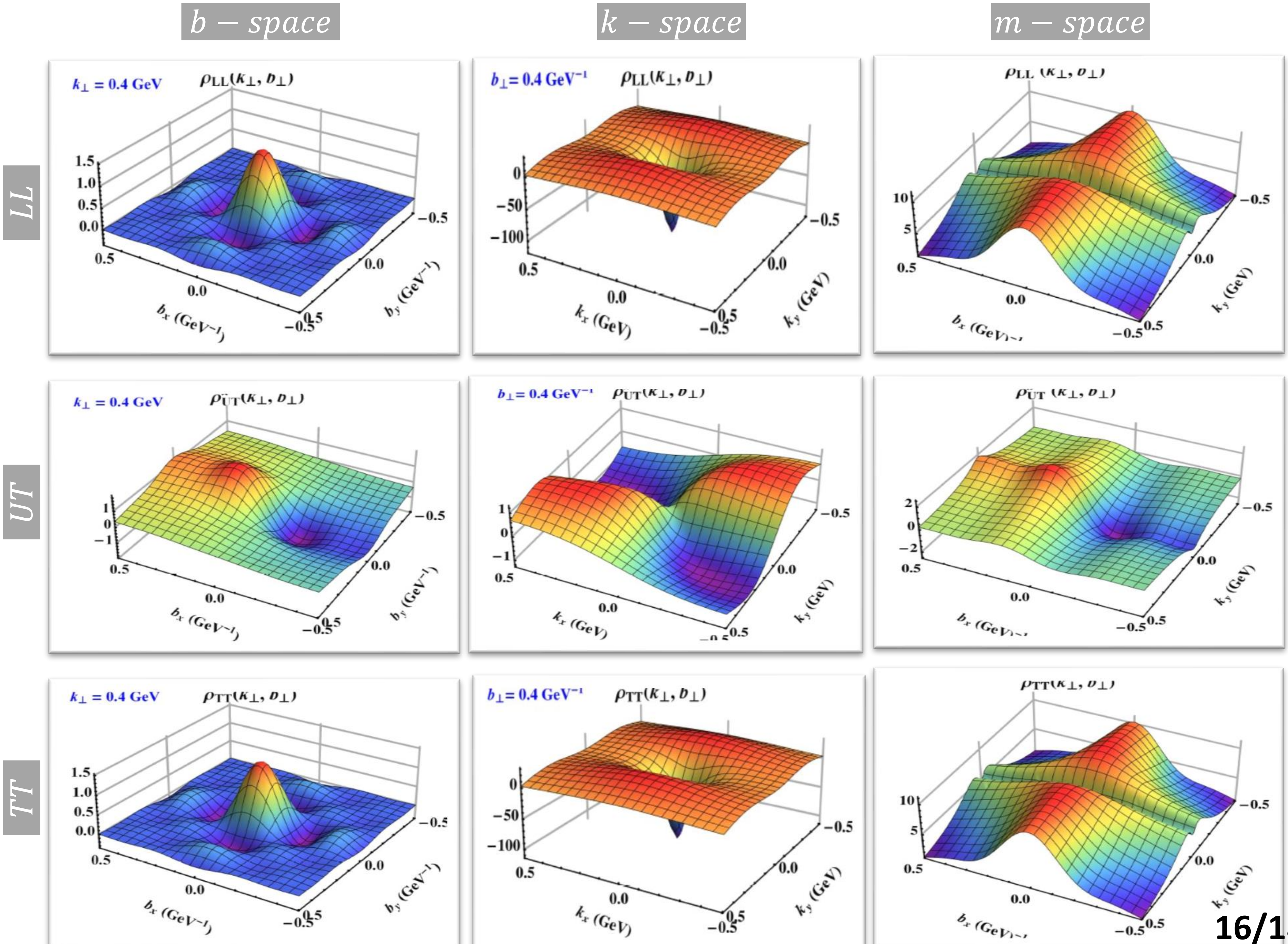
$m - space$



[S.N, A. Mukherjee & J.More PRD 95 (2017), 074039]

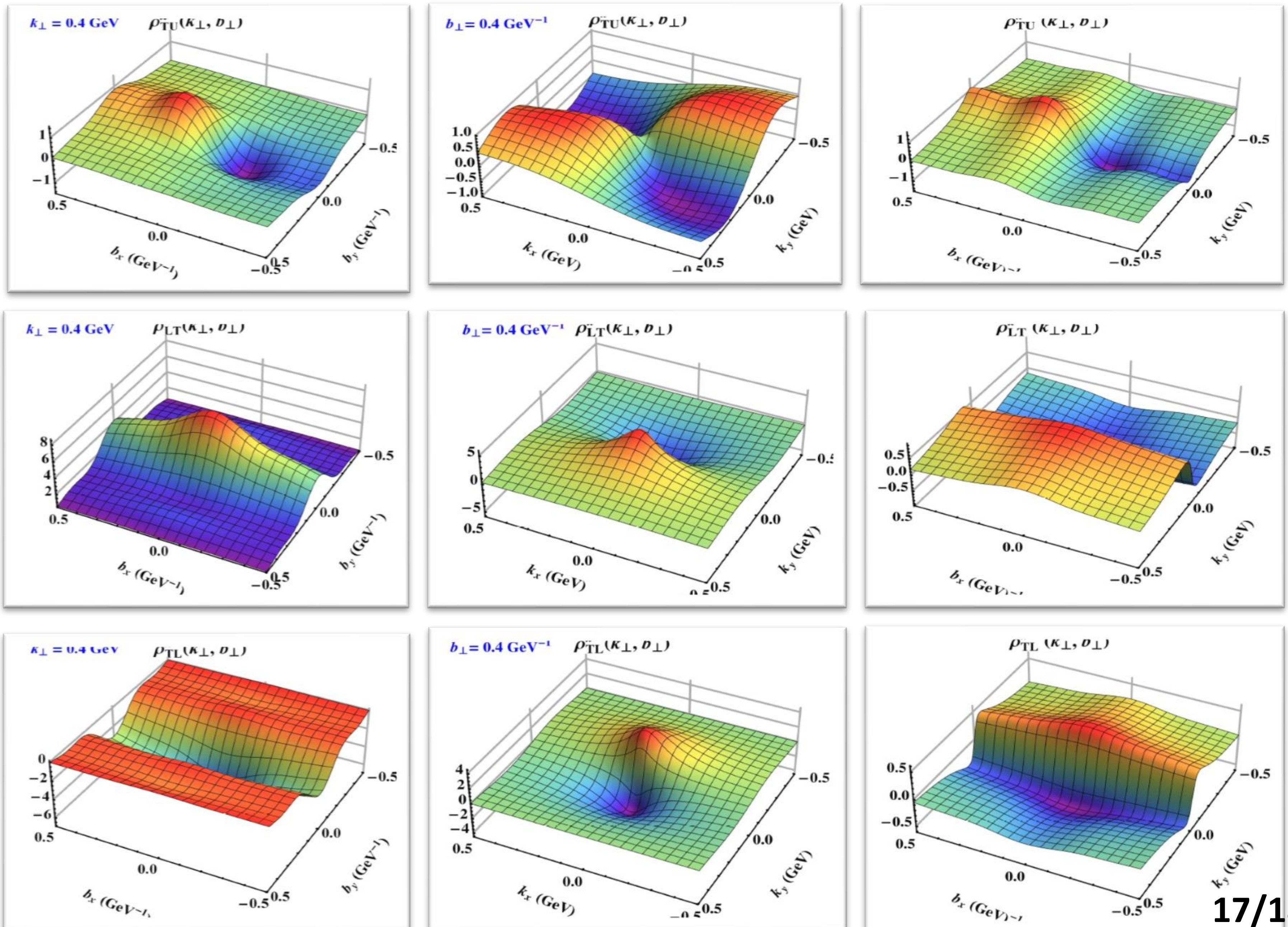
- dipole structure observed in b and k space.
- quadrapole structure observed in mixed space.
- qualitative agreement with other models ([1] & [2]).

[1] T. Liu and B. Q. Ma, Phys. Rev. D 91, 034019 (2015).
[2] C. Lorcé and B. Pasquini, Phys. Rev. D 93, 034040 (2016).



$b-space$
$k-space$
$m-space$
LL
UT
TT
$k_\perp = 0.4$ GeV
$\rho_{LL}(k_\perp, b_\perp)$
$b_\perp = 0.4$ GeV^{-1}
$\rho_{LL}(k_\perp, b_\perp)$
$\rho_{UT}(k_\perp, b_\perp)$
$\rho_{TT}(k_\perp, b_\perp)$
b_x (GeV^{-1})
b_y (GeV^{-1})
k_x (GeV)
k_y (GeV)

	$b - space$	$k - space$	$m - space$
TU			
LT			
TL			



Conclusion

- We calculated the quark Wigner distributions in the dressed quark model with all possible polarization configurations.

- These distributions can be related to the spin-orbit correlation and orbital angular momentum of quark.

- Our calculations provide a complementary study to other model calculations.

Thank you

For more on Wigner distributions
Talk by Chandan Mondal at 17:00

backup slides

$$\rho_{UU}(\boldsymbol{b}_\perp,\boldsymbol{k}_\perp,x) = N\int\frac{d^2\Delta_\perp}{2(2\pi)^2}\frac{\cos(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp)D(\boldsymbol{q'}_\perp)} \times \Big[\frac{\Big(4k_\perp^2-\Delta_\perp^2(1-x)^2\Big)(1+x^2)}{x^2(1-x)^3}+\frac{4m^2(1-x)}{x^2}\Big]$$

$$\rho_{LL}(\boldsymbol{b}_\perp,\boldsymbol{k}_\perp,x) = N\int\frac{d^2\Delta_\perp}{2(2\pi)^2}\frac{\cos(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp)D(\boldsymbol{q'}_\perp)} \times \Big[\frac{\Big(4k_\perp^2-\Delta_\perp^2(1-x)^2\Big)(1+x^2)}{x^2(1-x)^3}-\frac{4m^2(1-x)}{x^2}\Big]$$

$$\rho_{TT}(\boldsymbol{b}_\perp,\boldsymbol{k}_\perp,x) = N\int\frac{d^2\Delta_\perp}{2(2\pi)^2}\frac{\cos(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp)D(\boldsymbol{q'}_\perp)}\Big[\frac{2\Big(4k_\perp^2-\Delta_\perp^2(1-x)^2\Big)}{x(1-x)^3}\Big]$$

$$\rho_{UL}(\boldsymbol{b}_\perp,\boldsymbol{k}_\perp,x) = N\int\frac{d^2\Delta_\perp}{2(2\pi)^2}\frac{\sin(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp)D(\boldsymbol{q'}_\perp)}\Big[\frac{4\Big(k_y\Delta_x-k_x\Delta_y\Big)(1+x)}{x^2(1-x)}\Big]$$

$$\rho^x_{LT}(\boldsymbol{b}_\perp, \boldsymbol{k}_\perp, x) = N \int \frac{d^2\Delta_\perp}{2(2\pi)^2} \frac{\cos(\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q'}_\perp)} \left[\frac{8mk_x}{x^2(1-x)} \right]$$

$$\rho^x_{TU}(\boldsymbol{b}_\perp, \boldsymbol{k}_\perp, x) = N \int \frac{d^2\Delta_\perp}{2(2\pi)^2} \frac{\sin(\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q'}_\perp)} \left[\frac{4m\Delta_x}{x} \right]$$

$$\rho^x_{TL}(\boldsymbol{b}_\perp, \boldsymbol{k}_\perp, x) = N \int \frac{d^2\Delta_\perp}{2(2\pi)^2} \frac{\cos(\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q'}_\perp)} \left[\frac{-8mk_x}{x(1-x)} \right]$$

$$\rho^x_{UT}(\boldsymbol{b}_\perp, \boldsymbol{k}_\perp, x) = N \int \frac{d^2\Delta_\perp}{2(2\pi)^2} \frac{\sin(\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp)}{D(\boldsymbol{q}_\perp) D(\boldsymbol{q'}_\perp)} \left[\frac{4m\Delta_x}{x^2} \right]$$

Quark spin-orbit correlation

$$W^{[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\bar{u}(p',\lambda')\Big[F_{1,1} - \frac{i\sigma^{i+}k_{i\perp}}{P^+}F_{1,2} - \frac{i\sigma^{i+}\Delta_{i\perp}}{P^+}F_{1,3} + \frac{i\sigma^{ij}k_{i\perp}\Delta_{j\perp}}{M^2}F_{1,4}\Big]u(p,\lambda),$$

$$W^{[\gamma^+\gamma_5]}_{\lambda,\lambda'} = \frac{\bar{u}(p',\lambda')}{2M}\Big[\frac{-i\epsilon^{ij}_{\perp}k_{i\perp}\Delta_{j\perp}}{M^2}G_{1,1} - \frac{i\sigma^{i+}\gamma_5 k_{i\perp}}{P^+}G_{1,2} - \frac{i\sigma^{i+}\gamma_5\Delta_{i\perp}}{P^+}G_{1,3} + i\sigma^{+-}\gamma_5 G_{1,4}\Big]u(p,\lambda).$$

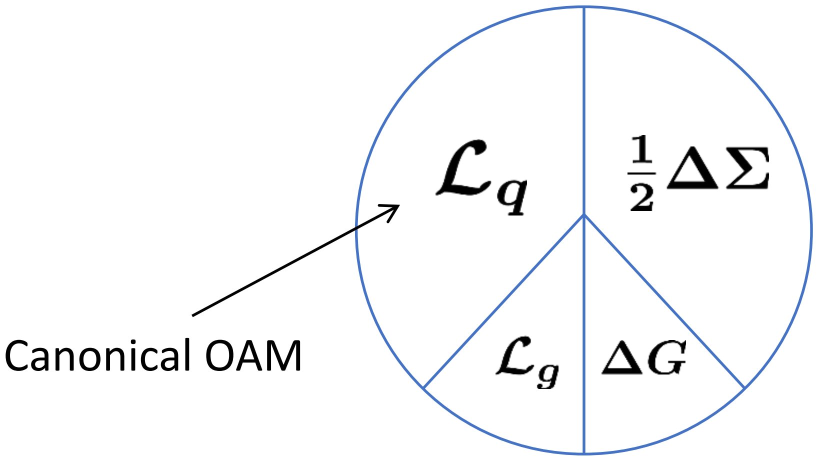
$$C_z^q = \int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\boldsymbol{k}_\perp^2}{M^2}\,G_{11}^q(x,0,\boldsymbol{k}_\perp^2,0,0).$$

Orbital Angular Momentum

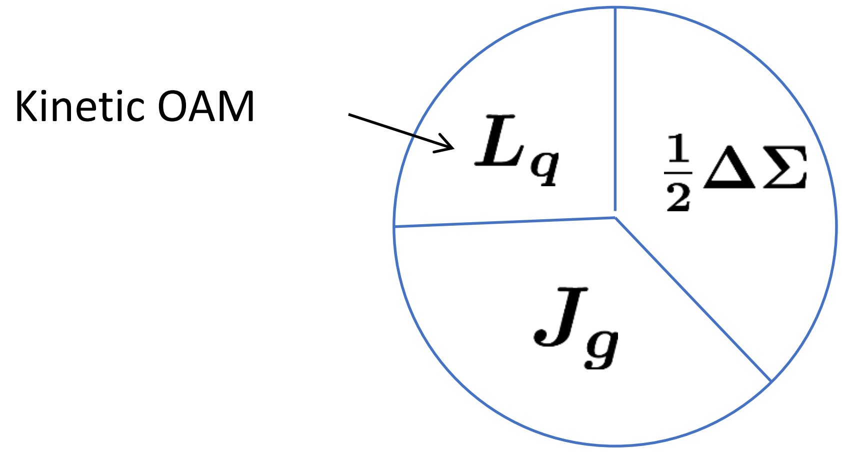
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q + \mathcal{L}_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

Jaffe-Manohar decomposition



Ji's decomposition



Quark OAM

Kinetic OAM

[X. Ji, PRL 78 (1997) 610]

$$L_z^q = \frac{1}{2}\int dx\{x[H^q(x,0,0) + E^q(x,0,0)] - \tilde{H}^q(x,0,0)\}$$

Canonical OAM

[C. Lorce & B. Pasquini, PRD 84 (2011) 014015]

$$l_z^q = -\int dx d^2k_\perp \frac{k_\perp^2}{m_q^2} F_{14}$$

GPDs

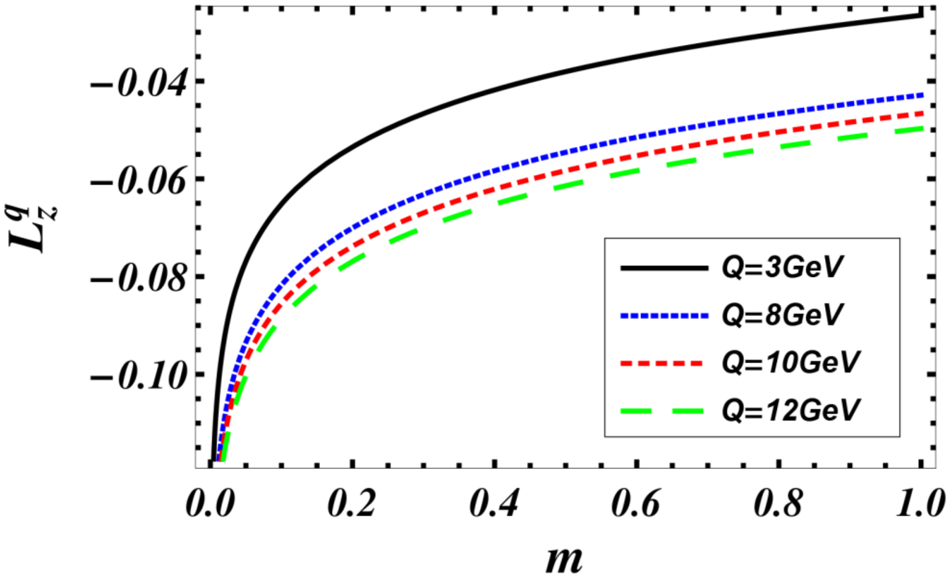
$$H(x,0,t) = \int d^2k_\perp F_{11}$$

$$E(x,0,t) = \int d^2k_\perp \Big[- F_{11} + 2\Big(\frac{k_\perp.\Delta_\perp}{\Delta_\perp^2} F_{12} + F_{13}\Big)\Big]$$

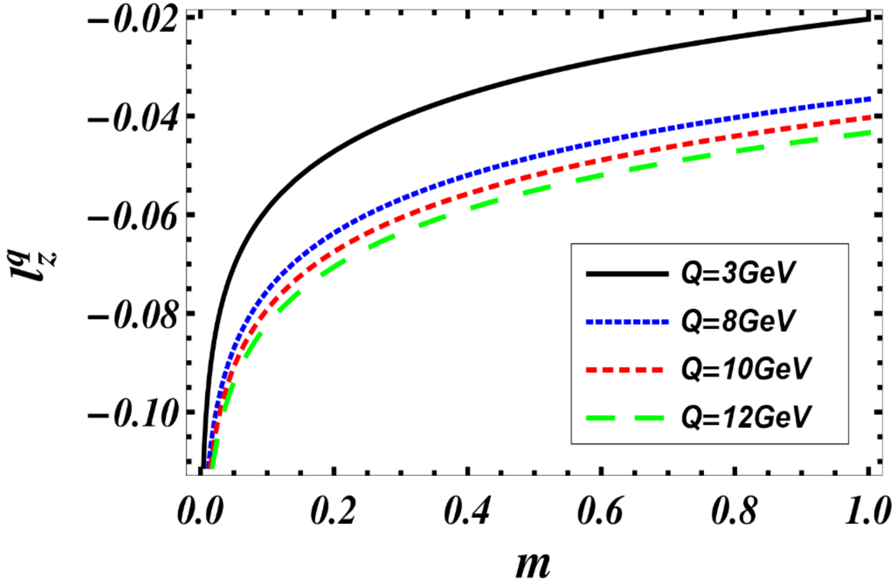
$$\tilde{H}(x,0,t) = \int d^2k_\perp G_{14}$$

GTMDs

Plots for OAM



Kinetic OAM



Canonical OAM

[S.N, A. Mukherjee & V. K. Ojha PRD 90 (2014) 1, 014024]