Pseudoscalar and vector meson production in heavy hadron weak decays

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Λ_b decays in external emission

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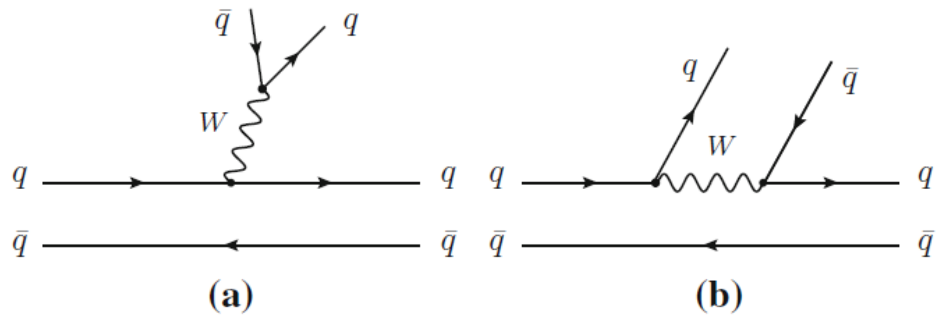


Fig. 1 Meson decay with external emission (**a**) or internal emission (**b**)

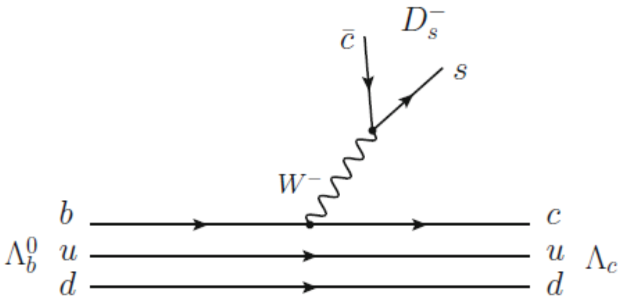


Fig. 2 Quark description in $\Lambda_b \to D_s^- \Lambda_c$ decay with external emission

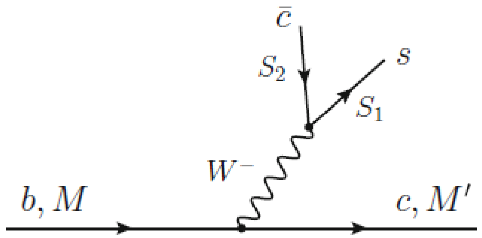


Fig. 3 Angular momenta for the b, c transition diagram of Fig. 2

$$t = \langle c|\gamma^{\mu}(1-\gamma_5)|b\rangle \, \langle s|\gamma_{\mu}(1-\gamma_5)|c'\rangle$$

$$u_r = \mathcal{A}\begin{pmatrix} \chi_r \\ \mathcal{B}\,(\vec{\sigma}\cdot\vec{p})\;\chi_r \end{pmatrix}, \quad \chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with

$$\mathcal{A} = \left(\frac{E_p + m}{2\,m}\right)^{1/2}, \qquad \mathcal{B} = \frac{1}{E_p + m},$$

$$v_r = \mathcal{A}\begin{pmatrix} \mathcal{B}\,(\vec{\sigma}\cdot\vec{p})\;\chi_r \\ \chi_r \end{pmatrix}$$

We take the Dirac representation for the γ^μ matrices,

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

To avoid using quark wave functions we use:

$$\frac{p_c^\mu}{m_c} = \frac{p_{\Lambda_c}^\mu}{M_{\Lambda_c}}, \qquad \frac{p_b^\mu}{m_b} = \frac{p_{\Lambda_b}^\mu}{M_{\Lambda_b}},$$

In Dai et al. EPJC 78 (2018) it is shown that the error made is of the order of $p_{int}^2/m_{\Lambda b}^2$

Quite good in our case, but exact only in the infinitely heavy quark mass limit. We implement implicitly the rules of heavy quark physics.

$$\mathcal{A} = \left(\frac{\frac{E_\Lambda}{M_\Lambda} + 1}{2}\right)^{1/2}, \quad \mathcal{B}_Q\; p_Q = \mathcal{B}\cdot p, \quad \mathcal{B} = \frac{1}{M_\Lambda(1 + \frac{E_\Lambda}{M_\Lambda})}$$

Evalaution of matrix elements:

We work in the D_s rest frame: Λ_b and Λ_c have then the same momentum

$$p = \frac{\lambda^{1/2}(M^2_{\Lambda_b}, M^2_{D_s}, M^2_{\Lambda_c})}{2\,M_{D_s}}$$

For the quarks of the D_s we can use $\gamma^0 \equiv 1,\ \gamma^i\gamma_5 \equiv \sigma^i$

$$\langle S_1|S_2\rangle\ \langle M'|\gamma^0 - \gamma^0\gamma_5|M\rangle + \langle S_1|\sigma^i|S_2\rangle\ \langle M'|\gamma^i - \gamma^i\gamma_5|M\rangle$$

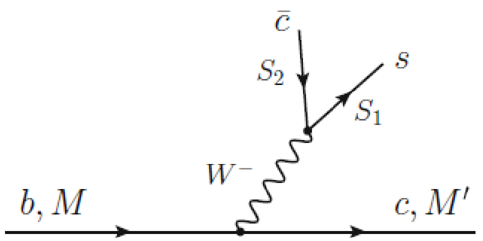


Fig. 3 Angular momenta for the b, c transition diagram of Fig. 2

External emission $\Lambda_b \to D_s^- \, (D_s^{*-}) \, \Lambda_c$

$$t = \mathcal{A}\mathcal{A}' \, [t_1 + t_2 + t_3 + t_4 + t_5 + t_6]$$

$$t_1 = (1 + \mathcal{B}\mathcal{B}' \, \vec{p}^{\,2}) \, \langle S_1 | S_2 \rangle \, \langle M' | M \rangle,$$
$$t_2 = -(\mathcal{B} + \mathcal{B}') \, \langle S_1 | S_2 \rangle \, \langle M' | \vec{\sigma} \cdot \vec{p} \, | M \rangle,$$
$$t_3 = (\mathcal{B} + \mathcal{B}') \, \langle S_1 | \vec{\sigma} \cdot \vec{p} \, | S_2 \rangle \, \langle M' | M \rangle,$$
$$t_4 = (-1 + \mathcal{B}\mathcal{B}' \, \vec{p}^{\,2}) \, \langle S_1 | \sigma^i | S_2 \rangle \, \langle M' | \sigma^i | M \rangle,$$
$$t_5 = -2\mathcal{B}\mathcal{B}' \, \langle S_1 | \vec{\sigma} \cdot \vec{p} \, | S_2 \rangle \, \langle M' | \vec{\sigma} \cdot \vec{p} \, | M \rangle,$$
$$t_6 = i \, (\mathcal{B} - \mathcal{B}') \, \epsilon_{ijk} \, p^j \, \langle S_1 | \sigma^i | S_2 \rangle \, \langle M' | \sigma^k | M \rangle$$

All these terms are evaluated with angular momentum algebra with the help of Racah coefficients

Λ_b decay in internal emission

$$\langle \bar{u}_s | \gamma^0 - \gamma^0 \gamma_5 | u_{c'} \rangle \, \langle \bar{u}_c | \gamma^0 - \gamma^0 \gamma_5 | u_b \rangle$$
$$- \, \langle \bar{u}_s | \gamma^i - \gamma^i \gamma_5 | u_{c'} \rangle \, \langle \bar{u}_c | \gamma^i - \gamma^i \gamma_5 | u_b \rangle$$

$$t = \mathcal{A}\mathcal{A}' \left[t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8\right]$$

$$\begin{aligned}
t_1 &= \langle M'|S_2\rangle\ \langle S_1|M\rangle, \\
t_2 &= -\mathcal{B}'\ \langle M'|\vec{\sigma}\cdot\vec{p}\,|S_2\rangle\ \langle S_1|M\rangle, \\
t_3 &= -\mathcal{B}\,\langle M'|S_2\rangle\ \langle S_1|\vec{\sigma}\cdot\vec{p}\,|M\rangle, \\
t_4 &= \mathcal{B}\mathcal{B}'\ \langle M'|\vec{\sigma}\cdot\vec{p}\,|S_2\rangle\ \langle S_1|\vec{\sigma}\cdot\vec{p}\,|M\rangle, \\
t_5 &= -\langle M'|\sigma^i|S_2\rangle\ \langle S_1|\sigma^i|M\rangle, \\
t_6 &= \mathcal{B}'\ \langle M'|\vec{\sigma}\cdot\vec{p}\,\sigma^i|S_2\rangle\ \langle S_1|\sigma^i|M\rangle, \\
t_7 &= \mathcal{B}\,\langle M'|\sigma^i|S_2\rangle\ \langle S_1|\sigma^i\,\vec{\sigma}\cdot\vec{p}\,|M\rangle, \\
t_8 &= -\mathcal{B}\mathcal{B}'\ \langle M'|\vec{\sigma}\cdot\vec{p}\,\sigma^i|S_2\rangle\ \langle S_1|\sigma^i\,\vec{\sigma}\cdot\vec{p}\,|M\rangle
\end{aligned}$$

The evaluation is similar for meson decays but we in addition we perform the couplings of the spins leading to vector mesons or pseudoscalars. Involved algebra which allows us to relate many processes

$$\Lambda_b \to D_s^- \Lambda_c \qquad \Gamma = \frac{1}{2\pi}\frac{M_{\Lambda_c}}{M_{\Lambda_b}} \overline{\sum}\sum |t|^2 \, P_{D_s^{(*)-}}$$

$$\overline{\sum}\sum |t|^2 = \begin{cases} 2(\mathcal{A}\mathcal{A}')^2 \left[(1+\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2})^2 + (\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2} \right], & \text{for } j=0; \\ 2(\mathcal{A}\mathcal{A}')^2 \left[3 + 3(\mathcal{B}^2+\mathcal{B}'^2)\,\vec{p}^{\,2} - 4\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2} + 3(\mathcal{B}\mathcal{B}')^2\,\vec{p}^{\,4} \right], & \text{for } j=1, \end{cases}$$

External emission in ***B*** decays $\qquad \bar{B}^0 \to D_s^- D^+,\ D_s^- D^{*+},\ D_s^{*-} D^+,\ D_s^{*-} D^{*+}$

(A) $j=0,\ j'=0$:

$$\overline{\sum}\sum |t|^2 = (\mathcal{A}\mathcal{A}')^2 \cdot 2\,(1+\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2})^2;$$

(B) $j=0,\ j'=1$:

$$\overline{\sum}\sum |t|^2 = (\mathcal{A}\mathcal{A}')^2 \cdot 2\,(\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2};$$

(C) $j=1,\ j'=0$:

$$\overline{\sum}\sum |t|^2 = (\mathcal{A}\mathcal{A}')^2 \cdot 2\,(\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2};$$

$$\Gamma = \frac{1}{8\pi}\frac{1}{M_B^2} \overline{\sum}\sum |t|^2 \, P_{D_s^{(*)}}$$

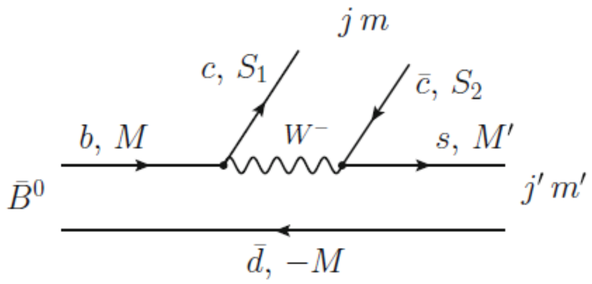
(D) $j=1,\ j'=1$:

$$\overline{\sum}\sum |t|^2 = (\mathcal{A}\mathcal{A}')^2 \left[6 + 4\mathcal{B}^2\,\vec{p}^{\,2} + 4\mathcal{B}'^2\,\vec{p}^{\,2} - 12\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2} + 6(\mathcal{B}\mathcal{B}')^2\,\vec{p}^{\,4} \right];$$

Λ_b decay in internal emission

$$\overline{\sum}\sum |t|^2 = \begin{cases} 2(\mathcal{A}\mathcal{A}')^2\,[\,(1+\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2})^2 + (\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2}\,], & \text{for } j=0; \\ 2(\mathcal{A}\mathcal{A}')^2\,[\,3+3(\mathcal{B}^2+\mathcal{B}'^2)\,\vec{p}^{\,2} - 4\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2} + 3(\mathcal{B}\mathcal{B}')^2\,\vec{p}^{\,4}\,], & \text{for } j=1, \end{cases}$$

Internal emission for meson decays



$$\overline{\sum}\sum |t|^2 = \begin{cases} 2\,(\mathcal{A}\mathcal{A}')^2\,(1+\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2})^2, & \text{for } j=0,\ j'=0; \\ 2\,(\mathcal{A}\mathcal{A}')^2\,(\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2}, & \text{for } j=0,\ j'=1; \\ 2\,(\mathcal{A}\mathcal{A}')^2\,(\mathcal{B}+\mathcal{B}')^2\,\vec{p}^{\,2}, & \text{for } j=1,\ j'=0; \\ 2\,(\mathcal{A}\mathcal{A}')^2\,[\,3+2\mathcal{B}^2\,\vec{p}^{\,2}+2\mathcal{B}'^2\,\vec{p}^{\,2}-6\mathcal{B}\mathcal{B}'\,\vec{p}^{\,2}+3(\mathcal{B}\mathcal{B}')^2\,\vec{p}^{\,4}\,], & \text{for } j=1,\ j'=1; \end{cases}$$

External emission

Decay process	BR (Theo.)	BR (Exp.)
$\Lambda_b \to D_s^- \Lambda_c$	(fit to the Exp.)	$(1.10 \pm 0.10) \times 10^{-2}$
$\Lambda_b \to D_s^{*-} \Lambda_c$	$(1.35 \pm 0.12) \times 10^{-2}$	
$\Lambda_b \to D^- \Lambda_c$ (Cabibbo suppressed)	$(6.89 \pm 0.62) \times 10^{-4}$	$(4.6 \pm 0.6) \times 10^{-4}$
$\Lambda_b \to D^{*-} \Lambda_c$ (Cabibbo suppressed)	$(8.19 \pm 0.74) \times 10^{-4}$	

Internal emission

Decay process	$BR(b \to \Lambda_b) \times$ BR (Theo.)	$BR(b \to \Lambda_b) \times$ BR (Exp.)
$\Lambda_b \to J/\psi \Lambda$	(fit to the Exp.)	$(5.8 \pm 0.8) \times 10^{-5}$
$\Lambda_b \to \eta_c \Lambda$	$(3.9 \pm 0.5) \times 10^{-5}$	
$\Lambda_b \to D^0 \Lambda$ (Cabibbo suppressed)	$(8.9 \pm 1.2) \times 10^{-6}$	
$\Lambda_b \to D^{*0} \Lambda$ (Cabibbo suppressed)	$(9.5 \pm 1.3) \times 10^{-6}$	

Table 3 Branching ratios for $\bar{B}^0$ decays in external emission

Decay process	BR (Theo.)	BR (Exp.)
$\bar{B}^0 \to D_s^- D^+$	$(1.31 \pm 0.10) \times 10^{-2}$	$(7.2 \pm 0.8) \times 10^{-3}$
$\bar{B}^0 \to D_s^{*-} D^+$	$(7.25 \pm 0.58) \times 10^{-3}$	$(7.4 \pm 1.6) \times 10^{-3}$
$\bar{B}^0 \to D_s^- D^{*+}$	$(7.68 \pm 0.61) \times 10^{-3}$	$(8.0 \pm 1.1) \times 10^{-3}$
$\bar{B}^0 \to D_s^{*-} D^{*+}$	(fit to the Exp.)	$(1.77 \pm 0.14) \times 10^{-2}$

Table 4 Branching ratios for B^- decays in external emission

Decay process	BR (Theo.)	BR (Exp.)
$B^- \to D_s^- D^0$	$(1.27 \pm 0.18) \times 10^{-2}$	$(9.0 \pm 0.9) \times 10^{-3}$
$B^- \to D_s^{*-} D^0$	$(7.03 \pm 0.99) \times 10^{-3}$	$(7.6 \pm 1.6) \times 10^{-3}$
$B^- \to D_s^- D^{*0}$	$(7.43 \pm 1.04) \times 10^{-3}$	$(8.2 \pm 1.7) \times 10^{-3}$
$B^- \to D_s^{*-} D^{*0}$	(fit to the Exp.)	$(1.71 \pm 0.24) \times 10^{-2}$

Table 5 Branching ratios for $\bar{B}_s^0$ decays in external emission

Decay process	BR (Theo.)	BR (Exp.)
$\bar{B}_s^0 \to D_s^- D_s^+$	$(1.06 \pm 0.14) \times 10^{-2}$	$(4.4 \pm 0.5) \times 10^{-3}$
$\bar{B}_s^0 \to D_s^{*-} D_s^+ + D_s^- D_s^{*+}$	$(1.08 \pm 0.14) \times 10^{-2}$	$(1.37 \pm 0.16) \times 10^{-2}$
$\bar{B}_s^0 \to D_s^{*-} D_s^{*+}$	(fit to the Exp.)	$(1.43 \pm 0.19) \times 10^{-2}$

Table 6 Rates of branching ratios for B_c^- decays in external emission

Rate	Theo.	Exp.
$\Gamma_{B_c^- \to D_s^- \eta_c} / \Gamma_{B_c^- \to D_s^- J/\psi}$	1.76	
$\Gamma_{B_c^- \to D_s^{*-} \eta_c} / \Gamma_{B_c^- \to D_s^- J/\psi}$	0.90	
$\Gamma_{B_c^- \to D_s^{*-} J/\psi} / \Gamma_{B_c^- \to D_s^- J/\psi}$	2.47	2.5 ± 0.5

Table 10 Branching ratios for $\bar{B}^0$ decays in internal emission

Decay process	BR (Theo.)	BR (Exp.)
$\bar{B}^0 \to \eta_c \bar{K}^0$	$(1.23 \pm 0.05) \times 10^{-3}$	$(8.0 \pm 1.2) \times 10^{-4}$
$\bar{B}^0 \to J/\psi \bar{K}^0$	(fit to the Exp.)	$(8.73 \pm 0.32) \times 10^{-4}$
$\bar{B}^0 \to \eta_c \bar{K}^{*0}$	$(4.53 \pm 0.17) \times 10^{-4}$	$(6.3 \pm 0.9) \times 10^{-4}$
$\bar{B}^0 \to J/\psi \bar{K}^{*0}$	$(1.31 \pm 0.05) \times 10^{-3}$	$(1.28 \pm 0.05) \times 10^{-3}$
$\bar{B}^0 \to \psi(2S) \bar{K}^0$	$(2.9 \pm 0.2) \times 10^{-4}$	$(5.8 \pm 0.5) \times 10^{-4}$
$\bar{B}^0 \to \psi(2S) \bar{K}^{*0}$	(fit to the Exp.)	$(5.9 \pm 0.4) \times 10^{-4}$

Table 11 Branching ratios for B^- decays in internal emission

Decay process	BR (Theo.)	BR (Exp.)
$B^- \to \eta_c K^-$	$(1.45 \pm 0.04) \times 10^{-3}$	$(9.6 \pm 1.1) \times 10^{-4}$
$B^- \to J/\psi K^-$	(fit to the Exp.)	$(1.026 \pm 0.031) \times 10^{-3}$
$B^- \to \eta_c K^{*-}$	$(5.32 \pm 0.16) \times 10^{-4}$	$(1.0^{+0.5}_{-0.4}) \times 10^{-3}$
$B^- \to J/\psi K^{*-}$	$(1.53 \pm 0.05) \times 10^{-3}$	$(1.43 \pm 0.08) \times 10^{-3}$
$B^- \to \psi(2S) K^-$	$(3.4 \pm 0.7) \times 10^{-4}$	$(6.26 \pm 0.24) \times 10^{-4}$
$B^- \to \psi(2S) K^{*-}$	(fit to the Exp.)	$(6.7 \pm 1.4) \times 10^{-4}$

Table 12 Branching ratios for $\bar{B}_s$ decays in internal emission

Decay process	BR (Theo.)	BR (Exp.)
$\bar{B}_s \to \eta_c \eta$	$(3.63 \pm 0.27) \times 10^{-4}$	
$\bar{B}_s \to J/\psi \eta$	$(2.55 \pm 0.19) \times 10^{-4}$	$(4.0 \pm 0.7) \times 10^{-4}$
$\bar{B}_s \to \eta_c \phi$	$(3.64 \pm 0.27) \times 10^{-4}$	
$\bar{B}_s \to J/\psi \phi$	(fit to the Exp.)	$(1.08 \pm 0.08) \times 10^{-3}$
$\bar{B}_s \to \eta_c \eta'$	$(3.88 \pm 0.29) \times 10^{-4}$	
$\bar{B}_s \to J/\psi \eta'$	$(2.24 \pm 0.17) \times 10^{-4}$	$(3.3 \pm 0.4) \times 10^{-4}$
$\bar{B}_s \to D^0 K^0$	(fit to the Exp.)	$(4.3 \pm 0.9) \times 10^{-4}$
$\bar{B}_s \to D^{*0} K^0$	$(3.4 \pm 0.7) \times 10^{-4}$	$(2.8 \pm 1.1) \times 10^{-4}$
$\bar{B}_s \to D^0 K^{*0}$	$(2.1 \pm 0.4) \times 10^{-4}$	$(4.4 \pm 0.6) \times 10^{-4}$
$\bar{B}_s \to D^{*0} K^{*0}$	$(3.0 \pm 0.6) \times 10^{-4}$	

Table 13 Rates of branching ratios for B_c^- decays in internal emission

Rate	Theo.	Exp.
$\Gamma_{B_c^- \to \eta_c D_s^-} / \Gamma_{B_c^- \to J/\psi D_s^-}$	2.03	
$\Gamma_{B_c^- \to \eta_c D_s^{*-}} / \Gamma_{B_c^- \to J/\psi D_s^-}$	0.98	
$\Gamma_{B_c^- \to J/\psi D_s^{*-}} / \Gamma_{B_c^- \to J/\psi D_s^-}$	3.44	2.5 ± 0.5
$\Gamma_{B_c^- \to D^0 D^-} / \Gamma_{B_c^- \to D^{*0} D^-}$	1.42	
$\Gamma_{B_c^- \to D^0 D^{*-}} / \Gamma_{B_c^- \to D^{*0} D^-}$	1.07	
$\Gamma_{B_c^- \to D^{*0} D^{*-}} / \Gamma_{B_c^- \to D^{*0} D^-}$	1.61	

Conclusions

The method assumes free spinors. Form factors stemming from the radial wave functions are not considered. They approximately cancel in ratios, except when the particles involved have very different mass.

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Decays involving pions and other light particles are not well described in this approach.
The D decays are also evaluated but they involve light particles in the final state, and the results are no so good.

But we prove that for heavy hadron decays into other heavy hadrons the approach is rather good and allows one to make many predictions for new rates not yet observed.