Considerations on Schmid theorem

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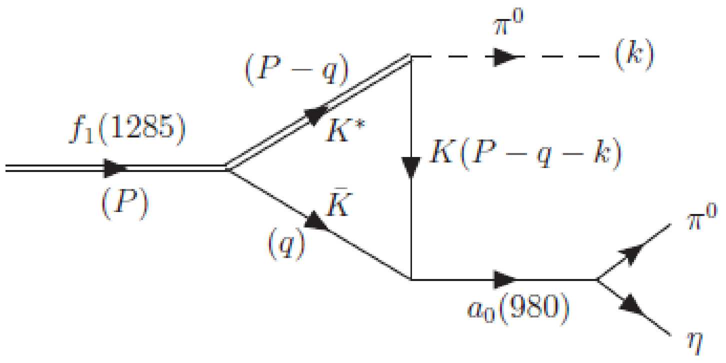
Triangle singularities
Schmid theorem
Dependence on the width
Final considerations

Triangle singularities

THE $\pi a_0(980)$ DECAY MODE OF THE $f_1(1285)$



$$t_T = i\int \frac{d^4q}{(2\pi)^4}\, \vec{\epsilon}_{f_1}\cdot\vec{\epsilon}_{K^*}\; \vec{\epsilon}_{K^*}\cdot(2\vec{k}+\vec{q})\, \frac{1}{q^2-m_K^2+i\epsilon}\, \frac{1}{(P-q)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}\, \frac{1}{(P-q-k)^2-m_K^2+i\epsilon}$$

$$\tilde{t}_T = \int \frac{d^3q}{(2\pi)^3}\left(2+\frac{\vec{k}\cdot\vec{q}}{\vec{k}^2}\right)\frac{1}{8\,\omega(q)\omega'(q)\omega^*(q)}\,\frac{1}{k^0-\omega'(q)-\omega^*(q)+i\epsilon}\,\frac{1}{P^0-\omega^*(q)-\omega(q)+i\epsilon}$$
$$\times\frac{2P^0\omega(q)+2k^0\omega'(q)-2(\omega(q)+\omega'(q))(\omega(q)+\omega'(q)+\omega^*(q))}{(P^0-\omega(q)-\omega'(q)-k^0+i\epsilon)(P^0+\omega(q)+\omega'(q)-k^0-i\epsilon)},$$

$$\omega(q) = \sqrt{\vec{q}^{\,2}+m_K^2},\quad \omega'(q) = \sqrt{(\vec{q}+\vec{k})^2+m_K^2},\quad \omega^*(q) = \sqrt{\vec{q}^{\,2}+m_{K^*}^2}$$

Poles in the integration

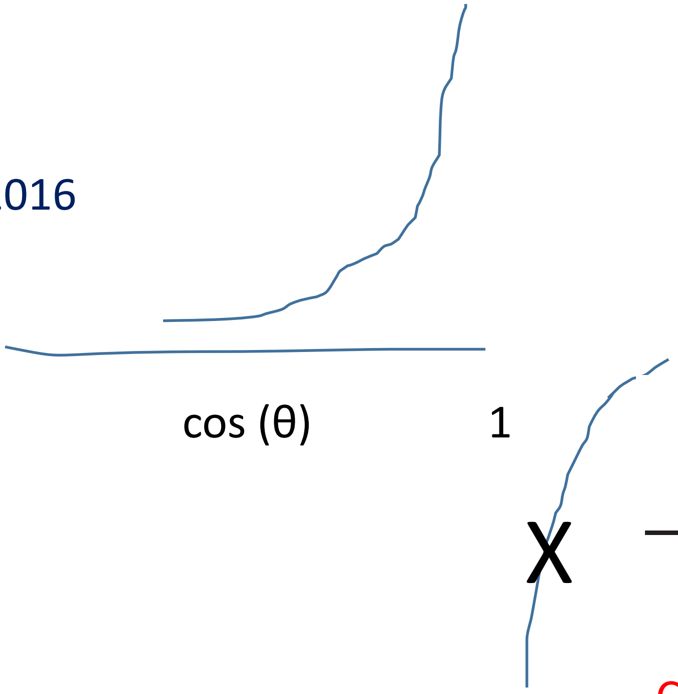
$$P^0-\omega^*(q)-\omega(q)+i\epsilon = 0, \qquad q_{\rm on+} = q_{\rm on}+i\,\epsilon \quad \text{with} \quad q_{\rm on} = \frac{1}{2M}\sqrt{\lambda(M^2,m_1^2,m_2^2)}$$

$$P^0-\omega(q)-\omega'(q)-k^0+i\epsilon = 0$$

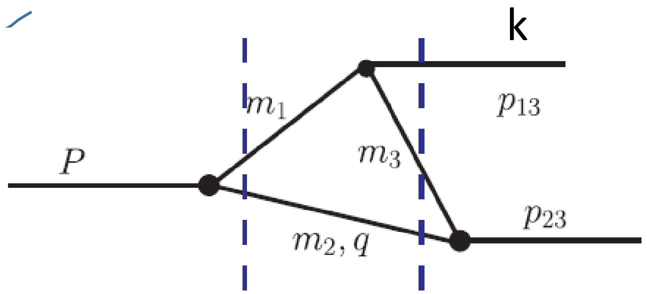
$$P^0 - \omega(q) - \omega'(q) - k^0 + i\epsilon \quad = \quad 0 \qquad \omega'(q) = \sqrt{(\vec{q}+\vec{k})^2 + m_K^2}$$

If we fix cos (θ)= ± 1 and we make this expression zero, then in the integral of cos(θ) one cannot cancel the divergence with the principal value, and the divergence remains. Θ is the angle between $\vec{k}$ and $\vec{q}$.

Bayar, Aceti, Guo, E. O, PRD 2016



For cos(θ)=-1



Calculate solution when p_{23} is at rest (p_2*) and make a boost

$$q_{a+} = \gamma\,(v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(v\,E_2^* - p_2^*) - i\,\epsilon$$

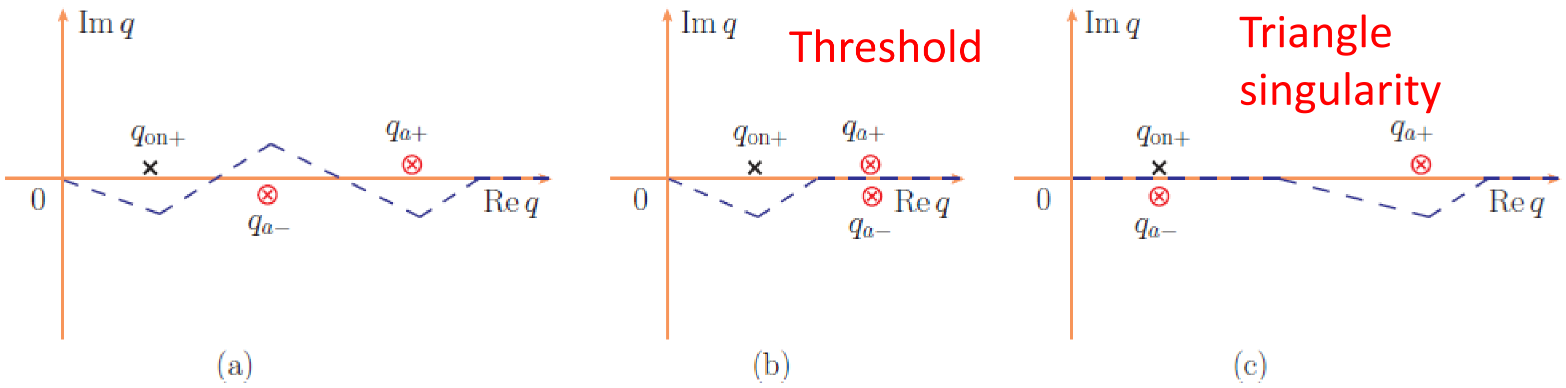
$$v = \frac{k}{E_{23}}\,, \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}}\,,$$

$$E_2^* = \frac{1}{2m_{23}}\left(m_{23}^2 + m_2^2 - m_3^2\right)\,, \qquad p_2^* = \frac{1}{2m_{23}}\sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}$$

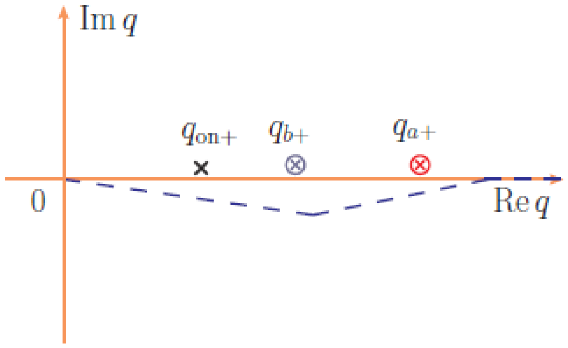
For cos(θ)=1

$$q_{b+} = \gamma\,(-v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{b-} = -\gamma\,(v\,E_2^* + p_2^*) - i\,\epsilon$$

For cos(θ)=-1

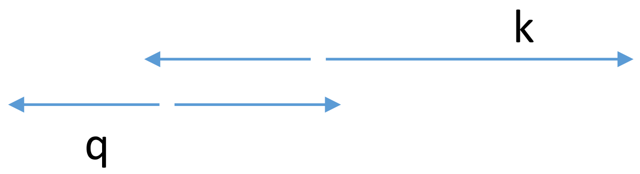


For cos(θ)=1



Triangle singularity

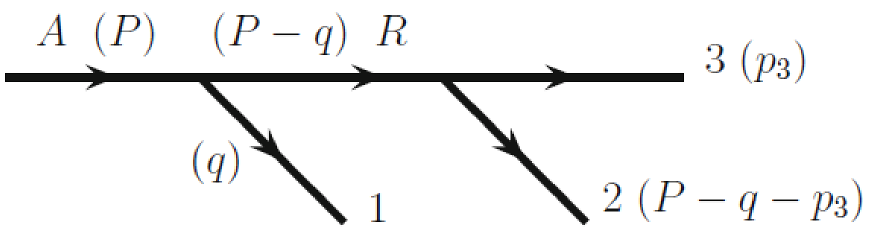
$$\lim_{\epsilon\to 0} (q_{on+} - q_{a-}) = 0$$

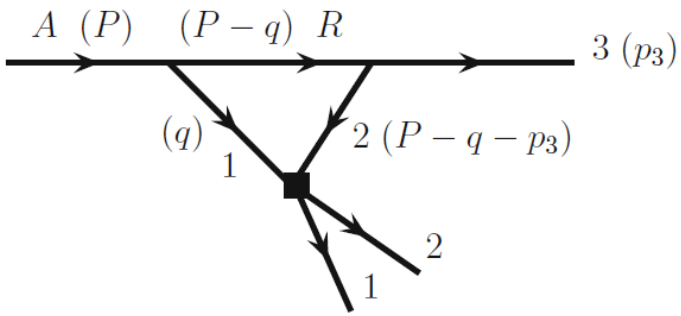


Very simple expression to see where the TS appears , and to explain the Coleman-Norton theorem, Nuovo Cim. 1965, (TS appears when the decays in the loop can occur at the classical level).

Considerations on the Schmid theorem for triangle singularities

V.R. Debastiani, S. Sakai, E. O.   EPJC 79, 69 (2019)





$t_t^{(0)}$ is the L=0 partial wave of the tree level amplitude

$$t_t^{(0)} + t_L = t_t^{(0)} e^{2i\delta}$$

t_L is the loop amplitude

$$\left|t_t^{(0)} + t_L\right|^2 = \left|t_t^{(0)} e^{2i\delta}\right|^2 = \left|t_t^{(0)}\right|^2$$

$$\begin{aligned}\int_{-1}^{1} d\cos\theta\, |t_t + t_L|^2 &= \int_{-1}^{1} d\cos\theta \left|t_t^{(\ell\neq 0)} + t_t^{(0)} + t_L\right|^2 \\ &= \int_{-1}^{1} d\cos\theta \left|t_t^{(\ell\neq 0)} + t_t^{(0)} e^{2i\delta}\right|^2 \\ &= \int_{-1}^{1} d\cos\theta \left(|t_t^{(\ell\neq 0)}|^2 + |t_t^{(0)}|^2\right) \\ &= \int_{-1}^{1} d\cos\theta |t_t|^2,\end{aligned}$$

C. Schmid, Final-state interactions and the simulation of resonances. Phys. Rev. **154**, 1363 (1967)

→ The rescattering mechanism does not change the width given by the tree level

$$\Gamma_A = \frac{1}{2M_A} \int \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2\omega_3} \frac{1}{8\pi} \int_{-1}^{1} d\cos\theta \, \tilde{p}_1 \frac{1}{M_{\rm inv}(12)} |t|^2$$

$$t_t = g_A \, g_R \frac{1}{2\omega_R(\vec{p}_3 - \vec{q}\,)} \frac{1}{\tilde{E}_A - \omega_1(q) - \omega_R(\vec{p}_3 - \vec{q}\,) + i\epsilon}$$

$$t_t^{(0)} = \frac{1}{2} \int_{-1}^{1} d\cos\theta \frac{1}{2\omega_R(\vec{p}_3 - \vec{q}\,)} \times \frac{g_A \, g_R}{\tilde{E}_A - \omega_1(q) - \omega_R(\vec{p}_3 - \vec{q}\,) + i\epsilon}$$

$$t_L = \frac{1}{(2\pi)^2} \int_0^{\infty} q^2 \, dq \frac{1}{2\omega_1(q)} \frac{1}{2\omega_2(q)} \times \frac{1}{\tilde{E}_A - \tilde{E}_3 - \omega_1(q) - \omega_2(q) + i\epsilon} \times 2 \frac{1}{2} \int_{-1}^{1} d\cos\theta \frac{1}{2\omega_R(\vec{p}_3 - \vec{q}\,)} \times \frac{1}{\tilde{E}_A - \omega_1(q) - \omega_R(\vec{p}_3 - \vec{q}\,) + i\epsilon} g_A \, g_R \, t_{12,12}$$

Picking up the singular part of the q integration we find

$$t_L = \frac{1}{(2\pi)^2} \frac{1}{4} |q_{\rm on}| \frac{1}{M_{\rm inv}(12)} g_A \, g_R \, t_{12,12} \, (-) 4\pi i \times \frac{1}{2} \int_{-1}^{1} d\cos\theta \frac{1}{2\omega_R(\vec{p}_3 - \vec{q}\,)} \times \frac{1}{\tilde{E}_A - \omega_1(q) - \omega_R(\vec{p}_3 - \vec{q}\,) + i\epsilon}.$$

$$t_L = -i \frac{1}{4\pi} |q_{\rm on}| \frac{1}{M_{\rm inv}(12)} t_{12,12} \, t_t^{(0)}$$

If iε becomes iΓ/2 the theorem is no longer exact

$$t_L = -i\,\frac{1}{4\pi}\,|q_{\rm on}|\,\frac{1}{M_{\rm inv}(12)}\,t_{12,12}\,t_t^{(0)}$$

$$f = -\frac{1}{8\pi}\,\frac{1}{M_{\rm inv}}\,t$$

$$t_L = 2\,i\,|q_{\rm on}|\,f\,t_t^{(0)}$$

$$f = \frac{\eta\,e^{2i\delta} - 1}{2\,i\,|q_{\rm on}|}$$

$$t_t^{(0)} + t_L = t_t^{(0)}(1 + 2\,i\,|q_{\rm on}|\,f) = \eta\,e^{2i\delta}\,t_t^{(0)}$$

$$\begin{aligned}\int_{-1}^{1} d\cos\theta\,|t_t+t_L|^2 &= \int_{-1}^{1} d\cos\theta\left\{\left|t_t^{(\ell\neq0)}\right|^2 + \eta^2\left|t_t^{(0)}\right|^2\right\}\\ &= \int_{-1}^{1} d\cos\theta\left\{\left|t_t^{(\ell\neq0)}\right|^2 + \left|t_t^{(0)}\right|^2 - (1-\eta^2)\left|t_t^{(0)}\right|^2\right\}\\ &= \int_{-1}^{1} d\cos\theta\,|t_t|^2\; - (1-\eta^2)\int_{-1}^{1} d\cos\theta\left|t_t^{(0)}\right|^2 \qquad (46)\end{aligned}$$

and since $t_t^{(0)}$ contains the same singularity as t_L, the singularity of the triangle diagram will show up with a strength of $1-\eta^2$.

A.V. Anisovich, V.V. Anisovich, Rescattering effects in three particle states and the Schmid theorem. Phys. Lett. B **345**, 321 (1995)

$$\frac{d\Gamma_A^{(L)}}{dM_{\rm inv}(12)} \propto \int_{-1}^{1} d\cos\theta \, \left|t_t^{(0)}\right|^2$$

$$\sim 2\left(\frac{g_A\, g_R}{4 p_3\, q}\right)^2 \left|\ln\left(\frac{\Gamma_R}{\omega_R(p_3-q)-\omega_R(p_3+q)}\right)\right|^2 . \quad (51)$$

$$\frac{d\Gamma_A^{(t)}}{dM_{\rm inv}(12)} \propto \frac{1}{4 p_3\, q} \, \frac{g_A^2\, g_R^2}{\widetilde{E}_A - \omega_1(q)} \, \frac{\pi}{\Gamma_R} .$$

1) First consequence: in the limit where Schmid theorem is exact, Γ =0, it is also irrelevant because the tree level is infinitely larger that the triangle singularity

2) The theorem does not hold when the 1+2 -> 1+2 amplitude has inelasticities

3) What happens when Γ is not small?

$M_A = 2154\,\mathrm{MeV}$

$M_R = 1600\,\mathrm{MeV}, \quad \Gamma_R = 30\,\mathrm{MeV}$

$M_1 = 500\,\mathrm{MeV}$

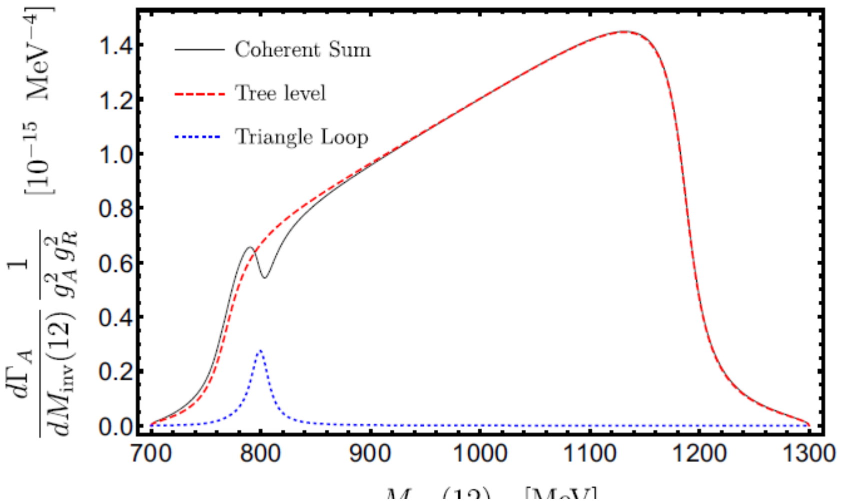
$M_2 = 200\,\mathrm{MeV}$

$M_3 = 900\,\mathrm{MeV}.$

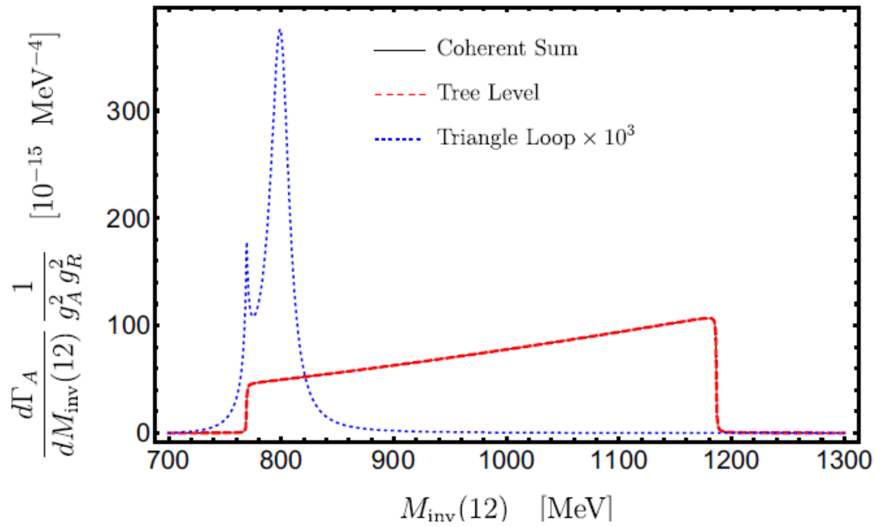
Example of relalistic calculation

$$t_{12,12} = \frac{g^2}{M_{\rm inv}^2(12) - M_{\rm BW}^2 + i\,\Gamma_{BW}(M_{\rm inv}(12))\,M_{\rm inv}(12)}$$

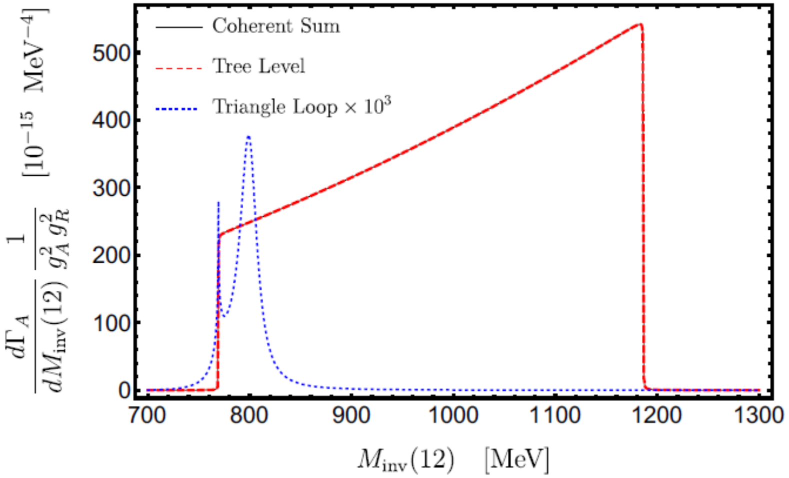
$M_A = 2200$ MeV, $\Gamma_R = 30$ MeV.



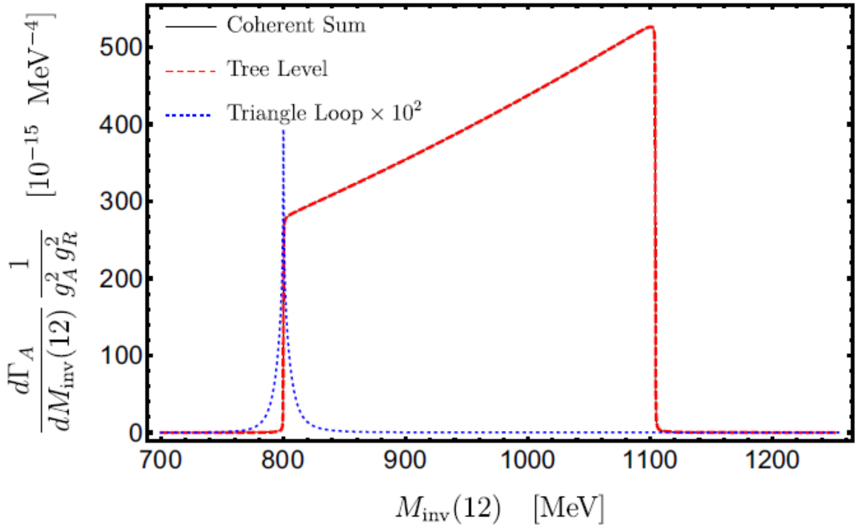
$M_A = 2200$ MeV, $\Gamma_R = 0.5$ MeV.



$M_A = 2200$ MeV, $\Gamma_R = 0.1$ MeV.



$M_A = 2154$ MeV, $\Gamma_R = 0.1$ MeV.



Conclusions:

1) First consequence: in the limit where Schmid theorem is exact, $\Gamma = 0$, it is also irrelevant because the tree level is infinitely larger that the triangle singularity

2) The theorem does not hold when the 1+2 -> 1+2 amplitude has inelasticities

3) For realistic widths do not rely upon Schmid theorem and evaluate the rescattering diagram

4) Even then, the coherent sum of tree level and TS rescattering changes the cross section, or differential width, less than an incoherent sum of the processes, and much less than a coherent sum of terms with the same phase. THE PROCESS STILL HAS SOME MEMORY OF SCHMID THEOREM