Triangle singularity in the $J/\psi \to K^+K^- f_0(980)/a_0(980)$ decays

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Guilin 2019/08/20

From threshold cusp to triangle singularity

Singularities systematized by landau [L. D. Landau (1959)]:

- Two-body threshold cusp,
- Triangle singularity…

were fashionable in the sixties, see references:

R. Karplus et al, Phys. Rev. 111, 1187 (1958);

R. F. Peierls, Phys. Rev. Lett. 6, 641 (1961);

I. J. R. Aitchison, Phys. Rev. 133, B1257 (1964);

Y. F. Chang and S. F. Tuan, Phys. Rev. 136, B741 (1964);

J. B. Bronzan, Phys. Rev. 134, B687 (1964);

S. Coleman and R. E. Norton, Nuovo Cim. 38, 438 (1965);

C. Schmid, Phys. Rev. 154, 1363 (1967);

N. E. Booth, Phys. Rev. Lett. 7, 35 (1961);

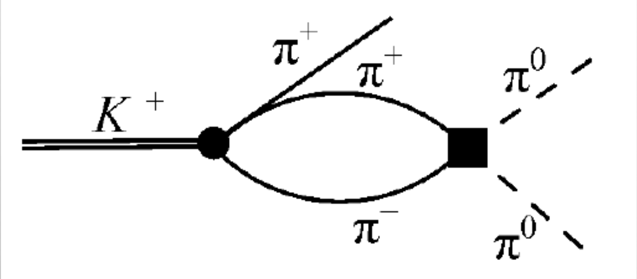
V. V. Anisovich, Phys. Lett. 10, 221 (1964)…

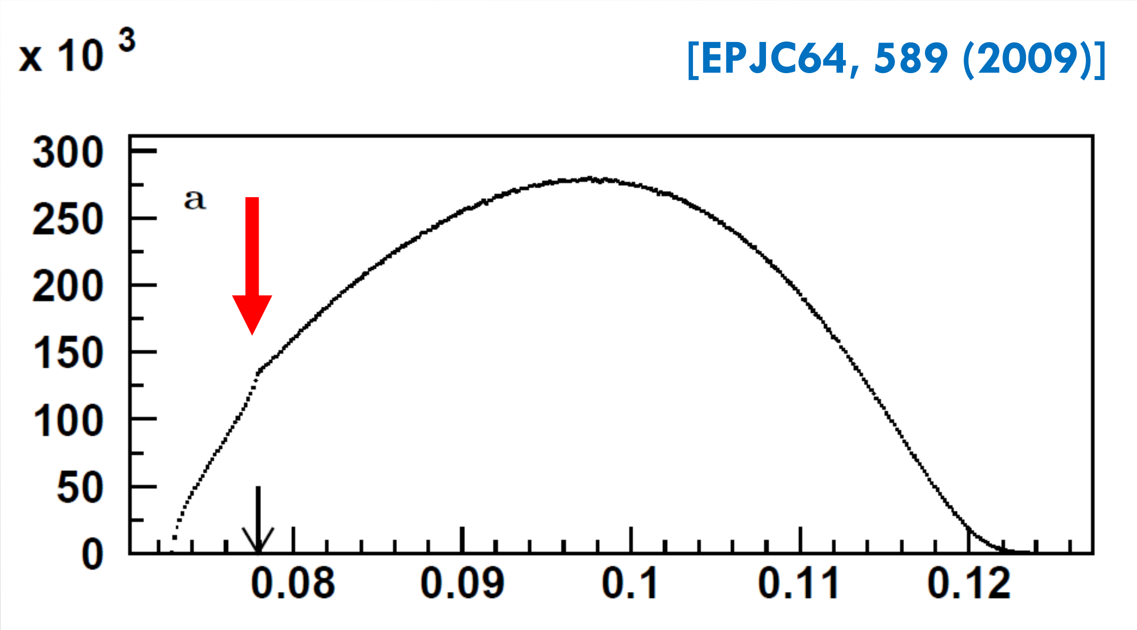
From threshold cusp to triangle singularity

- **A well-known example of the threshold cusp:**

the precise measurement of the $\pi\pi$ S-wave scattering length

from the cusp at the $\pi^+\pi^-$ threshold



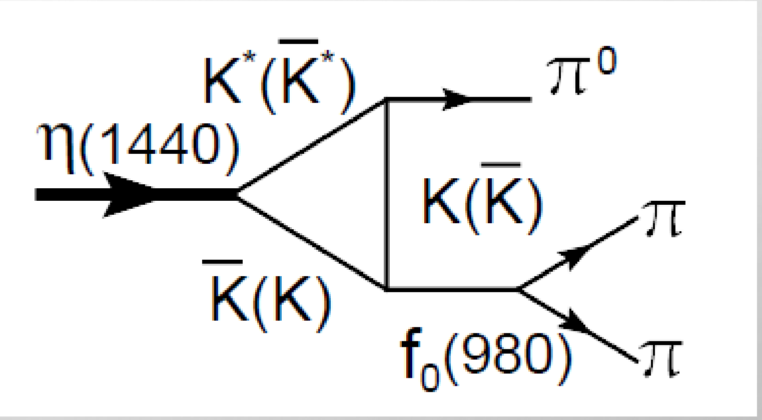


[Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); . . .]

From threshold cusp to triangle singularity

Anomalously large isospin breaking in $\eta(1405) \to \pi^0 f_0(980)$ reaction was found in [BESIII, Phys. Rev. Lett. 108, 182001 (2012)].

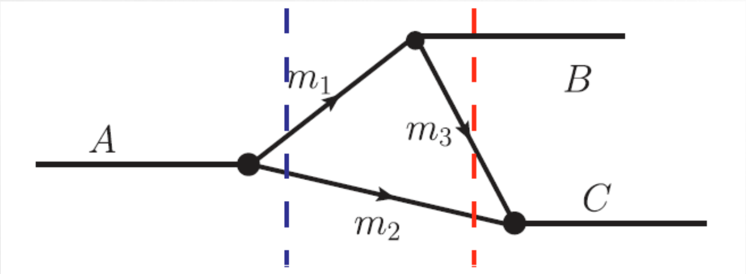
This was explained in terms of the $K\bar{K}K^*$ triangle singularity in
[J.-J. Wu et al, PRL108, 081803 (2012)],
[F. Aceti et al, PRD86, 114007 (2012)],
[X.-G. Wu et al, PRD87, 014023 (2013)],
[Achasov et al, PRD92, 036003 (2015)] ...



Coleman-Norton Theorem

The Coleman-Norton Theorem [Coleman, Norton (1965); Bronzan (1964)] tells that the triangle singularity is in the physical region only when the process can happen classically:

- All the intermediate states are on shell.
- The particle 3 emitted from the decay of the particle 1 moves along the same direction as the particle 2 with a larger speed than the particle 2 and can catch up with it to rescatter.



$$i\int \frac{d^4q}{(2\pi)^4}\frac{i}{(P-q)^2-m_1^2+i\varepsilon}\frac{i}{q^2-m_2^2+i\varepsilon}\frac{i}{(P-q-k)^2-m_3^2+i\varepsilon}$$

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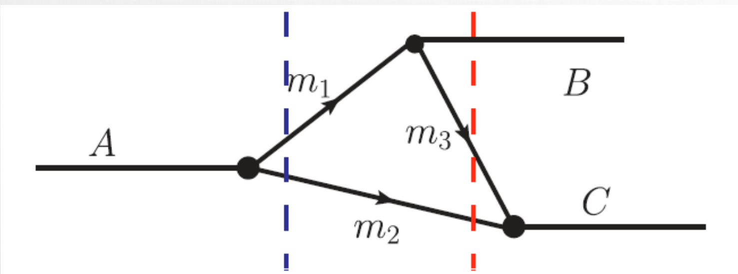

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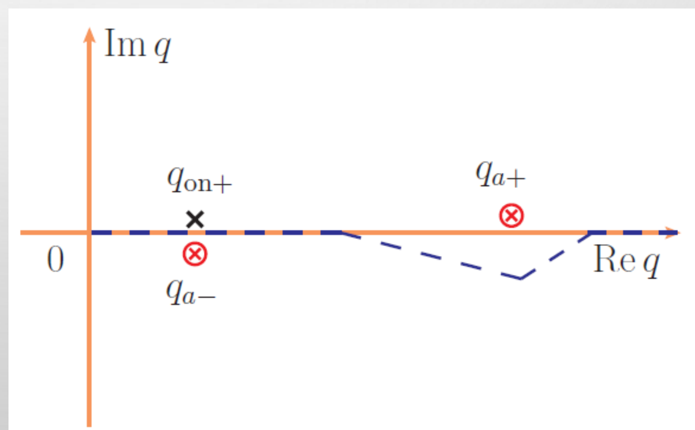
triangle singularity: $q_{on+} = q_{a-}$

$$q_{on+} = \frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} + i\varepsilon$$

$$q_{a+} = \gamma(vE_2^* + p_2^*) + i\varepsilon$$

$$q_{a-} = \gamma(vE_2^* - p_2^*) + i\varepsilon$$



only depends on kinematics

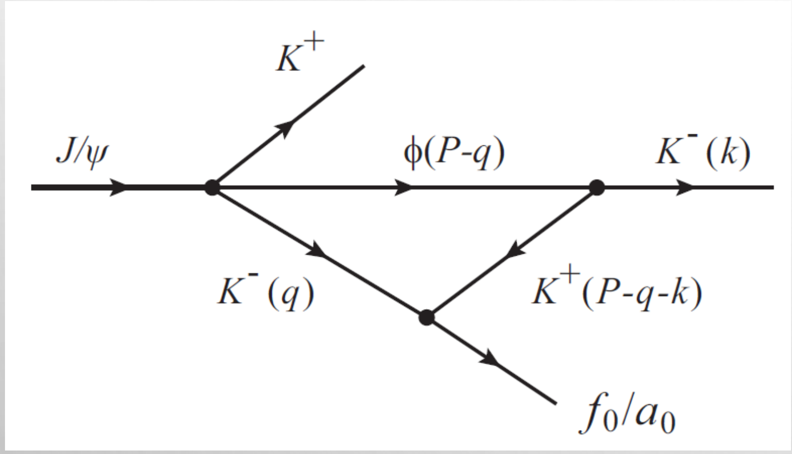
[M. Bayar, F. Aceti, F. K. Guo, and E. Oset, PRD94, 074039 (2016)]

References in recent years

M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD91, 094015 (2015);
A. P. Szczepaniak, PLB747, 410 (2015);
F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, PRD92, 071502 (2015);
X. H. Liu, M. Oka and Q. Zhao, PLB753, 297 (2016);
X.-H. Liu, Q. Wang and Q. Zhao, PLB757, 231 (2016);
A. E. Bondar and M. B. Voloshin, PRD93, 094008 (2016);
X. H. Liu and U. G. Meissner, EPJC77, 816 (2017);
R. Pavao, S. Sakai and E. Oset, EPJC77, 599 (2017);
J.-J. Xie, L.-S. Geng and E. Oset, Phys. Rev. D 95, 034004 (2017);
E. Wang, J.-J. Xie, W.-H. Liang, F.-K. Guo and E. Oset, PRC95, 015205 (2017);
J. J. Xie and F. K. Guo, PLB774, 108 (2017);
S. Sakai, E. Oset and A. Ramos, EPJA54, 10 (2018);
Q.-R. Gong, J.-L. Pang, Y.-F. Wang and H.-Q. Zheng, EPJC78, 276 (2018);
W. H. Liang, S. Sakai, J. J. Xie and E. Oset, CPC42, 044101 (2018);
J. J. Xie and E. Oset, PLB792, 450-453 (2019);
L. R. Dai, Q. X. Yu and E. Oset, PRD99, 016021 (2019);
V. R. Debastiani, S. Sakai and E. Oset, EPJC79, 69 (2019);
Z. Cao and Q. Zhao, PRD99, 014016 (2019);
F.-K. Guo, PRL122, 202002 (2019) …

$J/\psi \to K^+K^-f_0(980)/a_0(980)$ decays

A triangle singularity can be found at $M_{inv}(K^-f_0/K^-a_0) \approx 1515$ MeV:



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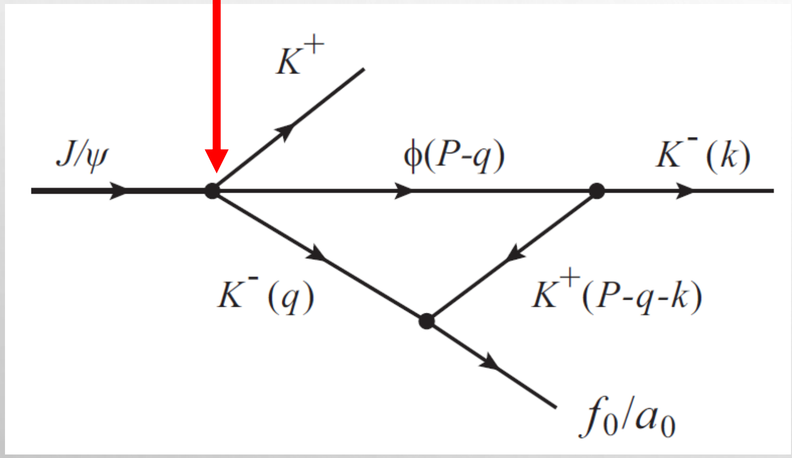

- $Br(J/\psi \to K^+K^-\phi) = (8.3 \pm 1.2) \times 10^{-4}$

$$t_{J/\psi \to K^+K^-\phi} = A\, \varepsilon(J/\psi) \cdot \varepsilon(\phi)$$

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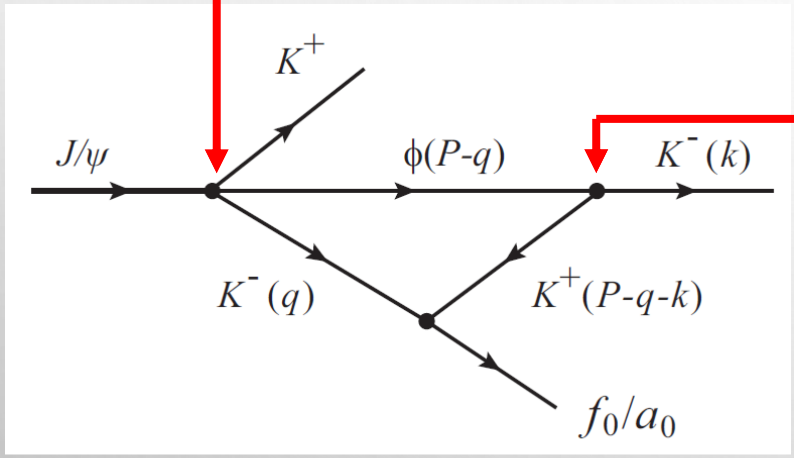

- $Br(J/\psi \to K^+K^-\phi) = (8.3 \pm 1.2) \times 10^{-4}$
- $\Gamma(\phi) = 4.25$ MeV

$$t_{\phi \to K^+K^-} = g_V\,(p^\mu_{K^+} - p^\mu_{K^-}) \cdot \varepsilon_\mu(\phi)$$

$J/\psi \to K^+K^- f_0(980)/a_0(980)$ decays

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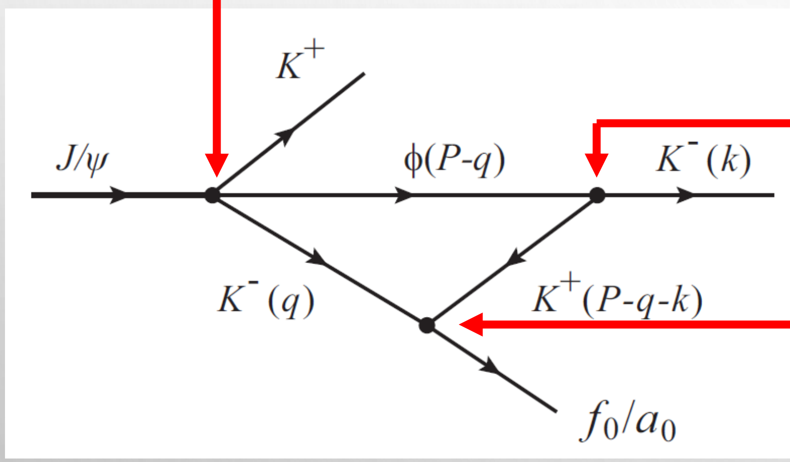
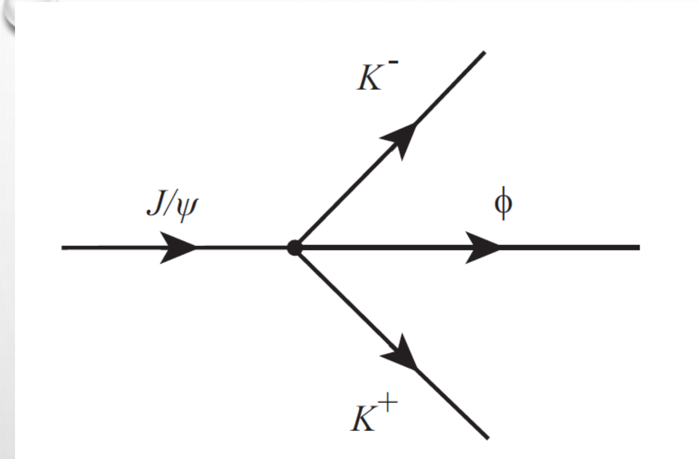


- $Br(J/\psi \to K^+K^-\phi) = (8.3 \pm 1.2) \times 10^{-4}$
- $\Gamma(\phi) = 4.25$ MeV
- **Well studied within chiral unitary model, see:**
  W. H. Liang and E. Oset, PLB 737, 70 (2014)
  J. J. Xie, L. R. Dai and E. Oset, PLB742, 363 (2015)

$g_{f_0/a_0, K^+K^-}$

Step 1: tree diagram



$$t_{J/\psi\to K^+K^-\phi} = \boldsymbol{A}\,\boldsymbol{\varepsilon}(J/\psi)\cdot\boldsymbol{\varepsilon}(\phi)$$

$$\frac{d\Gamma_{J/\psi\to K^+K^-\phi}}{dM_{inv}(K^-\phi)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{J/\psi}^2}p_{K^+}\tilde{p}_{K^-}\sum|t_{J/\psi\to K^+K^-\phi}|^2$$

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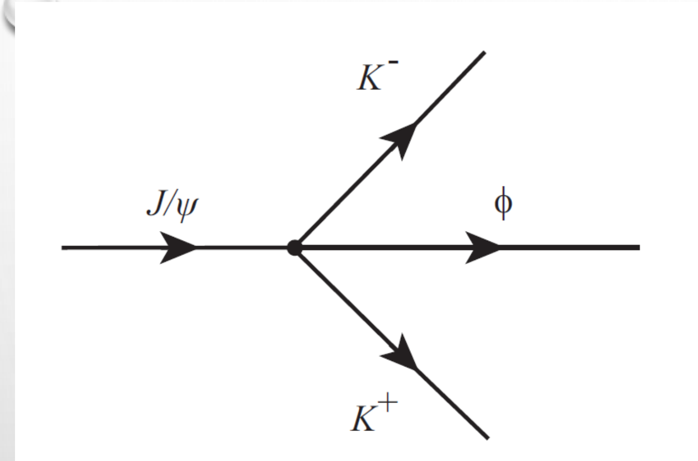

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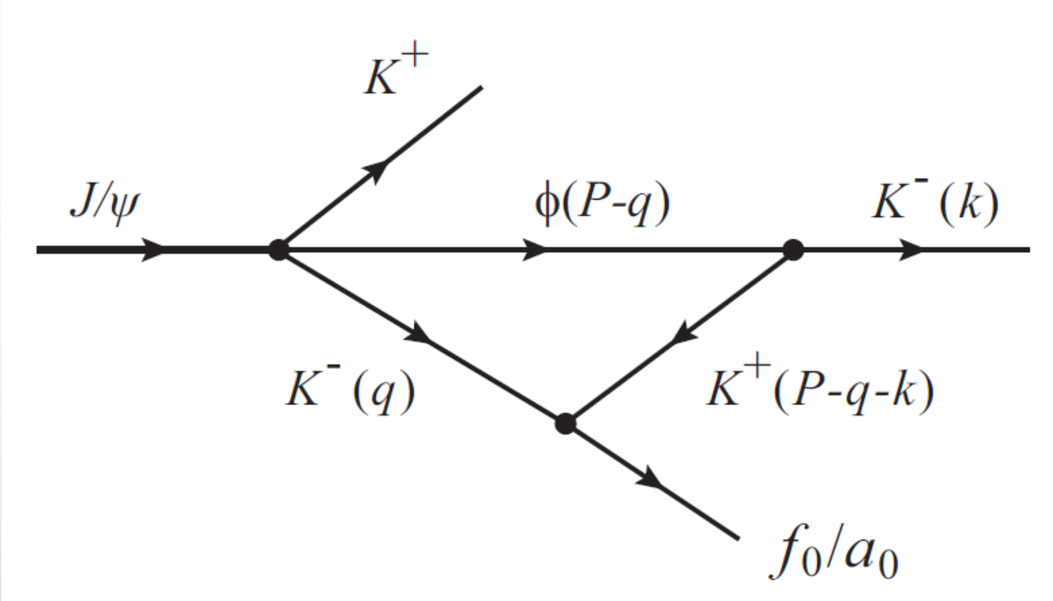
$$\frac{\boldsymbol{A}^2}{\Gamma_{J/\psi}} = \frac{Br(J/\psi \to K^+K^-\phi)}{\int \frac{1}{(2\pi)^3} \frac{1}{4M_{J/\psi}^2} p_{K^+} \tilde{p}_{K^-}\, dM_{inv}(K^-\phi)} = 0.018 \pm 0.003\ \mathrm{MeV}^{-1}$$

Step 2: triangle diagram

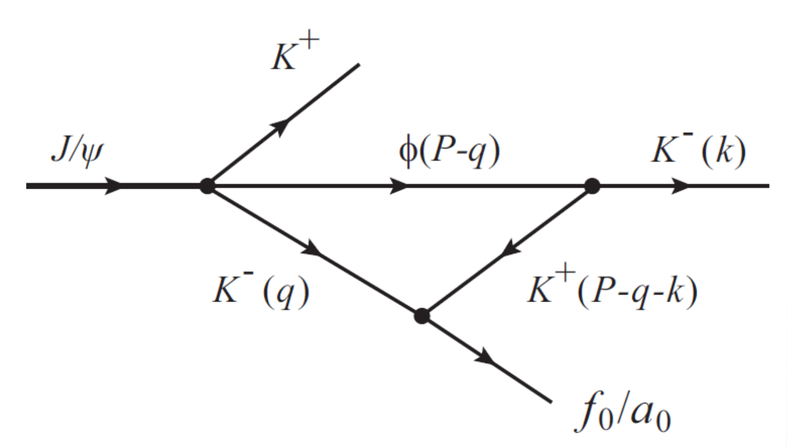


$$-\boldsymbol{it} = -i\boldsymbol{A}\int \frac{d^4q}{(2\pi)^4}\frac{i}{(P-q)^2-m_\phi^2+i\varepsilon}\frac{i}{q^2-m_{K^-}^2+i\varepsilon}\frac{i}{(P-q-k)^2-m_{K^+}^2+i\varepsilon}$$

$$\times\,\boldsymbol{\varepsilon}(J/\psi)\cdot\boldsymbol{\varepsilon}(\phi)\boldsymbol{\varepsilon}(\phi)\cdot(2\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{P})\times(-ig_V)\times(\boldsymbol{-ig_{f_0,K^+K^-}})$$

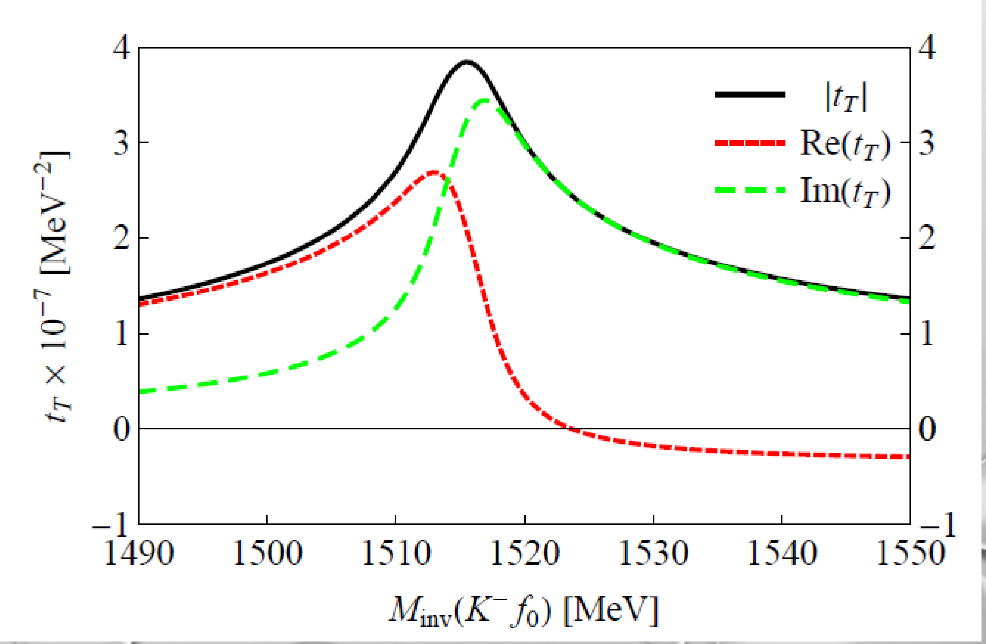
$$= \boldsymbol{-iA\,g_V\,g_{f_0,K^+K^-}\,\varepsilon(J/\psi)\cdot k\,t_T}$$

Step 2: triangle diagram



$$t = A\, g_V\, g_{f_0,K^+K^-}\, \varepsilon(J/\psi)\cdot k\, t_T$$

triangle amplitude t_T with
a peak at $M_{inv}(K^-f_0/K^-a_0) \approx 1515$ MeV



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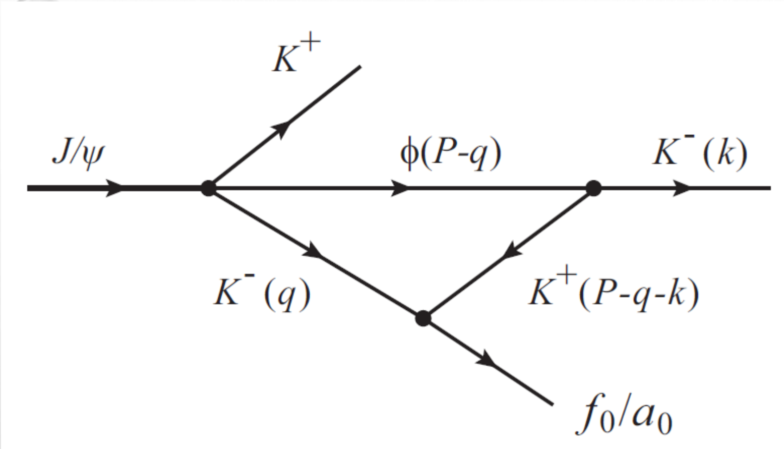

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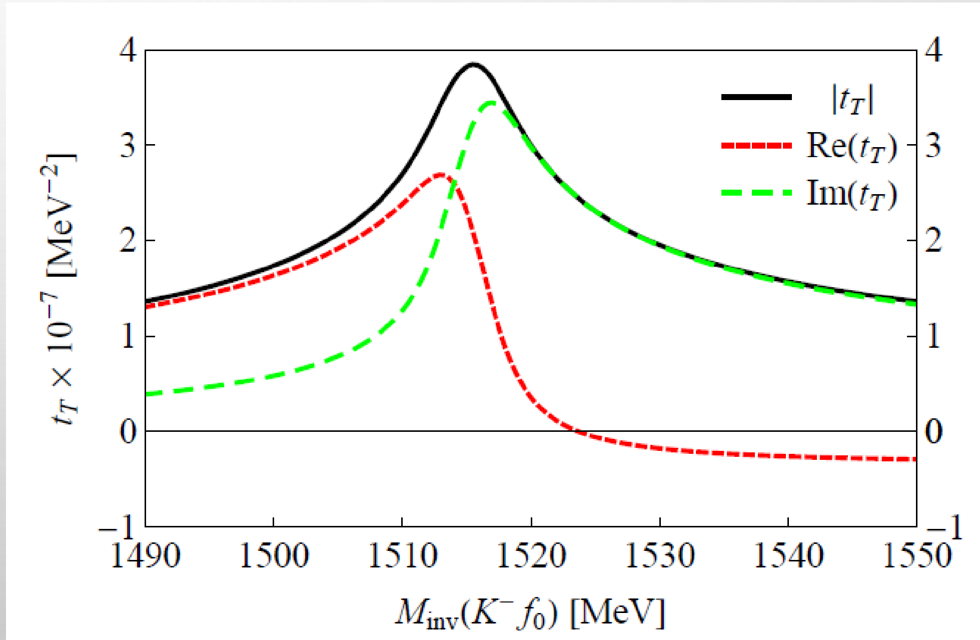
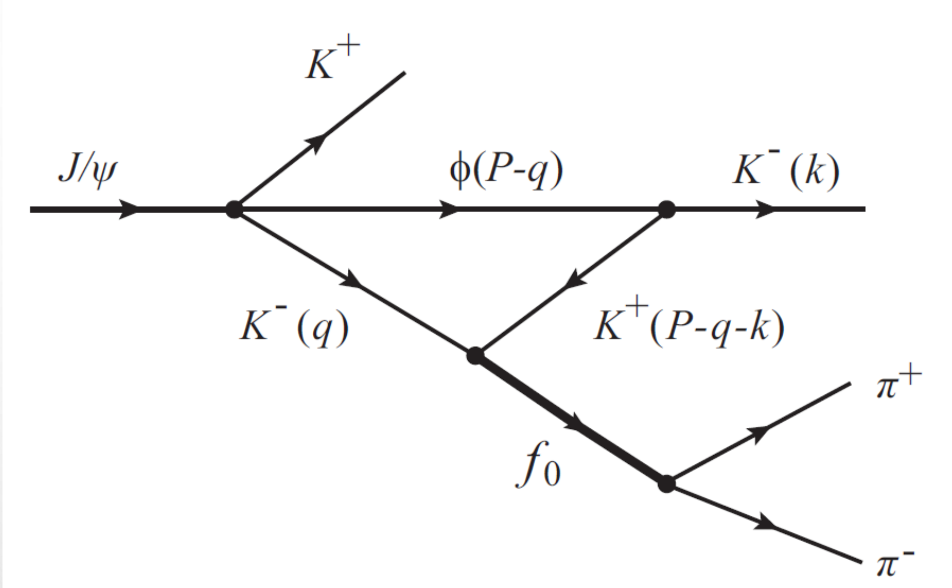


Step 3: final state interaction



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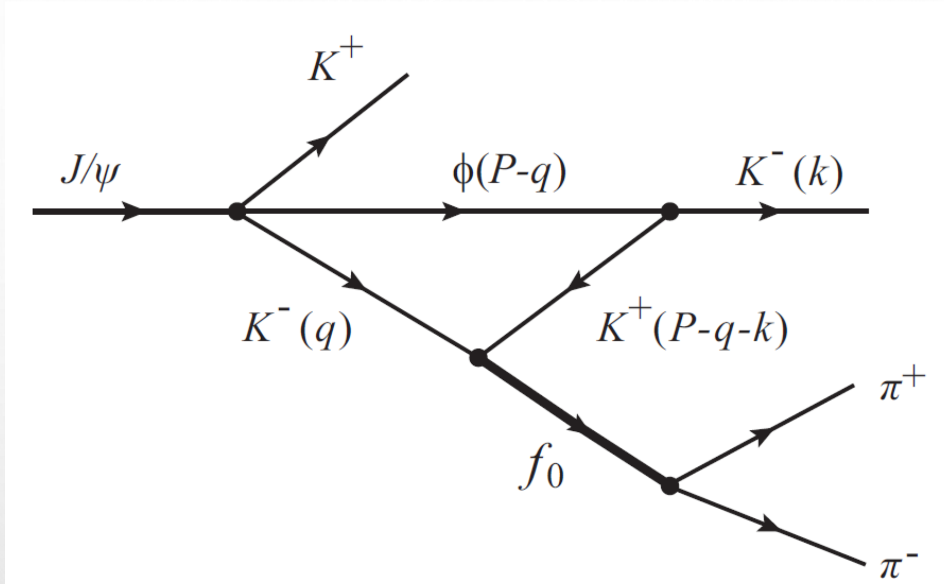


$$-it' = -iA\int \frac{d^4q}{(2\pi)^4}\frac{i}{(P-q)^2-m_\phi^2+i\varepsilon}\frac{i}{q^2-m_{K^-}^2+i\varepsilon}\frac{i}{(P-q-k)^2-m_{K^+}^2+i\varepsilon}$$

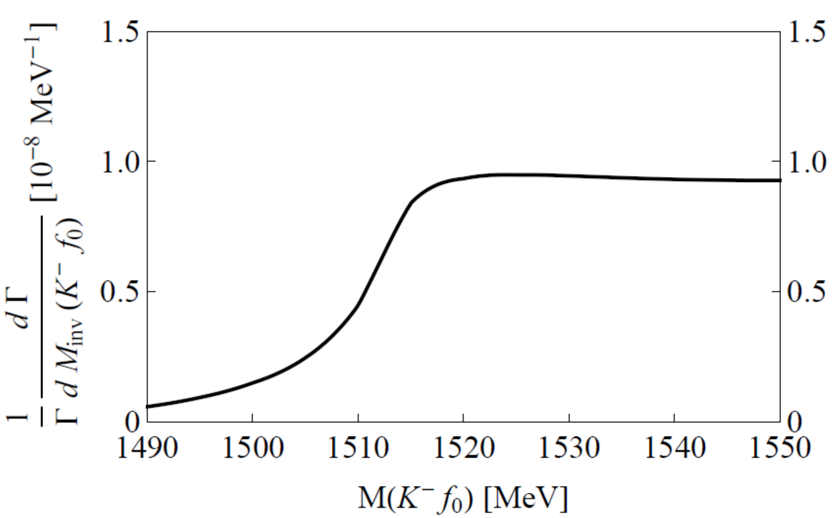
$$\times\, \boldsymbol{\varepsilon}(J/\psi)\cdot\boldsymbol{\varepsilon}(\phi)\boldsymbol{\varepsilon}(\phi)\cdot(2\boldsymbol{k}+\boldsymbol{q}-\boldsymbol{P})\times(-ig_V)\times(-ig_{f_0,K^+K^-})$$

$$(-i\, t_{K^+K^-,\pi^+\pi^-})$$

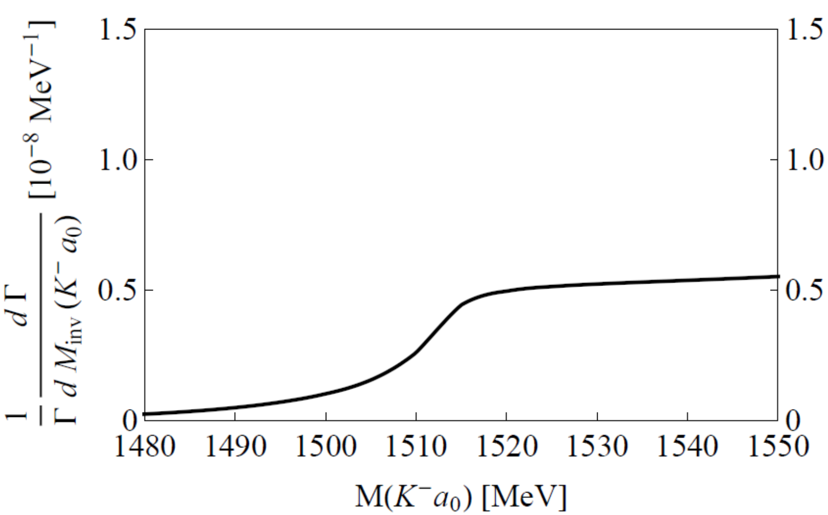
$$= -iA\, g_V\, t_{K^+K^-,\pi^+\pi^-}\, \boldsymbol{\varepsilon}(J/\psi)\cdot \boldsymbol{k}'\, t_T'$$

Step 3: final state interaction

$$\frac{1}{\Gamma_{J/\psi}}\frac{d^2\Gamma_{J/\psi\to K^+K^-f_0\to K^+K^-\pi^+\pi^-}}{dM_{inv}(K^-f_0)dM_{inv}(\pi^+\pi^-)} = \frac{\boldsymbol{A}^2}{\Gamma_{J/\psi}}\frac{1}{(2\pi)^5}\frac{1}{4M_{J/\psi}^2}\frac{1}{3}p''_{K^+}\tilde{p}''^3_{K^-}p''_{\pi^+}\, g_V^2|t_{J/\psi\to K^+K^-\phi}|^2\,|\boldsymbol{t'_T}|^2$$



(a)



(b)

$$\frac{1}{\Gamma_{J/\psi}}\frac{d^2\Gamma_{J/\psi\to K^+K^-f_0\to K^+K^-\pi^+\pi^-}}{dM_{inv}(K^-f_0)}$$

$$\frac{1}{\Gamma_{J/\psi}}\frac{d^2\Gamma_{J/\psi\to K^+K^-a_0\to K^+K^-\pi^0\eta}}{dM_{inv}(K^-a_0)}$$

Summary

- The following channels are accessible in present facilities:

$$Br(J/\psi \to K^+K^-f_0 \to K^+K^-\pi^+\pi^-) = 7.6 \times 10^{-6}$$
$$Br\left(J/\psi \to K^+K^-a_0 \to K^+K^-\pi^0\eta\right) = 5.2 \times 10^{-6}$$

- There is a sharp raise of this magnitude around $M_{inv}(K^-f_0/K^-a_0) \approx$ **1515 MeV**, where the triangle singularity appears.

- All these features are tied to the nature of the $f_0(980)$ and $a_0(980)$ as dynamically generated resonances.

Thank you very much!

谢谢