Hidden charm pentaquark states and $\Sigma_c^{(*)}\bar{D}^{(*)}$ interaction in ChPT

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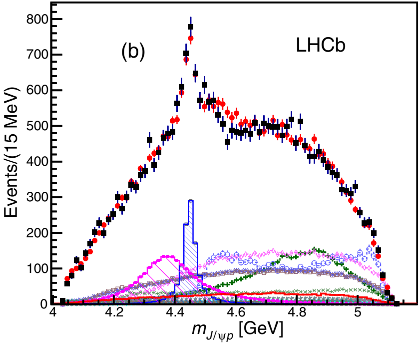
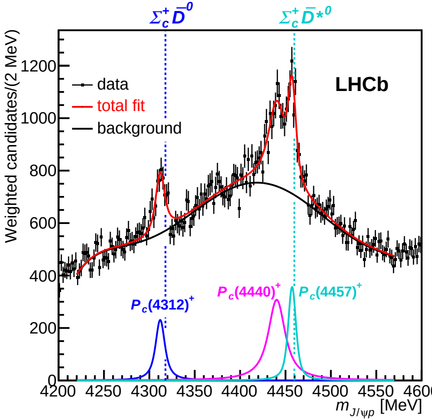
Contents

Introduction

P_c states in LHCb

- In 2015, $\Lambda_b \to J/\psi p K$, $P_c(4380)$ and $P_c(4450)$
- Recently, $P_c(4450) \Rightarrow P_c(4440) + P_c(4457)$; A new state $P_c(4312)$ with 7.3σ; $I = 1/2$?





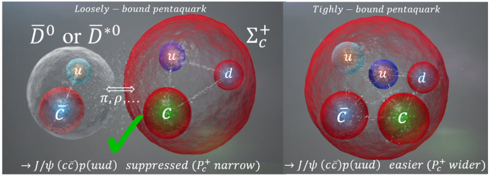
Phys.Rev.Lett. 115 (2015) 072001; Phys.Rev.Lett. 122 (2019) 222001 ;

Compact or molecular states ?

- Compact pentaquark states: tightly bounded states
- Molecular states: loosely bound states of two color singlet hadrons
- The three P_c states under the thresholds 9 MeV, 5 MeV and 22 MeV
- Three P_c states are the good candidates of molecular states
- Our work

  ⇒Obtain the $\Sigma_c^{(*)}\bar{D}^{(*)}$ potential in ChPT D-D,B-B interactions, Zhan-Wei Liu's talk

  ⇒Solve the Schrödinger Eq.



Phys.Rev.Lett. 122 (2019) 222001

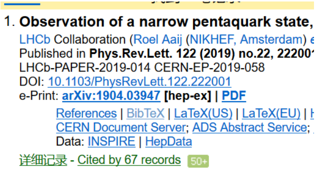
Why Chiral perturbation theory (ChPT)?

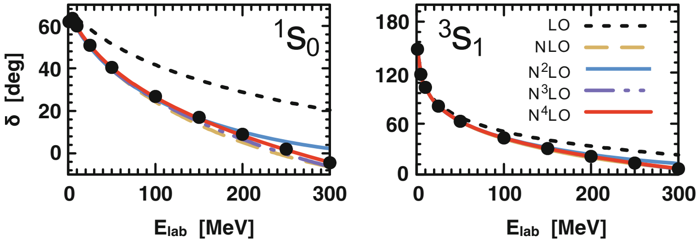
- Effective field theory, model independence

  ⇒ Systematically expansion, controllable and estimable error
- Loop diagrams bring novel effect

  ⇒ Heavy quark symmetry violation

  ⇒ Novel structure in potential
- Lattice QCD: chiral extrapolation
- Analogy between P_c states and deuteron
- Modern theory of nucleon force Phys. Rept. 503, 1 (2011).; Rev. Mod. Phys. 81, 1773 (2009).


1. **Observation of a narrow pentaquark state,**
LHCb Collaboration (Roel Aaij (NIKHEF, Amsterdam)
Published in **Phys.Rev.Lett. 122 (2019) no.22, 22200**
LHCb-PAPER-2019-014 CERN-EP-2019-058
DOI: 10.1103/PhysRevLett.122.222001
e-Print: **arXiv:1904.03947 [hep-ex] | PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) |
CERN Document Server; ADS Abstract Service;
Data: INSPIRE | HepData
详细记录 - Cited by 67 records 50+

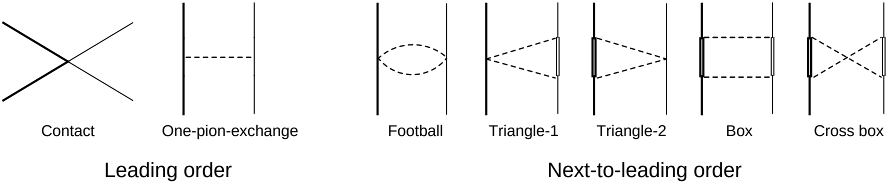


Lect.Notes Phys. 957 (2019) 1-396

$\Sigma_c^{(*)}\bar{D}^{(*)}$ interaction in ChPT

Feynman diagrams of $\Sigma_c^{(*)}\bar{D}^{(*)}$ to NLO

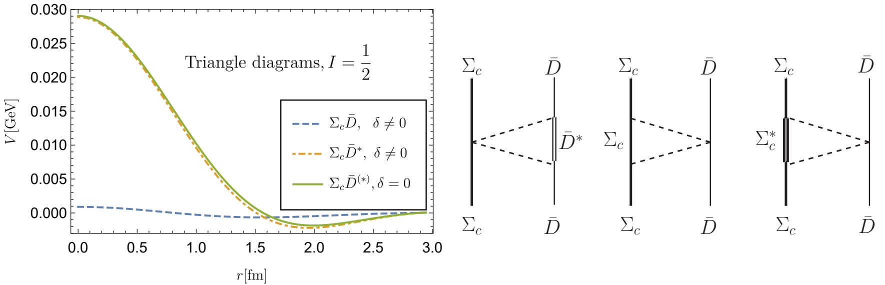
- Topological diagrams



- $\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$ and $\Sigma_c^*\bar{D}^*$
- All intermediate states: heavy quark symmetry (HQS) partner states and Λ_c
- Small-scale expansion: $\frac{m_\pi}{\Lambda_\chi}$, $\frac{p}{\Lambda_\chi}$ and $\frac{\delta}{M} \sim \frac{\epsilon}{\Lambda}$ J.Phys. G24 (1998) 1831-1859
- Keep mass splitting, HQS violation effect
- Unknown low energy constants (LECs): contact terms

Heavy quark symmetry: violation

- The heavy quark spin symmetry (HQSS) is approximate, which will be broken in the couple-channel calculation. David Entem's talk
- HQS violation effect is more significant for the $\Sigma_c^{(*)}\bar{D}$ systems than others



- Minimum of potential with the loosely bound state: -0.06-0.15 GeV.

The HQSS violation effect
may change the existence of bound states.

Heavy quark symmetry: quark model

- The heavy dof.: spectators; light dof.: interactions

$$V_{\text{quark-level}} = \left[V_a + \tilde{V}_a \boldsymbol{l}_1 \cdot \boldsymbol{l}_2\right] + \left[\frac{V_c}{m_c}\boldsymbol{l}_1 \cdot \boldsymbol{h}_2 + \frac{V_d}{m_c}\boldsymbol{l}_2 \cdot \boldsymbol{h}_1 + \frac{V_e}{m_c^2}\boldsymbol{h}_1 \cdot \boldsymbol{h}_2\right],$$

$$V_{\Sigma_c \bar{D}} = V_1, \quad V_{\Sigma_c \bar{D}^*} = V_2 + \tilde{V}_2 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2,$$

$$V_{\Sigma_c^* \bar{D}} = V_3, \quad V_{\Sigma_c^* \bar{D}^*} = V_4 + \tilde{V}_4 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2.$$

$$V_1 = V_2 = V_3 = V_4 = V_a$$

$$\tilde{V}_2 = \frac{2}{3}\tilde{V}_a, \quad V_4 = \frac{1}{3}\tilde{V}_a$$

- Ignoring mass splittings in loops, the HQS manifests itself

M. Z. Liu, et.al Phys.Rev.Lett. 122, 242001; C. W. Xiao,et.al Phys.Rev. D100, 014021;Y. Shimizu, et.al arXiv:1904.00587...

- In QM, the HQS violation vanishes for $\Sigma_c \bar{D}$ system

$$\langle \boldsymbol{l}_1 \cdot \boldsymbol{h}_2 \rangle = \langle \boldsymbol{l}_2 \cdot \boldsymbol{h}_1 \rangle = \langle \boldsymbol{h}_1 \cdot \boldsymbol{h}_2 \rangle = 0 \tag{1}$$

- QM: analytical terms; Loop diagrams: nonanalytical structures

Loops bring novel effects.

Novel structure in potential

- Structures for two-nucleon force

$$\{1, \quad \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \quad i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{q}\times\boldsymbol{k}, \quad \boldsymbol{\sigma}_1\cdot\boldsymbol{q}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}, \quad \boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{k}\} \quad \otimes \quad \{1, \quad \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}$$

- For S-wave, two spin structures left: $\{1, \quad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\}$
- Above results base on that the Pauli matrices are nilpotent.
  $\Rightarrow$ The power of the Pauli matrices is at most one.

$$[\sigma_i, \sigma_j] = i2\epsilon_{ijk}\sigma_k, \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{I}$$

- For the two-body systems with arbitrary spin

$$[S_i, S_j] = i\epsilon_{ijk}S_k, \quad \{S_i, S_j\} \not\propto \delta_{ij}\mathbb{I}$$

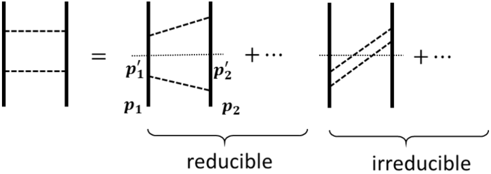
- For S-wave $\Sigma_c^*\bar{D}^*$: $\{1, \quad \boldsymbol{S}_1\cdot\boldsymbol{S}_2, \quad (\boldsymbol{S}_1\cdot\boldsymbol{S}_2)^2, ...\}$
- In the loops, $(\boldsymbol{S}_1\cdot\boldsymbol{S}_2)^2$ terms appear
  $\Rightarrow$ heavy quark spin symmetry violation effect

Loops bring novel effects.

Weinberg's formalism: NN system

- The box diagrams is enhanced by pinch singularity, power count fails
- Time-ordered perturbation theory

$$Amp = \langle NN|H_I|NN\rangle + \sum_{\psi} \frac{\langle NN|H_I|\psi\rangle\langle\psi|H_I|NN\rangle}{E_{NN} - E_{\psi}} \tag{2}$$

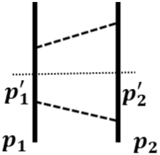


$$\begin{aligned} Amp_{\text{2PR}} &\sim (E_{inital} - E_{inter})^{-1} \\ &\sim \frac{m_N}{\boldsymbol{p^2}} \gg \frac{1}{\epsilon} \end{aligned}$$

- Only include the two particle irreducible (2PIR) graphs in potential
- Potential as the kernel of Lippmann-Schwinger Eq. or Schrödinger Eq.
- The tree level one-pion exchange diagrams would be iterated to generate the 2PR contributions automatically

Nucl.Phys. B363 (1991) 3-18; Phys.Lett. B291 (1992) 459-464; Nucl.Phys. A625 (1997) 758-788

Weinberg's formalism: $\Sigma_c^{(*)}\bar{D}^{(*)}$ system



$$Amp \sim (E_{inital} - E_{inter})^{-1}$$
$$\approx \left[(m_1 + m_2 + \frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2}) - (m_1' + m_2' + \frac{\boldsymbol{p}_1'^2}{2m_1} + \frac{\boldsymbol{p}_2'^2}{2m_2})\right]^{-1}$$

- When $m_1 = m_1'$, $m_2 = m_2'$

$$Amp \sim \frac{m}{\boldsymbol{p^2}} \gg \frac{1}{\epsilon}$$

$\Rightarrow$ Pinch singularity, power counting fails

$\Rightarrow$ Pole of heavy hadrons

$\Rightarrow$ Iterating $1-\pi$ exchange (elastic channel)

$\Rightarrow$ Need to be subtracted

- When $m_1 - m_1' = \delta_1$, $m_2 - m_2' = \delta_2$

$$Amp \sim \frac{1}{\delta_1 + \delta_2} \sim \frac{1}{\epsilon}$$

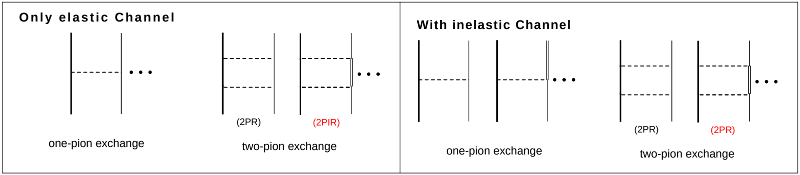
$\Rightarrow$ small scale expansion, power counting works

$\Rightarrow$ Pole of heavy hadrons

$\Rightarrow$ Iterating $1-\pi$ exchange (inelastic channel)

$\Rightarrow$ Depend on your scheme

Couple Channel effect



$\Rightarrow$ tree: $\Sigma_c \bar{D}^* - \Sigma_c \bar{D}^*$

$\Rightarrow$ tree : $\begin{cases} \Sigma_c \bar{D}^* - \Sigma_c \bar{D}^* \\ \Sigma_c \bar{D}^* - \Sigma_c^* \bar{D}^* \\ \Sigma_c^* \bar{D}^* - \Sigma_c^* \bar{D}^* \end{cases}$

$\Rightarrow$ box: $\Sigma_c \bar{D}^*$-$\Sigma_c \bar{D}^*$-$\Sigma_c \bar{D}^*$

subtract the 1-π iteration

$\Rightarrow$ box: $\Sigma_c \bar{D}^*$-$\Sigma_c^* \bar{D}^*$-$\Sigma_c \bar{D}^*$

no subtraction

$\Rightarrow$ Couple channel effect: loops

$\Rightarrow$ box: $\Sigma_c \bar{D}^*$-$\Sigma_c \bar{D}^*$-$\Sigma_c \bar{D}^*$

subtract the 1-π iteration

$\Rightarrow$ box: $\Sigma_c \bar{D}^*$-$\Sigma_c^* \bar{D}^*$-$\Sigma_c \bar{D}^*$

subtract the 1-π iteration

$\Rightarrow$Couple channel effect: iterating tree.

Numerical results: elastic channel

Contact terms

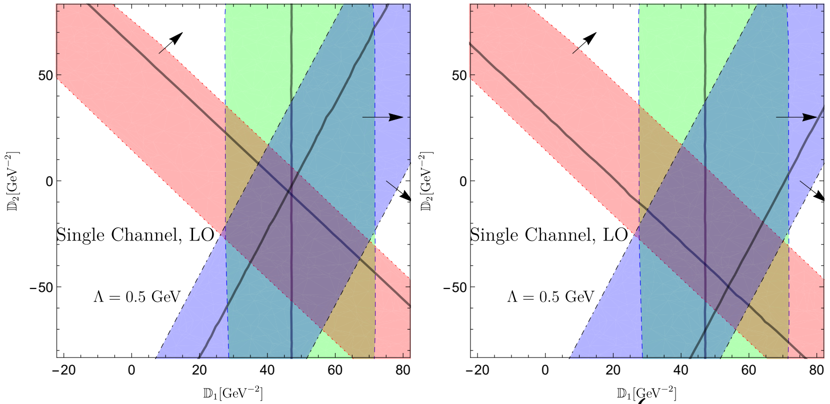
- For $I = 1/2$,

$$\mathcal{V}_{\Sigma_c \bar{D}} = -\mathbb{D}_1, \quad \mathcal{V}_{\Sigma_c \bar{D}^*} = -\mathbb{D}_1 + \frac{2}{3}\mathbb{D}_2 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \tag{3}$$

- Package heavy mesons exchanged interaction like ρ and ω
- Renormalization
  $\Rightarrow$ absorb the divergence in the loops
  $\Rightarrow$ remove the scale dependence.
- In our calculation, we omit the NLO contact terms
- The two LO contact LECs are varied to fit the three P_c states
- Some contribution from the NLO contact terms is incorporated
- Dimensional regularization, $\overline{MS}$-scheme, $\Lambda_\chi = 1.0$ GeV; Gaussian regulator Λ=0.5 GeV

$$V(r) = \frac{1}{(2\pi)^3} \int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mathcal{V}(\boldsymbol{q})\mathcal{F}(\boldsymbol{q}), \quad \mathcal{F}(\boldsymbol{q}) = \exp(-\boldsymbol{q}^{2n}/\Lambda^{2n}) \tag{4}$$

LO results: single channel

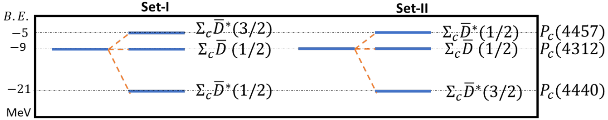


$$\mathcal{V}_{\Sigma_c \bar{D}} = V_c, \quad \mathcal{V}_{\Sigma_c \bar{D}^*} = V_c + V_{ss} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2; \quad \langle \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \rangle = \begin{cases} -1 & J = \frac{1}{2} \\ \frac{1}{2} & J = \frac{3}{2} \end{cases}$$

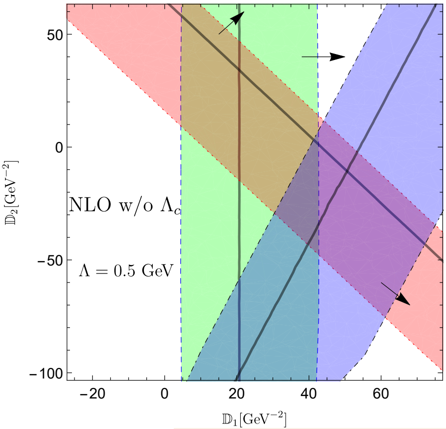
$1 - \pi$ exchange + contact

Keep HQS

Set-I is favored



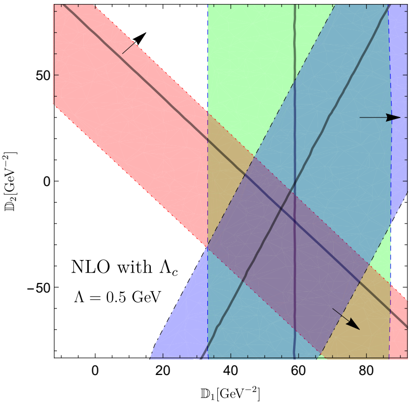
NLO: without Λ_c

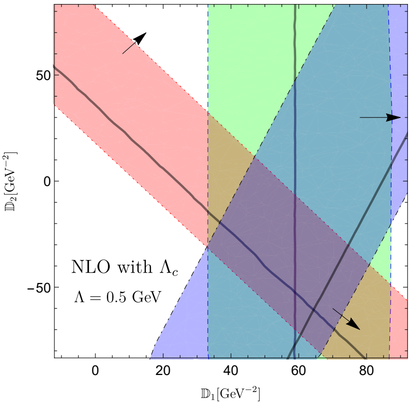


- The result deviate from those of LO which keep the HQS
- There is a very SMALL region where three states coexist as the molecular states
- Restricting the binding energy in exp., it is hard to reproduce three states as molecules simultaneously

Set-I: $\mathbb{D}_1 = 42$ GeV^{-2}, $\mathbb{D}_2 = -25$ GeV^{-2} (in units of MeV).				
$\Delta E \vert I = \frac{1}{2}; J\rangle$	$\Sigma_c \bar{D}$	$\Sigma_c \bar{D}^*$	$\Sigma_c^* \bar{D}$	$\Sigma_c^* \bar{D}^*$
$J = \frac{1}{2}$	−29.1	−6.7		−26.6
$J = \frac{3}{2}$		−2.9	−34.3	−2.6
$J = \frac{5}{2}$				×

NLO: with Λ_c





Set-I: $\mathbb{D}_1 = 55\ \mathrm{GeV}^{-2}$, $\mathbb{D}_2 = -10\ \mathrm{GeV}^{-2}$ (in units of MeV)

$\Delta E \vert I=\frac{1}{2};J\rangle$	$\Sigma_c\bar{D}$	$\Sigma_c\bar{D}^*$	$\Sigma_c^*\bar{D}$	$\Sigma_c^*\bar{D}^*$
$J=\frac{1}{2}$	-7.24	-23.92		-15.41
$J=\frac{3}{2}$		-4.77	-38.14	-4.40
$J=\frac{5}{2}$				-0.81

Set-II: $\mathbb{D}_1 = 60\ \mathrm{GeV}^{-2}$, $\mathbb{D}_2 = -60\ \mathrm{GeV}^{-2}$ (in units of MeV).

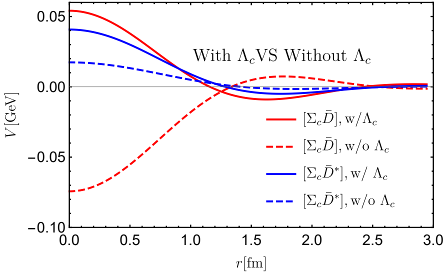
$\Delta E \vert I=\frac{1}{2};J\rangle$	$\Sigma_c\bar{D}$	$\Sigma_c\bar{D}^*$	$\Sigma_c^*\bar{D}$	$\Sigma_c^*\bar{D}^*$
$J=\frac{1}{2}$	-9.95	-2.86		$\times$
$J=\frac{3}{2}$		-18.53	-43.55	-0.68
$J=\frac{5}{2}$				-11.86

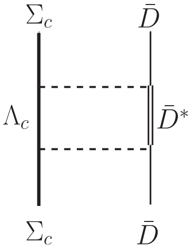
Why the contribution of Λ_c so significant?

- $g_{\Lambda_c \Sigma_c \pi}$ and $g_{\Sigma_c \Sigma_c \pi}$ are at the same order
- Accident degeneracy of $\Sigma_c \bar{D}$ and $\Lambda_c \bar{D}^*$ amplify the amplitude
  $\Rightarrow$ box diagrams $\Sigma_c \bar{D}$-$\Lambda_c \bar{D}^*$-$\Sigma_c \bar{D}$

$$Amp \sim (M_{\Sigma_c} + M_D - M_{\Lambda_c} - M_{D^*})^{-1} = (25 \text{ MeV})^{-1}$$

- The $2-\pi$ exchange potential for $[\Sigma_c \bar{D}]^{J=1/2}_{I=1/2}$ and $[\Sigma_c \bar{D}^*]^{J=3/2}_{I=1/2}$





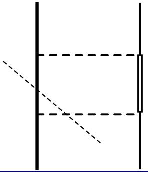
- The power counting is destroyed: $M_{\Sigma_c} + M_D - M_{\Lambda_c} - M_{D^*} \ll m_\pi \sim \epsilon$
- Scheme with inelastic channels can solve the problem

Comparison of NN and $\Sigma_c^{(*)}\bar{D}^{(*)}$

	NN	$\Sigma_c^{(*)}\bar{D}^{(*)}$
Chiral dynamics	$N-N$	(light diquark)-(light quark)
Inter. states	Δ	Λ_c
Inter. states	-	HQSS partner states e.g. $c(qq)^{I=1}_{s=1} = \Sigma_c + \Sigma_c^*$
Heavy quark limit	-	Pinch singularity in elastic channel Including inelastic channel
M_{inter}	$M_\Delta > M_N$	e.g. $M_{\Lambda_c} + M_\pi < M_{\Sigma_c}$
Image part	w/o	w/
small scale expansion	work	fails in $\Sigma_c\bar{D} - \Lambda_c\bar{D}^* - \Sigma_c\bar{D}$
spin structure	$\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}$	$\boldsymbol{S_1}\cdot\boldsymbol{S_2}, \quad (\boldsymbol{S_1}\cdot\boldsymbol{S_2})^2 \ldots$
Weinberg composite	$\lvert E_D\rvert \ll \frac{m_\pi^2}{2\mu}$	$\lvert E_{P_c}\rvert \sim \frac{m_\pi^2}{2\mu} \approx 9$ MeV

Phys.Rev. 137 (1965) B672-B678; Talks of Xian-Wei Kang, Tetsuo Hyodo

- In present calculation, we ignore the image part of potential
- The potentials with the inelastic channels have been obtained
- Solving LS equation with these potentials is on-going



Summary and Outlook

Summary and Outlook

- $\Sigma_c^{(*)}\bar{D}^{(*)}$ potential in ChPT to NLO

  $\Rightarrow$ contact, $1-\pi$, $2-\pi$

  $\Rightarrow$ Loops bring the novel effect

  $\Rightarrow$ HQS breaking effect, Significant !

  $\Rightarrow$ Novel spin structure in potential: $(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)^2$...

  $\Rightarrow$ Couple channel effect in two ways:

  i)elastic channels only

  ii) with inelastic channels

  $\Rightarrow$ Different treatments to box diagrams for two ways

  $\Rightarrow$ Λ_c, Important!

  $\Rightarrow$ Reproduce three P_c states simultaneously as molecular states

- Outlook

  $\Rightarrow$ Lattice QCD simulation on $\Sigma_c^{(*)}\bar{D}^{(*)}$ potential is called for

  $\Rightarrow$ Chiral extrapolation

Thanks for your attention!

Contact terms

- For $I = 1/2$,

$$\mathcal{V}_{\Sigma_c \bar{D}} = -\mathbb{D}_1, \quad \mathcal{V}_{\Sigma_c \bar{D}^*} = -\mathbb{D}_1 + \frac{2}{3}\mathbb{D}_2 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \tag{5}$$

- Package heavy mesons exchanged interaction like ρ and ω
- Renormalization

  $\Rightarrow$ absorb the divergence in the loops

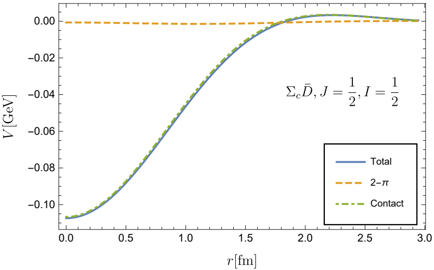
  $\Rightarrow$ remove the scale dependence.
- Contact or pion-exchange? depend on regularization schemes

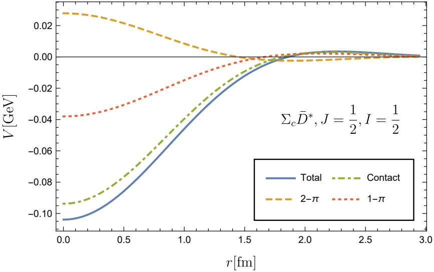
  Phys. Rev. D91, 034002 (2015).
- Depend on chiral truncation order; types of regulator and values of cutoff

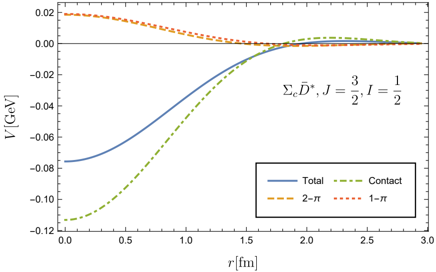
  Phys. Rept. 503, 1 (2011).
- Dimensional regularization, $\overline{MS}$-scheme, $\Lambda_\chi = 1.0$ GeV; Gaussian regulator Λ=0.5 GeV

$$V(r) = \frac{1}{(2\pi)^3} \int d^3\boldsymbol{q}\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mathcal{V}(\boldsymbol{q}) \mathcal{F}(\boldsymbol{q}), \quad \mathcal{F}(\boldsymbol{q}) = \exp(-\boldsymbol{q}^{2n}/\Lambda^{2n}) \tag{6}$$

Potential


$\Sigma_c\bar{D}, J=\frac{1}{2}, I=\frac{1}{2}$
Total
2−π
Contact
V[GeV]
r[fm]


$\Sigma_c\bar{D}^*, J=\frac{1}{2}, I=\frac{1}{2}$
Total
Contact
2−π
1−π
V[GeV]
r[fm]


$\Sigma_c\bar{D}^*, J=\frac{3}{2}, I=\frac{1}{2}$
Total
Contact
2−π
1−π
V[GeV]
r[fm]

Chiral perturbation theory

- QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f (i\not{D} - \mathcal{M} q_f) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

$$f = (u, d, s, c, b, t),$$

$$\mathcal{M} = diag(m_u, m_d, m_s, m_c, m_b, m_t)$$

S.Weinberg

- two approximate symmetry: chiral symmetry and heavy quark symmetry

$$m_u, m_d, m_s \ll 1\text{GeV}, \quad m_c, m_b \gg \Lambda_{QCD} \tag{7}$$

- Chiral perturbation theory (ChPT) and heavy quark effective theory (HQET)
- $\text{SU}(3)_L \times \text{SU(3)}_R \to \text{SU(3)}_V \quad \Rightarrow$ 8 Goldstone bosons
- Quark masses break the chiral symmetry explicitly: $m_\pi^2 \sim m_q$
- Freedom: Goldstone bosons and matter fields, e.g. N, D and Σ_c
- Expansion $\epsilon/\Lambda_\chi, \;\; \Lambda_\chi \approx 4\pi F_\pi \approx m_\rho$

  $\epsilon : m_\pi$, momentum of pion and residue momentum of matter fields

Contact terms of NN

C_S	C_T	Λ, channel	C_S	C_T	Λ, channel
Ordonez:1995rz			Epelbaum:2004fk		
112	13.5	$I=0$	-107.57	-11.64	$\{0.45, 0.5\}, np$
-26.6	-68.9	$I=1$	88.61	53.20	$\{0.6, 0.5\}, np$
Machleidt:2011zz			-121.08	-6.17	$\{0.45, 0.7\}, np$
-100.28	5.61	$\Lambda=0.5, np$	33.70	25.66	$\{0.6, 0.7\}, np$
-99.55	7.07	$\Lambda=0.6, np$			

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2}C_S\bar{N}N\bar{N}N - \frac{1}{2}C_T\bar{N}\boldsymbol{\sigma}N\cdot\bar{N}\boldsymbol{\sigma}N, \tag{8}$$

$$\mathcal{V}_{NN} = C_S + C_T\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}. \tag{9}$$

pinch singularities

$$V \sim \int \frac{d^d\lambda^{4-d}}{(2\pi)^d} \frac{1}{-v\cdot l + i\epsilon} \frac{1}{v\cdot l + i\epsilon} \frac{1}{l^2 - m_1^2 + i\epsilon} \frac{1}{(l+q)^2 - m_2^2 + i\epsilon} \quad (10)$$

$$\sim \int \frac{d^d\lambda^{4-d}}{(2\pi)^d} \frac{1}{-l^0 + i\epsilon} \frac{1}{l^0 + i\epsilon} \frac{1}{l^{02} - \omega_1^2 + i\epsilon} \frac{1}{l^{02} - \omega_2^2 + i\epsilon} \quad (11)$$

$$\frac{1}{-v\cdot l + i\epsilon} \frac{1}{v\cdot l + i\epsilon} \to \frac{1}{-v\cdot l - \frac{\boldsymbol{l}^2}{2M_1} + i\epsilon} \frac{1}{v\cdot l - \frac{\boldsymbol{l}^2}{2M_2} + i\epsilon} \quad (12)$$

$$\int dl^0 \frac{f(l^0)}{-l^0 - \frac{\boldsymbol{l}^2}{2M_1} + i\epsilon} \frac{1}{l^0 - \frac{\boldsymbol{l}^2}{2M_2} + i\epsilon} \quad (13)$$

$$\sim \frac{f(\frac{\boldsymbol{l}^2}{2M_2})}{-\frac{\boldsymbol{l}^2}{2M_2} - \frac{\boldsymbol{l}^2}{2M_1}} \sim f\frac{M}{\boldsymbol{l}^2} \quad (14)$$

power counting: $\frac{1}{l}$, our calculation $\frac{M}{\boldsymbol{l}^2}$

The Lagrangians

$$\Sigma_c = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^{+}}{\sqrt{2}} \\ \frac{\Sigma_c^{+}}{\sqrt{2}} & \Sigma_c^{0} \end{pmatrix}, \; \Sigma_c^{*\mu} = \begin{pmatrix} \Sigma_c^{*++} & \frac{\Sigma_c^{*+}}{\sqrt{2}} \\ \frac{\Sigma_c^{*+}}{\sqrt{2}} & \Sigma_c^{*0} \end{pmatrix}^{\mu}, \tag{15}$$

$$\tilde{P} = \begin{pmatrix} \bar{D}^0 \\ \bar{D}^- \end{pmatrix}, \quad \tilde{P}^{*\mu} = \begin{pmatrix} \bar{D}^{*0} \\ \bar{D}^{*-} \end{pmatrix}, \tag{16}$$

$$\psi^\mu = \mathcal{B}^{*\mu} - \sqrt{\tfrac{1}{3}}(\gamma^\mu + v^\mu)\gamma^5 \mathcal{B},$$

$$\tilde{H} = (\tilde{P}^*_\mu \gamma^\mu + i\tilde{P}\gamma_5)\tfrac{1-\not{v}}{2} \tag{17}$$

$$\begin{aligned} \mathcal{L}^{(0)}_{\Sigma_c\phi} &= -\mathrm{Tr}[\bar{\psi}^\mu iv\cdot D\psi_\mu] + ig_a\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[\bar{\psi}^\mu u^\rho v^\sigma \psi^\nu] + i\frac{\delta_a}{2}\mathrm{Tr}[\bar{\psi}^\mu \sigma_{\mu\nu}\psi^\nu]. \\ \mathcal{L}^{(0)}_{\bar{D}\phi} &= -i\langle \bar{\tilde{H}} v\cdot D\tilde{H}\rangle + g_b\langle \bar{\tilde{H}} u_\mu \gamma^\mu\gamma_5 \tilde{H}\rangle - \frac{\delta_b}{8}\langle \bar{\tilde{H}}\sigma^{\mu\nu}\tilde{H}\sigma_{\mu\nu}\rangle, \end{aligned} \tag{18}$$