What's left to learn from Mesons with Heavy Quarks?

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Outline

- Heavy-light Mesons $(Q\overline{q})$
- Quarkonium Mesons ($Q\overline{Q}'$)
- Disentangling States Near Threshold
- Summary

- QCD dynamics greatly simplifies for heavy quarks ($m_Q \gg \Lambda_{QCD}$)
- For systems with heavy quarks and light quarks:
 - HQET: systematic expansion in powers of $\Lambda_{\text{QCD}}/m_{\text{Q}}$
 - Heavy-light systems: $(c\overline{q})$, $(b\overline{q})$, (cqq), (bqq), (ccq), (cbq), (bbq) for q=u,d or s
 - HQS relations between excitation spectrum in $[(c\bar{q}), (b\bar{q}), (ccq), (bcq) and (bbq)]$ and between [(cqq) and (bqq)]
 - QED analog hydrogen atom (e-p)
- For non relativistic (QQ): bound states form with masses M near $2m_Q$:
 - NRQCD: systematic expansion in powers of v/c
 - Quarkonium systems: $(c\overline{c})$, $(b\overline{b})$, $(b\overline{c})$
 - heavy quark velocity: $p_Q/m_Q \approx v/c \ll 1$
 - binding energy: $2m_Q M \approx m_Q v^2/c^2$
 - QED analogs positronium (e^+e^-), (true) muonium ($\mu^-\mu^+$), muonium ($e^-\mu^+$)

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Heavy Quark Symmetry

- As $m_Q \rightarrow \infty$ the spectrum of heavy-light mesons becomes doubly degenerate.
 - $J = j_1 + s_Q$ where j_1 the total angular momentum of the light degrees of freedom.
 - Corrections of order $1/m_{\rm Q}$ do three things:
 - split the spin degeneracy: $C_1(j_l \cdot s_Q)/m_Q$
 - spin independent shift center of gravity: $C_2(\nabla_Q \cdot \nabla_Q)/m_Q$
 - mix states with same J^P different $j_1 : C_3[(j_1 \cdot r_1)(s_Q \cdot r_1) 1/3(j_1 \cdot s_Q)(r_1 \cdot r_1)]/m_Q$ eg. mix $j_1 = 1/2,3/2$ with J^P = 1⁺ P states
 - This symmetry doesn't depend on the QCD dynamics of the light degrees of freedom.
 - Implications for the P states of the heavy-light system
 - Independent of dynamic nature of the (0+, 1+) j_1 = 1/2 P states of the B, D system
 - HQS does not require the four P states to be degenerate as m_Q → ∞, as expected in leading order of NR limit.
 - Can see this behavior in the excitation spectrum of the the B, D systems

Observed States in D Spectrum



- HQS determines the ratios of hadronic transitions very useful in distinguishing excited states
- Various proposals for the shifts of the $D_s^*(2317)$ and $D_s(2460)$:
 - Influence of the nearby decay channels.
 - Chiral multiplets (0-,0+).
 - Threshold bound states of DK and D*K respectively.

Observed states in B meson system



- HQS relates the excitation spectrum in the D system to the B system.
- Various models will be disentangled when the narrow $B_s(j^p = \frac{1}{2})$ states are observed.

Important to observe the $B_s(j^p = \frac{1}{2})$ states

- Spin splittings (MeV)

Multiplet	D Mesons			B Mesons		
j_l^P	u	d	\mathbf{S}	u	d	S
$\frac{1}{2}^{-}$	142.0	140.6	144.0	45.4	45.2	48.5
$\frac{1}{2}^{+}$			141.7			
$\frac{\overline{3}}{2}$ +	39.9	41.3	34.0	11.3	13.4	11.2

- Shift of CoG (relative)

Multiplet	D Mesons			B Mesons		
j_l^P	u	d	\mathbf{S}	u	d	S
$\frac{1}{2}^{+}$			348			
$\frac{3}{2}^{+}$	479	479	484	423	421	434

- Need $j_1^P = \frac{1}{2}^+ B_s$ states to extrapolate to HQS limit for P states
- Will allow to distinguish various models of the QCD dynamics of these states.



Approach to Heavy Quark Symmetry

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- Mass predictions: Bs (ji^P=¹/₂+)
 - Approaches to QCD dynamics
 - · LQCD
 - Chiral multiplets (0⁺, 0⁻)
 - K matrix: B+K^(*) scattering
 - Tetraquarks
 - RQM with coupled channels
 - Splitting (from HQS)
 - $[m(1^+) m(0^+)]_b \approx [m(1^+) m(0^+)]_c M(J/\psi)/M(\Upsilon)$ $\approx 46 \text{ MeV}$
 - All valid approaches should agree [RQM looks low]
 - $CoG = [3 m(1^+) + m(0^+)]/4$
 - QCD dynamics dependent: C2, C3
 - See a wide variation in models as expected.
 - If C₂ same as for D system: CoG = 5837 MeV
 - Need experimental determination.
 [Belle 2, LHC] B_s B_{s0}* Threshold ~ 11.130

C. B. Lang, Daniel Mohler, Sasa Prelovsek, R. M. Woloshyn

[arXiv:1501.01646]

Table 5: Comparison of masses from this work to results from various model based calculations; all masses in MeV.

J^P	0^{+}	1+
Covariant (U)ChPT [24]	5726(28)	5778(26)
NLO UHMChPT [19]	5696(20)(30)	5742(20)(30)
LO UChPT [17, 18]	5725(39)	5778(7)
LOχ-SU(3) [16]	5643	5690
HQET + ChPT [20]	5706.6(1.2)	5765.6(1.2)
Bardeen, Eichten, Hill [15]	5718(35)	5765(35)
rel. quark model [5]	5804	5842
rel. quark model [22]	5833	5865
rel. quark model [23]	5830	5858
HPQCD [30]	5752(16)(5)(25)	5806(15)(5)(25)
this work	5713(11)(19)	5750(17)(19)

LQCD calculation includes the mixing of the two meson thresholds.

Table 2. Predicted masses of the 0^+ and 1^+ heavy-strange mesons, and poles given as $(M, \Gamma/2)$ for the 0^+ and 1^+ heavy-nonstrange mesons. Here *M* and Γ are the mass and the total decay width, respectively. For comparison, the RPP values [13] and latest lattice QCD results are also shown. All values are in units of MeV.

	Prediction		RPP	Lattice QCD
D_{s0}^{*}	2315^{+18}_{-28}		2317.7 ± 0.6	2348^{+7}_{-4} [44]
D_{s1}^{so}	$2456_{-21}^{+\overline{15}}$		2459.5 ± 0.6	2451 ± 4 [44]
B^*_{s0}	5720_{-23}^{+16}		-	5711 ± 23 [64]
B_{s1}^{s0}	5772_{-21}^{+15}		-	$5750 \pm 25 \ [64]$
D_0^*	$(2105^{+6}_{-8}, 102^{+10}_{-11}),$	$(2451^{+35}_{-26}, 134^{+7}_{-8})$	$(2318 \pm 29, 134 \pm 20)$	_
D_1	$(2247^{+5}_{-6}, 107^{+11}_{-10}),$	$(2555^{+47}_{-30}, 203^{+8}_{-9})$	$(2427 \pm 40, 192^{+65}_{-55})$	_
B_0^*	$(5535^{+9}_{-11}, 113^{+15}_{-17}),$	$(5852^{+16}_{-19}, 36 \pm 5)$	_	_
B_1	$(5584^{+9}_{-11}, 119^{+14}_{-17}),$	$(5912^{+15}_{-18}, 42^{+5}_{-4})$	-	_

- Branching fractions
 - Both D and B systems can be used to distinguish models.
 - Need to measure (H = D or B) branching fractions.
 - $R_1(H) = Br(H_s(0+) \rightarrow H_s(0-) + \gamma)/Br(H_s(0+) \rightarrow H_s(0-) + \pi^0)$
 - $R_2(H) = Br(H_s(1+) \rightarrow H_s(1-) + \gamma)/Br(H_s(1+) \rightarrow H_s(1-) + \pi^0)$
 - $R_3(H) = Br(H_s(1+) \rightarrow H_s(0-) + \gamma)/Br(H_s(1+) \rightarrow H_s(1-) + \pi^0)$
 - $R_4(H) = Br(H_s(1+) \rightarrow H_s(1-) + 2\pi)/Br(H_s(1+) \rightarrow H_s(1-) + \pi^0)$
 - Expect R₁(D) << R₁(B) cancelling c and s quark terms for E1 transition.
 - R₂(B) distinguishes models.
 - R₂(H)/R₃(H) measures mixing of the J=1 states.
 - HQS assures that the
 - $\Gamma(H_s(0+) \rightarrow H_s(0-) + \pi^0) = \Gamma(H_s(1+) \rightarrow H_s(1-) + \pi^0)$

Table 1	: The	hadronic	and el	lectromagnetic	transition	rates	for	narrow	j_l^P	$= 1/2^+$	heavy-
light P	states								U		

system	transition	Q(keV)	dependency	width (keV)	exp BR
$(c\bar{s})$	$0^+ \rightarrow 1^- + \gamma$	212	m_{s}^{*}, m_{c}^{*}	1.74	(<5)%
	$0^+ \rightarrow 0^- + \pi^0$	297	$G_A \delta_{\eta \pi 0}$	21.5	$(100^{+0}_{-20})\%$
	total			23.2	
$(c\bar{s})$	$1^+ \rightarrow 0^+ + \gamma$	323	m_s^*, m_c^*, θ_c	0.43	$(3.7^{+5.0}_{-2.4})\%$
	$1^+ \to 1^- + \gamma$	323	$m_s^*, m_c^*, heta_c$	3.49	(< 8)%
	$1^+ \rightarrow 0^- + \gamma$	442	$m_s^*, m_c^*, heta_c$	7.62	$(18 \pm 4)\%$
	$1^+ \rightarrow 1^- + \pi^0$	298	$G_A \delta_{\eta \pi 0}$	21.5	$(48 \pm 11)\%$
	$1^+ \to 1^- + 2\pi$	221	$g_A \delta_{\sigma_1 \sigma_3}$	9.7	
	total			42.7	
$(b\bar{s})$	$0^+ \to 1^- + \gamma$	293	m_s^*, m_b^*	58.3	
	$0^+ \rightarrow 0^- + \pi^0$	297	$G_A \delta_{\eta \pi 0}$	21.5	
	total			79.8	
$(b\bar{s})$	$1^+ \rightarrow 0^+ + \gamma$	335	$m_s^*, m_b^*, heta_b$	0.15	
	$1^+ \rightarrow 1^- + \gamma$	335	$m_s^*, m_b^*, heta_b$	42.3	
	$1^+ \rightarrow 0^- + \gamma$	381	$m_s^*, m_b^*, heta_b$	58.3	
	$1^+ \rightarrow 1^- + \pi^0$	298	$G_A \delta_{\eta \pi 0}$	21.5	
	$1^+ \rightarrow 1^- + 2\pi$	125	$g_A \delta_{\sigma_1 \sigma_3}$	0.24	
	total			123.8	

W.Bardeen, E.E., C. Hill PR D68 054024 (2003)

[hep-ph/0305049]

Hadronic and electromagnetic transition rates for narrow B_{s1} heavy-light P states. Results for various other models: B^*K bound state in Heavy Chiral Unitary [hep-ph/0801.1932,hep-ph/0803.1223] and Light-Cone QCD sum rules[hep-ph/0711.2559], $\mathcal{L}_{eff}(1)$ [hep-ph/0801.2232] and (2) [hep-ph/1405.2242], Bethe-Salpeter [arXiv:1906.09002] approaches; and Relativistic quark model with mixing to four quark (two meson) states[hep-ph/0711.2359]. (Results in keV)

Approach	$\Gamma(B_{s1} \to B_s^* \pi)$	$\Gamma(B_{s1} \to B_s \gamma)$	$\Gamma(B_{s1} \to B_s^* \gamma)$
Heavy Chiral Unitarity	10.36	3.2 - 15.8	0 6.1
Light Cone Sum Rules	5.3 - 20.7	106.5(60.7)	75.6(6.0)
Bethe-Salpeter	27.5 - 39.2	45.2 - 79.8	0.4 - 2.6
$\mathcal{L}_{eff}(1)$	57.0 - 94.0	2.0 - 2.67	0.04 - 0.18
$\mathcal{L}_{eff}(2)$	1.8 ± 1.8	4.1 ± 10.9	46.9 ± 33.6

- QCD dynamics greatly simplifies for heavy quarks ($m_Q \gg \Lambda_{QCD}$)
- For systems with heavy quarks and light quarks:
 - HQET: systematic expansion in powers of $\Lambda_{\text{QCD}}/m_{\text{Q}}$
 - Heavy-light systems: $(c\overline{q})$, $(b\overline{q})$, (cqq), (bqq), (ccq), (cbq), (bbq) for q=u,d or s
 - HQS relations between excitation spectrum in $[(c\bar{q}), (b\bar{q}), (ccq), (bcq) and (bbq)]$ and between [(cqq) and (bqq)]
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 - NRQCD: systematic expansion in powers of v/c
 - Quarkonium systems: (cc), (bb), (bc)
 - heavy quark velocity: $p_Q/m_Q \approx v/c \ll 1$
 - binding energy: $2m_Q M \approx m_Q v^2/c^2$
 - QED analogs positronium (e^+e^-), (true) muonium ($\mu^-\mu^+$), muonium ($e^-\mu^+$)

Narrow States Below Threshold

- For charmonium system:
 - experiment theory
 - Below threshold only ¹D₂ state unobserved.
 - Two body thresholds
 - Charmed mesons:
 Γ< 1 MeV



 $b\overline{b}$ states For bottomonium system: $B_s^* \bar{B}_{s1p}^P \\ B_s^* \bar{B}_{s1}^P$ $B_{s}\bar{B}_{s1}^{P} \ B_{s}^{*}\bar{B}_{s0}^{P} \ B_{s}\bar{B}_{s1p}^{P}$ • experiment theory $\bar{B}^{P}_{\cdot 0}$ 11.0 $\Upsilon(10993)$ · Below threshold $6^{1}S_{0}$ $\Upsilon(10890)$ $5^{1}S_{0}$ 14 state unobserved. Two body thresholds В $\overline{B}\overline{B}^*$ $B\overline{B}$ $\Upsilon(10579)$ $\chi_{b2}(10524)$ $4^{1}S_{0}$ 10.5 $\chi_{b1}(10513)$ Mass (GeV) $3^{1}P_{1}$ • Beauty mesons: $2^{3}D_{1,2,3}$ $3^{3}P_{0}$ $2^{1}D_{2}$ $\cdot 1^3 F_{2,3,4}$ $\Upsilon(10355)$ $\chi_{b2}(10269) \ \chi_{b1}(10256) \ \chi_{b0}(10232)$ Γ<1 MeV $1^{1}F_{3}$ $3^{1}S_{0}$ ${ \overset{1^{3}D_{3}}{\Upsilon_{2}(10164)}}$ $h_b(10\overline{260})$ $1^{1}D_{2}$ • Pattern of thresholds $-\Upsilon(10023)$ 10.0 affected by smaller $\chi_{b2}(9912)$ $\eta_b(9999)$ $\chi_{b1}(9893)$ $h_b(9899)$ spin-splitings $\chi_{b0}(9859)$ 9.5 $\Upsilon(9460)$ $\eta_b(9399)$ thresholds S Ρ D F

B_c System

- Only other quarkonium system with narrow states is the $(b\bar{c})$
 - Has been studied theoretically for many years. I will report on a recent update. (E.E and C. Quigg [1902.09735])
 - Starting point a QCD inspired variation of Cornell potential:
 - V(r) = -4/3 $\alpha_s(r)/r$ + r/a² with a = 2.34 GeV⁻¹; m_c = 1.84 GeV, m_b = 5.18 GeV
 - $\alpha_s(r)$ the four loop QCD running coupling at short distance but becoming frozen at large distance [as suggest by Gribov for light mesons: $\alpha_s \rightarrow (3\pi/4) (1 \int (2/3)) \approx 0.44)$]



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- Results
 - potential
 - Frozen alpha (red)
 - original Cornell (green)
 - Richardson (blue)
 - The spin-dependent terms determined from fits to observed (cc) and (bb) systems



- $b\bar{c}$ States $B_{s}^{*}\bar{D}_{s1}^{P} D_{s}^{*}\bar{B}_{s0}^{P} B_{s}\bar{D}_{s1p}^{P} = \blacksquare \blacksquare \blacksquare B_{s}^{*}\bar{D}_{s0}^{P}, B_{s}^{*}\bar{D}_{s1p}^{P}$ 8.0 $D_s^* B_{s0}^* - D_s \overline{B}_{s1p}^P = B_s \overline{D}_{s0}^* D_s \overline{B}_{s0}^P$ $,B_{s}^{*}\bar{D}_{s0}^{P}$ $= = = B_s^* \bar{D}_s^*$ $= = B_s^* \bar{D}_s^*$ 7.5 $\begin{array}{c} B^*_s \bar{D}_s \\ B^*_s \bar{D}^*_s, B_s \bar{D}_s \\ B \bar{D}^* \end{array}$ $3^{3}S_{1}$ Jass (GeV) $3^{1}S_{0}$ $B \underline{B} \overline{D} \\ B \overline{D}$ $= \frac{2^{3} P_{2}}{2^{3} P_{0}}$ 7.0 $-1^3 D_{1,2,3}$ $1D_{2}'$ $B_c^*(6897)$ $B_{c}(6866)$ $= \begin{bmatrix} 1^{3} P_{2} \\ P_{2} \\ 1^{3} P_{0} \end{bmatrix}$ $1P'_{1}$ 6.5 $-B_c^*$ (6329)(lqcd) $B_{c}(6275)$
 - EE & C. Quigg [arXiv:1902.09735]

D

Ρ

thresholds

- B_c is the unique heavy-heavy meson that only has weak decays.
 - A rich excitation spectrum of states.
 - All the excited states below BD threshold decay to B_{c.}
 - Bc(2S) -> Bc(1S) + $\pi\pi$.
 - The P-states have photon transitions to lower S and D states
 - Many states observable at the LHC and a future TevaZ factory.

HADRON2019 Guilin

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- Spin splittings unequal mass
 - General form

$$H_{\text{eff}} = \left[\frac{L \cdot s_1}{2m_1^2} + \frac{L \cdot s_2}{2m_2^2}\right]T_1 + \frac{L \cdot (s_1 + s_2)}{m_1 m_2}T_2 + \frac{s_1 \cdot s_2}{m_1 m_2}T_3 + \frac{S_{12}}{m_1 m_2}T_4$$

- The spin singlet state is shifted from the center of gravity of the spin triplets states.

$$H(spin \ orbit) = \begin{bmatrix} \frac{1}{4}(\frac{1}{m_1^2} + \frac{1}{m_2^2})T_1 + \frac{1}{m_1m_2}T_2 \end{bmatrix} \begin{pmatrix} L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -(L+1) \end{pmatrix} + \\ \frac{1}{4}(\frac{1}{m_1^2} - \frac{1}{m_2^2})T_1 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{L(L+1)} & 0 \\ 0 & \sqrt{L(L+1)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Can be measured from the 1P spin splittings

- Decay rates of excited (bc) narrow states: •
 - 12 to 15 narrow states depending exact position of the 2P states
 - notable 2π hadronic transitions



- notable photon transitions



Decay Mode	$k_{\gamma} [\text{keV}]$	Branching Fraction (%)	Decay Mode	$k_{\gamma} [\text{keV}]$	Branching Fraction
	$1^{1}S_{0}$ (6275) :	weak decays		$3^{3}P_{0}$ (7104) : Γ	= 60.9 keV
	$1^{3}S_{1}$ (6329) · Γ	= 0 144 keV	$1^{3}S_{1}$ (6329)	733	46.4
${}^{1}S_{0} + \gamma$	54	100	$2^{3}S_{1}$ (6897)	204	45.0
1 D0 T 1	0 ³ D (((00)) D	59.1 L.V	$3^{3}D_{1}$ (7006)	97	8.44
³ S. (6220)	$2 P_0 (0092) : 1$ 254	= 53.1 KeV		$3P_1$ (7135) : Γ =	= 87.1 keV
51 (0329)	304	100	$1^{3}S_{1}$ (6329)	761	32.6
9	$2P_1$ (6730) : Γ	= 72.5 keV	$1^{1}S_{0}$ (6275)	809	6.24
$^{3}S_{1}$ (6329)	389	86.2	$2^{3}S_{1}$ (6897)	234	40.4
$^{1}S_{0}$ (6275)	440	13.7	$2^{1}S_{0}$ (6866)	264	8.39
	$2P'_1$ (6738) : Γ	= 99.9 keV	$3D_2$ (7005)	129	4.74
${}^{1}S_{0}$ (6275)	448	92.4	$3D'_{2}(7015)$	119	4.55
$^{13}S_1$ (6329)	397	7.51	$3^{3}D_{1}$ (7006)	128	2.88
	$2^{3}P_{2}$ (6750) : Γ	= 79.7 keV		$3P'(7143) \cdot \Gamma$	- 113 keV
${}^{3}S_{1}$ (6329)	409	100	$1^{1}S_{2}$ (6275)	816	32.0
01 (0020)	olg (cocc) D	79.1 h-W	$1^{3}S_{1}$ (6220)	769	2.9
10	$2 S_0 (0800) : 1$	= 73.1 KeV	$2^{1}C$ (6866)	272	3.88
$S_0 + \pi \pi$	100	81.1	$2 S_0 (0800)$ $2^3 C (6807)$	212	47.0
$P_1(6738)$	126	16.5	$2^{\circ}S_{1}(0897)$	242	0.10
$P_1(0730)$	134	2.24	$3D_2$ (7015) 2D (7005)	127	4.22
2	$2^{3}S_{1}$ (6897) : Γ	= 76.8 keV	$3D_2(1003)$	137	0.72
${}^{3}S_{1} + \pi\pi$		65.0	3~ ()	$3^{\circ}P_2$ (7154) : 1	= 100 keV
$^{3}P_{0}$ (6692)	201	7.66	$1^{3}S_{1}$ (6329)	777	35.2
P_1 (6730)	165	11.5	$2^{3}S_{1}$ (6897)	252	49.2
P'_1 (6738)	157	1.13	$3D_2$ (7005)	147	1.09
$^{3}P_{2}$ (6750)	145	14.6	$3D_2'$ (7015)	137	1.18
	$3D_2$ (7005) : Γ	= 93.7 keV	$3^{3}D_{1}$ (7006)	146	0.16
$^{1}S_{0} + \pi\pi$		12.2	$3^{3}D_{3}$ (7010)	142	13.0
$^{3}S_{1} + \pi\pi$		9.0		$4F_3$ (7221) : Γ =	= 77.6 keV
$2P_1$ (6730)	270	29.1	$3D_2$ (7005)	213	52.4
$2P'_1$ (6738)	262	42.3	$3D'_2$ (7015)	203	42.8
$2^{3}P_{2}$ (6750)	250	7.24	$3^{3}D_{3}$ (7010)	208	4.71
	$3^{3}D_{1}$ (7006) : I	r = 117 keV		$4^{3}F_{4}$ (7223) : Γ :	= 79.9 keV
$^{3}S_{1} + \pi\pi$	- ()	17.0	$3^{3}D_{3}$ (7010)	210	100
$2^{3}P_{0}$ (6692)	306	53.4	······	$4^{3}F_{2}$ (7233) · Γ	= 95.3 keV
$2P_1$ (6730)	270	25.3	$3D_{2}$ (7005)	225	6.73
$2P'_1$ (6738)	262	2.62	$3D'_{2}$ (7015)	215	7.96
$2^{3}P_{2}(6750)$	251	1.51	$3^{3}D_{1}$ (7006)	224	84.8
	$3^{3}D_{2}$ (7010) · F	-87.2 keV	$3^{3}D_{2}$ (7010)	220	0.42
${}^{3}S_{1} \pm \pi\pi$	0 23 (1010) . 1	22.9		4E' (7927) · Γ -	- 80.0 koV
$^{3}P_{2}$ (6750)	255	77.0	3D' (7015)	918	- 03.3 KeV 47 4
212 (0100)	200	00.1.1.37	$3D_2$ (7015) $3D_2$ (7005)	210	47.4 52.5
10	$3D_2(7015):1^\circ$	= 92.1 KeV	0.02 (1000)	220	52.0
$S_0 + \pi \pi$		9.2			
$D_1 + \pi \pi$	070	12.4			
$P_1(6738)$	272	37.2			
$x_1(0(30))$	219	41.0			

Decay Mode	$k_{\gamma} [\mathrm{keV}]$	Branching Fraction (%)
	$3^{3}P_{0}$ (7104) : Γ =	= 60.9 keV
$1^{3}S_{1}$ (6329)	733	46.4
$2^{3}S_{1}$ (6897)	204	45.0
$3^{3}D_{1}$ (7006)	97	8.44
	$3P_1$ (7135) : $\Gamma =$	87.1 keV
$1^{3}S_{1}$ (6329)	761	32.6
$1^{1}S_{0}$ (6275)	809	6.24
$2^{3}S_{1}$ (6897)	234	40.4
$2^{1}S_{0}$ (6866)	264	8.39
$3D_2$ (7005)	129	4.74
$3D'_{2}$ (7015)	119	4.55
$3^{3}D_{1}$ (7006)	128	2.88
	$3P'_1$ (7143) : Γ =	= 113 keV
$1^{1}S_{0}$ (6275)	816	32.9
$1^{3}S_{1}$ (6329)	768	3.88
$2^{1}S_{0}$ (6866)	272	47.0
$2^{3}S_{1}$ (6897)	242	5.15
$3D'_2$ (7015)	127	4.22
$3D_2$ (7005)	137	6.72
	$3^{3}P_{2}$ (7154) : Γ :	= 100 keV
$1^{3}S_{1}$ (6329)	777	35.2
$2^{3}S_{1}$ (6897)	252	49.2
$3D_2$ (7005)	147	1.09
$3D'_2$ (7015)	137	1.18
$3^{3}D_{1}$ (7006)	146	0.16
$3^{3}D_{3}$ (7010)	142	13.0
	$4F_3$ (7221) : $\Gamma =$	77.6 keV
$3D_2$ (7005)	213	52.4
$3D'_2$ (7015)	203	42.8
$3^{3}D_{3}$ (7010)	208	4.71
	$4^{3}F_{4}$ (7223) : Γ =	= 79.9 keV
$3^{3}D_{3}$ (7010)	210	100
	$4^{3}F_{2}$ (7233) : Γ =	= 95.3 keV
$3D_2$ (7005)	225	6.73
$3D'_2$ (7015)	215	7.96
$3^{3}D_{1}$ (7006)	224	84.8
$3^{3}D_{3}$ (7010)	220	0.42
	$4F'_3$ (7237) : $\Gamma =$	89.9 keV
$3D'_2$ (7015)	218	47.4
$3D_2$ (7005)	228	52.5

TABLE VI. Total widths Γ and branching fractions \mathcal{B} for principal decay modes of $(c\bar{b})$ states below threshold, updating

• Strong Decays for states above threshold.

- 35 states above threshold and have significant decay widths



FIG. 9. Strong decay widths of the 3^1S_0 ($c\bar{b}$) level near openflavor threshold. The shaded band on the mass axis indicates ± 20 MeV around our nominal value for the mass of this state, 7253 MeV.



- 2P states just below threshold and may have significant mixing



- Production at hadron colliders
 - Use BCVEGPY 2.2 [hep-ph/05504017] Chao-Hsi Chang, Jian-Xiong Wang, Zing-Gang Wu
 - No calculation for D states (expected small)
 - p_T and rapidity distributions nearly state independent.

TABLE VIII. Production rates (in nb) for $(c\bar{b})$ states in pp collisions at the LHC. The production rates were calculated using the BCVEGPY2.2 generator of Ref. [45], extended to include the production of 3P states. Color-octet contributions to *s*-wave production are small; we show them (following |) only for the 1*S* states.

$(c\overline{b})$ level	$\sigma(\sqrt{s}=8~{\rm TeV})$	$\sigma(\sqrt{s}=13~{\rm TeV})$	$\sigma(\sqrt{s} = 14 \text{ TeV})$
$1^{1}S_{0}$	46.8 1.01	$80.3 \mid 1.75$	88.0 1.90
$1^{3}S_{1}$	$123.0 \mid 4.08$	$219.1 \mid 6.97$	$237.0 \mid 7.55$
$2^{3}P_{0}$	1.113	1.959	2.108
$2^{3}P_{1}$	2.676	4.783	5.214
$2^{1}P_{1}$	3.185	5.702	6.166
$2^{3}P_{2}$	6.570	11.57	12.64
$2^{1}S_{0}$	9.58	16.94	18.45
$2^{3}S_{1}$	23.46	41.72	45.53
$3^{3}P_{0}$	0.915	1.642	1.806
$3^{3}P_{1}$	2.263	4.082	4.478
$3^{1}P_{1}$	2.695	4.817	5.287
$3^{3}P_{2}$	5.53	9.98	10.90
$3^{1}S_{0}$	4.23	7.53	8.08
$3^{3}S_{1}$	10.16	18.21	19.83



FIG. 5. Transverse momentum distribution of B_c produced in pp collisions at $\sqrt{s} = 8$ TeV (dotted blue curve), $\sqrt{s} = 13$ TeV (solid black curve), and $\sqrt{s} = 14$ TeV (dashed red curve), calculated using BCVEGPY2.2 [45] Small shape variations are statistical fluctuations.



FIG. 4. Rapidity distribution for production of B_c^* in pp collisions at $\sqrt{s} = 8$ TeV (dotted blue curve), $\sqrt{s} = 13$ TeV (solid black curve), and $\sqrt{s} = 14$ TeV (dashed red curve), calculated using BCVEGPY2.2 [45]. The bin width is $\Delta y = 0.5$. The mild asymmetries are statistical fluctuations.



- B_c spectrum
- 2S -> ππ + 1S transitions has been observed ATLAS, CMS, LHCb
- theory
 - Combine theoretically expected production rates with the branching fractions for the $\pi\pi$ transitions.
 - results in expectations for $\pi\pi$ transition rates.
- experiment
 - 2S -> ππ + 1S transitions
 has been observed ATLAS,
 CMS, LHCb
 - the gamma transition $B_c^* \rightarrow Y B_c$ is unobserved



EE & C. Quigg [arXiv:1902.09735]



- The excited P states
 - The 2P states very close to decay thresholds.
 - Various models tested as analogs of X(3872).
 - Combining theoretically expected production rates and decay fractions yield the rates for the photon transitions.
- E1 transition photons observable: CMS, LHCb, Atlas



EE & C. Quigg [arXiv:1902.09735]

Why it works so well

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• Lattice calculation V(r), then SE

$$-\frac{1}{2\mu}\frac{d^2u(r)}{dr^2} + \left\{\frac{\langle \boldsymbol{L}_{Q\bar{Q}}^2\rangle}{2\mu r^2} + V_{Q\bar{Q}}(r)\right\}u(r) = E u(r)$$

- What about the gluon and light quark degrees of freedom of QCD?
- Two thresholds:
 - Usual $(Q\bar{q})+(q\bar{Q})$ decay threshold
 - Excite the string hybrids
- Tetraquarks may appear.
- Hybrid states will appear in the spectrum associated with the potential Π_u , ...
- In the static limit this occurs at separation:
 r ≈ 1.2 fm.
- Between 3S-4S in (cc̄) ; near the 5S in (bb̄).



- Present status of states that don't fit as ordinary charmonium states
 - "?" states need info
 - if J^{PC} = 0⁺⁺
 X(3915) ? 2³P₂ state
 - ψ(4660) ? 55 state
 - ψ(4230), ψ(4360)?
 hybrids?
 - "near" states possible molecules
 - All the states at at or above threshold



X(3872)

- X(3872) $J^{PC} = 1^{++}$ M= 3871.69 ± 0.17 T< 1.2 MeV from $J/\psi \pi \pi$ mode
 - $M_X M_D M_{D^*} = 0.01 \pm 0.17 \text{ MeV}$
 - Large isospin violation $\pi^+\pi^- J/\psi(1S)$ large Isospin violation > 3.2%- Decays observed: $\rho J/\psi(1S)$ $\omega J/\psi(1S)$ > 2.3% $D^0 \overline{D}^0 \pi^0$ > 40%suggests molecule $\bar{D}^{*0} D^{0}$ > 30% $\gamma J/\psi$ > 0.7% $\gamma\psi(2S)$ > 4%suggests 2P state $\frac{\mathcal{B}(X(3872) \rightarrow \psi(2S)\gamma)}{\mathcal{B}(X(3872) \rightarrow J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29$ - LHCb [arXiv:1404.0275]
 - Mixed state with sizable quarkonium component likely.
 - BES III (Chang-Zheng Yuan talk LATTICE 2019)

mode $D^{*0}\overline{D^0} + c.c.$	$\gamma J/\psi$	$\gamma\psi'$	$\gamma D^+ D^-$	$\omega J/\psi$	$\pi^0 \chi_{c1}$
ratio 14.81 ± 3.80	0.79 ± 0.28	< 0.42	< 0.99	$1.7^{+0.4}_{-0.3} \pm 0.2$ [27]	$0.88^{+0.33}_{-0.27} \pm 0.10$ [37]

- For LQCD: Where is the $\chi'_{c0}(2^{3}P_{0})$ state?

• LQCD approaches need to include the $2^{3}P_{1}(cc)$ state to observe a X(3872) state



Alexandrou et al. arXiv:1212.1418 Prelovsek et al. arXiv:1405.7623 * Guerrieri et al. arXiv:1411.2247 Padmanath et al. arXiv:1503.03257 Francis et al. arXiv:1607.05214

Prelovsek et al. arXiv:1405.7623 Caveats:

- (1) m_{π} = 266 MeV
- (2) limited spacings and volumes
- (3) must include all states below and in region of possible new 4Q states

• BESIII [arXiv:1901.03992]

- X(3872) -> π⁰ ³P_J

TABLE I. Final results for the normalization and search channels and their ratios. Individual efficiencies are reported without considering ISR in the MC (no ISR) and are for illustration only. Efficiency ratios are for the search channels divided by the normalization channel and include effects due to ISR (with ISR), which nearly cancel in the ratio. Numbers in parentheses are 90% C.L. upper limits. The first errors are statistical and the second are systematic.

	$\pi^{+}\pi^{-}I/a/$	$\pi^0 \chi_{0}$	$\pi^0 \chi$.	$\pi^0 \chi$
		<i>n</i> <u><i>A</i></u> <u><i>c</i></u> 0	<u> </u>	<u> </u>
Event yield	$84.1^{+10.1}_{-9.4}$	$1.9^{+1.9}_{-1.3}$	$10.8^{+3.8}_{-3.1}$	$2.5^{+2.3}_{-1.7}$
Signal significance (σ)	16.1	1.6	5.2	1.6
Efficiency (no ISR) $(\%)$	32.3	8.8	14.1	12.8
Efficiency ratio (with ISR)		0.272	0.435	0.392
$\mathcal{B}(\chi_{cJ} \to \gamma J/\psi) \times \mathcal{B}(\pi^0 \to \gamma \gamma) \ (\%)$		1.3	33.5	19.0
Total systematic error $(\%)$		17.0	11.9	9.4
$\mathcal{B}(X \to \pi^0 \chi_{cJ}) / \mathcal{B}(X \to \pi^+ \pi^- J/\psi)$		$6.6^{+6.5}_{-4.5} \pm 1.1 \ (19)$	$0.88^{+0.33}_{-0.27} \pm 0.10$	$0.40^{+0.37}_{-0.27} \pm 0.04 \ (1.1)$

- Rate for a conventional 2³P₁ expected to be small but unknown.

- Ratios :	J=O	J=1	J=2	S. Dubynskiy and M. B. Voloshin, Phys. Rev. D 77, 014013 (2008).
• 2 ³ P ₁ -> ³ P _J	0	5	2.14	
 D⁰ D^{*0} molecule 	7.76	3	3.56	

- Might be a better discriminator
- Can use polarization in prompt production (p p) of X(3872) to distinguish 2³P₁ or molecule. Model suggests dominantly 2³P₁ production.

M. Butenschoen, Zhi-Guo He, B. Kniehl, [arXiv:1906.0853]

• After 15 years we still are not sure of the mixture: $X(3872) = cos(\theta) |D^0D^{*0} + sin(\theta) |2^3P_1 > 1$

Summary

- Much left to understand in mesons containing heavy quarks
- A few of the most exciting opportunities are:
- For heavy-light systems:
 - Fully characterized the nature of the $j_1^{P}=\frac{1}{2}$ narrow (Qs) P states.
 - masses: (B_{s0}^*, B_{s1}) Belle 2, LHCb
 - decay branching fractions: (B_{s0}^*, B_{s1}) , (D_{s0}^*, D_{s1}) Belle 2, LHCb, BES III
 - relative strengths of photon and π^0 transitions are sensitive probes
- For quarkonium systems:
 - The B_c excited states provides new insights: LHC, TeraZ
 - States below threshold are all extremely narrow.
 - Probe the spin structure for unequal masses.
 - 2 P states very near threshold possible X(3872) analogs
 - Disentangling states near threshold offers best hope for understanding the new exotic states in QCD.