Comprehensive study of light mesons in nuclear matter with three-flavor extended linear sigma model

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Based on D. Suenaga, P. Lakaschus, in preparation

1. Introduction

- **Mesons in nuclear matter**
  - Understanding of modifications of mesons in nuclear matter is related to...

(Theoretically)

Partial restoration of chiral symmetry,
Change of axial anomaly, etc...

$$\langle \bar{q}q \rangle_{\rm vac} \to \langle \bar{q}q \rangle_{\rm med} \ ??$$

$$\partial_\mu j_A^\mu = \ ??$$

(Experimentally)

Modification of vector mesons experiments at J-PARC, J-Lab, ...
Mesic nuclei at GSI, the University of Bonn, Spring-8...

- It is worth exploring the changes of light mesons in nuclear matter comprehensively within a chiral model in a unified way

1. Introduction

- **Extended Linear Sigma Model (eLSM)**
  - The extended Linear Sigma Model (eLSM) is a chiral model which can successfully reproduce light meson properties (masses and widths)

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

Includes 9 scalars, 9 pseudo-scalars、9 vectors, and 9 axial-vectors (and 1 dilaton)

  - We applied the eLSM into cold nuclear matter reproducing saturation properties and studied mass changes of the mesons in nuclear matter comprehensively

- **The eLSM**

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

– (2 quark state) meson field is schematically written by

$$\Phi^{ij} \sim \bar{q}_R^j q_L^i \qquad \text{(scalar and pseudo-scalar)}$$

$$R_\mu^{ij} \sim \bar{q}_R^j \gamma_\mu q_R^i \quad L_\mu^{ij} \sim \bar{q}_L^j \gamma_\mu q_L^i \qquad \text{(vector and axial vector)}$$

– $U(3)_L \times U(3)_R$ chiral transformation laws and particle assignment are

$$\Phi \to g_L \Phi g_R^\dagger$$
$$R_\mu \to g_R R_\mu g_R^\dagger$$
$$L_\mu \to g_L L_\mu g_L^\dagger$$

$$\Phi = \sum_{i=0}^{8}(S_i + iP_i)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix},$$

$$L^\mu = \sum_{i=0}^{8}(V_i^\mu + A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu,$$

$$R^\mu = \sum_{i=0}^{8}(V_i^\mu - A_i^\mu)T_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu,$$

2. eLSM

- **Lagrangian**

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

– The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{dil}} + \text{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2\text{Tr}(\Phi^\dagger\Phi) - \lambda_1[\text{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\text{Tr}(\Phi^\dagger\Phi)^2 - \frac{1}{4}\text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$
$$+ \text{Tr}\left[\left(\left(\frac{G}{G_0}\right)^2\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \text{Tr}[H(\Phi + \Phi^\dagger)] + c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\}$$
$$+ \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) + \frac{h_1}{2}\text{Tr}(\Phi^\dagger\Phi)\text{Tr}(L_\mu^2 + R_\mu^2) + h_2\text{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2] + 2h_3\text{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)$$
$$+ g_3[\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\text{Tr}(L_\mu L^\mu L_\nu L^\nu) + \text{Tr}(R_\mu R^\mu R_\nu R^\nu)]$$
$$+ g_5\text{Tr}(L_\mu L^\mu)\text{Tr}(R_\nu R^\nu) + g_6[\text{Tr}(L_\mu L^\mu)\text{Tr}(L_\nu L^\nu) + \text{Tr}(R_\mu R^\mu)\text{Tr}(R_\nu R^\nu)],$$

$$R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$$
$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu$$
$$D_\mu\Phi = \partial_\mu\Phi - ig_1(L_\mu\Phi - \Phi R_\mu)$$
$$\Delta = \text{diag}(\delta_N, \delta_N, \delta_S)$$
$$H = \text{diag}(h_{0N}, h_{0N}, h_{0S})$$

– Scale invariance

– Chiral symmetry (approximately)

– Axial anomaly

- **Lagrangian**

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

  – By assuming the large-N_c suppression to the interactions including spin-1 mesons

$$
\begin{aligned}
\mathcal{L}^{\rm red}_{\rm eLSM} = & \,{\rm Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2{\rm Tr}[\Phi^\dagger\Phi] - \lambda_1\left({\rm Tr}[\Phi^\dagger\Phi]\right)^2 \\
& -\lambda_2{\rm Tr}[(\Phi^\dagger\Phi)^2] + {\rm Tr}[H(\Phi^\dagger+\Phi)] - \frac{1}{4}{\rm Tr}[L_{\mu\nu}L^{\mu\nu} \\
& +R_{\mu\nu}R^{\mu\nu}] + {\rm Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2+R_\mu^2)\right] \\
& +c_1\left({\rm det}\Phi - {\rm det}\Phi^\dagger\right)^2 + h_2{\rm Tr}[L_\mu^2\Phi\Phi^\dagger + R_\mu^2\Phi^\dagger\Phi] \\
& +2h_3{\rm Tr}[L_\mu\Phi R^\mu\Phi^\dagger] \\
& +g_{4p}\Big({\rm Tr}[L_\mu L_\nu L^\mu L^\nu] + {\rm Tr}[R_\mu R_\nu R^\mu R^\nu]\Big) \\
& +g_{4p}\Big({\rm Tr}[L_\mu L^\mu L_\nu L^\nu] + {\rm Tr}[R_\mu R^\mu R_\nu R^\nu]\Big)\,. \qquad (6)
\end{aligned}
$$

– This Lagrangian is fundamental to describe mesons in our approach

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

- **Parameters**
  - Parameters are determined by reproducing decay widths and masses

Observable	Fit (MeV)	Experiment (MeV)
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

Parameter		Value
C_1 [GeV^2]	$C_1 = m_0^2 + \lambda_1(\phi_N^2 + \phi_S^2)$	-0.9183 ± 0.0006
C_2 [GeV^2]	$C_2 = m_1^2$	0.4135 ± 0.0147
c_1 [GeV^{-2}]		450.5420 ± 7.0339
δ_S [GeV^2]		0.1511 ± 0.0038
g_1		5.8433 ± 0.0176
g_2		3.0250 ± 0.2329
ϕ_N [GeV]	(VEV of σ_N in the vacuum)	0.1646 ± 0.0001
ϕ_S [GeV]	(VEV of σ_S in the vacuum)	0.1262 ± 0.0001
h_2		9.8796 ± 0.6627
h_3		4.8667 ± 0.0864
λ_2		68.2972 ± 0.0435

2. eLSM

• Parameters

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

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$\Gamma_{\phi\to\bar{K}K}$		
$\Gamma_{a_1\to\rho\pi}$		
$\Gamma_{a_1\to\pi\gamma}$		
$\Gamma_{f_1(1420)\to K^\star}$		
Γ_{a_0}		
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h_2		9.8796 ± 0.6627
h_3		4.8667 ± 0.0864
λ_2		68.2972 ± 0.0435

– λ_1 still remains as free

⇒ will be fixed by nuclear matter properties later

- **Parity Doublet Model**
  - Two-flavor Parity Doublet Model (PDM) is employed to describe meson-nucleon interactions

$$
\begin{aligned}
\mathcal{L}_N = {} & \bar{\psi}_{1r}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{R\!\!\!/})\psi_{1r} + \bar{\psi}_{1l}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{L\!\!\!/})\psi_{1l} + \bar{\psi}_{2r}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{L\!\!\!/})\psi_{2r} + \bar{\psi}_{2l}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{R\!\!\!/})\psi_{2l} \\
& + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{1r}\gamma^\mu\psi_{1r} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{1l}\gamma^\mu\psi_{1l}\right) + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{1l}\gamma^\mu\psi_{1l} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{1r}\gamma^\mu\psi_{1r}\right) \\
& + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{2r}\gamma^\mu\psi_{2r} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{2l}\gamma^\mu\psi_{2l}\right) + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{2l}\gamma^\mu\psi_{2l} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{2r}\gamma^\mu\psi_{2r}\right) \\
& - M_0\left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] - k_1(\mathrm{det}\tilde{\Phi} + \mathrm{det}\tilde{\Phi}^\dagger)\left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] \\
& - k_2\left(\mathrm{det}\tilde{\Phi} - \mathrm{det}\tilde{\Phi}^\dagger\right)\left[\bar{\psi}_{1l}\psi_{2r} + \bar{\psi}_{1r}\psi_{2l} + \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] \\
& - G_1\left[\bar{\psi}_{1r}\tilde{\Phi}^\dagger\psi_{1l} + \bar{\psi}_{1l}\tilde{\Phi}\psi_{1r}\right] - G_2\left[\bar{\psi}_{2r}\tilde{\Phi}^\dagger\psi_{2l} + \bar{\psi}_{2l}\tilde{\Phi}\psi_{2r}\right]
\end{aligned}
$$

$$
\begin{aligned}
\psi_{1l(2r)} &\to g_L\psi_{1l(2r)} \\
\psi_{1r(2l)} &\to g_R\psi_{1r(2l)} \\
\tilde{\Phi} &= \sigma_N + i\pi^a\tau^a \\
\tilde{V}_\mu &= \frac{\tilde{L}_\mu + \tilde{R}_\mu}{2} = \frac{1}{2}(\omega_N + \rho^a\tau^a)_\mu \\
\tilde{A}_\mu &= \frac{\tilde{L}_\mu - \tilde{R}_\mu}{2} = \frac{1}{2}(f_{1N} + a_1^a\tau^a)_\mu
\end{aligned}
$$

• Parity Doublet Model

– Two-flavor Parity Doublet Model (PDM) is employed to describe meson-nucleon interactions

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\begin{aligned}
\mathcal{L}_N = & \bar{\psi}_{1r}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{R\!\!\!/})\psi_{1r} + \bar{\psi}_{1l}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{L\!\!\!/})\psi_{1l} + \bar{\psi}_{2r}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{L\!\!\!/})\psi_{2r} + \bar{\psi}_{2l}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{R\!\!\!/})\psi_{2l} \\
& + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{1r}\gamma^\mu\psi_{1r} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{1l}\gamma^\mu\psi_{1l}\right) + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{1l}\gamma^\mu\psi_{1l} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{1r}\gamma^\mu\psi_{1r}\right) \\
& + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{2r}\gamma^\mu\psi_{2r} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{2l}\gamma^\mu\psi_{2l}\right) + \tilde{g}\left(\mathrm{Tr}[\tilde{R}_\mu]\bar{\psi}_{2l}\gamma^\mu\psi_{2l} + \mathrm{Tr}[\tilde{L}_\mu]\bar{\psi}_{2r}\gamma^\mu\psi_{2r}\right) \\
& - M_0\left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] - k_1(\det\tilde{\Phi} + \det\tilde{\Phi}^\dagger)\left[\bar{\psi}_{1l}\psi_{2r} - \bar{\psi}_{1r}\psi_{2l} - \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] \\
& - k_2\left(\det\tilde{\Phi} - \det\tilde{\Phi}^\dagger\right)\left[\bar{\psi}_{1l}\psi_{2r} + \bar{\psi}_{1r}\psi_{2l} + \bar{\psi}_{2l}\psi_{1r} + \bar{\psi}_{2r}\psi_{1l}\right] \\
& - G_1\left[\bar{\psi}_{1r}\tilde{\Phi}^\dagger\psi_{1l} + \bar{\psi}_{1l}\tilde{\Phi}\psi_{1r}\right] - G_2\left[\bar{\psi}_{2r}\tilde{\Phi}^\dagger\psi_{2l} + \bar{\psi}_{2l}\tilde{\Phi}\psi_{2r}\right]
\end{aligned}
$$

$$\psi_{1l(2r)} \to g_L\psi_{1l(2r)}$$

$$\psi_{1r(2l)} \to g_R\psi_{1r(2l)}$$

$$\tilde{\Phi} = \sigma_N + i\pi^a\tau^a$$

$$\tilde{V}_\mu = \frac{\tilde{L}_\mu + \tilde{R}_\mu}{2} = \frac{1}{2}(\omega_N + \rho^a\tau^a)_\mu$$

$$\tilde{A}_\mu = \frac{\tilde{L}_\mu - \tilde{R}_\mu}{2} = \frac{1}{2}(f_{1N} + a_1^a\tau^a)_\mu$$

– Mass formulae are

$$m_\pm = \frac{1}{2}\left[\sqrt{(G_1+G_2)^2\phi_N^2 + 4\left(M_0 + \frac{k_1}{2}\phi_N^2\right)^2} \mp (G_2 - G_1)\phi_N\right]$$

(M_0: chiral invariant mass)

3. eLSM in nuclear matter

D. Suenaga, P. Lakaschus, in preparation

- **Parity Doublet Model**
  - Two-flavor Parity Doublet Model (PDM) is employed to describe meson-nucleon interactions

$$
\begin{aligned}
\mathcal{L}_N = {} & \bar\psi_{1r}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{R}\!\!\!/)\psi_{1r} + \bar\psi_{1l}(i\partial\!\!\!/ + \mu_B\gamma_0 + g_V \tilde{L}\!\!\!/)\psi_{1l} + \bar\psi_{2r}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{L}\!\!\!/)\psi_{2r} + \bar\psi_{2l}(i\partial\!\!\!/ + \mu_B\gamma_0 + h_V \tilde{R}\!\!\!/)\psi_{2l} \\
& + \tilde g\left(\mathrm{Tr}[\tilde R_\mu]\bar\psi_{1r}\gamma^\mu\psi_{1r} + \mathrm{Tr}[\tilde L_\mu]\bar\psi_{1l}\gamma^\mu\psi_{1l}\right) + \tilde g\left(\mathrm{Tr}[\tilde R_\mu]\bar\psi_{1l}\gamma^\mu\psi_{1l} + \mathrm{Tr}[\tilde L_\mu]\bar\psi_{1r}\gamma^\mu\psi_{1r}\right) \\
& + \tilde g\left(\mathrm{Tr}[\tilde R_\mu]\bar\psi_{2r}\gamma^\mu\psi_{2r} + \mathrm{Tr}[\tilde L_\mu]\bar\psi_{2l}\gamma^\mu\psi_{2l}\right) + \tilde g\left(\mathrm{Tr}[\tilde R_\mu]\bar\psi_{2l}\gamma^\mu\psi_{2l} + \mathrm{Tr}[\tilde L_\mu]\bar\psi_{2r}\gamma^\mu\psi_{2r}\right) \\
& - M_0\left[\bar\psi_{1l}\psi_{2r} - \bar\psi_{1r}\psi_{2l} - \bar\psi_{2l}\psi_{1r} + \bar\psi_{2r}\psi_{1l}\right] - \underline{k_1(\det\tilde\Phi + \det\tilde\Phi^\dagger)\left[\bar\psi_{1l}\psi_{2r} - \bar\psi_{1r}\psi_{2l} - \bar\psi_{2l}\psi_{1r} + \bar\psi_{2r}\psi_{1l}\right]} \\
& \underline{- k_2\left(\det\tilde\Phi - \det\tilde\Phi^\dagger\right)\left[\bar\psi_{1l}\psi_{2r} + \bar\psi_{1r}\psi_{2l} + \bar\psi_{2l}\psi_{1r} + \bar\psi_{2r}\psi_{1l}\right]} \\
& - G_1\left[\bar\psi_{1r}\tilde\Phi^\dagger\psi_{1l} + \bar\psi_{1l}\tilde\Phi\psi_{1r}\right] - G_2\left[\bar\psi_{2r}\tilde\Phi^\dagger\psi_{2l} + \bar\psi_{2l}\tilde\Phi\psi_{2r}\right]
\end{aligned}
$$

$$
\begin{aligned}
\psi_{1l(2r)} &\to g_L\psi_{1l(2r)} \\
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\tilde\Phi &= \sigma_N + i\pi^a\tau^a \\
\tilde V_\mu &= \frac{\tilde L_\mu + \tilde R_\mu}{2} = \frac{1}{2}(\omega_N + \rho^a\tau^a)_\mu \\
\tilde A_\mu &= \frac{\tilde L_\mu - \tilde R_\mu}{2} = \frac{1}{2}(f_{1N} + a_1^a\tau^a)_\mu
\end{aligned}
$$

  - k_1, k_2 terms violate $U(1)_A$ symmetry explicitly

    (direct coupling of axial anomaly to the nucleons)

L. Olbrich, M. Zetenyi, F. Giacosa and D. H. Rischke, Phys. Rev. D 97, no. 1, 014007 (2018)

3. eLSM in nuclear matter

D. Suenaga, P. Lakaschus, in preparation

- **Construction of nuclear matter**
  - Thermodynamic potential Ω (effective action) is constructed by nucleon one-loop with meson mean fields at zero temperature combining the eLSM and the PDM
  - Density dependences of $\phi_N, \phi_S, \bar{\omega}_N$ are determined by gap equations

$$\frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = \frac{\partial\Omega}{\partial\bar{\omega}_N} = 0$$

  - Parameters are fixed by vacuum and nuclear matter properties

Inputs in the vacuum	Values
$\hat{m}_+$ [GeV]	0.939
$\hat{m}_-$ [GeV]	1.535
$\hat{g}_A^{N_+}$	1.267
$\hat{g}_A^{N_-}$	0.2±0.3 [61]
$\hat{\Gamma}_{N^*(1535)\to N\eta}$ [GeV]	0.065

Inputs in nuclear matter	Values
$P\|_{\rho_0}$	0
$\mu_B\|_{\rho_0}$ [GeV]	0.923
K [GeV]	0.24

  - Only M_0 and k_1 are left as free parameters in our approach

3. eLSM in nuclear matter

D. Suenaga, P. Lakaschus, in preparation

- **Meson masses in nuclear matter**
  - Nucleon loop corrections should be included to calculate meson masses in nuclear matter to respect the original chiral symmetry

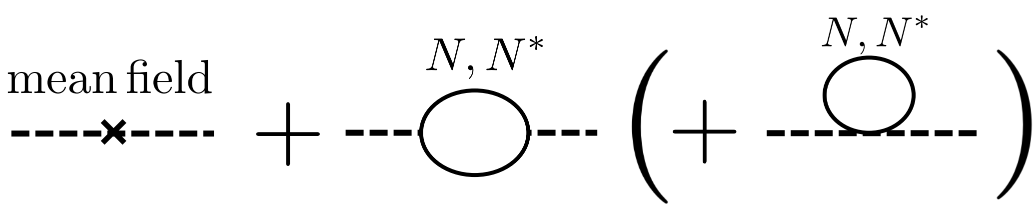
D. Suenaga, M. Harada, and S. Yasui PRC96 (2017)

  - Meson masses in nuclear matter is defined by the following formula in the present analysis

m_X : mean field level

$\Pi_X(q_0, \vec{q})$: one loop corrections

$$\tilde{m}_X^2 \equiv m_X^2 + \Pi_X(m_X, \vec{0})$$



  - If the meson has mixing with the other one, then we need to solve it

4. Results

- **Determined parameters**

Constraints by nuclear matter properties are very strong. . .

  – The allowed ranges of M_0, $\tilde{k}_1(= k_1\hat{\phi}_N)$ are

$$0.6\,\mathrm{GeV} \lesssim M_0 \lesssim 0.8\,\mathrm{GeV} \quad \text{and} \quad -5 \lesssim \tilde{k}_1 \lesssim 15$$

  – The remaining parameters for given $M_0, \tilde{k}_1$ are fixed as

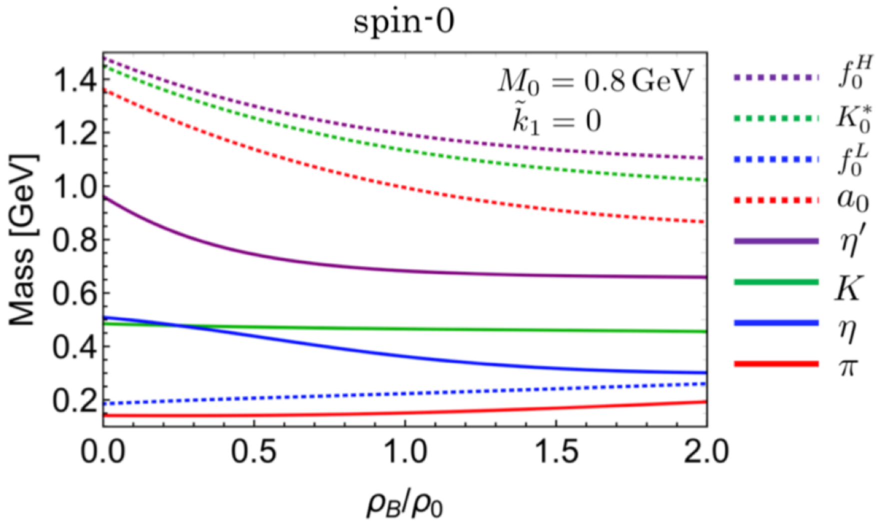
M_0 [GeV]	$\tilde{k}_1$	G_1	G_2	$\tilde{k}_2$	g_V	h_V	$\tilde{g}$	λ_1	g_{4p}
0.8	5	-0.2924	3.329	-19.60	21.10	8.063	-6.333	-22.73	39.09
0.8	0	3.922	7.542	-0.2467	3.847	-9.186	-2.623	-22.98	42.01
0.8	-5	5.324	8.945	0.8113	2.625	-10.41	-2.668	-22.67	2.442
0.7	5	1.488	5.109	-4.392	8.432	-4.601	-4.350	-22.54	114.4
0.7	0	4.386	8.007	0.1265	3.381	-9.652	-3.683	-22.84	171.9
0.7	-5	5.497	9.118	0.9509	2.506	-10.53	-3.537	-22.48	106.7

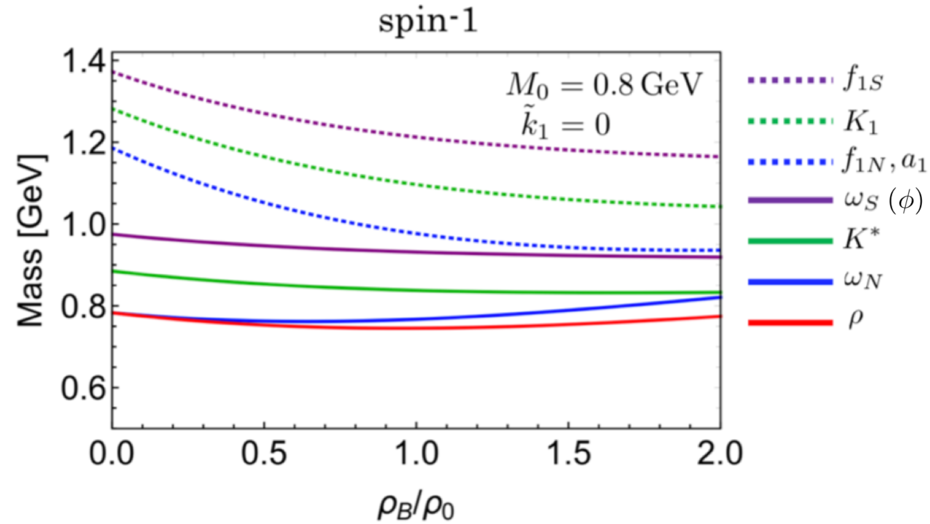
$$\tilde{k}_1 = k_1\hat{\phi}_N$$
$$\tilde{k}_2 = k_2\hat{\phi}_N$$

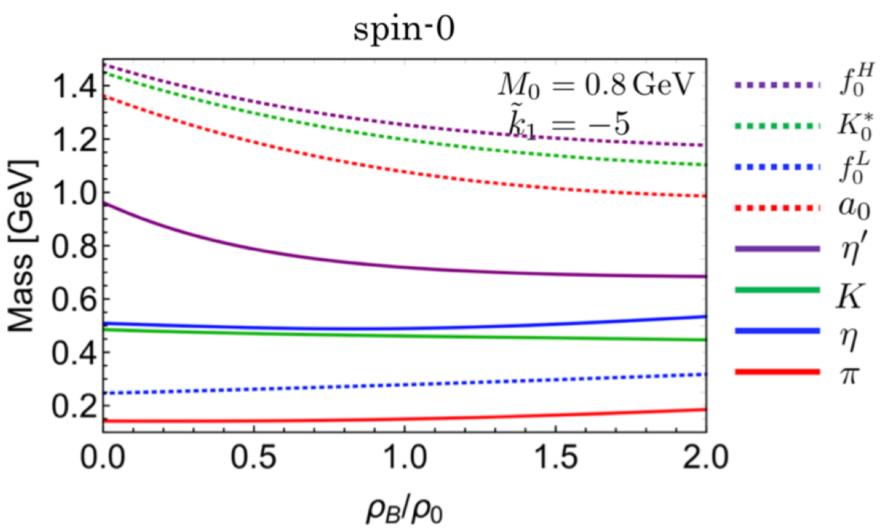
  – The results of meson masses in medium shown in the next slides suggest that $M_0 \approx 0.8\,\mathrm{GeV}$ and $\tilde{k}_1 \approx 0$ is preferable

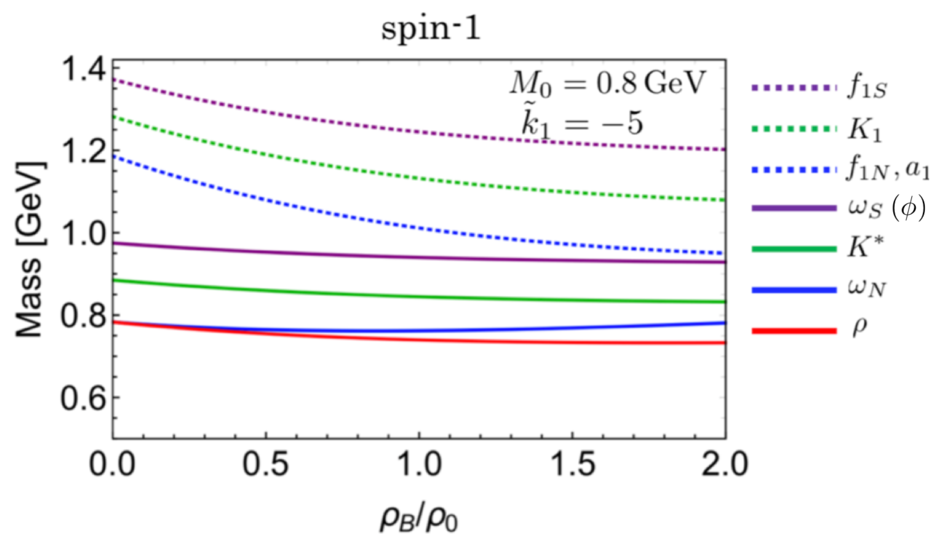
4. Results

- **Results with** $M_0 = 0.8\,\mathrm{GeV}$ **and** $\tilde{k}_1 = 0, -5$





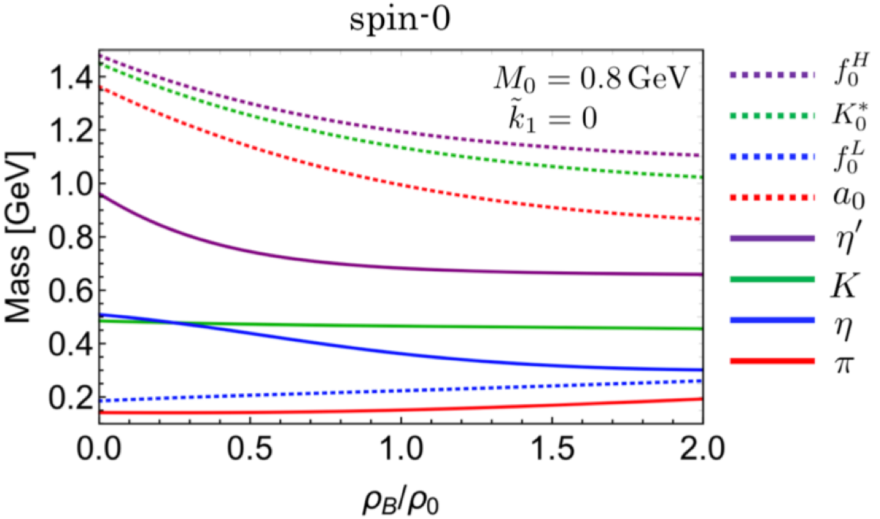


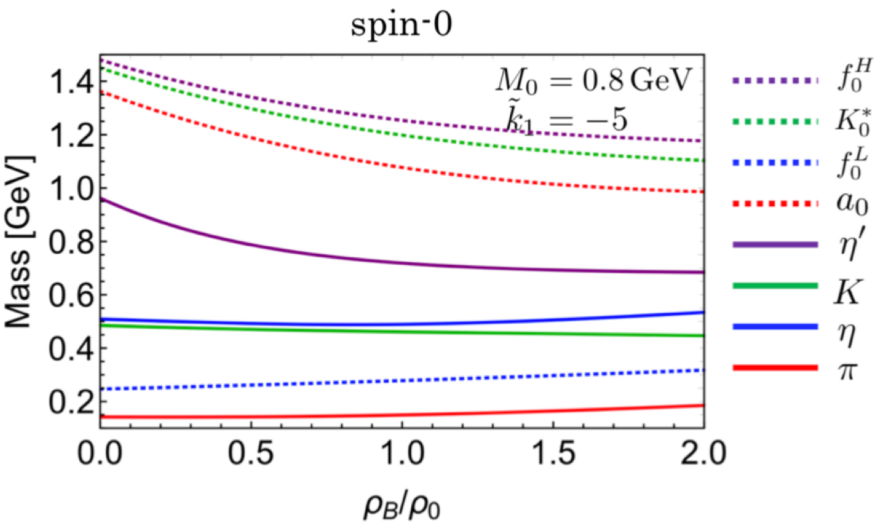


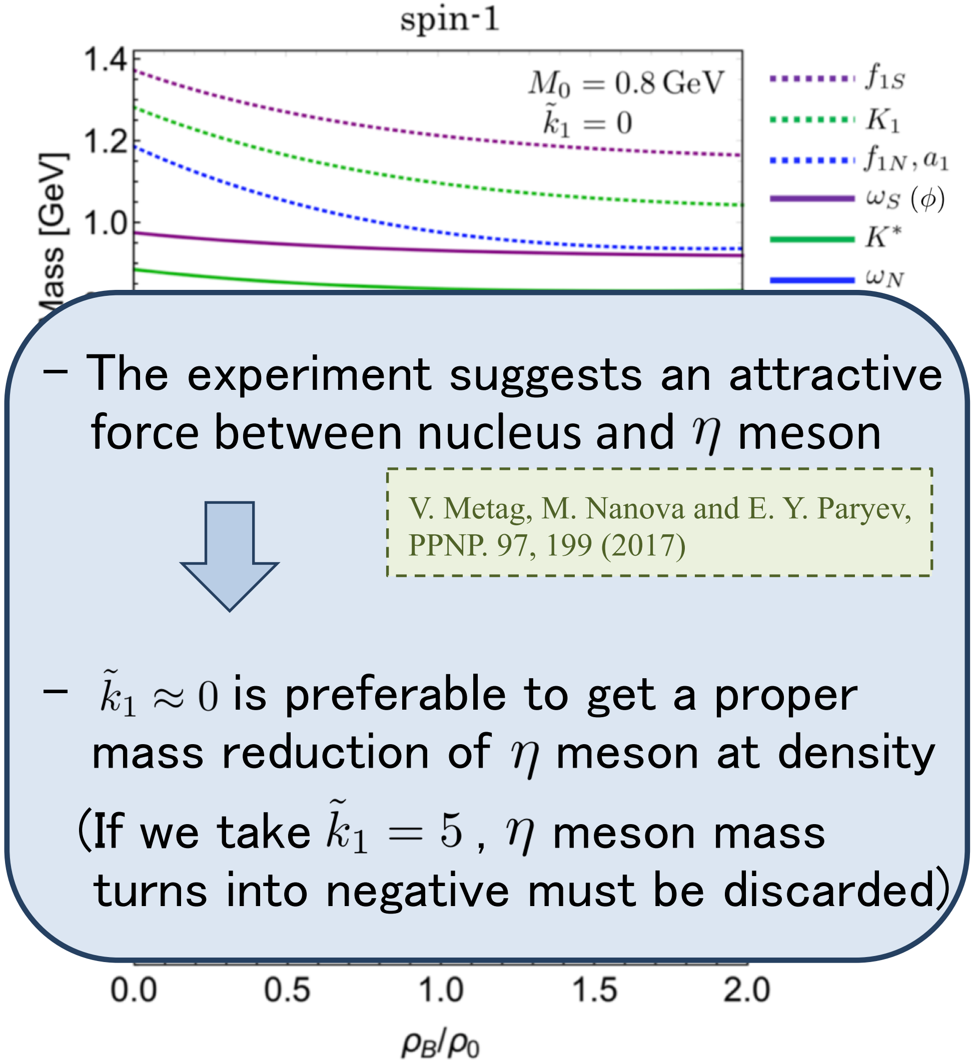
4. Results

D. Suenaga, P. Lakaschus, in preparation

- **Results with $M_0 = 0.8\,\mathrm{GeV}$ and $\tilde{k}_1 = 0, -5$**







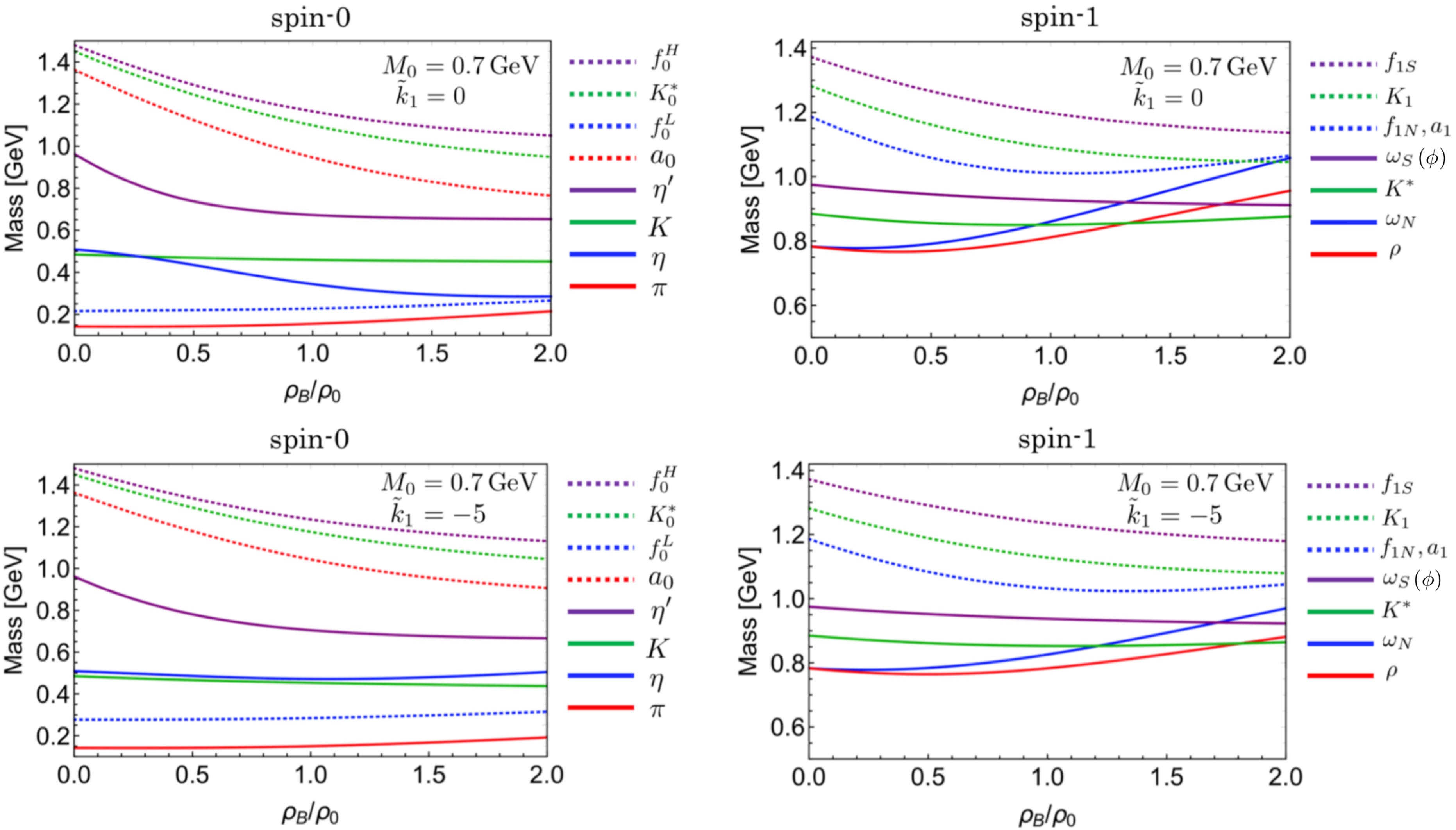
– The experiment suggests an attractive force between nucleus and η meson

V. Metag, M. Nanova and E. Y. Paryev, PPNP. 97, 199 (2017)

– $\tilde{k}_1 \approx 0$ is preferable to get a proper mass reduction of η meson at density

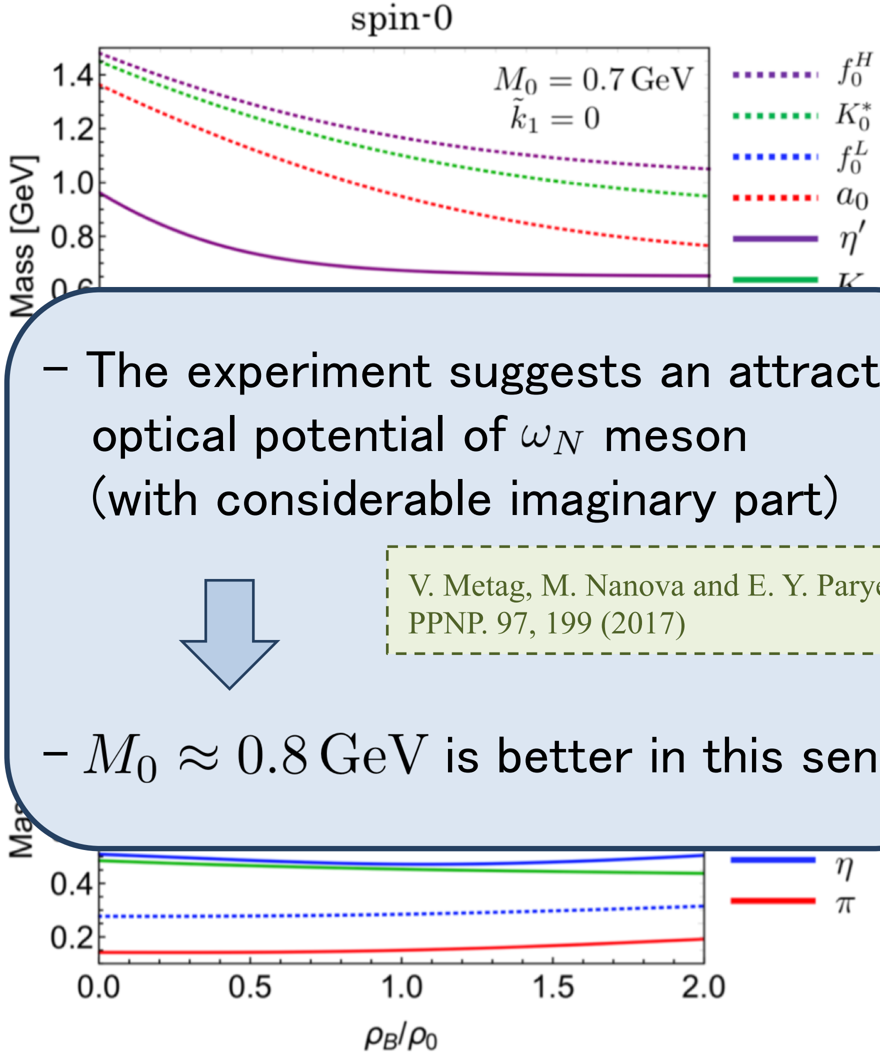
(If we take $\tilde{k}_1 = 5$, η meson mass turns into negative must be discarded)

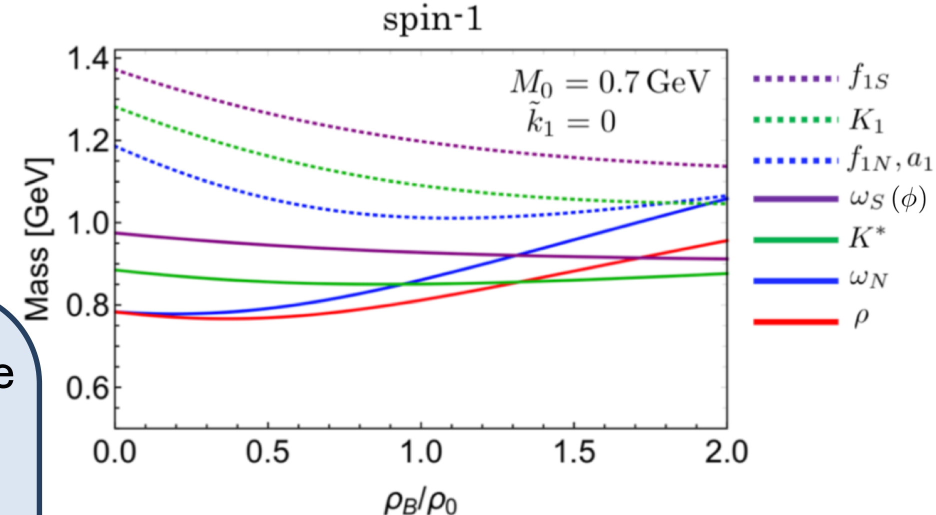
- **Results with $M_0 = 0.7\,\mathrm{GeV}$ and $\tilde{k}_1 = 0, -5$**

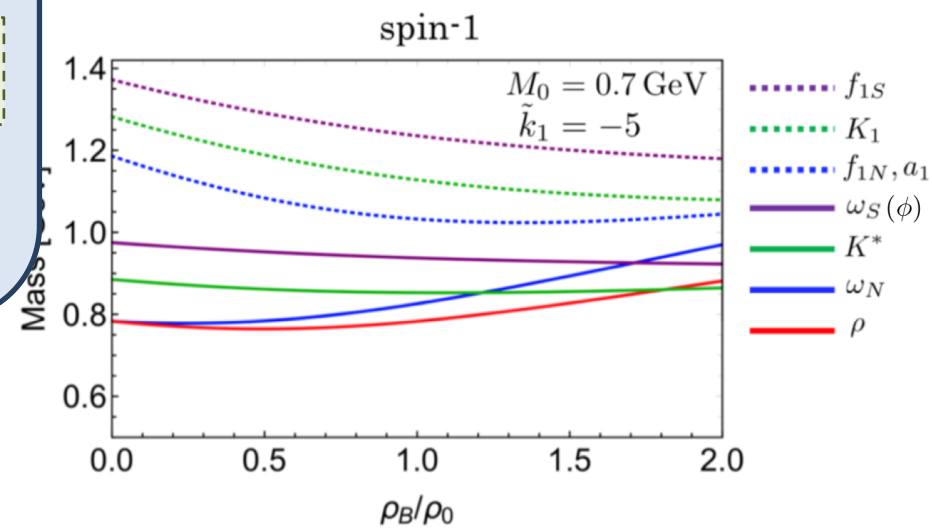


4. Results

- **Results with $M_0 = 0.7\,\mathrm{GeV}$ and $\tilde{k}_1 = 0, -5$**







- The experiment suggests an attractive optical potential of ω_N meson (with considerable imaginary part)

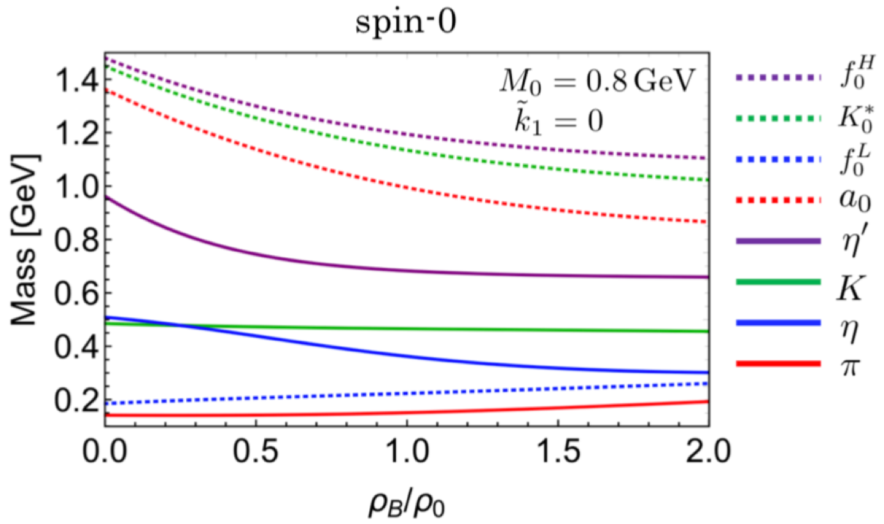
V. Metag, M. Nanova and E. Y. Paryev, PPNP. 97, 199 (2017)

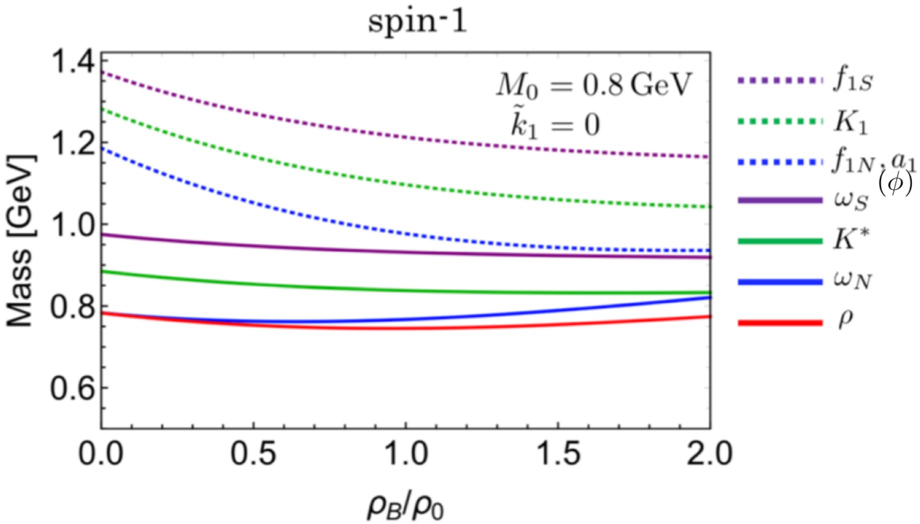
⇓

- $M_0 \approx 0.8\,\mathrm{GeV}$ is better in this sense

5. Conclusions

- **Summary**
  - We employed the three-flavor extended Linear Sigma Model (eLSM) and two-flavor Parity Doublet Model (PDM) to study meson masses in nuclear matter comprehensively
  - The density dependences of mesons (particularly η and ω_N) suggest $M_0 \approx 0.8\,\mathrm{GeV}$ and $k_1 \approx 0$ is preferable





5. Conclusions

- **Discussions**
  - To study the nature of $f_0(0^+)$ mesons more reliably, we need to incorporate tetraquark states into the model
  - Inclusion of imaginary part by a large decay width (if necessary) and a broadening in nuclear matter is sufficient to study meson mass more precisely
  - Direct interactions between matter and open/hidden strange mesons were missing ($\longrightarrow$ e.g. three-flavor PDM ?)

Back up

- **Notes on PRD 87, 014011 (2013)**
  - The values of λ_1, h_1 are not determined
  - λ_1, h_1 could be fixed by scalar–isoscalar (f_0) mesons properties
  - f_0 mesons are not used as input since they include many uncertainties

$$\mathcal{L} = \mathcal{L}_{\rm dil} + \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2\mathrm{Tr}(\Phi^\dagger\Phi) - \underline{\lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2} - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$
$$+ \mathrm{Tr}\left[\left(\left(\frac{G}{G_0}\right)^2\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] + c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\}$$
$$+ \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) + \underline{\frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2)} + h_2\mathrm{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)$$
$$+ g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu R_\nu R^\nu)]$$
$$+ g_5\mathrm{Tr}(L_\mu L^\mu)\mathrm{Tr}(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\mathrm{Tr}(R_\nu R^\nu)],$$

- # Input parameters

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

  - Scalar and pseudo-scalar sector

$$m_{a_0(980)} = (980 \pm 49)\ \mathrm{MeV},$$
$$m_{a_0(1450)} = (1474 \pm 74)\ \mathrm{MeV},$$
$$m_{K_0^\star(800)} = (676 \pm 40)\ \mathrm{MeV},$$
$$m_{K_0^\star(1430)} = (1425 \pm 71)\ \mathrm{MeV},$$
$$\Gamma_{a_0(1450)} = (265 \pm 13.3)\ \mathrm{MeV},$$
$$\Gamma_{K_0^\star(800)\to K\pi} = (548 \pm 27.4)\ \mathrm{MeV},$$
$$\Gamma_{K_0^\star(1430)\to K\pi} = (270 \pm 80)\ \mathrm{MeV},$$
$$|\mathcal{M}_{a_0(980)\to KK}| = (3590 \pm 440)\ \mathrm{MeV}\,[37],$$
$$|\mathcal{M}_{a_0(980)\to \eta\pi}| = (3300 \pm 166.5)\ \mathrm{MeV}\,[37].$$

$$f_\pi = (92.2 \pm 4.6)\ \mathrm{MeV},$$
$$f_K = (155.6/\sqrt{2} \pm 5.5)\ \mathrm{MeV}$$
$$m_\pi = (138 \pm 6.9)\ \mathrm{MeV},$$
$$m_K = (495.6 \pm 24.8)\ \mathrm{MeV},$$
$$m_\eta = (547.9 \pm 27.4)\ \mathrm{MeV},$$
$$m_{\eta'} = (957.8 \pm 47.9)\ \mathrm{MeV},$$

- # Input parameters

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

  - Vector and axial-vector sector

$$
\begin{aligned}
m_\rho &= (775.5 \pm 38.8)\ \mathrm{MeV}, \\
m_{K^\star} &= (893.8 \pm 44.7)\ \mathrm{MeV}, \\
m_\phi &= (1019.5 \pm 51)\ \mathrm{MeV}, \\
\Gamma_{\rho\to\pi\pi} &= (149.1 \pm 7.4)\ \mathrm{MeV}, \\
\Gamma_{K^\star\to K\pi} &= (46.2 \pm 2.3)\ \mathrm{MeV}, \\
\Gamma_{\phi\to KK} &= (3.54 \pm 0.178)\ \mathrm{MeV},
\end{aligned}
$$

$$
\begin{aligned}
m_{a_1} &= (1230 \pm 61.5)\ \mathrm{MeV}, \\
m_{f_1(1420)} &= (1426.4 \pm 71.3)\ \mathrm{MeV}, \\
\Gamma_{a_1\to\rho\pi} &= (425 \pm 175)\ \mathrm{MeV}, \\
\Gamma_{a_1\to\pi\gamma} &= (0.640 \pm 0.250)\ \mathrm{MeV}, \\
\Gamma_{f_1(1420)\to K^\star K} &= (43.9 \pm 2.2)\ \mathrm{MeV},
\end{aligned}
$$

  - The experimental error is increased to 5% artificially if the error is less than 5%, since isospin breaking effects are not included

- χ^2 **fitting**

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, and D. H. Rischke; PRD 87, 014011 (2013)

Observable	Fit (MeV)	Experiment (MeV)
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

Parameter		Value
C_1 [GeV^2]	$C_1 = m_0^2 + \lambda_1(\phi_N^2 + \phi_S^2)$	-0.9183 ± 0.0006
C_2 [GeV^2]	$C_2 = m_1^2$	0.4135 ± 0.0147
c_1 [GeV^{-2}]		450.5420 ± 7.0339
δ_S [GeV^2]		0.1511 ± 0.0038
g_1		5.8433 ± 0.0176
g_2		3.0250 ± 0.2329
ϕ_N [GeV]		0.1646 ± 0.0001
ϕ_S [GeV]		0.1262 ± 0.0001
h_2		9.8796 ± 0.6627
h_3		4.8667 ± 0.0864
λ_2		68.2972 ± 0.0435

- **Mean field mass formula**
  - Spin-0

$$m_{\sigma_N}^2 = m_0^2 + \left(3\lambda_1 + \frac{3}{2}\lambda_2\right)\phi_N^2 + \lambda_1\phi_S^2 - \frac{1}{2}(h_2+h_3)\bar{\omega}_N^2 \ ,$$

$$m_{\sigma_S}^2 = m_0^2 + \lambda_1\phi_N^2 + 3(\lambda_1+\lambda_2)\phi_S^2 \ ,$$

$$m_{\sigma_S\sigma_N}^2 = 2\lambda_1\phi_N\phi_S \ ,$$

$$m_{a_0}^2 = m_0^2 + \left(\lambda_1 + \frac{3}{2}\lambda_2\right)\phi_N^2 + \lambda_1\phi_S^2 - \frac{1}{2}(h_2+h_3)\bar{\omega}_N^2$$

$$m_{K_0^*}^2 = m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^2 + (\lambda_1+\lambda_2)\phi_S^2 + \frac{\lambda_2}{\sqrt{2}}\phi_N\phi_S - g_1^2\frac{\bar{\omega}_N^2}{4} - \frac{1}{4}(h_2+h_3)\bar{\omega}_N^2$$

$$m_{\eta_N}^2 = m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^2 + \lambda_1\phi_S^2 + c_1\phi_N^2\phi_S^2 - \frac{1}{2}(h_2+h_3)\bar{\omega}_N^2 \ ,$$

$$m_{\eta_S}^2 = m_0^2 + \lambda_1\phi_N^2 + (\lambda_1+\lambda_2)\phi_S^2 + \frac{c_1}{4}\phi_N^4 \ ,$$

$$m_{\eta_N\eta_S}^2 = \frac{c_1}{2}\phi_N^3\phi_S \ ,$$

$$m_\pi^2 = m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^2 + \lambda_1\phi_S^2 - \frac{1}{2}(h_2+h_3)\bar{\omega}_N^2 \ ,$$

$$m_K^2 = m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^2 + (\lambda_1+\lambda_2)\phi_S^2 - \frac{\lambda_2}{\sqrt{2}}\phi_N\phi_S - g_1^2\frac{\bar{\omega}_N^2}{4} - \frac{1}{4}(h_2+h_3)\bar{\omega}_N^2$$

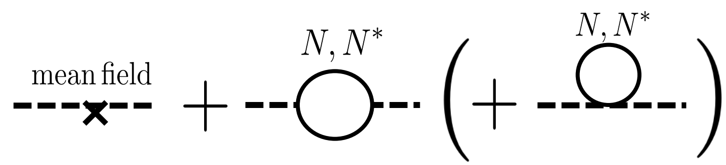
- **Mean field mass formula**
  - Spin-1

$$m^2_{\omega_N} = m_1^2 + \frac{1}{2}(h_2 + h_3)\phi_N^2 + 2g_{4p}\bar{\omega}_N^2 \ ,$$
$$m^2_{\omega_S} = m_1^2 + (h_2 + h_3)\,\phi_S^2 + 2\delta_S \ ,$$
$$m^2_{\rho} = m^2_{\omega_N} \ ,$$
$$m^2_{K^*} = m_1^2 + \frac{1}{4}(g_1^2 + h_2)\phi_N^2 + \frac{1}{2}(g_1^2 + h_2)\phi_S^2 + \frac{1}{\sqrt{2}}(h_3 - g_1^2)\phi_N\phi_S + \delta_S + g_{4p}\bar{\omega}_N^2$$

$$m^2_{f_{1N}} = m_1^2 + \frac{1}{2}(2g_1^2 + h_2 - h_3)\phi_N^2 + 2g_{4p}\bar{\omega}_N^2 \ ,$$
$$m^2_{f_{1S}} = m_1^2 + (2g_1^2 + h_2 - h_3)\phi_S^2 + 2\delta_S \ ,$$
$$m^2_{a_1} = m^2_{f_{1N}} \ ,$$
$$m^2_{K_1} = m_1^2 + \frac{1}{4}(g_1^2 + h_2)\phi_N^2 + \frac{1}{2}(g_1^2 + h_2)\phi_S^2 - \frac{1}{\sqrt{2}}(h_3 - g_1^2)\phi_N\phi_S + \delta_S + g_{4p}\bar{\omega}_N^2$$

• Mass formula in medium

– By defining $\tilde{m}_X^2 \equiv m_X^2 + \Pi_X(m_X, \vec{0})$



$$(m_{a_0}^2)^{\rm med} = \tilde{m}_{a_0}^2\ , \ (m_{\omega_N}^2)^{\rm med} = \tilde{m}_{\omega_N}^2\ , \ (m_{\omega_S}^2)^{\rm med} = \tilde{m}_{\omega_S}^2$$
$$(m_{\rho}^2)^{\rm med} = \tilde{m}_{\rho}^2,\ (m_{K^*}^2)^{\rm med} = \tilde{m}_{K^*}^2\ , \ (m_{f_{1N}}^2)^{\rm med} = \tilde{m}_{f_{1N}}^2$$
$$(m_{f_{1S}}^2)^{\rm med} = \tilde{m}_{f_{1S}}^2\ , \ (m_{a_1}^2)^{\rm med} = \tilde{m}_{a_1}^2\ , \ (m_{K_1}^2)^{\rm med} = \tilde{m}_{K_1}^2$$

and

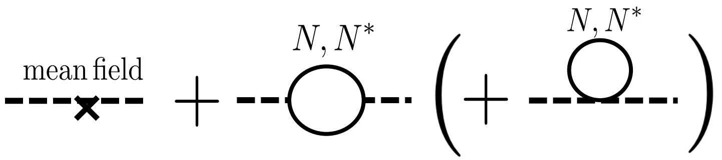
$$(m_\pi^2)^{\rm med} = Z_\pi^2 \tilde{m}_\pi^2\ , \ (m_{K_0^*}^2)^{\rm med} = Z_{K_0^*}^2 \tilde{m}_{K_0^*}^2$$
$$(m_K^2)^{\rm med} = Z_K^2 \tilde{m}_K^2\ ,$$

with

$$Z_\pi = \frac{\tilde{m}_{a_1}}{\sqrt{\tilde{m}_{a_1}^2 - g_1^2\phi_N^2}}\ ,$$
$$Z_{K_0^*} = \frac{2\tilde{m}_{K^*}}{\sqrt{4\tilde{m}_{K^*}^2 - g_1^2(\phi_N - \sqrt{2}\phi_S)^2}}$$
$$Z_K = \frac{2\tilde{m}_{K_1}}{\sqrt{4\tilde{m}_{K_1}^2 - g_1^2(\phi_N + \sqrt{2}\phi_S)^2}}$$

• Mass formula in medium

– By defining $\tilde{m}_X^2 \equiv m_X^2 + \Pi_X(m_X, \vec{0})$



$$(m^2_{f_0^H/f_0^L})^{\rm med} = \frac{1}{2}\Big(\tilde{m}^2_{\sigma_N} + \tilde{m}^2_{\sigma_S} \pm\sqrt{(\tilde{m}^2_{\sigma_N} - \tilde{m}^2_{\sigma_S})^2 + 4\tilde{m}^4_{\sigma_N\sigma_S}}\Big)$$

$$(m^2_{\eta'/\eta})^{\rm med} = \frac{1}{2}\Big((m^2_{\eta_N})^{\rm med} + (m^2_{\eta_S})^{\rm med} \pm\sqrt{\big((m^2_{\eta_N})^{\rm med} - (m^2_{\eta_S})^{\rm med}\big)^2 + 4(m^4_{\eta_N\eta_S})^{\rm med}}\Big) , \quad \text{(B5)}$$

where $(m^2_{\eta_N})^{\rm med} = Z^2_{\eta_N}\tilde{m}^2_{\eta_N}$, $(m^2_{\eta_S})^{\rm med} = Z^2_{\eta_S}\tilde{m}^2_{\eta_S}$ and $(m^4_{\eta_N\eta_S})^{\rm med} = Z^2_{\eta_N}Z^2_{\eta_S}\tilde{m}^4_{\eta_N\eta_S}$ with

$$Z_{\eta_N} = \frac{\tilde{m}_{f_{1N}}}{\sqrt{\tilde{m}^2_{f_{1N}} - g_1^2\phi_N^2}} ,$$

$$Z_{\eta_S} = \frac{\tilde{m}_{f_{1S}}}{\sqrt{\tilde{m}^2_{f_{1S}} - 2g_1^2\phi_S^2}} , \quad \text{(B6)}$$

- **Construction of nuclear matter**
  - Thermodynamic potential Ω (effective action) is constructed by nucleon one-loop with meson mean fields at zero temperature

$$\Omega/V = -\frac{1}{4\pi^2}\left\{\frac{2}{3}\sqrt{k_F^2+m_+^2}k_F^3 - \sqrt{k_F^2+m_+^2}k_F m_+^2 + m_+^4 \ln\left(\frac{k_F+\sqrt{k_F^2+m_+^2}}{m_+}\right)\right\}$$
$$+\left(\frac{m_0^2}{2}(\phi_N^2+\phi_S^2)+\frac{\lambda_1}{4}(\phi_N^2+\phi_S^2)^2+\frac{\lambda_2}{8}(\phi_N^4+2\phi_S^4)-h_{0N}\phi_N-h_{0S}\phi_S-\frac{m_{\omega_N}^2}{2}\bar{\omega}_N^2-\frac{g_{4p}}{2}\bar{\omega}_N^4\right)$$
$$-\left(\frac{m_0^2}{2}(\hat{\phi}_N^2+\hat{\phi}_S^2)+\frac{\lambda_1}{4}(\hat{\phi}_N^2+\hat{\phi}_S^2)^2+\frac{\lambda_2}{8}(\hat{\phi}_N^4+2\hat{\phi}_S^4)-h_{0N}\hat{\phi}_N-h_{0S}\hat{\phi}_S\right),$$

where $\mu_B^* \equiv \mu_B - g_\omega \bar{\omega}_N$ with $\mu_B^* = \sqrt{k_F^2+m_+^2}$ and $g_\omega = -\frac{1}{2}(g_V\cos^2\theta + h_V\sin^2\theta) - 2\tilde{g}$

- **Results with another choice of $\hat{m}_-$**

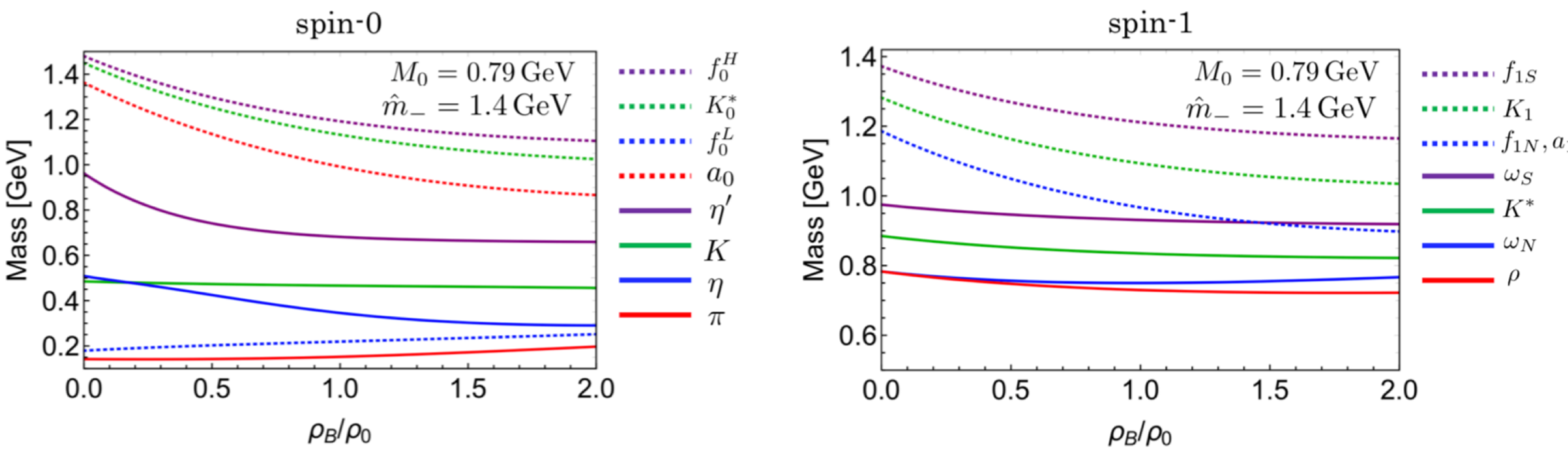


FIG. 6. (color online) The density dependence of spin-0 (left) and spin-1 (right) meson masses with $\hat{m}_- = 1.4$ MeV, $M_0 = 0.79$ GeV, and $k_1 = k_2 = 0$.

- Results with another choice of $\hat{m}_-$

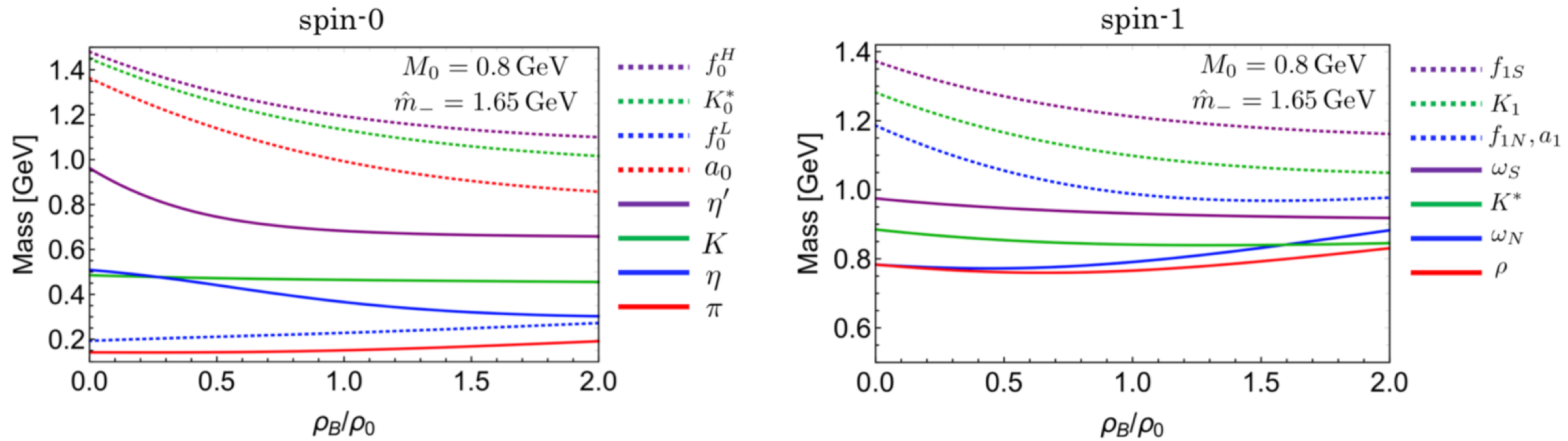


FIG. 7. (color online) The density dependence of spin-0 (left) and spin-1 (right) meson masses with $\hat{m}_- = 1.65$ MeV, $M_0 = 0.8$ GeV, and $k_1 = k_2 = 0$.