Possible interpretation of N(1685) and others

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nucleon resonance below 2 Gev

Table : Nucleon resonances below 2 Gev (Experimental parameters (exp) are from PDG).

Resonance	Status	J^P	M_{BW}^{exp} MeV	Γ_{BW}^{exp} MeV
$N(1440)$	****	$\frac{1}{2}^+$	1410-1470	250-450
$N(1680)$	****	$\frac{3}{2}^+$	1665-1680	115-130
$N(1710)$	****	$\frac{1}{2}^+$	1680-1740	80-200
$N(1720)$	****	$\frac{5}{2}^+$	1680-1750	150-400
$N(1860)$	**	$\frac{5}{2}^+$	1800-1980	220-410
$N(1880)$	***	$\frac{1}{2}^+$	1830-1930	200-400
$N(1900)$	****	$\frac{3}{2}^+$	1890-1950	100-320
$N(1990)$	**	$\frac{1}{2}^+$	1950-2100	200-400
$N(1520)$	****	$\frac{3}{2}^-$	1510-1520	100-120
$N(1535)$	****	$\frac{1}{2}^-$	1525-1545	125-175
$N(1650)$	****	$\frac{1}{2}^-$	1645-1670	100-150
$N(1675)$	****	$\frac{5}{2}^-$	1670-1680	130-160
$N(1685)$	*	$\frac{1}{2}^-?$	1665-1675	15-45
$N(1700)$	***	$\frac{3}{2}^-$	1650-1750	100-300
$N(1875)$	***	$\frac{3}{2}^-$	1820-1920	120-250
$N(1895)$	****	$\frac{1}{2}^-$	1880-1910	80-200

N(1685)

- $N(1685)$ was firstly reported in the photoproduction of η meson off the quasi-free neutron. Its Breit-Wigner (BW) width is less than $30\ \mathrm{MeV}$ much less than the lowest BW widths of other low-lying nucleon resonances. [Phys. Lett. B **647**: 23-29 (2007)]

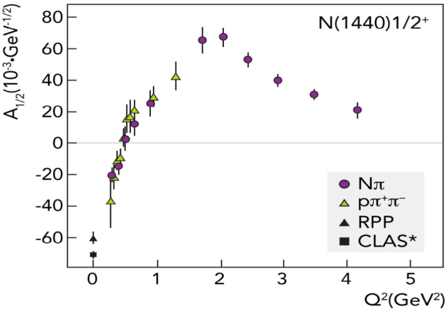
- The one-star isospin $1/2$ nucleon resonance N(1685) was introduced into PDG since 2012 after the observation of a narrow enhancement at $W \sim 1.68$ in the $\gamma n \to \eta n$ excitation function at GRAAL, CBELSA/TAPS, LNS, and A2@MAMI.

- Though removed from the listings at 2016, the new date on the invariant mass spectra of $\gamma N \to \pi\eta N$ reactions from GRAAL still reveal the N(1685) resonance.[JETP Letters **106**: 693-699(2017)]

- The interpretation of the $N(1685)$ is still an open question.

Roper Resonance $N_{1/2+}(1440)$: Current experimental status

The transverse helicity amplitudes for Roper-resonance electroproduction. Circles -analysis of single-pion final states(Aznauryan et al., 2008, 2009); triangles - analysis of $ep \to e^{'}\pi^{+}\pi^{-}p^{'}$ (Mokeev et al., 2012, 2016); square- CLAS Collaboration result $\gamma^{*}p \to N(1440)$(Dugger et al., 2009); and triangle - global average of this value (Patrignani et al., 2016).



- The sign change in the helicity amplitude as a function of Q^2 suggests a node in the wave function and thus mainly a first radial excited state.

$N_{1/2+}(1440)$, $N_{3/2-}(1520)$ and $N_{1/2-}(1535)$

- In the traditional q^3 picture, the Roper $N_{1/2+}(1440)$ usually gets a mass ~ 100 MeV above the $N_{3/2-}(1520)$ and $N_{1/2-}(1535)$, but not 100 MeV below it.
- In the $N_{3/2-}(1520)$ decay modes, branching ratio of $\Gamma_{\eta N}/\Gamma_{tot}$ is almost zero.
- In the $N_{1/2-}(1535)$ decay modes, the branching ratio of $N(1535) \to N\eta$ process is as large as $N(1535) \to N\pi$.
- In this work, $N_{1/2+}(1440)$ is supposed to be a q^3 first radial excitation;
- $N_{3/2-}(1520)$ may contain a large ground-state $(l = 0)$ non-strange pentaquark component.
- $N_{1/2-}(1535)$ may contain a large ground-state $(l = 0)$ $uuds\bar{s}$ component.

Constituent quark model with cornell like potential

- Realistic Hamiltonian for a N-quark system:

$$\begin{aligned} H &= H_0 + H_{hyp}^{OGE}, \\ H_0 &= \sum_{k=1}^{N}(m_k + \frac{p_k^2}{2m_k}) + \sum_{i<j}^{N}(-\frac{3}{8}\lambda_i^C \cdot \lambda_j^C)(A_{ij}r_{ij} - \frac{B_{ij}}{r_{ij}}), \\ H_{hyp}^{OGE} &= -C_{OGE}\sum_{i<j}\frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j, \end{aligned} \quad (1)$$

- where A_{ij} and B_{ij} are mass dependent coupling parameters, taking the form,

$$A_{ij} = a\sqrt{\frac{m_{ij}}{m_u}}, \;\; B_{ij} = b\sqrt{\frac{m_u}{m_{ij}}} \quad (2)$$

- with m_{ij} being thereduced mass of i^{th} and j^{th}quarks, defined as $m_{ij} = \frac{2m_i m_j}{m_i+m_j}$. $C_{OGE} = C_m\, m_u^2$, with m_u being the constituent u quark mass and C_m a constant. λ_i^C in the above equations are the generators of color SU(3) group.

Model parameters

- The model parameters are determined by fitting the theoretical results to the experimental data of the mass of the 8 baryon isospin states, namely, $N(938)$, $\Lambda(1115)$, $\Sigma^0(1193)$, $\Xi^0(1315)$, $\Delta(1232)$, $\Sigma^*(1385)$, $\Xi^*(1530)$, and $\Omega^-(1672)$.
- And the first radial excitation state $N(1440)$ (Roper resonance) and a number of orbital excitation $l = 1$ and $l = 2$ baryons. All these baryons are believed to be mainly $3q$ states whose masses were taken from Particle Data Group.
- The model parameters are predetermined as

$$\begin{aligned} &m_u = m_d = 350 \text{ MeV}\,, \quad m_s = 525 \text{ MeV}\,, \\ &C_m = 18 \text{ MeV}, \quad a = 42000 \text{ MeV}^2, \quad b = 0.72 \end{aligned} \tag{3}$$

Mass of excited low-lying q^3 states with the model parameters

For q^3 light baryon states in SU(3) flavor symmetry and do a quark mass correction for strange quark.

Baryon	PDG (MeV)	M(Cal.) (MeV)	Error (MeV)
Proton	938	940	2
Δ	1232	1228	4
Λ	1115	1133	18
Σ	1193	1165	-28
Σ^*	1385	1373	-12
Ξ	1315	1340	25
Ξ^*	1530	1521	-9
Ω	1672	1673	1

Resonance of nucleon with model parameters

States $\Psi(N, L^P)$	Status	J^P	PDG	M(Cal.) (MeV)
$N(56, {}^2 8, 0, 0^+)$	****	$\frac{1}{2}^+$	938	940
$N(56, {}^2 8, 2, 0^+)$	****	$\frac{1}{2}^+$	N(1440)	1449
$N(70, {}^2 8, 2, 0^+)$	****	$\frac{1}{2}^+$	N(1710)	1571
$N(70, {}^4 8, 2, 0^+)$	****	$\frac{3}{2}^+$	N(1900)	1859
$N(56, {}^2 8, 2, 2^+)$	****	$\frac{3}{2}^+$	N(1720)	1593
$N(56, {}^2 8, 2, 2^+)$	****	$\frac{5}{2}^+$	N(1680)	1593
$N(70, {}^2 8, 2, 2^+)$	-	$\frac{3}{2}^+$	$missing$	1637
$N(70, {}^2 8, 2, 2^+)$	**	$\frac{5}{2}^+$	N(1860)	1637
$N(70, {}^4 8, 2, 2^+)$	****	$\frac{1}{2}^+$	N(1880)	1925
$N(70, {}^4 8, 2, 2^+)$	****	$\frac{3}{2}^+$	N(1900)	1925
$N(70, {}^4 8, 2, 2^+)$	**	$\frac{5}{2}^+$	N(2000)	1925
$N(70, {}^4 8, 2, 2^+)$	**	$\frac{7}{2}^+$	N(1990)	1925
$N(70, {}^2 8, 1, 1^-)$	****	$\frac{3}{2}^-$	N(1535)	1345
$N(70, {}^2 8, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1520)	1345
$N(70, {}^4 8, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1650)	1633
$N(70, {}^4 8, 1, 1^-)$	****	$\frac{5}{2}^-$	N(1700)	1633
$N(70, {}^4 8, 1, 1^-)$	***	$\frac{3}{2}^-$	N(1675)	1633

$q^4\bar{q}$ pentaquark wave functions

- The ground state pentaquark wave functions take the general form,

$$\Psi = \frac{1}{\sqrt{3}}\left[\psi^{o}_{[5]}\left(\psi^{c}_{[211]_\lambda}\psi^{sf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{sf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{sf}_{[31]_\eta}\right)\right]$$

- Permutation groups are applied to work out the spin-flavor [31] configurations of q^4 cluster of pentaquark, the wave functions are constructed systematically in the form of Yamanouchi basis.

$[31]_{FS}$			
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	

- The pentaquark spatial wave functions of various symmetries, which are derived in the harmonic-oscillator interaction, are applied to the system with cornell-like potential.[arXiv:1908.04972 (2019)]

Mass of ground state pentaquark $q^4\bar{q}$

$q^4\bar{q}$ Configurations	J^P	$\Delta M(q^4\bar{q})$(MeV)	$M(q^4\bar{q})$ (MeV)
$\Psi^{sf}_{[4]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	528, 240	2537, 2249
$\Psi^{sf}_{[31]_F[4]_S}(q^4\bar{q})$	$\frac{3}{2}^-, \frac{5}{2}^-$	0, 240	2009, 2249
$\Psi^{sf}_{[31]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	96, 24	2105, 2033
$\Psi^{sf}_{[31]_F[22]_S}(q^4\bar{q})$	$\frac{1}{2}^-$	0	2009
$\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	-336, 24	1673, 2033

- We just choose $\Psi^{sf}_{[22]_F[31]_S}(q^4\bar{q})$ $\frac{3}{2}^-$ state to mix with $N_{3/2^-}(1520)$.
- In our calculation one non-strange pentaquark with mass 1673 MeV, $1/2^-$ in $\Psi^{sf}_{[31]_{FS}[22]_F[31]_S}(q^4\bar{q})$ configuration was got, but where is this state?

Mass of ground state pentaquark $q^3s\bar{s}$

$q^4\bar{q}$ configurations	J^P	$\Delta M(q^4\bar{q})$(MeV)	$M(q^4\bar{q})$ (MeV)
$\Psi^{sf}_{[4]_F[31]_S}(q^3s\bar{s})$	$\frac{1}{2}^-, \frac{3}{2}^-$	397, 221	2756, 2580
$\Psi^{sf}_{[31]_F[4]_S}(q^3s\bar{s})$	$\frac{3}{2}^-, \frac{5}{2}^-$	56, 183	2415, 2542
$\Psi^{sf}_{[31]_F[31]_S}(q^3s\bar{s})$	$\frac{1}{2}^-, \frac{3}{2}^-$	85, 51	2444, 2410
$\Psi^{sf}_{[31]_F[22]_S}(q^3s\bar{s})$	$\frac{1}{2}^-$	30	2389
$\Psi^{sf}_{[211]_F[31]_S}(q^3s\bar{s})$	$\frac{1}{2}^-, \frac{3}{2}^-$	-329, -117	2030, 2242
$\Psi^{sf}_{[211]_F[22]_S}(q^3s\bar{s})$	$\frac{1}{2}^-$	-195	2164
$\Psi^{sf}_{[22]_F[31]_S}(q^3s\bar{s})$	$\frac{1}{2}^-, \frac{3}{2}^-$	-227, -7	2132, 2352

Baryons masses assumption

- In the picture that baryons consist of the q^3 as well as $q^4\,\bar{q}$ pentaquark component, the wave function of baryons may take the form,

$$\begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |q^3\rangle \\ |q^4\bar{q}\rangle \end{pmatrix}, \quad (4)$$

where θ is the mixing angle between q^3 and $q^4\,\bar{q}$ components of the corresponding states.

- q^3 $N(70,1,1^-)$ states $N_{3/2-}(1520)$ and $N_{1/2-}(1535)$ wave function can be expressed in terms of q^3 component and the $q^4\,\bar{q}$ pentaquark component,

$$\begin{pmatrix} |N_{3/2-}(1520)\rangle \\ |N_{3/2-}(1906)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} |N^{q^3}(1345)\rangle \\ |N^{q^4\bar{q}}(2033)\rangle \end{pmatrix}, \quad (5)$$

$$\begin{pmatrix} |N_{1/2-}(1535)\rangle \\ |N_{1/2-}(1890)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} |N^{q^3}(1345)\rangle \\ |N^{q^3 s\bar{s}}(2030)\rangle \end{pmatrix}, \quad (6)$$

- The mixing angles are $\theta_1 = 27.7^\circ$ and $\theta_2 = 29.1^\circ$.

negative parity states below 2 Gev

Table : Nucleon resonances of negative parity in q^3 spectrum.

Resonance	Status	J^P	M^{exp} MeV	$M(Cal.)$ MeV
$N(1520)$	****	$\frac{3}{2}^-$	1510-1520	1345
$N(1535)$	****	$\frac{1}{2}^-$	1525-1545	1345
$N(1650)$	****	$\frac{1}{2}^-$	1645-1670	1633
$N(1675)$	****	$\frac{5}{2}^-$	1670-1680	1633
$N(1685)$	*	$-$	-	-
$N(1700)$	***	$\frac{3}{2}^-$	1650-1750	1633
$N(1875)$	***	$\frac{3}{2}^-$	1820-1920	1906
$N(1895)$	****	$\frac{1}{2}^-$	1880-1910	1890

- $N_{3/2-}(1875)$ and $N_{1/2-}(1895)$ could be expressed as the higher mixture state of q^3 and $q^4\,\overline{q}$ states corresponding to $N_{3/2-}(1520)$ and $N_{1/2-}(1535)$.
- We can't locate $N(1685)$ in the q^3 negative party spectrum.

Discussion of N(1685)

- The narrow $N(1685)$ is just like three narrow pentaquark-like states, $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ reported by LHCb Collaboration.
- For the mass spectrum considering only, the state with the $[31]_{FS}[22]_F[31]_S(q^4\overline{q})$ configuration and quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ is predicted to be the lowest pentaquark state whose mass is derived as 1673MeV.
- We expect that the $N(1685)$ could be the lowest pentaquark state.

Summary

- We apply the pentaquark wave functions we got in our previous work [arXiv:1908.04972 (2019)] to evaluate the light pentaquark mass of all quark configurations as an example.
- The decay width of $N(1685)$ as the ground state pentaquark state could be calculated in the future.

Thank You Very Much For Your Attentions!

All q^3 octet and decuplet wave function

q^3 baryons wave function and possible J^P, (not including LS coupling for all the cases yet)

(1) $N, L = (0,0)$, $[56, 0^+]$

$$\begin{aligned} \Psi(q^3)_{Octet} &= \frac{1}{2}\psi^c_{[111]}[\phi^2_{00s}(\Phi_\lambda\chi_\rho + \Phi_\rho\chi_\lambda),\ J^P = \frac{1}{2}^+ \\ \Psi(q^3)_{Decuplet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^2_{00\lambda}\chi_\lambda + \phi^2_{00\rho}\chi_\rho),\ J^P = \frac{3}{2}^+ \end{aligned} \tag{7}$$

(2) $N, L = (1,1)$, $[70, 1^-]$ (for non-strange nucleon state singlet state doesn't exist)

$$\begin{aligned} \Psi^{(q^3)}_{Singlet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_A(\phi^1_{1m\lambda}\chi_\rho - \phi^1_{1m\rho}\chi_\lambda),\ J^P = \frac{1}{2}^-, \frac{3}{2}^- \\ \Psi^{(q^3)}_{Octet1} &= \frac{1}{2}\psi^c_{[111]}[\phi^1_{1m\rho}(\Phi_\lambda\chi_\rho + \Phi_\rho\chi_\lambda),\ J^P = \frac{1}{2}^-, \frac{3}{2}^- \\ &\quad +\phi^1_{1m\lambda}(\Phi_\rho\chi_\rho - \Phi_\lambda\chi_\lambda)], \\ \Psi^{(q^3)}_{Octet2} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^1_{1m\lambda}\Phi_\lambda + \phi^1_{1m\rho}\Phi_\rho),\ J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^- \\ \Psi^{(q^3)}_{Decuplet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^1_{1m\lambda}\chi_\lambda + \phi^1_{1m\rho}\chi_\rho),\ J^P = \frac{1}{2}^-, \frac{3}{2}^- \end{aligned} \tag{8}$$

All q^3 octet and decuplet wave function

(3) $N, L = (2,0)$, $[56, 0^+]$ spatial part symmetric

$$\begin{aligned}
\Psi^{(q^3)}_{Octet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{00S}(\Phi_\rho\chi_\rho + \Phi_\lambda\chi_\lambda),\ J^P = \frac{1}{2}^+ \\
\Psi^{(q^3)}_{Decuplet} &= \psi^c_{[111]}\phi^2_{00S}\Phi_S\chi_S,\ J^P = \frac{3}{2}^+
\end{aligned} \tag{9}$$

$N, L = (2,0)$, $[70, 0^+]$ spatial part mixed symmetric

$$\begin{aligned}
\Psi^{(q^3)}_{Singlet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_A(\phi^2_{00\lambda}\chi_\rho - \phi^2_{00\rho}\chi_\lambda),\ J^P = \frac{1}{2}^+ \\
\Psi^{(q^3)}_{Octet1} &= \frac{1}{2}\psi^c_{[111]}[\phi^2_{00\rho}(\Phi_\lambda\chi_\rho + \Phi_\rho\chi_\lambda),\ J^P = \frac{1}{2}^+ \\
&\quad +\phi^2_{00\lambda}(\Phi_\rho\chi_\rho - \Phi_\lambda\chi_\lambda)], \\
\Psi^{(q^3)}_{Octet2} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{00\lambda}\Phi_\lambda + \phi^2_{00\rho}\Phi_\rho),\ J^P = \frac{3}{2}^+ \\
\Psi^{(q^3)}_{Decuplet} &= \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^2_{00\lambda}\chi_\lambda + \phi^2_{00\rho}\chi_\rho),\ J^P = \frac{1}{2}^+
\end{aligned} \tag{10}$$

All q^3 octet and decuplet wave function

(4) $N, L = (2,2)$, $[56, 2^+]$ spatial part symmetric

$$\Psi^{(q^3)}_{Octet} = \frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{2mS}(\Phi_\rho\chi_\rho + \Phi_\lambda\chi_\lambda),\ J^P = \frac{3}{2}^+, \frac{5}{2}^+$$

$$\Psi^{(q^3)}_{Decuplet} = \psi^c_{[111]}\phi^2_{2mS}\Phi_S\chi_S,\ J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+ \quad (11)$$

$N, L = (2,2)$, $[70, 2^+]$ spatial part mixed symmetric (for non-strange nucleon state singlet state doesn't exist)

$$\Psi^{(q^3)}_{Singlet} = \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_A(\phi^2_{2m\lambda}\chi_\rho - \phi^2_{2m\rho}\chi_\lambda),\ J^P = \frac{3}{2}^+, \frac{5}{2}^+$$

$$\Psi^{(q^3)}_{Octet1} = \frac{1}{2}\psi^c_{[111]}[\phi^2_{2m\rho}(\Phi_\lambda\chi_\rho + \Phi_\rho\chi_\lambda),\ J^P = \frac{3}{2}^+, \frac{5}{2}^+$$
$$+\phi^2_{2m\lambda}(\Phi_\rho\chi_\rho - \Phi_\lambda\chi_\lambda)],$$

$$\Psi^{(q^3)}_{Octet2} = \frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{2m\lambda}\Phi_\lambda + \phi^2_{2m\rho}\Phi_\rho),\ J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$$

$$\Psi^{(q^3)}_{Decuplet} = \frac{1}{\sqrt{2}}\psi^c_{[111]}\Phi_S(\phi^2_{2m\lambda}\chi_\lambda + \phi^2_{2m\rho}\chi_\rho),\ J^P = \frac{3}{2}^+, \frac{5}{2}^+ \quad (12)$$

7 charm baryon and 6 beauty baryon states

For the heavy baryon, we still assume that the three quarks as identical particles in spatial wave function, even though still in SU(3) flavor symmetry, but the quark configuration is q^2Q. And we correct the quark mass for c and b quark as s quark.

Charmed Baryon	PDG Data (MeV)	M(Cal.) (MeV)	Error (MeV)
Λ_C	2286	2285	-1
Σ_C	2455	2437	-18
Σ_C^*	2520	2497	-23
Ξ_C	2470	2473	-3
Ξ_C^*	2645	2618	-27
Ω_C	2695	2768	-27
Ω_C^*	2766	2808	42

Beauty Baryon	PDG Data (MeV)	M(Cal.) (MeV)	Error (MeV)
Λ_B	5620	5594	-26
Σ_B	5811	5773	-38
Σ_B^*	5832	5793	-39
Ξ_B	5794	5781	-13
Ξ_B^*	5945	5915	-30
Ω_B	6046	6097	-51

q^3 spatial wave function

NLM	Wave function
0 00	$\Psi^{0}_{00S} = \Psi_{000}(\rho)\Psi_{000}(\lambda)$
2 00	$\Psi^{2}_{00S} = \frac{1}{\sqrt{2}}[\Psi_{100}(\rho)\Psi_{000}(\lambda) + \Psi_{000}(\rho)\Psi_{100}(\lambda)]$
4 00	$\Psi^{4}_{00S} = \frac{\sqrt{5}}{4}[\Psi_{200}(\rho)\Psi_{000}(\lambda) + \sqrt{\frac{6}{5}}\Psi_{100}(\rho)\Psi_{100}(\lambda) + \Psi_{000}(\rho)\Psi_{200}(\lambda)]$
6 00	$\Psi^{6}_{00S} = \frac{\sqrt{14}}{8}[\Psi_{300}(\rho)\Psi_{000}(\lambda) + \frac{3}{\sqrt{7}}\Psi_{200}(\rho)\Psi_{100}(\lambda) + \frac{3}{\sqrt{7}}\Psi_{100}(\rho)\Psi_{200}(\lambda)] + \Psi_{000}(\rho)\Psi_{300}(\lambda)]$
8 00	$\Psi^{8}_{00S} = \frac{\sqrt{42}}{16}[\Psi_{400}(\rho)\Psi_{000}(\lambda) + \frac{2}{\sqrt{3}}\Psi_{300}(\rho)\Psi_{100}(\lambda) + \sqrt{\frac{10}{7}}\Psi_{200}(\rho)\Psi_{200}(\lambda)] + \frac{2}{\sqrt{3}}\Psi_{100}(\rho)\Psi_{300}(\lambda) + \Psi_{000}(\rho)\Psi_{400}(\lambda)]$
10 00	$\Psi^{10}_{00S} = \frac{\sqrt{33}}{16}[\Psi_{500}(\rho)\Psi_{000}(\lambda) + \sqrt{\frac{15}{11}}\Psi_{400}(\rho)\Psi_{100}(\lambda) + 5\sqrt{\frac{2}{33}}\Psi_{300}(\rho)\Psi_{200}(\lambda)] + 5\sqrt{\frac{2}{33}}\Psi_{200}(\rho)\Psi_{300}(\lambda) + \sqrt{\frac{15}{11}}\Psi_{100}(\rho)\Psi_{400}(\lambda) + \Psi_{000}(\rho)\Psi_{500}(\lambda)]$
12 00	$\Psi^{12}_{00S} = \frac{\sqrt{429}}{64}[\Psi_{600}(\rho)\Psi_{000}(\lambda) + 3\sqrt{\frac{2}{13}}\Psi_{500}(\rho)\Psi_{100}(\lambda) + \frac{15}{\sqrt{143}}\Psi_{400}(\rho)\Psi_{200}(\lambda)] + 10\sqrt{\frac{7}{429}}\Psi_{300}(\rho)\Psi_{300}(\lambda) + \frac{15}{\sqrt{143}}\Psi_{200}(\rho)\Psi_{400}(\lambda) + 3\sqrt{\frac{2}{13}}\Psi_{100}(\rho)\Psi_{500}(\lambda)] + \Psi_{000}(\rho)\Psi_{600}(\lambda)]$
14 00	$\Psi^{14}_{00S} = \frac{\sqrt{1430}}{128}[\Psi_{700}(\rho)\Psi_{000}(\lambda) + \sqrt{\frac{7}{5}}\Psi_{600}(\rho)\Psi_{100}(\lambda) + \sqrt{\frac{21}{13}}\Psi_{500}(\rho)\Psi_{200}(\lambda)] + 7\sqrt{\frac{5}{143}}\Psi_{400}(\rho)\Psi_{300}(\lambda) + 7\sqrt{\frac{5}{143}}\Psi_{300}(\rho)\Psi_{400}(\lambda) + \sqrt{\frac{21}{13}}\Psi_{200}(\rho)\Psi_{500}(\lambda)] + \sqrt{\frac{7}{5}}\Psi_{100}(\rho)\Psi_{600}(\lambda) + \Psi_{000}(\rho)\Psi_{700}(\lambda)]$

$q^4\bar{q}$ spatial wave function

NLM	Wave function
0 00	$\Psi^{0}_{00S} = \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta)\Psi_{000}(\xi)$
2 00	$\Psi^{2}_{00S} = \frac{1}{\sqrt{3}}[\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta)]\Psi_{000}(\xi)$
4 00	$\Psi^{4}_{00S} = \sqrt{\frac{5}{33}}[\Psi_{200}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{200}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{200}(\eta)$ $+\sqrt{\frac{6}{5}}\Psi_{100}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \sqrt{\frac{6}{5}}\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta) + \sqrt{\frac{6}{5}}\Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{100}(\eta)]\Psi_{000}(\xi)$
6 00	$\Psi^{6}_{00S} = \sqrt{\frac{35}{429}}[\Psi_{300}(\rho)\Psi_{000}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{300}(\lambda)\Psi_{000}(\eta) + \Psi_{000}(\rho)\Psi_{000}(\lambda)\Psi_{300}(\eta)$ $+\frac{3}{\sqrt{7}}\Psi_{200}(\rho)\Psi_{100}(\lambda)\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}\Psi_{200}(\rho)\Psi_{000}(\lambda)\Psi_{100}(\eta) + \frac{3}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{200}(\lambda)\Psi_{100}(\eta)$ $+\frac{3}{\sqrt{7}}\Psi_{100}(\rho)\Psi_{200}(\lambda)\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}\Psi_{100}(\rho)\Psi_{000}(\lambda)\Psi_{200}(\eta) + \frac{3}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{100}(\lambda)\Psi_{200}(\eta)$ $+3\sqrt{\frac{6}{35}}\Psi_{100}(\rho)\Psi_{100}(\lambda)\Psi_{100}(\eta)]\Psi_{000}(\xi)$