Tri-hadron bound state with heavy flavor

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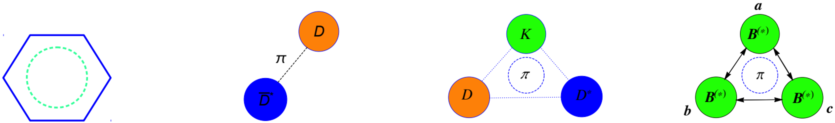
21/08/2019

The delocalized π bond in the DD^*K



Localized σ bond | Delocalized π bond | Delocalized π bond

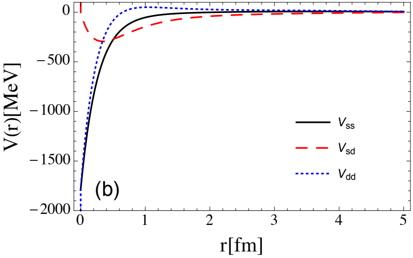
(a) (b) (c) (d) (e)

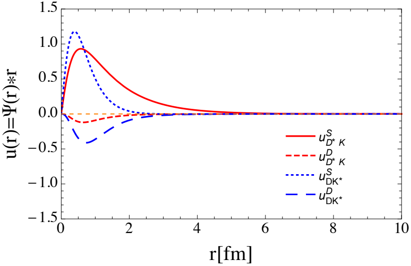
Figure: Diagrams (a), (b) and (c) are the leading OPE diagrams for the transitions among the relevant three-body channels, i.e. DD^*K, DDK^* and D^*DK channels. The TPE diagrams, i.e. (d) and (e), are the next-to-leading order contributions.

Monopole form factor $\mathcal{F}(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - q^2}$, with q the four-momentum of the pion and Λ the cutoff parameter. Λ_{D^*K}, Λ_{DD^*}.

Fix Λ_{D^*K} by reproducing the mass of $D_{s1}(2460)$

- $\mathrm{SU}(2)$ flavor symmetry. To the order $\mathcal{O}(\frac{p_K}{m_K})$. S-D wave mixing, and the coupled channels D^*K and DK^*. With $\Lambda_{D^*K} = 803.2\ \mathrm{MeV}$, we find a D^*K bound state with mass at $D_{s1}(2460)$.
- G-parity rule, i.e., $V_{A\bar{B}} = (-1)^{I_G} V_{AB}$ [1]. $V_{D^*K} = V_{\bar{D}^*K}$.
- $B^*\bar{K}$ with the mass $5772\ \mathrm{MeV}$ [2]. $\Lambda_{B^*\bar{K}} = 1451.0\ \mathrm{MeV}$.



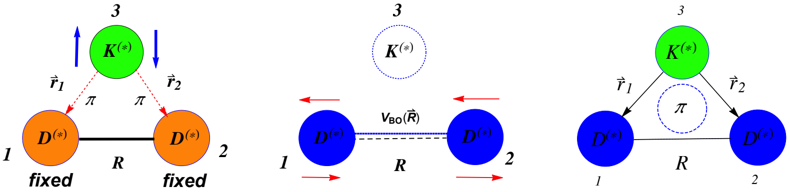


[1] E. Klempt, F. Bradamante, A. Martin and J. M. Richard, Phys. Rept. **368**, 119 (2002).

[2] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007).

Born-Oppenheimer (BO) approximation

- Firstly, we keep the two heavy mesons, i.e. D and D^*, at a given fixed location R and study the dynamical behavior of the light kaon.
- Then, we solve Schrödinger equation of the DD^* system with the effective BO potential created from the interaction with the kaon.

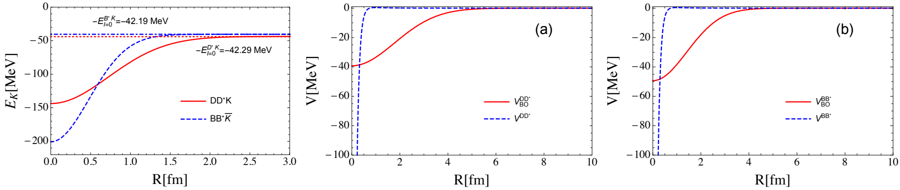


- The BO approximation is based on the factorized wave function

$$|\Psi_T(\vec{R},\vec{r})\rangle = |\Phi(\vec{R})\Psi(\vec{r}_1,\vec{r}_2)\rangle \ ,$$

with the two charmed mesons and the light kaon located at $\pm\vec{R}/2$ and $\vec{r}$. Here, $\vec{r}_1 = \vec{r} + \vec{R}/2$ and $\vec{r}_2 = \vec{r} - \vec{R}/2$ are the coordinates of the kaon relative to the first and second interacting D^*.

Numerical results for the Double heavy tri-meson bound states

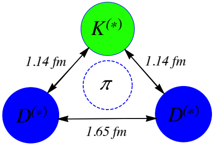


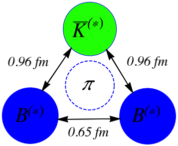
When the distance R is larger than a certain value, the kaon energy of the three-body system equal to the binding energy of the isosinglet D^*K or $B^*\bar{K}$ two-body system. The two-body binding energies are from the calculations [3].

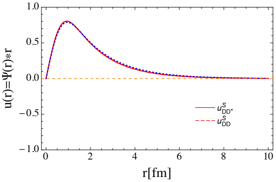
Table: Numerical results for the Double heavy tri-meson bound states.

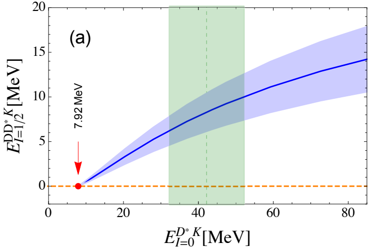
(MeV)	DD^*K	$BB^*\bar{K}$	$D\bar{D}^*K$	$B\bar{B}^*\bar{K}$
Binding energy	$8.29^{+4.32}_{-3.66}$	$41.76^{+8.84}_{-8.49}$	$8.29^{+6.55}_{-6.13}$	$41.76^{+9.02}_{-8.68}$
Mass	$4317.92^{+3.66}_{-4.32}$	$11013.65^{+8.49}_{-8.84}$	$4317.92^{+6.13}_{-6.55}$	$11013.65^{+8.68}_{-9.02}$

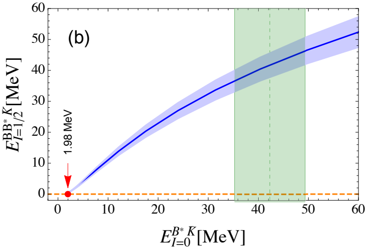
[3] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007)







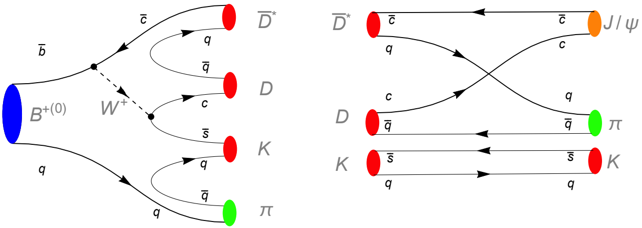




The red point is the critical point which indicates the lower limit of the required binding energy of the isosinglet D^*K or $B^*\bar{K}$ to form a three-body bound state. The vertical dashed lines and bands are the central values and uncertainties of the binding energies of the two-body subsystems from the analysis [4].

[4] F. K. Guo, P. N. Shen and H. C. Chiang, Phys. Lett. B **647**, 133 (2007)

Detection from B meson decays in the $J/\psi\pi\pi K$ channel



Aiming at the $X(3872)$, LHCb, Belle and BABAR have collected quite numerous data for B decays in the $J/\psi\pi\pi K$ channel. The existence of the $D\bar{D}^*K$ bound state could be checked from the experimental side by analyzing the current world data on the channels $J/\psi\pi^+K^0$, $J/\psi\pi^0K^+$, $J/\psi\pi^0K^0$, and $\boldsymbol{J/\psi\pi^-K^+}$.

Tri-meson bound state BBB^*

The OPEP indicates that there is only one virtual pion exchanged by any two constituents as shown in the following.

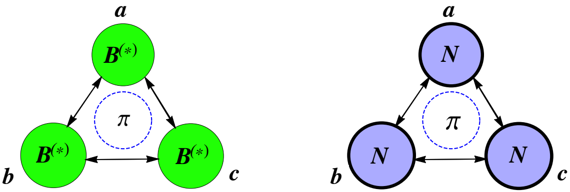


Figure: Dynamical illustration of the BBB^* system with a circle describing the delocalized π *bond* inside. Since the three constituents have the same probabilities to be the B and B^*, one can rewrite the system as $B_a^{(*)}B_b^{(*)}B_c^{(*)}$.

The OPE interaction for the BBB^*

Under SU(2) chiral symmetry, the OPE interaction is of order $\mathcal{O}(p^0)$ for the three-body system.

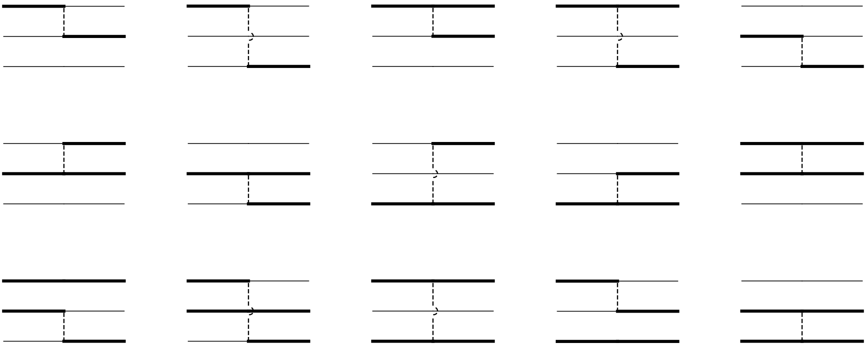
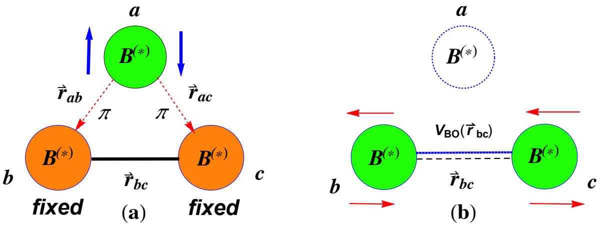


Figure: The leading order OPE diagrams for the transitions among the relevant three-body channels, i.e. $B_a^* B_b B_c$, $B_a B_b^* B_c$, $B_a B_b B_c^*$, $B_a^* B_b^* B_c$, $B_a^* B_b B_c^*$ and $B_a B_b^* B_c^*$.

Born-Oppenheimer potential

- Considering that the particle b and c are static with the separation r_{bc}, one can separate the degree of freedom of a from the three-body system.
- We assume the distance r_{bc} is a parameter. The mesons b and c are static, and have one-pion interactions with meson a, which can be viewed as two static sources.
- We explore the dynamics for the meson a in the limit $r_{bc} \to \infty$, and subtract the binding energy for the break-up state which is trivial for the three-body bound state.



Interpolating wave function of meson a

- $\frac{1}{\sqrt{2}}\psi(\vec{r}_{ab})|B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}}\psi(\vec{r}_{ab})|B_a B_b^* B_c\rangle + \psi'(\vec{r}_{ab})|B_a^* B_b^* B_c\rangle$
  Pion exchanged between a and b.
- $\frac{1}{\sqrt{2}}\psi(\vec{r}_{ac})|B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}}\psi(\vec{r}_{ac})|B_a B_b B_c^*\rangle + \psi'(\vec{r}_{ac})|B_a^* B_b B_c^*\rangle$
  Pion exchanged between a and c.

The final wave function for the meson a could be the superposition of these two components

$$\begin{aligned}\psi(\vec{r}_{ab}, \vec{r}_{ac}) &= C\Big\{[\frac{1}{\sqrt{2}}\psi(\vec{r}_{ab}) + \frac{1}{\sqrt{2}}\psi(\vec{r}_{ac})]|B_a^* B_b B_c\rangle + \frac{1}{\sqrt{2}}\psi(\vec{r}_{ab})|B_a B_b^* B_c\rangle \\ &+ \frac{1}{\sqrt{2}}\psi(\vec{r}_{ac})|B_a B_b B_c^*\rangle + \psi'(\vec{r}_{ab})|B_a^* B_b^* B_c\rangle + \psi'(\vec{r}_{ac})|B_a^* B_b B_c^*\rangle\Big\}.\end{aligned}$$

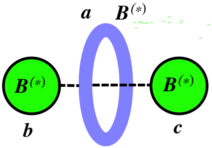
Accordingly, one can obtain the energy eigenvalue of the meson a

$$E_a(\Lambda,\ \vec{r}_{bc}) = \langle\psi(\vec{r}_{ab}, \vec{r}_{ac})|H_a|\psi(\vec{r}_{ab}, \vec{r}_{ac})\rangle$$

BO potential and Its physical meaning (Intensity of "glue")

We define the BO potential as

$$V_{BO}(\Lambda,\ \vec{r}_{bc}) = E_a(\Lambda,\ \vec{r}_{bc}) - E_2(\Lambda).$$



The BO potential can describe the contribution for the one meson on the dynamics of the two remaining mesons. The meson a here works like a kind of "glue".

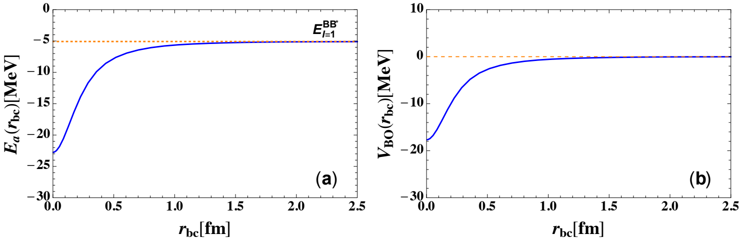


Figure: Here we chose the parameter $\Lambda = 1440$ MeV. $E^{BB*}_{I=1} = -5.08$ MeV.

$$\Psi_T = \alpha\Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}).$$

The configurations of the three-body systems

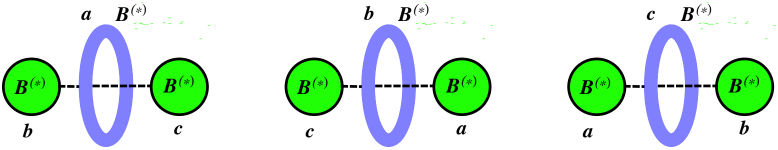


Figure: Every meson can be considered to be a lighter one and separated from the three-body system. Each of them can generate the "glue" for the remaining mesons.

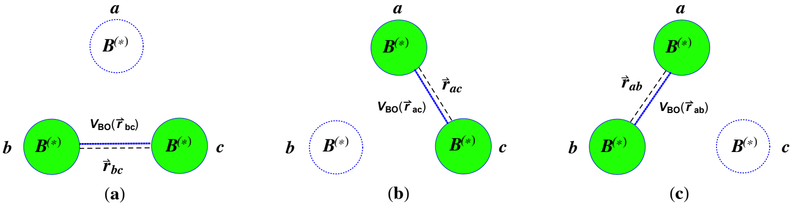


Figure: (a), (b) and (c) correspond to the wave functions $\psi_{\not a} = \Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac})$, $\psi_{\not b} = \Phi(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc})$ and $\psi_{\not c} = \Phi(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac})$, respectively.

Interpolating wave functions

The basis constitute a configuration space $\{\psi_{\not a},\ \psi_{\not b},\ \psi_{\not c}\}$.

$$
\begin{aligned}
\Psi_T &= \alpha\Phi(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}) + \beta\Phi(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc}) + \gamma\Phi(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac}) \\
&= \alpha\psi_{\not a} + \beta\psi_{\not b} + \gamma\psi_{\not c} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix},
\end{aligned}
$$

Expand $\Phi(\vec{r}_{bc})$, $\Phi(\vec{r}_{ac})$ and $\Phi(\vec{r}_{ab})$ as a set of Laguerre polynomials

$$
\chi_{nl}(r) = \sqrt{\frac{(2\lambda)^{2l+3} n!}{\Gamma(2l+3+n)}} r^l e^{-\lambda r} L_n^{2l+2}(2\lambda r), \qquad n = 1, 2, 3...
$$

$$
\psi_{\not a} = \sum_i \phi_i(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac}), \psi_{\not b} = \sum_i \phi_i(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc}), \psi_{\not c} = \sum_i \phi_i(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac})
$$

Here the subscript i is the order of Laguerre polynomials. We define the i^{th} order of the configuration functions as $\psi^i_{\not a} = \phi_i(\vec{r}_{bc})\psi(\vec{r}_{ab},\ \vec{r}_{ac})$, $\psi^i_{\not b} = \phi_i(\vec{r}_{ac})\psi(\vec{r}_{ab},\ \vec{r}_{bc})$ and $\psi^i_{\not c} = \phi_i(\vec{r}_{ab})\psi(\vec{r}_{bc},\ \vec{r}_{ac})$.

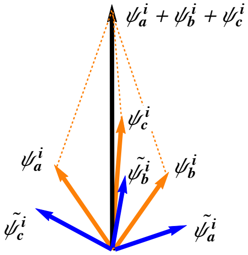
Orthonormalization

We orthonormalize the $\{\psi_{\not{a}},\ \psi_{\not{b}},\ \psi_{\not{c}}\}$ into a new basis $\{\tilde{\psi}_{\not{a}},\ \tilde{\psi}_{\not{b}},\ \tilde{\psi}_{\not{c}}\}$.

$$\tilde{\psi}^i_{\not{a}} = \frac{1}{N_i}\left[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{a}}\right],$$

$$\tilde{\psi}^i_{\not{b}} = \frac{1}{N_i}\left[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{b}}\right],$$

$$\tilde{\psi}^i_{\not{c}} = \frac{1}{N_i}\left[(\psi^i_{\not{a}} + \psi^i_{\not{b}} + \psi^i_{\not{c}}) - \sum_i x_{ij}\psi^j_{\not{c}}\right],$$

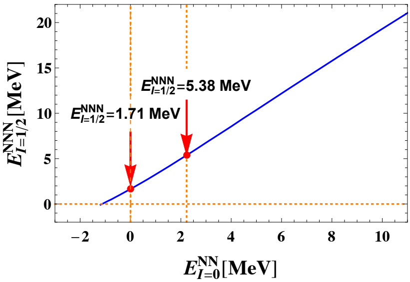


where the x_{ij} is a parameter matrix which will be determined later. The N_i are normalization coefficients.

Then the eigenvector for the three-body system $B_a^{(*)}B_b^{(*)}B_c^{(*)}$ can be written as a vector in the configuration space $\{\tilde{\psi}_{\not{a}},\ \tilde{\psi}_{\not{b}},\ \tilde{\psi}_{\not{c}}\}$. Therefore, we have

$$\Psi_T = \sum_i \tilde{\alpha}_i \tilde{\psi}^i_{\not{a}} + \sum_j \tilde{\beta}_j \tilde{\psi}^j_{\not{b}} + \sum_k \tilde{\gamma}_k \tilde{\psi}^k_{\not{c}},$$

Application to the *NNN* system (Triton or Helium-3 nucleus)



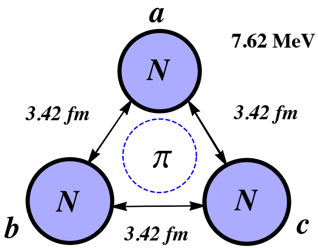


Figure: Dependence of the reduced three-body binding energy on the binding energy of its two-body subsystem (the deuteron). The result is comparable with the empirical binding energies of the triton (8.48 MeV) and helium-3 (7.80 MeV) nuclei.

Numerical results for the *NNN* system (Triton or Helium-3 nucleus)

Λ(MeV)	E_2(MeV)	E_3(MeV)	E_T(MeV)	$V_{BO}(0)$(MeV)	S wave(%)	D wave(%)	r_{rms}(fm)
830.00	-0.18	-1.93	-2.11	-4.54	94.01	5.99	4.21
850.00	-0.67	-2.71	-3.38	-5.36	93.36	6.64	4.00
870.00	-1.23	-3.65	-4.88	-6.32	92.68	7.32	3.78
890.00	-1.88	-4.77	-6.66	-7.42	91.99	8.01	3.54
899.60	-2.23	-5.38	-7.62	-8.00	91.66	8.34	3.42
900.00	-2.25	-5.41	-7.66	-8.03	91.64	8.36	3.42
920.00	-3.05	-6.85	-9.90	-9.35	90.97	9.03	3.18
940.00	-3.98	-8.51	-12.49	-10.83	90.35	9.65	2.95
960.00	-5.03	-10.42	-15.45	-12.46	89.76	10.24	2.74
980.00	-6.21	-12.57	-18.78	-14.23	89.23	10.77	2.54
1000.00	-7.55	-14.97	-22.51	-16.14	88.73	11.27	2.37
1020.00	-9.04	-17.61	-26.65	-18.19	88.27	11.73	2.23
1040.00	-10.69	-20.51	-31.20	-20.37	87.84	12.16	2.10

Table: Bound state solutions for the *NNN* system with isospin $I_3 = 1/2$. E_2 is the energy eigenvalue of its subsystem. E_3 is the reduced three-body energy eigenvalue relative to the break-up state of the *NNN* system. E_T is the total three-body energy eigenvalue relative to the *NNN* threshold.

Numerical results for the tri-meson bound state BBB^*

Table: Bound state solutions of the BBB^* with the isospin $I_3 = 3/2$. α and β are the probabilities for the components BBB^* and BB^*B^*, respectively.

E_2(MeV)	E_3(MeV)	E_T(MeV)	$V_{BO}(0)$(MeV)	S wave(%)	D wave(%)	r_{rms}(fm)	α(%)	β(%)
-0.18	-0.19	-0.38	-3.43	99.76	0.24	3.98	97.50	2.50
-0.48	-0.45	-0.93	-4.88	99.68	0.32	3.34	96.39	3.61
-0.89	-0.85	-1.74	-6.62	99.59	0.41	2.67	95.02	4.98
-1.43	-1.42	-2.85	-8.56	99.49	0.51	2.11	93.78	6.22
-2.11	-2.20	-4.31	-10.65	99.41	0.59	1.71	91.91	8.09
-2.94	-3.17	-6.11	-12.87	99.34	0.66	1.43	90.22	9.78
-3.93	-4.33	-8.26	-15.21	99.29	0.71	1.24	88.47	11.53
-5.08	-5.67	-10.75	-17.65	99.25	0.75	1.09	86.69	13.31
-6.40	-7.18	-13.58	-20.19	99.22	0.78	0.98	84.91	15.09
-7.88	-8.83	-16.71	-22.83	99.20	0.80	0.90	83.14	16.86
-9.54	-10.61	-20.16	-25.55	99.18	0.82	0.83	81.40	18.60
-11.38	-12.51	-23.89	-28.36	99.17	0.83	0.77	79.70	20.30
-13.39	-14.62	-28.01	-31.24	99.16	0.84	0.72	78.07	21.93
-15.59	-16.75	-32.34	-34.20	99.15	0.85	0.68	76.50	23.50
-17.97	-18.99	-36.95	-37.24	99.14	0.86	0.65	75.01	24.99

Numerical results for the tri-meson bound state BBB^*

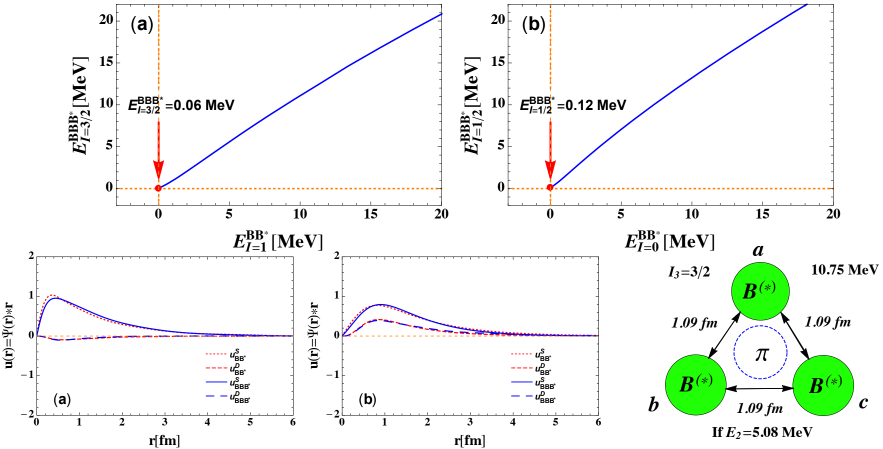


Figure: Here we chose the parameter $\Lambda = 1440$ MeV in (a) and $\Lambda = 1107.7$ MeV in (b) for a better comparison of all the cases, since they have the same two-body binding energy of 5.08 MeV.

Summary

- Based on the attractive force of the isosinglet D^*K and $B^*\bar{K}$ systems and the BOA method, we predict four double heavy tri-meson bound states, i.e DD^*K, $D\bar{D}^*K$, $BB^*\bar{K}$ and $B\bar{B}^*\bar{K}$ bound states.
- Hopefully the future analysis on the B meson decay data and NN collisions may unveil the existence of the tri-meson structures.
- We also predict that a triple heavy tri-meson molecular state for the BBB^* system is probably existent as long as the molecular states of its two-body subsystem BB^* exist.
- In our calculations, we use the Born-Oppenheimer potential method to construct our interpolating wave functions, which can be regarded as a version of the variational principle which always gives an upper limit of the energy of a system.

Thank you very much!