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The pseudoscalar meson and baryon octet interaction  
with strangeness zero in the unitary coupled-channel approximation

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•  $\Lambda(1405)$  : Single-pole •

F. Y. Dong, B. X. Sun and J. L. Pang, Chin. Phys. C , 41(7), 074108, (2017).  
The Kaon-nucleon interaction is studied in the unitary coupled-channel approximation, and the relativistic kinetic effect and off-shell corrections are taken into account in the calculation of the loop function. Only one peak of the amplitude is found in the 1400MeV region.

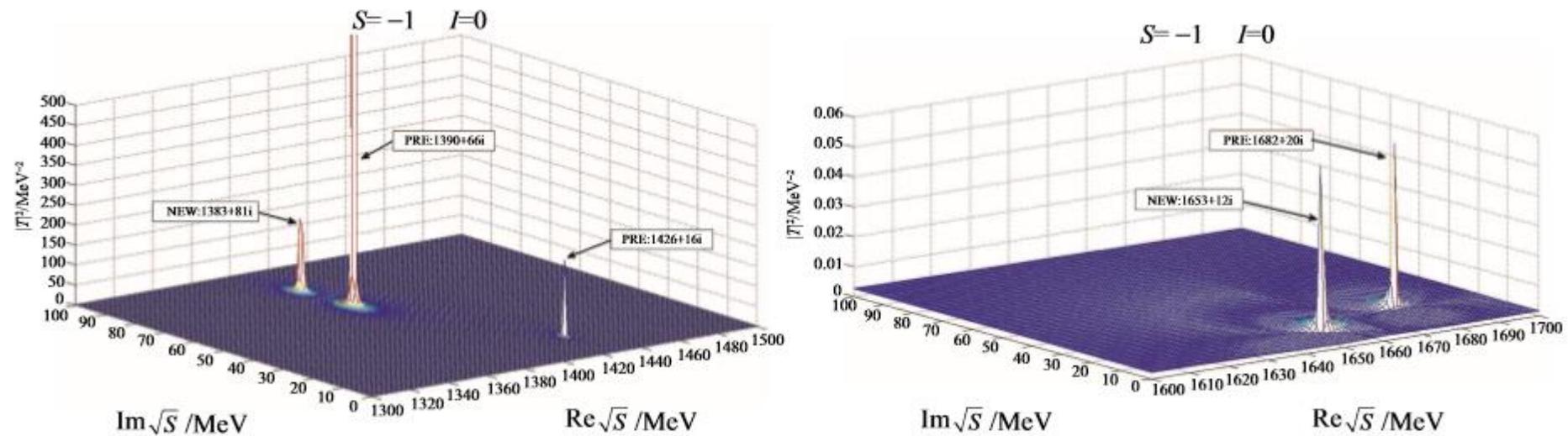


Fig. 1. Comparison of poles in the strangeness  $S=-1$  and isospin  $I=0$  sector. *NEW* denotes the case calculated from the loop function in Eq. (16), while *PRE* stands for the case of the loop function in the on-shell factorization approximation in Eq. (13).

## • $\Lambda(1405)$ : Double-pole •

Ulf-G Meissner and T. Hyodo, Chin. Phys. C , 40(10), 100001, (2016), p815.

### 100. Pole Structure of the $\Lambda(1405)$ Region

Written November 2015 by Ulf-G. Meißner (Bonn Univ. / FZ Jülich) and Tetsuo Hyodo (YITP, Kyoto Univ.).

The  $\Lambda(1405)$  resonance emerges in the meson-baryon scattering amplitude with the strangeness  $S = -1$  and isospin  $I = 0$ . It is the archetype of what is called a dynamically generated resonance, as pioneered by Dalitz and Tuan [1]. The most powerful and systematic approach for the low-energy regime of the strong interactions is chiral perturbation theory (ChPT), see e.g. Ref. 2. A perturbative calculation is, however, not applicable to this sector because of the existence of the  $\Lambda(1405)$  just below the  $\bar{K}N$  threshold. In this case, ChPT has to be combined with a non-perturbative resummation technique, just as in the case of the nuclear forces. By solving the Lippmann-Schwinger equation with the interaction kernel determined by ChPT and using a particular regularization, in Ref. 3 a successful description of the low energy  $K^-n$  scattering data is

# Hyperon I: Study of the $\Lambda(1405)$

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May 15, 2019

**Abstract.** Low-energy data on the three charge states in  $\gamma p \rightarrow K^+(\Sigma\pi)$  from CLAS at JLab, on  $K^-p \rightarrow \pi^0\pi^0\Lambda$  and  $\pi^0\pi^0\Sigma$  from the Crystal Ball at BNL, bubble chamber data on  $K^-p \rightarrow \pi^-\pi^+\pi^\pm\Sigma^\mp$ , low-energy total cross sections on  $K^-$  induced reactions, and data on the  $K^-p$  atom are fitted with the BnGa partial-wave-analysis program. We find that the data can be fitted well with just one isoscalar spin-1/2 negative-parity pole, the  $\Lambda(1405)$ , and background contributions.

the quark model. The data were fitted in [29], the best fit was achieved with two low-mass isovector states ( $\Sigma^*$ 's) and one isoscalar state  $\Lambda(1405)$ . A reanalysis of these data showed that the data are also compatible with a standard single-pole  $\Lambda(1405)$  [31]. Dong, Sun and Pang [32] solved the Bethe-Salpeter equation in an unitary coupled-channel ansatz taking relativistic effects and off-shell corrections into account. In their model, the authors found that the off-shell corrections are very important. Without these, the authors reproduced the two-pole structure. Yet one pole disappeared when the off-shell corrections were switched on, and only one  $\Lambda(1405)$  survived. This contradicts [12,33]; in their ansatz, off-shell effects were found to be small and two poles were present. Myint *al.* [34] used a chiral model and found two poles in the  $\Lambda(1405)$

## • Research Background •

There are two excited states in the  $S=0$  and  $I=1/2$  channel,  $N(1535)$  and  $N(1650)$ , which are difficult to be described in the framework of the consistent quark model.

However, in the Unitary coupled-channel approximation, these excited states are treated as hadronic resonances.

N. Kaiser, P. B. Siegel and W. Weise, PLB, 362, 23, (1995).

In the generation of  $N(1535)$ , the  $K\Lambda$  and  $K\Sigma$  channels play an important role.

# N(1535) N(1650)

## N(1535) BREIT-WIGNER MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>1525 to 1545 (<math>\approx 1535</math>) OUR ESTIMATE</b>			
1517 $\pm 4$	SOKHOYAN	15A	DPWA Multichannel
1526 $\pm 2$	SHKLYAR	13	DPWA Multichannel
1547.0 $\pm 0.7$	ARNDT	06	DPWA $\pi N \rightarrow \pi N, \eta N$
1550 $\pm 40$	CUTKOSKY	80	IPWA $\pi N \rightarrow \pi N$
1526 $\pm 7$	HOEHLER	79	IPWA $\pi N \rightarrow \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1519 $\pm 5$	ANISOVICH	12A	DPWA Multichannel
1538 $\pm 1$	SHRESTHA	12A	DPWA Multichannel
1553 $\pm 8$	BATINIC	10	DPWA $\pi N \rightarrow N\pi, N\eta$
1546.7 $\pm 2.2$	ARNDT	04	DPWA $\pi N \rightarrow \pi N, \eta N$
1526 $\pm 2$	PENNER	02C	DPWA Multichannel
1530 $\pm 10$	BAI	01B	BES $J/\psi \rightarrow p\bar{p}\eta$
1522 $\pm 11$	THOMPSON	01	CLAS $\gamma^* p \rightarrow p\eta$
1542 $\pm 3$	VRANA	00	DPWA Multichannel
1532 $\pm 5$	ARMSTRONG	99B	DPWA $\gamma^* p \rightarrow p\eta$

## N(1535) BREIT-WIGNER WIDTH

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>125 to 175 (<math>\approx 150</math>) OUR ESTIMATE</b>			
120 $\pm 10$	SOKHOYAN	15A	DPWA Multichannel
131 $\pm 12$	SHKLYAR	13	DPWA Multichannel
188.4 $\pm 3.8$	ARNDT	06	DPWA $\pi N \rightarrow \pi N, \eta N$
240 $\pm 80$	CUTKOSKY	80	IPWA $\pi N \rightarrow \pi N$
120 $\pm 20$	HOEHLER	79	IPWA $\pi N \rightarrow \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •			
128 $\pm 14$	ANISOVICH	12A	DPWA Multichannel
141 $\pm 4$	SHRESTHA	12A	DPWA Multichannel
182 $\pm 25$	BATINIC	10	DPWA $\pi N \rightarrow N\pi, N\eta$
129 $\pm 8$	PENNER	02C	DPWA Multichannel
95 $\pm 25$	BAI	01B	BES $J/\psi \rightarrow p\bar{p}\eta$
143 $\pm 18$	THOMPSON	01	CLAS $\gamma^* p \rightarrow p\eta$
112 $\pm 19$	VRANA	00	DPWA Multichannel
154 $\pm 20$	ARMSTRONG	99B	DPWA $\gamma^* p \rightarrow p\eta$

## N(1650) BREIT-WIGNER MASS

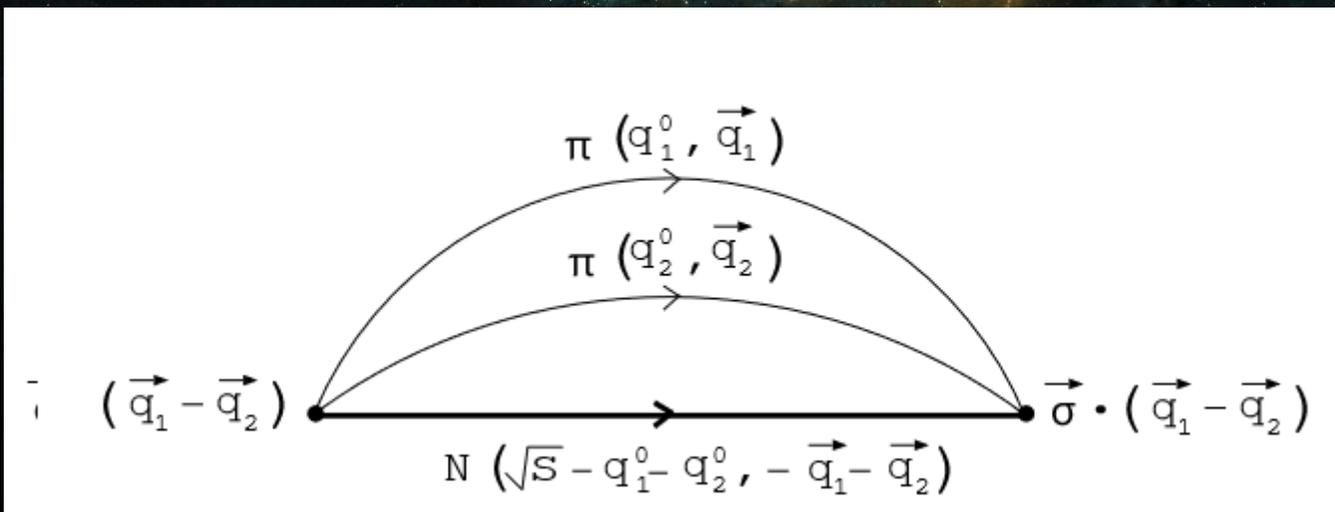
VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>1645 to 1670 (<math>\approx 1655</math>) OUR ESTIMATE</b>			
1654 $\pm 6$	SOKHOYAN	15A	DPWA Multichannel
1665 $\pm 2$	SHKLYAR	13	DPWA Multichannel
1634.7 $\pm 1.1$	ARNDT	06	DPWA $\pi N \rightarrow \pi N, \eta N$
1650 $\pm 30$	CUTKOSKY	80	IPWA $\pi N \rightarrow \pi N$
1670 $\pm 8$	HOEHLER	79	IPWA $\pi N \rightarrow \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •			
1651 $\pm 6$	ANISOVICH	12A	DPWA Multichannel
1664 $\pm 2$	SHRESTHA	12A	DPWA Multichannel
1652 $\pm 9$	BATINIC	10	DPWA $\pi N \rightarrow N\pi, N\eta$
1665 $\pm 2$	PENNER	02C	DPWA Multichannel
1647 $\pm 20$	BAI	01B	BES $J/\psi \rightarrow p\bar{p}\eta$
1689 $\pm 12$	VRANA	00	DPWA Multichannel

## N(1650) BREIT-WIGNER WIDTH

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
<b>110 to 170 (<math>\approx 140</math>) OUR ESTIMATE</b>			
102 $\pm 8$	SOKHOYAN	15A	DPWA Multichannel
147 $\pm 14$	SHKLYAR	13	DPWA Multichannel
115.4 $\pm 2.8$	ARNDT	06	DPWA $\pi N \rightarrow \pi N, \eta N$
150 $\pm 40$	CUTKOSKY	80	IPWA $\pi N \rightarrow \pi N$
180 $\pm 20$	HOEHLER	79	IPWA $\pi N \rightarrow \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •			
104 $\pm 10$	ANISOVICH	12A	DPWA Multichannel
126 $\pm 3$	SHRESTHA	12A	DPWA Multichannel
202 $\pm 16$	BATINIC	10	DPWA $\pi N \rightarrow N\pi, N\eta$
138 $\pm 7$	PENNER	02C	DPWA Multichannel
145 $\pm 80$ -45	BAI	01B	BES $J/\psi \rightarrow p\bar{p}\eta$
202 $\pm 40$	VRANA	00	DPWA Multichannel

# Background

T. Inoue, E. Oset and M. J. Vicente Vacas, PRC, 65, 035204, (2002).  
The  $\pi\pi N$  three-body force is taken into account in the final state interaction when the BS equation is solved in the unitary coupled-channel approximation. However, six parameters are included in the real part of the intermediate  $\pi\pi N$  loop function.



## Background

E. J. Garzon and E. Oset, PRC, 91, 025201 (2015).

The  $\rho N$  and  $\pi\Delta$  channels are taken into account in a non-relativistic approximation. The  $\rho N$  channel mainly gives a contribution to the generation of  $N(1650)$ , moreover, in the nonelastic scattering channels, the Kroll-Ruderman term supplies a constant potential, and the one-pion exchange potential is very weak.

$$t_{\rho N(s) \rightarrow \pi N(s)} = -2\sqrt{6}g \frac{D + F}{2f} \left\{ \frac{\frac{2}{3} \vec{q}_{\pi N}^2}{(P_V + q_{\pi N})^2 - m_\pi^2} + 1 \right\},$$

# Background

P. C. Bruns, M. Mai and U. G. Meissner, PLB, 697, 254, (2011).  
The NNLO terms is considered. By fitting the amplitude under 1.56GeV in the  $S_{11}$  channel, the resonance states corresponding to  $N(1535)$  and  $N(1650)$  can be generated simultaneously.

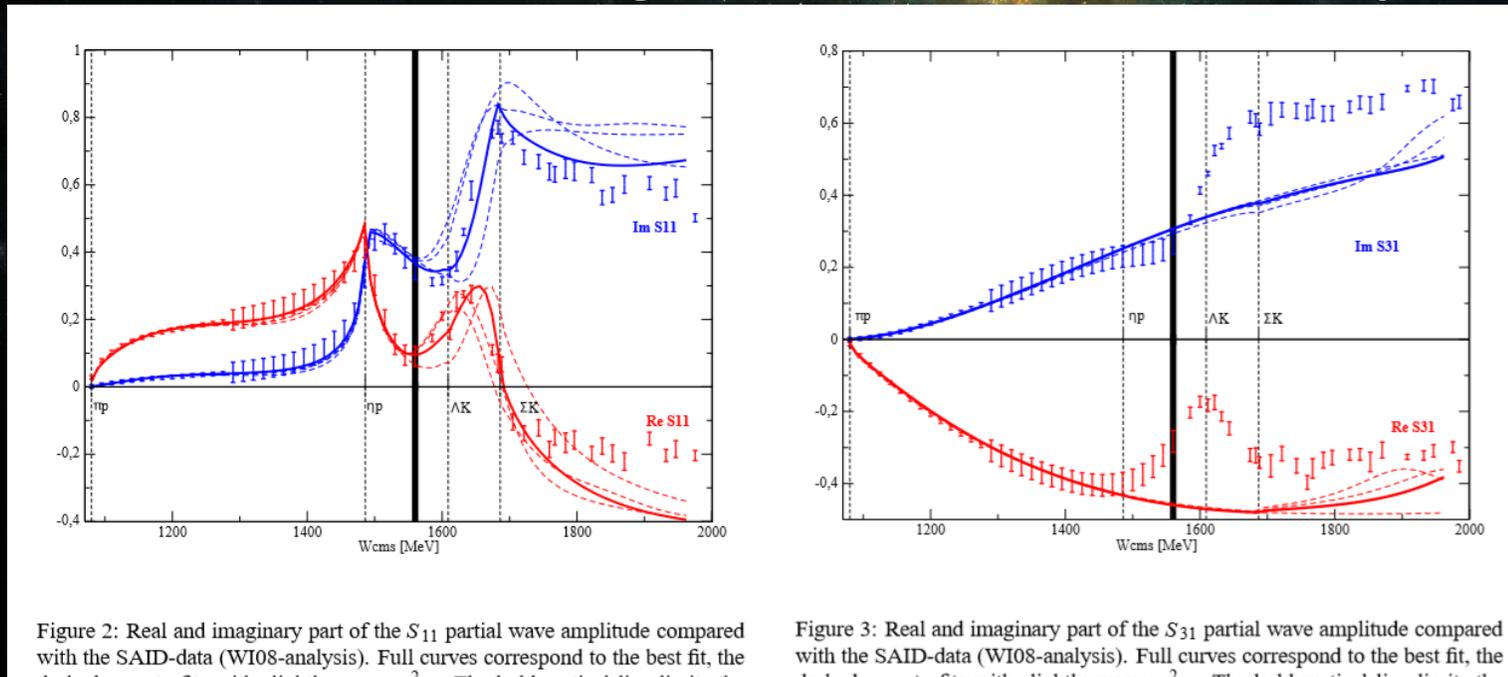


Figure 2: Real and imaginary part of the  $S_{11}$  partial wave amplitude compared with the SAID-data (WI08-analysis). Full curves correspond to the best fit, the dashed lines represent individual resonance contributions.

Figure 3: Real and imaginary part of the  $S_{31}$  partial wave amplitude compared with the SAID-data (WI08-analysis). Full curves correspond to the best fit, the dashed lines represent individual resonance contributions.

# Background

Z W Wei, W. Kamleh D. B. Leinweber et al., PRL, 116, 082004, (2016).

The Hamiltonian is divided into two parts: the non-interaction part and the interaction part, and then the relativistic Lippmann-Schwinger equation is solved to study the properties of  $N(1535)$ .

Y. F. Wang, D. L. Yao and H. Q. Zheng, EPJC, 78(7), 543, (2018).

Chin. Phys. C , 43(6), 064110, (2019).

The phase shift of different pion-nucleon partial waves is analyzed systematically in the framework of K-matrix.

T. Sekihara, T. Arai et al., PRC, 93(3), 035204, (2016).

The wave functions of composite hadron states of  $\Delta(1232)$ ,  $N(1535)$  and  $N(1650)$  are studied, and it is announced that the components of  $\pi N$ ,  $\eta N$ ,  $K\Lambda$  and  $K\sigma$  can be neglected.

## Pseudoscalar meson and baryon octet interaction with strangeness zero in the unitary coupled-channel approximation

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**Abstract:** The interaction of the pseudoscalar meson and the baryon octet is investigated by solving the Bethe-Salpeter equation in the unitary coupled-channel approximation. In addition to the Weinberg-Tomozawa term, the contribution of the  $s$ - and  $u$ - channel potentials in the  $S$ -wave approximation are taken into account. In the sector of isospin  $I = 1/2$  and strangeness  $S = 0$ , a pole is detected in a reasonable region of the complex energy plane of  $\sqrt{s}$  in the center-of-mass frame by analyzing the behavior of the scattering amplitude, which is higher than the  $\eta N$  threshold and lies on the third Riemann sheet. Thus, it can be regarded as a resonance state and might correspond to the  $N(1535)$  particle of the Particle Data Group (PDG) review. The coupling constants of this resonance state to the  $\pi N$ ,  $\eta N$ ,  $K\Lambda$  and  $K\Sigma$  channels are calculated, and it is found that this resonance state couples strongly to the hidden strange channels. Apparently, the hidden strange channels play an important role in the generation of resonance states with strangeness zero. The interaction of the pseudoscalar meson and the baryon octet is repulsive in the sector of isospin  $I = 3/2$  and strangeness  $S = 0$ , so that no resonance state can be generated dynamically.

**Keywords:** chiral Lagrangian, pion-nucleon interaction, Bethe-Salpeter equation

**PACS:** 12.40.Vv, 13.75.Gx, 14.20.Gk **DOI:** 10.1088/1674-1137/43/6/064111

# Chiral Lagrangian

$$L = \langle \bar{B}(i\gamma_\mu D^\mu - M)B \rangle + \frac{D/F}{2} \langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm \rangle.$$

$$\Phi = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

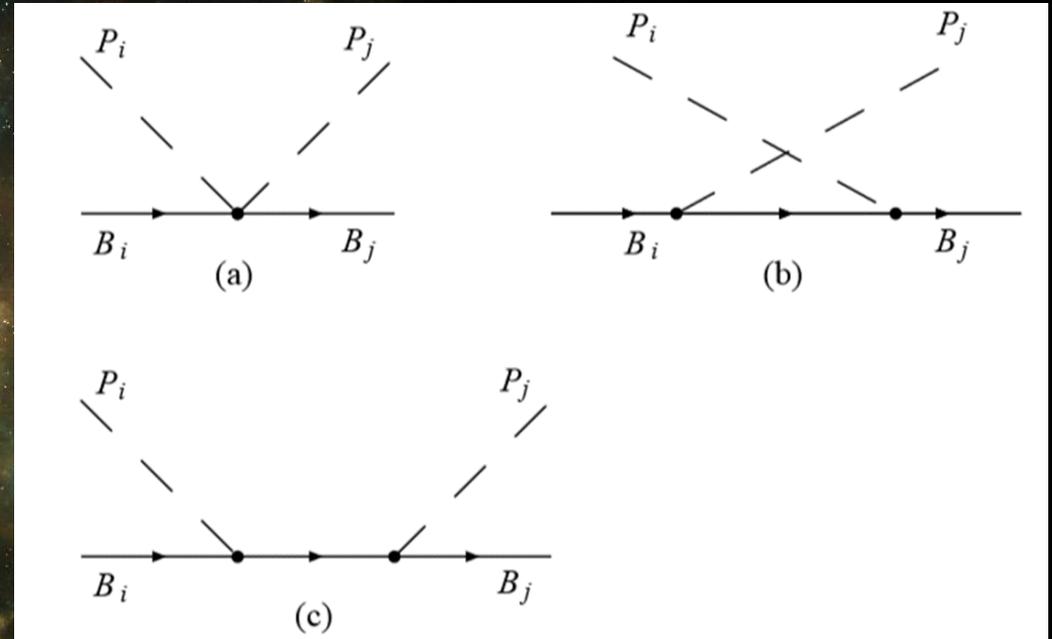
$$D = 0.80, \quad F = 0.46$$

# Potential of the pseudoscalar meson and the baryon

$$V_{ij}^{\text{con}} = -C_{ij} \frac{1}{4f_i f_j} \bar{U}(p_j, \lambda_j) \gamma_0 U(p_i, \lambda_i) (k_i^0 + k_j^0).$$

$$V_{ij}^s \approx AA' \frac{(\sqrt{s} - E)(\sqrt{s} - E')}{\sqrt{s} + M},$$

$$V_{ij}^u(S) = AA' \frac{(\sqrt{s} - E)(E + E' - \sqrt{s} - M)(\sqrt{s} - E')}{M_i^2 + m_j^2 - M^2 - 2E(\sqrt{s} - E')} \\ \times \frac{-1}{2\alpha} \ln \left( \frac{1 - \alpha}{1 + \alpha} \right).$$



$$\alpha = \frac{2|\vec{p}_i| |\vec{k}_j|}{M_i^2 + m_j^2 - M^2 - 2E(\sqrt{s} - E')}$$

# Bethe-Salpeter Equation

$$\tilde{T} = [1 - \tilde{V}\tilde{G}]^{-1}\tilde{V}$$

$$\begin{aligned}\tilde{V} &= V \sqrt{M_i M_j}, \\ \tilde{G}_l &= G_l / M_l,\end{aligned}$$

$$G_l = i \int \frac{d^d q}{(2\pi)^4} \frac{q + M_l}{q^2 - M_l^2 + i\epsilon} \frac{1}{(P - q)^2 - m_l^2 + i\epsilon},$$

$$\begin{aligned}&= \frac{\gamma_\mu P^\mu}{32P^2\pi^2} \left[ (a_l + 1)(m_l^2 - M_l^2) + \left( m_l^2 \ln \frac{m_l^2}{\mu^2} - M_l^2 \ln \frac{M_l^2}{\mu^2} \right) \right] \\ &+ \left( \frac{\gamma_\mu P^\mu [P^2 + M_l^2 - m_l^2]}{4P^2 M_l} + \frac{1}{2} \right) G'_l,\end{aligned}$$

# Loop function

$$G_l = i \int \frac{d^4 q}{(2\pi)^4} \frac{\gamma_\mu q^\mu + M_l}{q^2 - M_l + i\varepsilon} \frac{1}{(p - q)^2 - m_l^2 + i\varepsilon}$$

$$\frac{\gamma_\mu P^\mu}{32P^2 \pi^2} \left[ (a_l + 1)(m_l^2 - M_l^2) + (m_l^2 \ln \frac{m_l}{\mu^2} - M_l^2 \ln \frac{M_l^2}{\mu^2}) \right] + \left( \frac{s + m_l^2 - M_l^2}{4M_l \sqrt{s}} + \frac{1}{2} \right) G_l'$$

Since the total three-momentum in the center of mass system is zero,  $\gamma^\mu P_\mu \rightarrow \gamma^0 P_0$ . The external mesons and baryons are all on shell, so the  $\gamma_0$  matrix is replaced by a unit matrix in the low energy region.

$$G_l = \frac{\sqrt{s}}{32s\pi^2} \left[ (a_l + 1)(m_l^2 - M_l^2) + (m_l^2 \ln \frac{m_l}{\mu^2} - M_l^2 \ln \frac{M_l^2}{\mu^2}) \right] + \left( \frac{s + m_l^2 - M_l^2}{4M_l \sqrt{s}} + \frac{1}{2} \right) G_l'$$

# Loop function

$$\begin{aligned}
 G'_l(s) &= i2M_l \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - M_l^2 + i\varepsilon} \frac{1}{(P-q)^2 - m_l^2 + i\varepsilon} \\
 &= \frac{2M_l}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{\bar{q}l}{\sqrt{s}} \left[ \ln(s - (M_l^2 - m_l^2) + 2\bar{q}l\sqrt{s}) \right. \right. \\
 &\quad \left. \left. + \ln(s + (M_l^2 - m_l^2) + 2\bar{q}l\sqrt{s}) + \frac{\bar{q}_l}{\sqrt{s}} - \ln(-s + (M_l^2 - m_l^2) + 2\bar{q}l\sqrt{s}) \right. \right. \\
 &\quad \left. \left. - \ln(-s - (M_l^2 - m_l^2) + 2\bar{q}l\sqrt{s}) \right] \right\}
 \end{aligned}$$

## S=0, I=1/2 channel

$$|\pi N; \frac{1}{2}, -\frac{1}{2}\rangle = -\sqrt{\frac{2}{3}}|\pi^- p\rangle + \sqrt{\frac{1}{3}}|\pi^0 n\rangle,$$

$$|\eta N; \frac{1}{2}, -\frac{1}{2}\rangle = |\eta n\rangle,$$

$$|K\Lambda; \frac{1}{2}, -\frac{1}{2}\rangle = |K^0\Lambda\rangle,$$

$$|K\Sigma; \frac{1}{2}, -\frac{1}{2}\rangle = -\sqrt{\frac{1}{3}}|K^0\Sigma^0\rangle + \sqrt{\frac{2}{3}}|K^+\Sigma^-\rangle.$$

Table 2. The coefficients  $C_{ij}$  in the pseudoscalar meson and baryon octet interaction with isospin  $I = \frac{1}{2}$  and strangeness  $S = 0$ ,  $C_{ji} = C_{ij}$ .

$C_{ij}$	$\pi N$	$\eta N$	$K\Lambda$	$K\Sigma$
$\pi N$	2	0	$\frac{3}{2}$	$-\frac{1}{2}$
$\eta N$		0	$-\frac{3}{2}$	$-\frac{3}{2}$
$K\Lambda$			0	0
$K\Sigma$				2

## S=0, I=1/2 channel

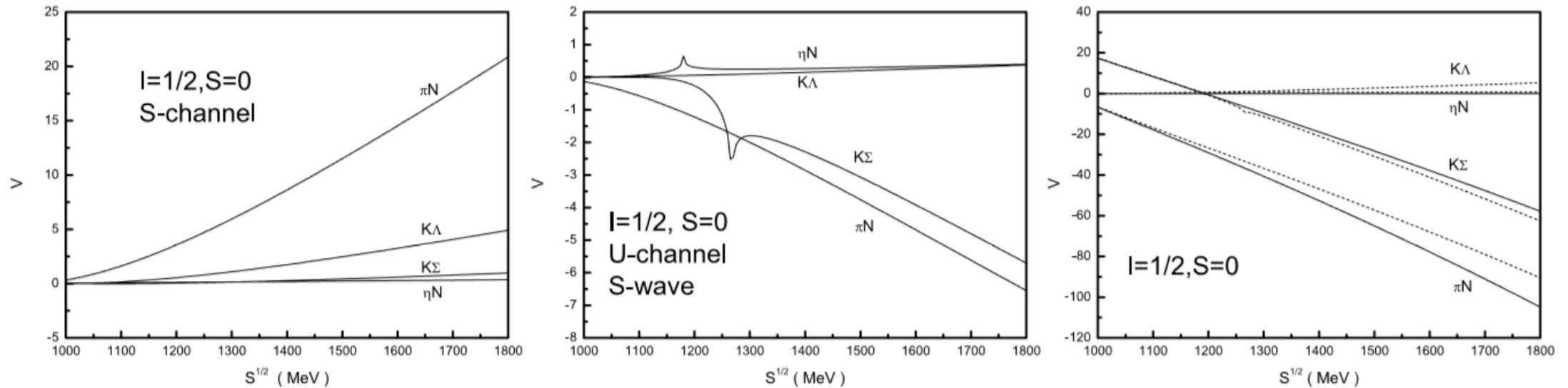


Fig. 2. Potentials of the pseudoscalar meson and baryon octet interaction as functions of the total energy of the system  $\sqrt{s}$  in the sector of isospin  $I = 1/2$  and strangeness  $S = 0$ . (left):  $s$ - channel. (middle):  $u$ - channel in the S-wave approximation. (right): The solid lines denote the Weinberg-Tomozawa contact interaction, while the dashed lines stand for the total S-wave potential from Eq. (18).

$S=0, I=1/2$  channel

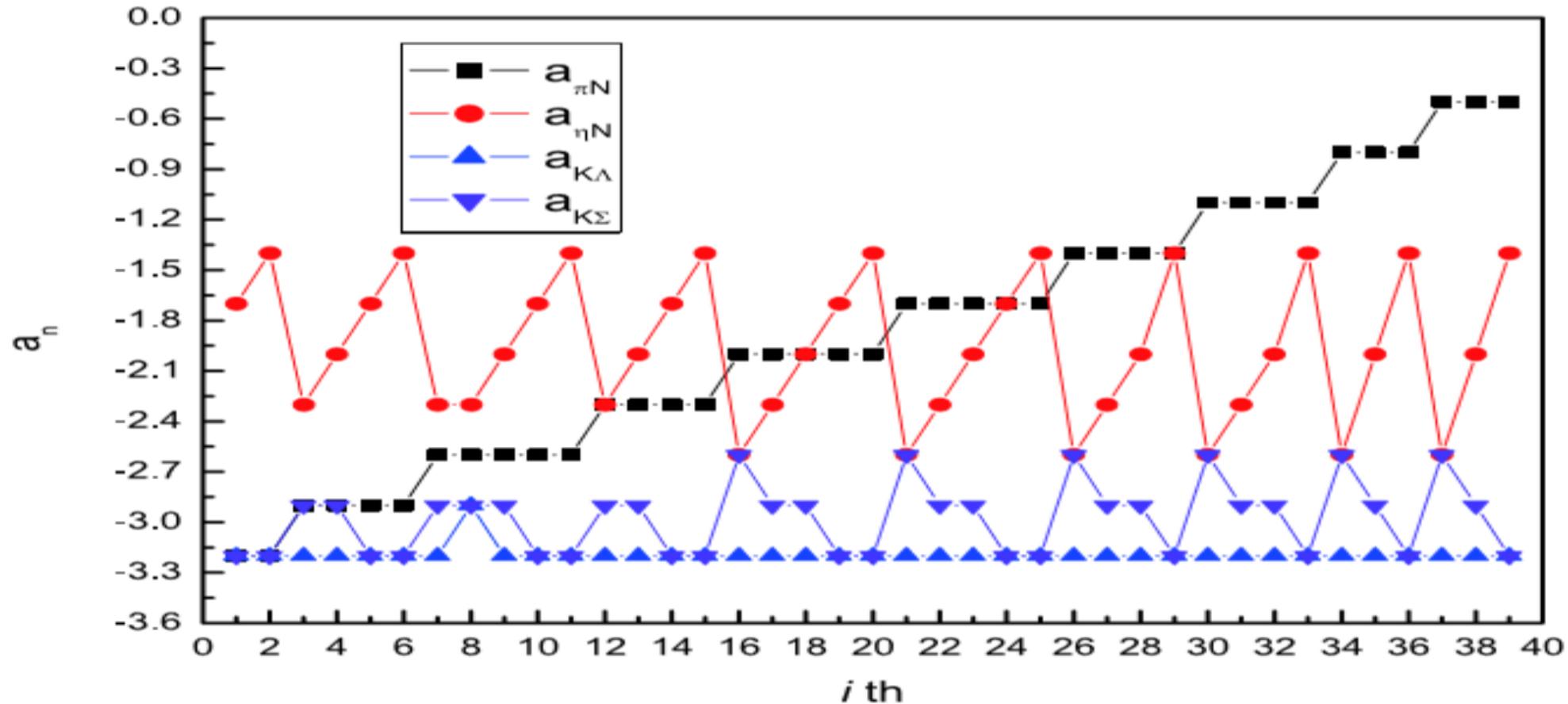


Fig. 3. (color online) The subtraction constants  $a_{\pi N}$ ,  $a_{\eta N}$ ,  $a_{K\Lambda}$ ,  $a_{K\Sigma}$  with the regularization scale  $\mu = 630$  MeV in the loop function in Eq. (24).

## S=0, I=1/2 channel

$$a_{\pi N} = -2.0,$$

$$a_{\eta N} = -1.7,$$

$$a_{K\Lambda} = -3.2$$

$$a_{K\Sigma} = -3.2.$$

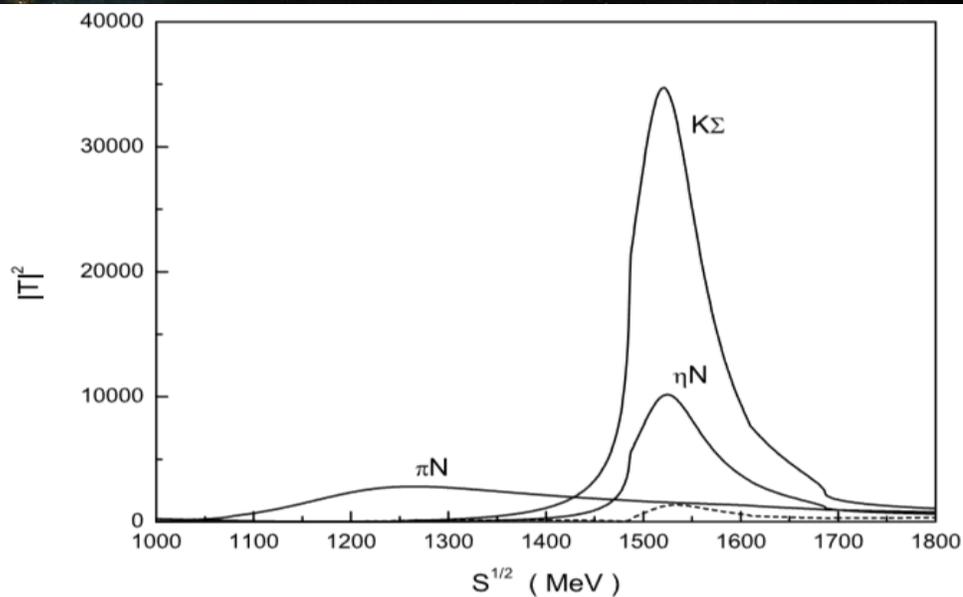


Fig. 4. The amplitude squared  $|T|^2$  as a function of the total energy  $\sqrt{s}$  for different channels with isospin  $I = 1/2$  and strangeness  $S = 0$ . The  $\pi N$ ,  $\eta N$  and  $K\Sigma$  channels are labeled in the figure, while the  $K\Lambda$  channel is drawn with the dashed line.

$g_{\pi N}$	$ g_{\pi N} $	$g_{\eta N}$	$ g_{\eta N} $	$g_{K\Lambda}$	$ g_{K\Lambda} $	$g_{K\Sigma}$	$ g_{K\Sigma} $
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$-2 + 3i$	5	$-65 + 25i$	70	$42 + 1i$	42	$94 - 27i$	98
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# Conclusion

1. The S-wave potential of the pseudoscalar meson and baryon octet is investigated, where the t-, s-, and u-channels are taken into account.
2. The relativistic kinetic effect is considered when the loop function is calculated in the dimensional regularization scheme.
3. A resonance state is generated dynamically by solving the Bethe-Salpeter equation, which might correspond to the N(1535) particle and couples strongly to the  $\eta N$ ,  $K \Lambda$ , and  $K \Xi$  channels.



THANK YOU!

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