Estimation of the low-lying tetraquark mass spectrum

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Outline

- Introduction
- Estimation of the low-lying tetraquark mass spectrum
  - Fix parameters
  - Construct tetraquark wave function
  - Calculate tetraquark mass spectrum
- Summary

These charged charmoniumlike states go beyond conventional .$c\bar{c}$-meson picture and could be tetraquark systems $c\bar{c}u\bar{d}$ due to carrying one charge.

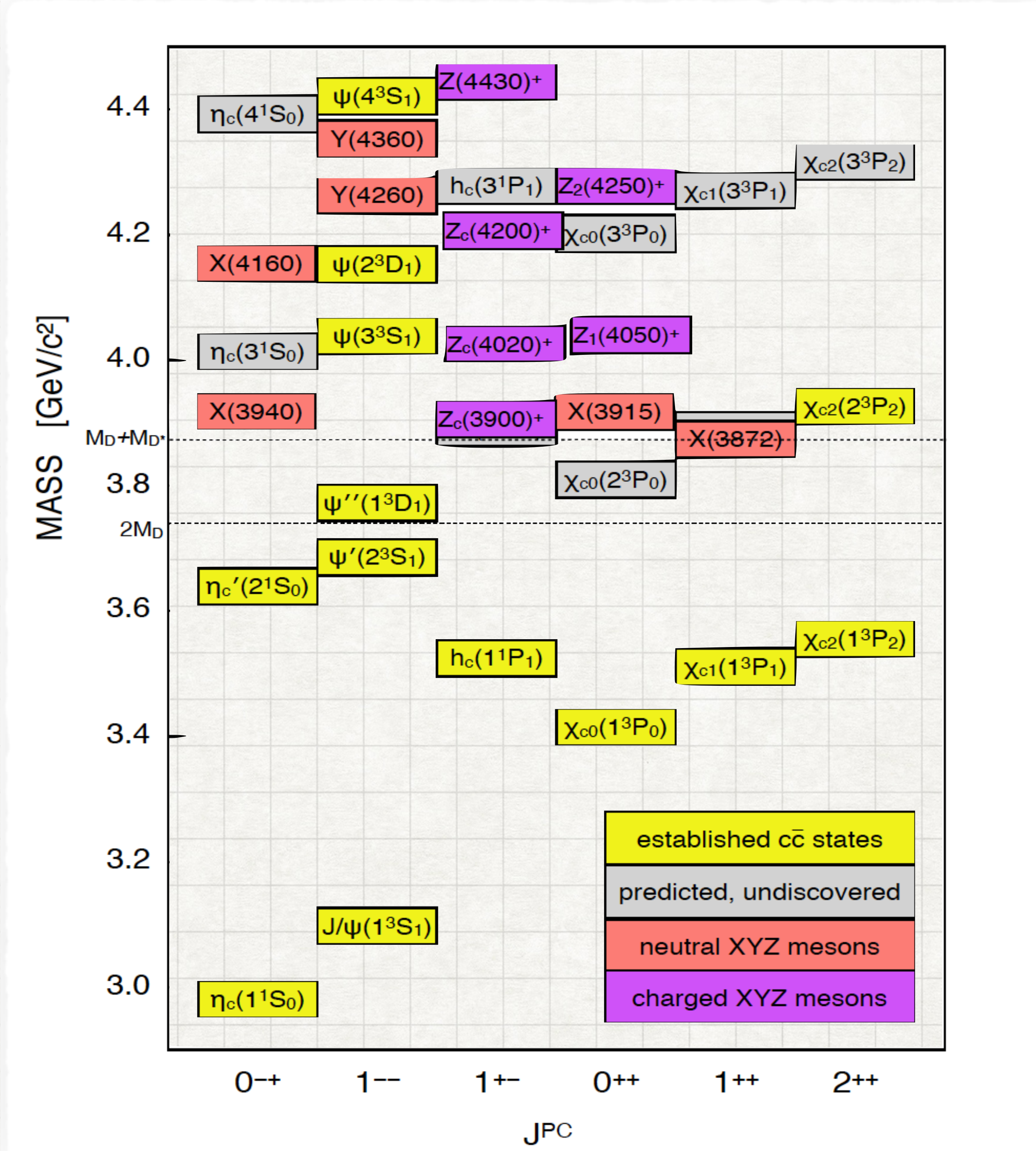


Figure 1: Charmonium meson spectrums include some charmonium-like XYZ states (Olsen, 2015).

$$H = 2M_{ave} + \frac{P^2}{2m_r} + \sigma r - \frac{A}{r} + H_{hyp} \qquad A_{ij} = A_c\sqrt{\frac{m_c}{m_{ij}}} \qquad \sigma_{ij} = \sigma_c\sqrt{\frac{m_{ij}}{m_c}} \qquad m_r = m_{ij} = 2\frac{m_i m_j}{m_i + m_j} \qquad (1)$$

$$H_{hyp} = -C_{ij}\frac{\lambda_i\lambda_j}{m_i m_j}\sigma_i\sigma_j \qquad C_{ij} = C_{gc}\left(\frac{m_{ij}}{m_c}\right)^{t_h} \qquad t_h = 1.26348 \qquad (2)$$

$$m_u = 350MeV \qquad m_s = 450MeV \qquad m_c = 1416MeV \qquad m_b = 4800MeV \qquad (3)$$

Meson mass spectrum result

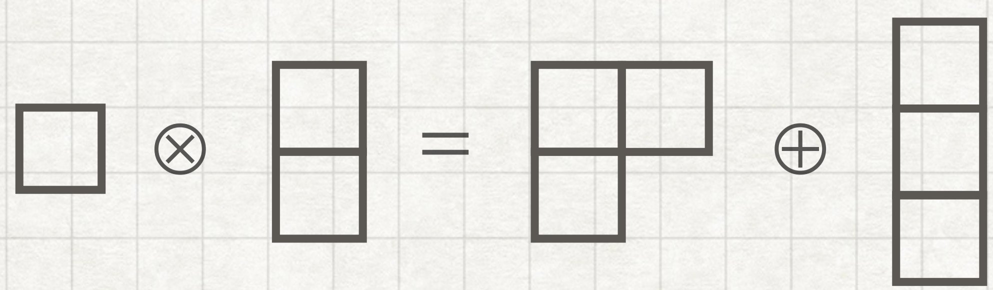
Meson (MeV)	Mave	A	$\sigma(GeV^2)$	pseudo meson n=0, s=0 Exp.	pseudo meson n=0, s=0 Ours	pseudo meson n=1, s=0 Exp.	pseudo meson n=1, s=0 Ours	Gap s=0 n=0&n=1 Exp.	vector meson n=0, s=1 Exp.	vector meson n=0, s=1 Ours	vector meson n=1, s=1 Exp.	vector meson n=1, s=1 Ours	Gap s=1 n=0&n=1 Exp.	Gap s=1 n=0&n=1 ours
$b\bar{b}$	4722	0.323711	0.316678	9399	9411	9999	10001	600	9460	9457	10023	10047	563	590
$c\bar{c}$	1534	0.596	0.172	2984	2984	3638	3574	654	3097	3097	3686	3688	589	591
$D_s(c\bar{s})$	1038	0.858185	0.119452	1968	1970	–	2561	–	2112	2112	2708	2703	596	591
$D(c\bar{u})$	986	0.946657	0.108288	1865	1866	–	2457	–	2007	2008	–	2599	–	591
$B(u\bar{b})$	2657	0.878035	0.116752	5279	5277	–	5868	–	5325	5328	–	5918	–	590
$B_s(s\bar{b})$	2702	0.781836	0.131117	5367	5364	–	5955	–	5415	5417	–	6008	–	591
$B_c(c\bar{b})$	3159	0.479586	0.213751	6275	6275	6842	6866	567	–	6333	–	6924	–	591
$q\bar{q}$	350	1.19879	0.0855127	–	463	–	1053	–	770	780	1450	1370	680	590
$s\bar{s}$	480	1.05724	0.0969623	–	763	–	1354	–	1020	1026	1680	1617	660	591

(M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018))

- Tetraquark are states of two quarks and two antiquarks, $q\bar{q}q\bar{q}$
- The construction of tetraquark color wave function is guided by:
  - The tetra-quark wave function should be a color singlet.
  - It demands that the color part of tetra-quark wave function must be [222] singlet.

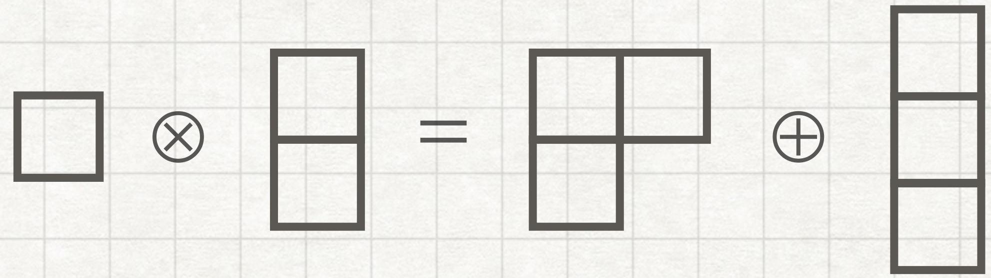
$$\psi^{c}_{[222]}(q^2\overline{q}^2) =$$

- A meson state is the direct product of quark and anti-quark states,



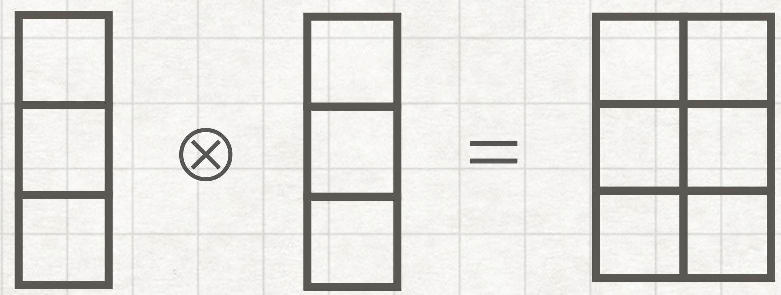
with the corresponding dimensions being: $3 \otimes \overline{3} = 8 \oplus 1$

- A meson state is the direct product of quark and anti-quark states,

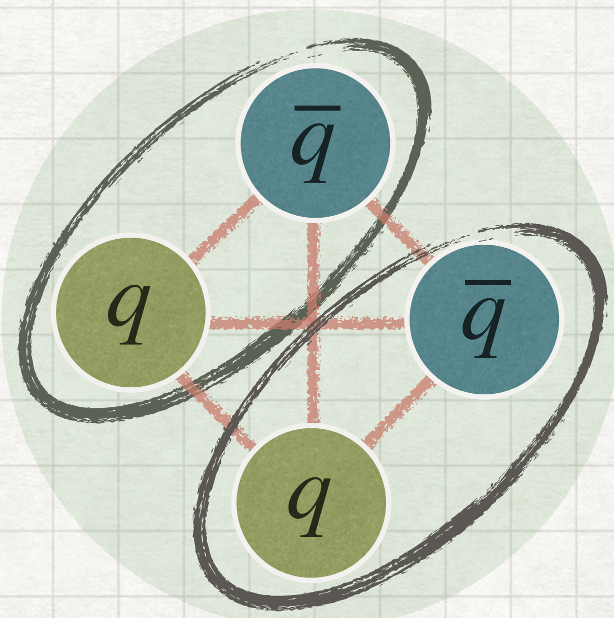


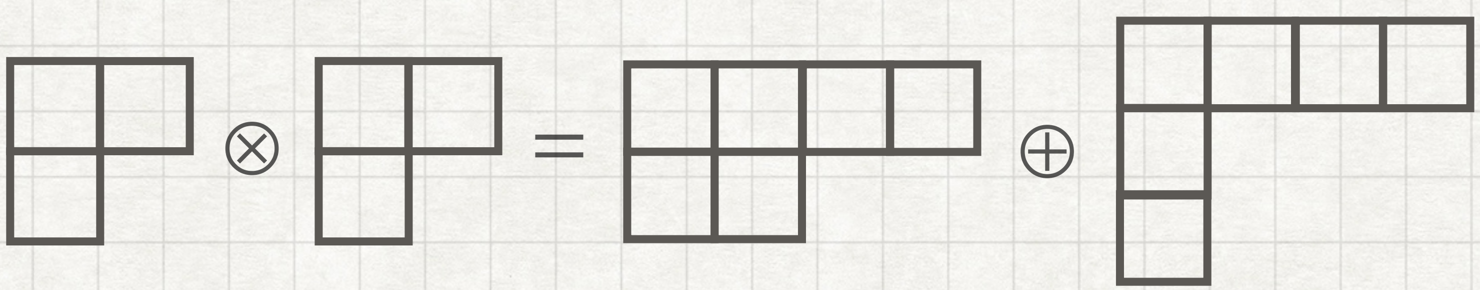
with the corresponding dimensions being: $3 \otimes \bar{3} = 8 \oplus 1$

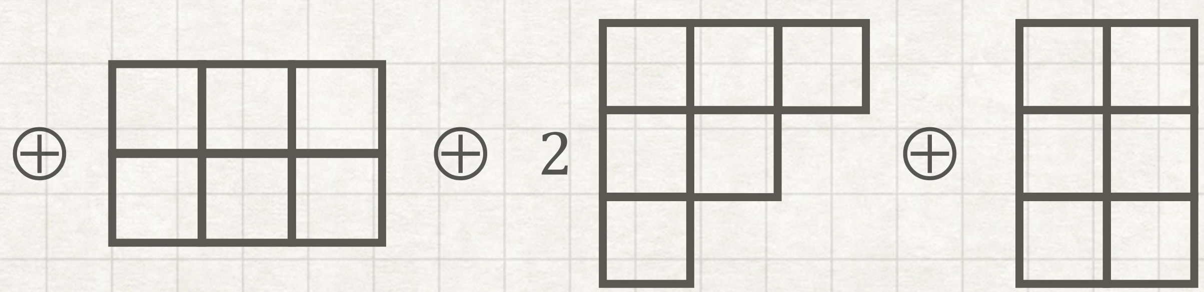
- The color wave function should be a color singlet (antisymmetric)



with the corresponding dimensions being: $1 \otimes 1 = 1$







with the corresponding dimensions being: $8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$

Color wave function

Singlet-Singlet state: $\psi^c_{q\bar{q}Q\bar{Q}}[1] = \frac{1}{3}(r\bar{r} + g\bar{g} + b\bar{b})(r\bar{r} + g\bar{g} + b\bar{b})$

Octet-Octet state:

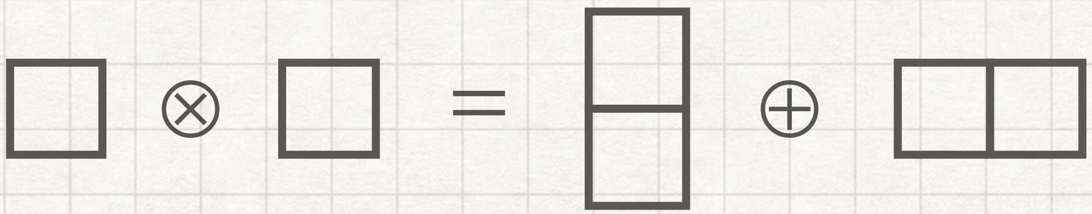
$$\psi^c_{q\bar{q}Q\bar{Q}}[8] = \frac{1}{\sqrt{8}}[r\bar{g}g\bar{r} + g\bar{r}r\bar{g} + r\bar{b}b\bar{r} + g\bar{b}b\bar{g} + b\bar{r}r\bar{b} + b\bar{g}g\bar{b}$$
$$+\frac{1}{2}(g\bar{g} - r\bar{r})(g\bar{g} - r\bar{r}) + \frac{1}{6}(r\bar{r} + g\bar{g} - 2b\bar{b})(r\bar{r} + g\bar{g} - 2b\bar{b})]$$

Color and spin matrix element $q\bar{q}q\bar{q}$

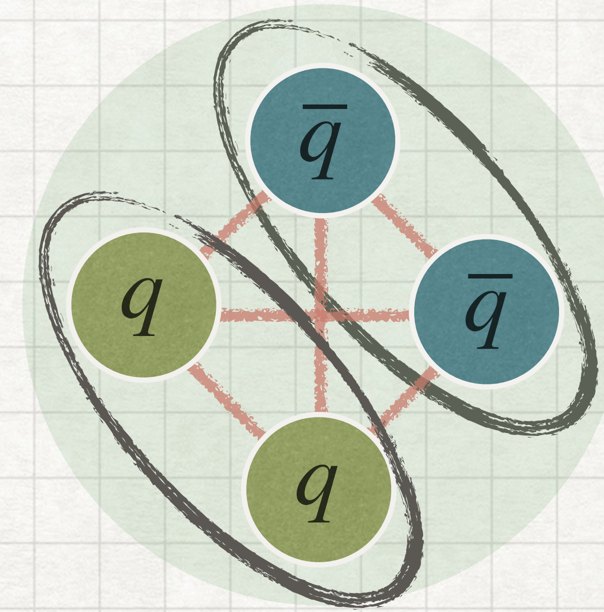
Color	$\lambda_1\lambda_2$	$\lambda_1\lambda_3$	$\lambda_1\lambda_4$	$\lambda_2\lambda_3$	$\lambda_2\lambda_4$	$\lambda_3\lambda_4$	$\sum \lambda_i\lambda_j$
$< Octet \vert \hat{O} \vert Octet >$	2/3	–4/3	–14/3	–14/3	–4/3	2/3	–32/3
$< Singlet \vert \hat{O} \vert Singlet >$	–16/3	0	0	0	0	–16/3	–32/3

Spin	$\sigma_1\sigma_2$	$\sigma_1\sigma_3$	$\sigma_1\sigma_4$	$\sigma_2\sigma_3$	$\sigma_2\sigma_4$	$\sigma_3\sigma_4$	$\sum \sigma_i\sigma_j$
$< S = 0 \vert \hat{O} \vert S = 0 >$	–3	0	0	0	0	–3	–6
$< S = 1 \vert \hat{O} \vert S = 1 >$	1	0	0	0	0	–3	–2
$< S = 2 \vert \hat{O} \vert S = 2 >$	1	1	1	1	1	1	6

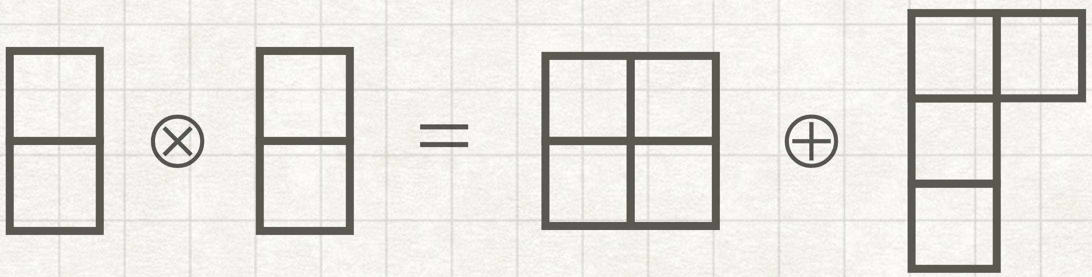
- The color part for two quarks in tetra-quark states is



$$3 \otimes 3 = \bar{3} \oplus 6$$

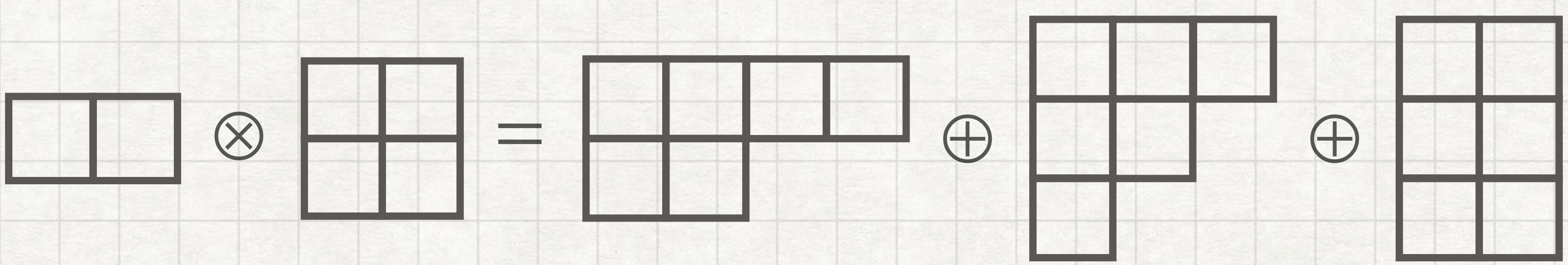


- The color part for two anti-quarks in tetra-quark states is

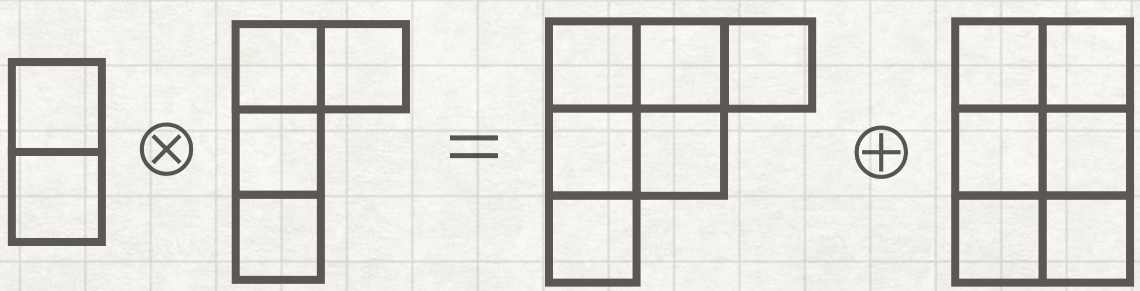


$$3 \otimes 3 = \bar{6} \oplus 3$$

- The color wave function should be a color singlet (antisymmetric)



$$6 \otimes \bar{6} = 27 \oplus 8 \oplus 1$$



$$\bar{3} \otimes 3 = 8 \oplus 1$$

Color wave function

Triplet-Triplet state: $\psi^c_{qQ\bar{q}\bar{Q}}[3] = \frac{1}{3}[\frac{1}{2}(rg - gr)(\bar{r}\bar{g} - \bar{g}\bar{r}) + \frac{1}{2}(br - rb)(\bar{b}\bar{r} - \bar{r}\bar{b}) + \frac{1}{2}(gb - bg)(\bar{g}\bar{b} - \bar{b}\bar{g})]$

Sextet-Sextet state:

$$\psi^c_{qQ\bar{q}\bar{Q}}[6] = \frac{1}{\sqrt{6}}[rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b} + \frac{1}{2}(rg + gr)(\bar{r}\bar{g} + \bar{g}\bar{r}) + \frac{1}{2}(rb + br)(\bar{r}\bar{b} + \bar{b}\bar{r}) + \frac{1}{2}(gb + bg)(\bar{g}\bar{b} + \bar{b}\bar{g})]$$

Color and spin matrix element $qq\bar{q}\bar{q}$

Color	$\lambda_1\lambda_2$	$\lambda_1\lambda_3$	$\lambda_1\lambda_4$	$\lambda_2\lambda_3$	$\lambda_2\lambda_4$	$\lambda_3\lambda_4$	$\sum \lambda_i\lambda_j$
$< Triplet \mid \hat{O} \mid Triplet >$	–8/3	–4/3	–4/3	–4/3	–4/3	–8/3	–32/3
$< Sextet \mid \hat{O} \mid Sextet >$	4/3	–10/3	–10/3	–10/3	–10/3	4/3	–32/3

Spin	$\sigma_1\sigma_2$	$\sigma_1\sigma_3$	$\sigma_1\sigma_4$	$\sigma_2\sigma_3$	$\sigma_2\sigma_4$	$\sigma_3\sigma_4$	$\sum \sigma_i\sigma_j$
$< S = 0 \mid \hat{O} \mid S = 0 >$	–3	0	0	0	0	–3	–6
$< S = 1 \mid \hat{O} \mid S = 1 >$	1	0	0	0	0	–3	–2
$< S = 2 \mid \hat{O} \mid S = 2 >$	1	1	1	1	1	1	6

Jacobi coordinate for $q\bar{q}q\bar{q}$

$$\sigma = \frac{1}{\sqrt{2}}(r_1 - r_2)$$

$$\sigma' = \frac{1}{\sqrt{2}}(r_3 - r_4)$$

$$\lambda = \frac{m_1 r_1 + m_2 r_2}{m_1 m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 m_4}$$

Reduced mass:

$$m_\sigma = \frac{2m_1 m_2}{m_1 + m_2} \quad m_{\sigma'} = \frac{2m_3 m_4}{m_3 + m_4} \quad m_\lambda = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}$$

for $q\bar{q}q\bar{q}$

$$m_\sigma = m_u \quad m_{\sigma'} = m_u \quad m_\lambda = m_u$$

(Elena Santopinto and Giuseppe Galatà, Phys. Rev. C75, 045206)

The hamiltonian for tetraquark

$$H = 4M_{ave} + \frac{P_{\sigma 1}^2}{2m_{\sigma 1}} + \frac{P_{\sigma 2}^2}{2m_{\sigma 2}} + \frac{P_{\lambda}^2}{2m_{\lambda}} + (-\frac{3}{8})\lambda_i^C . \lambda_j^C(\sigma r_{ij} - \frac{A}{r_{ij}}) + H_{hyp}$$

- Calculate the eigenvalues from the Hamiltonian: $< \psi_{total} | H | \psi_{total} > = E | \psi_{total} >$

$$A_{ij} = A_c\sqrt{\frac{m_c}{m_{ij}}} \qquad \sigma_{ij} = \sigma_c\sqrt{\frac{m_{ij}}{m_c}} \qquad m_r = m_{ij} = 2\frac{m_i m_j}{m_i + m_j}$$

$$m_u = 350 MeV \qquad m_s = 450 MeV \qquad m_c = 1416 MeV \qquad m_b = 4800 MeV$$

The hyperfine interaction term takes the form:

$$H_{hyp} = < \psi | - C_{ij} \sum_{i<j} \frac{\lambda_i^c \cdot \lambda_j^c}{m_i m_j} \vec{\sigma_i} \cdot \vec{\sigma_j} | \psi >$$

$$C_{ij} = C_{gc}(\frac{m_{ij}}{m_c})^{t_h} \qquad C_{gc} = 86.8 \qquad t_h = 1.26348 \qquad C_{ij} = 86.8 \times (\frac{350}{1416})^{1.26348}$$

Harmonic oscillator basis for tetraquark system

$NLM = 000$		$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	
$NLM = 200$	$\Psi_{100}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{100}(\lambda)$
$NLM = 400$	$\Psi_{200}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{200}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{200}(\lambda)$
	$\Psi_{100}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{100}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{100}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{100}(\lambda)$
$NLM = 600$	$\Psi_{300}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{300}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{300}(\lambda)$
	$\Psi_{000}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{200}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{200}(\sigma_2)\Psi_{100}(\lambda)$	$\Psi_{100}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{200}(\lambda)$
	$\Psi_{100}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{100}(\lambda)$	$\Psi_{100}(\sigma_1)\Psi_{200}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{200}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{100}(\lambda)$
		$\Psi_{200}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{000}(\lambda)$	

Hyperfine Interaction(MeV)	S=0	S=1	S=2
$<Triplet\|\hat{O}\|Triplet>$	−106.627	−35.5425	80.1789
$<Sextet\|\hat{O}\|Sextet>$	53.3137	17.7712	93.8199
$<Octet\|\hat{O}\|Octet>$	40.2978	0.709181	66.538
$<Singlet\|\hat{O}\|Singlet>$	−322.383	−5.67345	107.461

Tetraquark mass(MeV) $q\bar{q}c\bar{c}$ $qc\bar{q}\bar{c}$	S=0 $0^+0^{++}/1^-0^{++}$		S=1 $0^-1^{+-}/1^+1^{+-}$		S=2 $0^+2^{++}/1^-2^{++}$	
	Ours	Exp.	Ours	Exp.	Ours	Exp.
$<Octet\|\hat{O}\|Octet>$	4084.67	X(4050)	4045.08	X(4055)	4110.91	–
	4210.83	–	4171.24	Zc(4200)	4237.07	X(4250)
$<Singlet\|\hat{O}\|Singlet>$	3541.68	–	3858.39	Zc(3900)	3971.53	X(3940)
	3696.06	–	4012.77	X(4020)	4125.9	–
$<Sextet\|\hat{O}\|Sextet>$	4186.73	X(4160)	4151.19	–	4227.24	–
	4327.39	–	4291.85	–	4367.9	X(4350)
$<Triplet\|\hat{O}\|Triplet>$	3916.44	X(3915)	3987.53	–	4103.25	X(4100)
	4278.07	–	4349.15	–	4464.87	–

(M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018))

Hyperfine Interaction(MeV)	S=0	S=1	S=2
$< Triplet \| \hat{O} \| Triplet >$	−237.534	−79.178	158.356
$< Sextet \| \hat{O} \| Sextet >$	118.767	39.589	158.356
$< Octet \| \hat{O} \| Octet >$	59.383	19.7943	158.355
$< Singlet \| \hat{O} \| Singlet >$	−475.064	−158.355	158.355

Tetraquark mass(MeV) $q\bar{q}q\bar{q}$ $qq\bar{q}\bar{q}$	S=0 $0^{+}0^{++}/1^{-}0^{++}$		S=1 $0^{-}1^{+-}/1^{+}1^{+-}$		S=2 $0^{+}2^{++}/1^{-}2^{++}$	
	Ours	Exp.	Ours	Exp.	Ours	Exp.
$< Octet \| \hat{O} \| Octet >$	1794.74	–	1755.16	–	1893.72	f2(1910)
	1992.19	a0(1950)	1952.6	–	2091.16	–
$< Singlet \| \hat{O} \| Singlet >$	1076.58	–	1393.29	h1(1415)	1710	a2(1700)
	1302.81	–	1619.52	h1(1595)	1936.23	–
$< Triplet \| \hat{O} \| Triplet >$	1550.26	f0(1500)	1708.62	–	1946.15	f2(1950)
	1813.03	–	1971.39	–	2208.92	fj(2220)
$< Sextet \| \hat{O} \| Sextet >$	1826.45	–	1747.27	–	1866.04	f2(1810)
	2006.13	f0(2020)	1926.95	–	2045.72	f2(2010)

(M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018))

- All model parameters were predetermined by comparing the theoretical and experimental masses of light, charmed and bottom mesons.

- We constructed the tetraquark wave function. We derived color wave function in the Yamanouchi basis framework with permutation group.

- Some observed tetraquark states could be accommodated by our tetraquark mass spectra, and we also predicted some non-observed tetraquark states.

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