Track ID: 181

Relativistic effects in radiative charmonium transitions: A covariant quark model approach

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August 21, 2019,
XVIII International Conference on Hadron Spectroscopy and Structure (HADRON 2019), Guilin, China

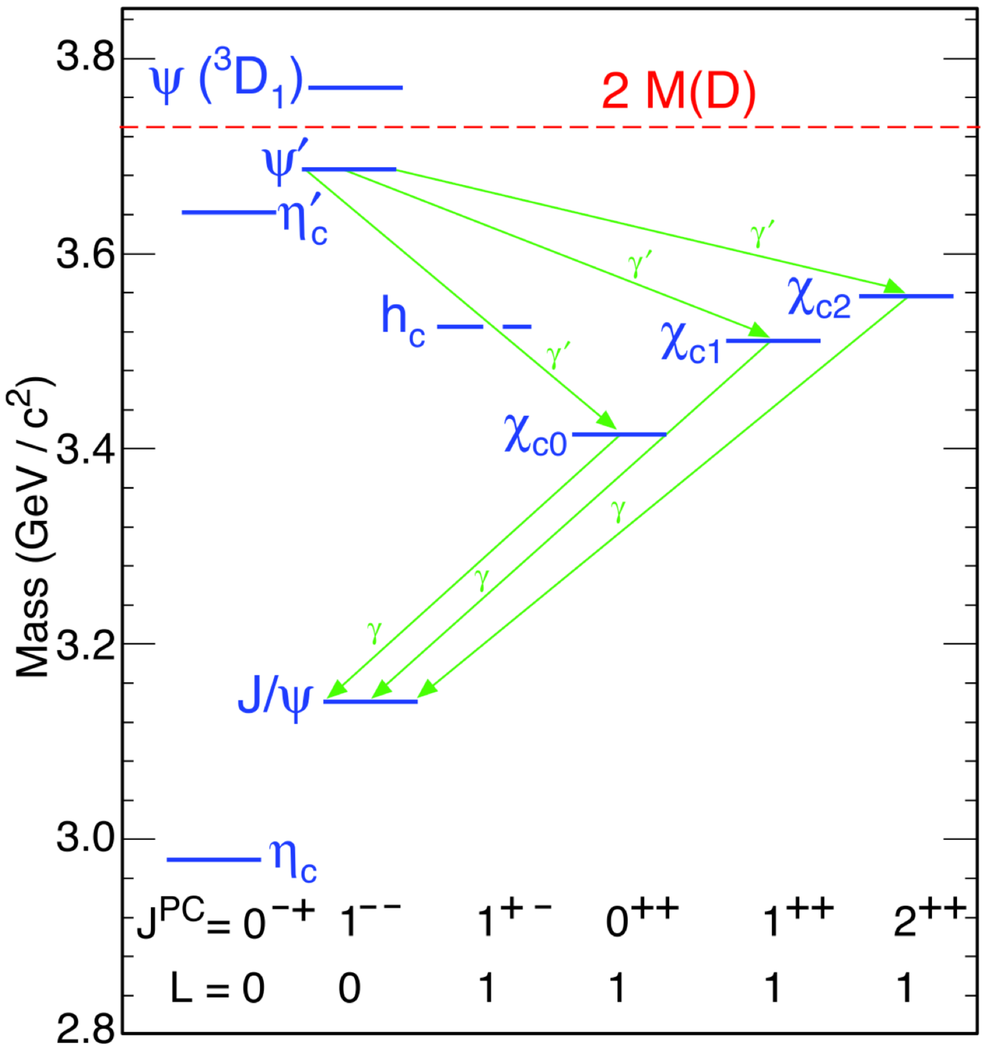
Introduction

General background

- In order to clarify the structure of hadrons, one of the appropriate way is to study the response to electromagnetic (EM) interactions, since the fundamental EM interaction is better known than strong one.

- In this meaning, the radiative transitions has been an important problem for a long time. It is believed that most reliable calculations are those of the c$\underline{c}$/b$\underline{b}$ since non-relativistic treatment permissible. In this line of thought, thus far many authors have applied non-relativistic quark models to study this problem.

- However, recent experiments clearly show that the quark model with relativistic correction /couple-channel effect reproduces the data better than the simple non-relativistic quark model (even below DD threshold).



CLEO Collaboration (M. Artuso *et al.*) PRD80 (2009)

Introduction

Motivation and purpose of this work

- As a matter of fact, our collaboration have tackled this problem about 20 years ago (S. Ishida, M. Morikawa, M. Oda, PTP99 (1988)). However, at that time, the long wavelength approximation was used and the relativistic effect of the form factor was not evaluated properly.

- Motivated by the recent experimental progress for the charmonium states, we study the radiative transitions of the $c\underline{c}$ system in the covariant quark model. In particular, we will focus on the relativistic effect in our approach and clarify how it affects the calculated widths.

Outline

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- Meaning of ``relativistic'' effects in this work

3. Transitions

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- 1^1P_1 to 1^1S_0 transitions
- $1^3P_{0,1,2}$ to 1^3S_1 transitions and higher-order multipole amplitudes
- 2^3S_1 to $1^3P_{0,1,2}$ transitions and higher-order multipole amplitudes

4. Summary

- Summary and remarks
- Future subjects

2. The Model : Boosted LS coupling scheme

Basic framework of the BLS scheme (1)

- In this study, we adopt a covariantly formulated quark-model, known as the ``boos-LS (bLS) coupling scheme''.

S. Ishida, K. Yamada, M. Oda, PRD 40 (1989)
S. Ishida, M. Morikawa, M. Oda, PTP99 (1988))

- In this scheme, cc̲ mesons are described as a bi-local WF

$$\Psi(x_1^\mu, x_2^\mu)_\alpha^\beta = \Psi(X^\mu, x^\mu)_\alpha^\beta \qquad \left(X^\mu = \frac{x_1^\mu + x_2^\mu}{2}, \quad x^\mu = x_1^\mu - x_2^\mu \right)$$

- The internal wave function is determined as a solution of the Klein-Gordon equation which leads to the squared-mass spectrum, instead of the Schrodinger equation which does the mass spectrum.

$$\Psi(X^\mu, x^\mu)_\alpha{}^\beta = \sqrt{\frac{2M}{2P_0 V}} e^{\mp i P_\mu X^\mu} \Phi(v, x)_\alpha{}^{\beta(\pm)}$$

satisfies $\left(-\frac{\partial^2}{\partial X_\mu \partial X^\mu} - \mathcal{M}^2(x)\right) \Psi(X, x)_\alpha{}^\beta = 0$

here, $\mathcal{M}(x)^2 = 4m\left(\frac{1}{m}\frac{\partial^2}{\partial x_\mu x^\mu} + U(x)\right)$, $U(x) = -\frac{1}{2} K x_\mu x^\mu + U_{\mathrm{OGE}}$

Basic framework of the BLS scheme (2)

- The internal wave functions of relevant mesons are described by a direct product of space-time and spin part, separately extended to the covariant form.

$$\Phi(v,x)_{\alpha}^{(\pm)\beta} = f(v,x)^{(nL)\mu\nu\cdots} \otimes \left(W_{\alpha}^{(\pm)\beta}(v)\right)_{\mu\nu\cdots} \qquad (\, P^{\mu} = M v^{\mu} \,)$$

- Space-time part:
  Definite-type 4dim. SHO

$$f_G(v,x) = \frac{\beta^2}{\pi} \exp\left[\frac{\beta^2}{2}(x^2 - 2(v\cdot x)^2)\right]$$

T. Takabayasi (1979) $P^{\mu} a_{\mu}^{\dagger} f(v,x) = 0$

$$f_{\mu_1\mu_2\cdots}(v,x) = a_{\mu_1}^{\dagger} a_{\mu_2}^{\dagger} \cdots f_G(v,x), \quad a_{\mu}^{\dagger} = \frac{1}{\sqrt{2\beta^2}}(\beta^2 x_{\mu} + \partial_{x\mu})$$

- Spin part:
  Bargmann Wigner spinors

$$W_{\alpha}^{(+)\beta}(v) \sim u_{\alpha}(v)\bar{v}^{\beta}(v), \quad W_{\alpha}^{(-)\beta}(v) \sim v_{\alpha}(v)\bar{u}^{\beta}(v)$$

Concrete form of WF

$$\Phi(v,x)_{1S}^{(+)} = f_G(v,x) \otimes (\gamma_5 - \epsilon_{\rho}(v)\gamma^{\rho}) \frac{1-\not{v}}{2\sqrt{2}}$$

$$\Phi(v,x)_{1P}^{(+)} = \sqrt{2\beta^2} x^{\kappa} f_G(v,x) \otimes (\gamma_5 \epsilon_{\kappa} - \epsilon_{\kappa\rho}(v)\gamma^{\rho}) \frac{1-\not{v}}{2\sqrt{2}}$$

$$\Phi(v,x)_{2S}^{(+)} = \frac{1}{\sqrt{6}} \left(3 + 2\beta^2 \left(g_{\kappa_1\kappa_2} - v_{\kappa_1} v_{\kappa_2}\right) x^{\kappa_1} x^{\kappa_2}\right) f_G(v,x) \otimes (\gamma_5 - \epsilon_{\rho}(v)\gamma^{\rho}) \frac{1-\not{v}}{2\sqrt{2}}$$

Basic framework of the BLS scheme (3)

- Following to the usual procedure, the photon coupling is obtained by minimal substitution on the action which leads the basic Klein-Gordon equation.

$$S_{\text{Int}} = -\int d^4x_1 \int d^4x_2 \; j^{(1)\mu}(x_1, x_2) A_\mu(x_1) + (1 \leftrightarrow 2),$$

$$j^{(i)\mu}(x_1, x_2) = \frac{ie}{2m} \overline{\Psi}(x_1, x_2) Q_i \left(\overleftrightarrow{\partial}_i^{\mu} - i g_M^{(i)} \sigma^{\mu\nu} \left(\overrightarrow{\partial_{i\nu}} + \overleftarrow{\partial_{i\nu}} \right) \right) \Psi(x_1, x_2)$$

- The lowest matrix element of the S matrix is given by

$$S_{fi} = \delta_{fi} + ie(2\pi)^4 \delta^4(P_I - P_F - q) \sqrt{\frac{2M_I}{2P_{I0}}} \sqrt{\frac{2M_F}{2P_{F0}}} \sqrt{\frac{1}{2q_0}} \mathcal{M}_{fi}$$

$$\mathcal{M}_{fi} = -i \frac{Q_1}{2m} \int d^4x \langle \overline{\Phi}(v_F, x)^{(-)} \left(\overleftrightarrow{\partial}_x^{\mu} - g_M^{(1)} \sigma^{\mu\nu} q_\nu \right) \Phi(v_I, x)^{(+)} \rangle e^{i \frac{q^\rho}{2} x_\rho} e_\mu^*(q) \; + (1 \leftrightarrow 2)$$

$$= -i \frac{eQ_1}{2m} \int d^4x \langle \overline{\Phi}(v_F, x)^{(-)} \left(-\frac{M_I^2 - M_F^2}{4} x^\mu - g_M^{(1)} \sigma^{\mu\nu} q_\nu \right) \Phi(v_I, x)^{(+)} \rangle e^{i \frac{q^\rho}{2} x_\rho} e_\mu^*(q) \; + (1 \leftrightarrow 2)$$

- This expression corresponds to the well-known expression of the non-relativistic quark model.

Features of the BLS scheme

- An important feature of this scheme is that the hadrons are described with covariant way to deal with the relativistic retardation (recoil) effect of the form factor.

(1) Space-time WF Overlapping Integral (OI)

e.g. 1S to 1S

$$F_{\mathrm{bLS}} = \int d^4x \; f_{\mathrm{G}}(v_F, x) f_{\mathrm{G}}(v_I, x) e^{\frac{i}{2} q_\mu x^\mu}$$

at $\boldsymbol{v}_I = 0$

$$\to \frac{4{\beta_F}^2{\beta_I}^2}{\sqrt{\frac{({\beta_F}^2+{\beta_I}^2)^2((b^2-1){\beta_F}^4-2(b^2+1){\beta_F}^2{\beta_I}^2+(b^2-1){\beta_I}^4)}{b^2-1}}} \exp\left[\frac{(b+1)q^2}{4(b-1){\beta_F}^2-4(b+1){\beta_I}^2}\right] \quad (b=\frac{V}{c})$$

Corresponding to $$F_{\mathrm{Non-Rel.}} = \frac{\left(\frac{2{\beta_F}^2{\beta_I}^2}{{\beta_F}^2+{\beta_I}^2}\right)^{3/2}}{{\beta_F}^{3/2}{\beta_I}^{3/2}} \exp\left[-\frac{q^2}{8\left({\beta_F}^2+{\beta_I}^2\right)}\right]$$

(2) Spin WF Overlapping

e.g. S=0 to S=0

$$\mathrm{tr}\left(\overline{W}(v_F)W(v_I)\right) = \frac{1+v_I\cdot v_F}{2} \geq 1 \,, \qquad W_{\mathrm{S=0}}(v) = \frac{\gamma_5}{\sqrt{2}}\frac{1-\not{v}}{2}$$

Corresponding to $$\mathrm{tr}\left(\chi_{00}^\dagger \chi_{00}\right) = 1 \,, \qquad \chi_{00} = \frac{\chi_+{\chi'}_-^\dagger - \chi_-{\chi'}_+^\dagger}{\sqrt{2}}$$

Meaning of ``relativistic effects'' in this work

- The boost LS coupling scheme is reduced to a simple non-relativistic quark model if we put the boost velocity zero and the ``relative time'' degree of freedom is ignored to replace 4d SHO with a 3d SHO.

- In this study, within the framework of our model, both decay widths of the non-relativistic and relativistic versions are calculated, and we shall compere them to elucidate how the relativistic contribution appears in each process of charmonium radiative transitions.

- Here, the differences between the non-relativistic and relativistic calculation are classified into 3 categories.

1) due to space(-time) WF overlapping

2) due to spin WF overlapping

3) due to Phase Space

Non-rel: $\frac{q}{2\pi}$

Rel. (bLS): $\frac{q}{2\pi} \cdot \frac{E_F}{M_I} \cdot \frac{M_F}{E_F} = \frac{q}{2\pi} \frac{M_F}{M_I}$

Parameters

- The main purpose of this study is to examine how the relativistic correction affects, so we will take a typical set of parameters without completely tuning them to reproduce the experimental values.

- quark anomalous moment; (set to 0)

$$g_M^{(c)} = 1 + \kappa^{(c)} \equiv 1 \quad (\kappa^{(c)} \equiv 0).$$

- quark mass;

$$m_c = \frac{M_{J/\psi}}{2} = 1.55\mathrm{GeV}.$$

- The SHO wave function scales;

In standard quark model, these values are set to reproduce the RMS radii of the respective Hamiltonian eigen states. Resulting typical values (adopted here) are listed in following Table. (See, e.g. F. E. Close, E. S. Swanson, PRD72(2005).)

RMS-equivalent β-values (in GeV).

1^1S_0	1^3S_1	1^1P_1	$1^3P_{0,1,2}$	2^1S_0	2^3S_1	$1^3D_{1,2,3}$
0.71	0.66	0.50	0.49	0.48	0.47	0.45

3. Transitions

1^3S_1 to 1^1S_0 transition (1)

- $J/\psi \to \eta_c(1S)\gamma$ is the most typical M1 transition.

  The obtained result shows a tiny relativistic effect reduce the width. This is a natural result because the transition with a small mass difference cause almost no recoil.

Calculated partial width for $J/\psi \to \eta_c(1S)\gamma$ (in keV).

Initial	Final	Non-rel. → Rel.	Rate of change	Experiment (PDG2019)	BGS2005[1)]	DLGZ2017[2)]
J/ψ	$\eta_c(1S)$	$\mathbf{2.43 \to 2.41}$	$\mathbf{-0.7}$ %	$\mathbf{1.58 \pm 0.37}$	2.9 (NR) 2.9 (NR(j_0)) 2.4 (GI)	2.39 (LP) 2.44 (SP)

Slightly reduced by 0.7%

breakdown

- Spin WF booting (amplitude): $+\mathbf{1.9}$%
- Space-time OI (amplitude): $-\mathbf{0.38}$ %
- Phase Space (width): $-\mathbf{3.7}$ %

1) T. Barnes, S. Godfrey, E. S. Swanson, PRD72(2005)
2) W. Deng, H. Liu, L. Gui, X. Zhong, PRD95(2017)

- Note that the calculated value seems to deviate from the experimental value. Similar results are obtained with other quark models.

1^3S_1 to 1^1S_0 transition (2)

- Furthermore, the recent measurement at KEDR gets a large partial decay width.

Measurement of $J/\psi \to \gamma\eta_c$ decay rate and η_c parameters at KEDR

Physics Letters B 738 (2014) 391–396

A B S T R A C T

Using the inclusive photon spectrum based on a data sample collected at the J/ψ peak with the KEDR detector at the VEPP-4M e^+e^- collider, we measured the rate of the radiative decay $J/\psi \to \gamma\eta_c$ as well as η_c mass and width. Taking into account an asymmetric photon lineshape we obtained $\Gamma^0_{\gamma\eta_c} = 2.98 \pm 0.18^{+0.15}_{-0.33}$ keV, $M_{\eta_c} = 2983.5 \pm 1.4^{+1.6}_{-3.6}$ MeV/c^2, $\Gamma_{\eta_c} = 27.2 \pm 3.1^{+5.4}_{-2.6}$ MeV.

$$\Gamma^0_{\gamma\eta_c} = 2.98 \pm 0.18^{+0.15}_{-0.33} \text{ keV.}$$

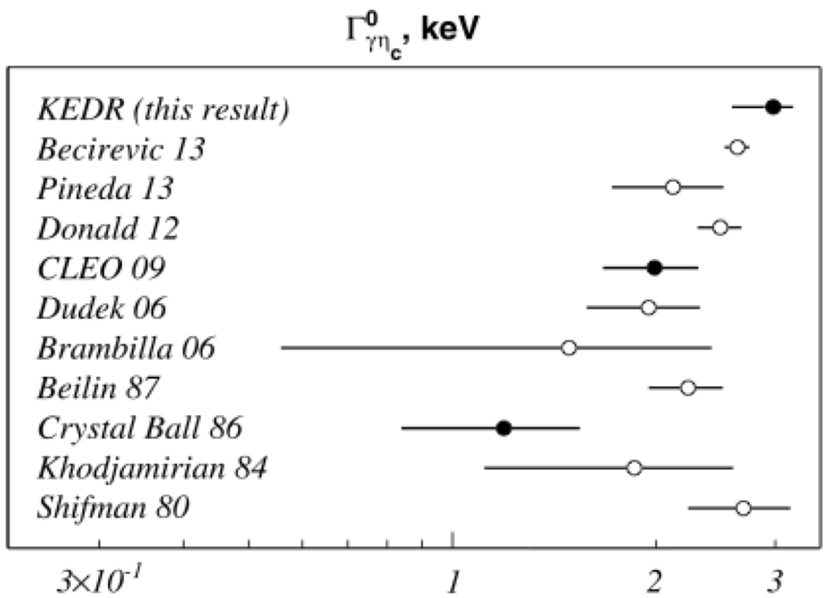

Fig. 3. Results of measurements (close circles) and theoretical predictions (open circles) on $\Gamma^0_{\gamma\eta_c}$.

- Note that similar results are reported in several lattice calculations.
  J. J. Dudek, *et al*., (2006, 2009)
  Y. Chen *et al*., (2011)
  D. Becirevic and F. Sanfilippo, (2012)

Comment on 2^3S_1 to 1^1S_0 transition

- Here we make a brief comment about 2^3S_1 to 1^1S_0 transition. It is basically same M1 transition process. In this process, we expect the overlap integral to be quite small since the 1S and 2S wavefunctions are orthogonal in dipole approximation. However, it changes significantly with the variation of SHO parameters, hence the numerical result is not stable. Therefore, we show only in the case that we choose moderate beta values.

Calculated partial width for $\psi(2S) \to \eta_c(1S)\gamma$ (in keV).

Initial	Final	Non-rel. → Rel.	Rate of change	Experiment (PDG2019)	BGS2005[1)]	DLGZ2017[2)]
$\psi(2S)$	$\eta_c(1S)$	$\mathbf{1.60 \to 3.11}$	**+95 %**	$\mathbf{1.00 \pm 0.15}$	4.6 (NR) 9.7 (NR(j_0)) 9.6 (GI)	8.08 (LP) 7.80 (SP)

($\beta_I = \beta_F = 0.54\text{GeV}$)

Here we have supposed $\psi(3686) \equiv \psi(2S)$.

- The numerical value itself is highly parameter-dependent and will not be discussed here. Note that the result including the recoil factor (OI) increase. This trend is the same as BGS2005.

1^1P_1 to 1^1S_0 transition

- The spin-current does not contribute to this transition since it does not cause spin-flip, so that it is a typical E1 transition.

Calculated partial width for $h_c(1P) \to \eta_c(1S)$ (in keV).

Initial	Final	Non-rel. → Rel.	Rate of change	Experiment (PDG2019)	BGS2005[1)]	DLGZ2017[2)]
$h_c(1P)$	$\eta_c(1S)$	$\mathbf{445 \to 330}$	$\mathbf{-26}\ \%$	$\mathbf{357 \pm 208}$	498 (NR) 352 (GI)	361 (LP) 373 (SP)

breakdown

- Spin WF booting (amplitude): $-\mathbf{6.4}\%$
- Space-time OI (amplitude): $-\mathbf{7.0}\ \%$
- Phase Space (width): $-\mathbf{15}\%$

1) T. Barnes, S. Godfrey, E. S. Swanson, PRD72(2005)
2) W. Deng, H. Liu, L. Gui, X. Zhong, PRD95(2017)

1^3P_J to 1^3S_1 (J=0,1,2) transitions

- In these processes, in addition to the dominant E1-contribution, the contribution of M2 is also possible in the sense of multiple music expansion. In the quark model, the M2 is suppressed relative to E1 by one power of V/c.
- We found that the spin-current term cause destructive / constructive interference with the convection-current term for the chi_{c2} / $\mathrm{chi}_{c1,0}$ transitions, respectively. Effects of spin-current to widths are about 10 ~ 20 %.

Calculated partial width for $\chi_{cJ}(1S) \to J/\psi(1S)\gamma$ (in keV).

Initial	Final	Non-rel. → Rel.	Rate of change	Experiment (BES2017 [0], PDG2019)	BGS2005 [1] (E1)	DLGZ2017 [2] (E+M)
$\chi_{c2}(1S)$	$J/\psi(1S)$	$\mathbf{373 \to 252}$	$\mathbf{-32\ \%}$	$\mathbf{363 \pm 41}$	424(NR) 313 (GI)	284 (LP) 292 (SP)
$\chi_{c1}(1S)$		$\mathbf{274 \to 254}$	$\mathbf{-7\ \%}$	$\mathbf{306 \pm 23}$	314 (NR) 239 (GI)	306 (LP) 319 (SP)
$\chi_{c0}(1S)$		$\mathbf{113 \to 132}$	$\mathbf{+16\ \%}$	$\mathbf{151 \pm 10}$	152 (NR) 114 (GI)	172 (LP) 179 (SP)

[0] M. Ablikim *et al*. (BESIII Collaboration), PRD96(2017)
[1] T. Barnes, S. Godfrey, E. S. Swanson, PRD72(2005)
[2] W. Deng, H. Liu, L. Gui, X. Zhong, PRD95(2017)

2^3S_1 to 1^3P_J (J=0,1,2) transitions

- These process are basically the same as the previous process. In the single photon emission by respective quark, only E1 + M2 is possible
 (If there is a state-mixing with the 1D-state in the initial 2S-state, E3 is also possible.)
- In this case the spin-current term cause constructive / destructive interference with the convection-current term for the chi_{c2} / $\mathrm{chi}_{c1,0}$ transitions, respectively, so that their width increase / decrease. Effects of spin-current to widths are about 3 ~ 15 %.

Calculated partial width for $\psi(2S) \to \chi_{cJ}(1S)\gamma$ (in keV).

Initial	Final	Non-rel. → Rel.	Rate of change	Experiment (BESIII2017 [0])	BGS2005 [1] (E1)	DLGZ2017 [2] (E+M)
$\psi(2S)$	$\chi_{c2}(1S)$	$\mathbf{24.3 \to 24.9}$	$\mathbf{+2.5\ \%}$	$\mathbf{27.5 \pm 1.7}$	38 (NR) 24 (GI)	38 (LP) 46 (SP)
	$\chi_{c1}(1S)$	$\mathbf{34.2 \to 31.8}$	$\mathbf{-7\ \%}$	$\mathbf{28.3 \pm 1.9}$	54 (NR) 29 (GI)	42 (LP) 45 (SP)
	$\chi_{c0}(1S)$	$\mathbf{40.7 \to 30.3}$	$\mathbf{-25\ \%}$	$\mathbf{26.9 \pm 1.8}$	63 (NR) 26 (GI)	22 (LP) 22 (SP)

[0] M. Ablikim *et al*. (BESIII Collaboration), PRD96(2017)
[1] T. Barnes, S. Godfrey, E. S. Swanson, PRD72(2005)
[2] W. Deng, H. Liu, L. Gui, X. Zhong, PRD95(2017)

Multipole amplitudes for 2S to 1P / 1P to 1S

- Experimentally, it has been found that the contribution of M2 to these processes is evidently not zero. The ratios of M2 contributions of chi_c1 to chi_c2 are independent of the quark mass and the anomalous magnetic moment at leading order in q/m.
  In latest experiment [1], they are determined to be

$$b_2^1/b_2^2 = 1.35 \pm 0.72, \qquad \text{(2P to 1S)}$$
$$a_2^1/a_2^2 = 0.617 \pm 0.083. \qquad \text{(1P to 1S)}$$

The corresponding results in our calculation are

$$b_2^1/b_2^2 = 1.05 \qquad \text{(2P to 1S)}$$
$$a_2^1/a_2^2 = 0.475 \qquad \text{(1P to 1S)}$$

the agreement is not bad, implicating that the order of our relativistic effect is appropriate.

[1] M. Ablikim *et al*. (BESIII Collaboration), PRD95(2017)

4. Summary

Summary

Summary and remarks

- Stimulated by the recent precise measurements for the charmonium, we study their radiative transitions in a covariant quark-model (based on bLS scheme).
- Results show, in most cases, the over/under-estimated widths of the non-relativistic quark model can be reduced to the desired width by relativistic effects.
- On the other hand, there are many reports[1] that experimental values are reproduced well as a result of incorporating the effects of couple channels (unquenched quark model). It is interesting that both contributions (relativistic correction and the coupled channel effect) play an effective role to reproduce the experiment.

Future subjects

- In Ref.[2], the radiative transitions of the psi(3770) ($\sim 1^3D_1$) has already been reported. Calculating this process by our model is an urgent task for us.

[1] e.g. E. Eichten, K. Lane, C. Quigg, PRD69 (2004),
G. Li, Q. Zhao, PLB670(2008).

[2] M. Ablikim et al. (BES III collaboration), PLB753 (2016),
M. Ablikim et al. (BES III collaboration), PRD91 (2015).