A possible prescription for incorporating the Nambu-Goldstone pions within the quark model

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INTRODUCTION (1)

The quark model has been applied with considerable success to mass spectra, strong and electromagnetic decays of hadrons. In these approaches only the degrees of freedom of the valence quark are retained, thus the pions are simply treated as interacting quark-antiquark pairs, which are identical to the $\rho(770)$ mesons, except for the internal spin structure of the quarks.

INTRODUCTION (2)

Meanwhile, from the point of view of QCD the pions are pseudo-Nambu-Goldstone (NG) bosons associated with the spontaneous breaking of chiral symmetry. The NG boson nature of the pions is not taken into consideration in the quark model at all.

INTRODUCTION (3)

In this work we present a possible prescription for incorporating the NG boson nature into the pions within the quark model by extending the Pauli spinors of quarks to the Dirac spinors, where there are two independent covariant spin wave functions of the pseudscalar, γ^5 and $\gamma^5(\gamma{\cdot}v)$, which belong to distinct irreducible representations $(1/2, 1/2)$ and $(0, 1) + (1, 0)$, respectively, of the chiral symmetry group $SU(2)_L \times SU(2)_R$.

INTRODUCTION (4)

Here we will take the chiral representation (1/2, 1/2) for the pion spin wave function as an ideal case. In order to examine the validity of this chiral spin wave function of the pion, it will be applied to the magnetic moment and $\pi\gamma$ decays of the $\rho(770)$ meson.

THE COVARIANT OSCILLATOR QUARK MODEL (1)

- Basic framework of the COQM

In the COQM quark-antiquark meson systems are described by the bilocal field

$$\Psi(x_1, x_2)_\alpha{}^\beta = \Psi(X, x)_\alpha{}^\beta$$

where x_1^μ (x_2^μ) is the space-time coordinate, α (β) the Dirac spinor index of the constituent quark (antiquark), and the center-of-mass and relative coordinates are defined, respectively, by

$$X^\mu = \frac{m_1 x_1^\mu + m_2 x_2^\mu}{m_1 + m_2}, \ \ x^\mu = x_1^\mu - x_2^\mu$$

THE COVARIANT OSCILLATOR QUARK MODEL (2)

with the constituent quark (antiquark) mass m_1 (m_2).

The bilocal meson field is required to satisfy the Klein-Gordon-type equation

$$\left(-\frac{\partial^2}{\partial X_\mu \partial X^\mu} - \mathcal{M}^2(x)\right)\Psi(X,x)_\alpha{}^\beta = 0$$

with the squared-mass operator, in the pure confining force limit,

$$\mathcal{M}^2(x) = 2(m_1+m_2)\left(\frac{1}{2\mu}\frac{\partial^2}{\partial x_\mu x^\mu} + U(x)\right),\ U(x) = -\frac{1}{2}Kx_\mu x^\mu + \text{const.}$$

THE COVARIANT OSCILLATOR QUARK MODEL (3)

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass and K is the spring constant.

A solution of the above Klein-Gordon-type equation can be written as

$$\Psi(X, x)_{\alpha}^{\pm\beta} = N e^{\mp i P_\mu X^\mu} \Phi(v, x)_{\alpha}^{\pm\beta}, \quad v^\mu = P^\mu/M$$

where N is the normalization constant for the plane wave, P^μ and M are the center-of-mass momentum and mass, respectively, of the whole meson system, and $\Phi(v, x)_{\alpha}^{\pm\beta}$ is the covariant internal wave function which is

THE COVARIANT OSCILLATOR QUARK MODEL (4)

given by the direct product of eigen-functions of the squared-mass operator and the covariant spinor wave functions, defined by the direct tensor product of respective Dirac spinors, with the meson four velocity v^{μ}, for the constituent quark and antiquark.

THE COVARIANT OSCILLATOR QUARK MODEL (5)

- Covariant spin wave functions of quark-antiquark meson systems

The covariant spin wave functions of quark-antiquark meson systems are expressed in terms of the individual Dirac spinor functions with the meson four velocity v^μ, as

$$W_{\alpha}^{+\,\beta}(v) \sim u_\alpha(v)\bar{v}^\beta(v), \quad W_{\alpha}^{-\,\beta}(v) \sim v_\alpha(v)\bar{u}^\beta(v)$$

where $u(v)$ $(v(v))$ is the free Dirac spinor with the positive (negative) energy. The wave function $W(v)$ can be covariantly decomposed on the basis of the Dirac matrices,

THE COVARIANT OSCILLATOR QUARK MODEL (6)

$(\mathbf{1}, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma^5, \gamma^5)$. The definite J^{PC} states with $0^{-+}, 1^{--}$ in the nonrelativistic quark model correspond to the covariant wave functions

$$\frac{1}{2\sqrt{2}}\gamma^5(1-\gamma\cdot v)\,, \qquad \frac{1}{2\sqrt{2}}(-\gamma^\mu)(1-\gamma\cdot v)$$

respectively.

Here in order to incorporate the NG boson nature of the pion, in addition to the above pseudoscalar wave function, we introduce

THE COVARIANT OSCILLATOR QUARK MODEL (7)

another spin wave function which is given by

$$-\frac{1}{2\sqrt{2}}\gamma^5(1+\gamma\cdot v)$$

being orthogonal to the above pseudoscalar wave function. These two orthogonal states can be represented in terms of another basis,

$$\frac{\gamma^5}{2}\ , \quad -\frac{\gamma^5}{2}\gamma\cdot v$$

THE COVARIANT OSCILLATOR QUARK MODEL (8)

which correspond to the irreducible representations $(1/2, 1/2)$ and $(0, 1) + (1, 0)$ of the chiral symmetry group $SU(2)_L \times SU(2)_R$.

Hereafter we ideally consider $\gamma^5/2$ for the spin wave function of the pion as a reflection of the NG boson nature. More generally it is a mixture of the two chiral representations.

THE COVARIANT OSCILLATOR QUARK MODEL (9)

- Key features of the COQM

In order to freeze the redundant freedom of relative time for the four-dimensional harmonic oscillator, which gives here the squared-mass operator, the definite-metric-type subsidiary condition is adopted. The space-time wave functions satisfying this condition is normalizable and leads to the desirable asymptotic behavior of electromagnetic form factors of hadrons.

THE COVARIANT OSCILLATOR QUARK MODEL (10)

The eigenvalues of the squared-mass operator are given by

$$M_N^2 = M_0^2 + N\Omega, \quad \Omega = 2(m_1 + m_2)\sqrt{\frac{K}{\mu}}$$

where $N = 2N_r + L$, N_r (L) being the radial (orbital angular momentum) quantum number. This squared-mass formula gives linear Regge trajectories with the slope Ω^{-1}, in accord with the well-known experimental fact.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (1)

- Lagrangians for the free bilocal meson fields

The Klein-Gordon-type equation mentioned above is rewritten in terms of the quark and antiquark coordinates as

$$2(m_1+m_2)\left(\sum_{i=1}^{2}\frac{-1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}\partial x_i^{\mu}}-U(x_1,x_2)\right)\Psi(x_1,x_2)_{\alpha}{}^{\beta}=0$$

or, equivalently,

$$\left(\sum_{i=1}^{2}\frac{-1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}\partial x_i^{\mu}}-U(x_1,x_2)\right)\Psi(x_1,x_2)_{\alpha}{}^{\beta}=0$$

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (2)

This equation is derived from either of the actions

$$S_{\text{free}}^{(\text{KG,S})} = \int d^4x_1 \int d^4x_2 \; \mathcal{L}_{\text{free}}^{(\text{KG,S})}(\Psi, \partial_{1\mu}\Psi, \partial_{2\mu}\Psi)$$

with the Klein-Gordon-like Lagrangian

$$\mathcal{L}_{\text{free}}^{(\text{KG})} = \overline{\Psi}(x_1, x_2) 2(m_1 + m_2) \left(\sum_{i=1}^{2} \frac{-1}{2m_i} \frac{\overleftarrow{\partial}}{\partial x_{i\mu}} \frac{\overrightarrow{\partial}}{\partial x_i^{\mu}} - U(x_1, x_2) \right) \Psi(x_1, x_2)$$

and the Schrödinger-like Lagrangian

$$\mathcal{L}_{\text{free}}^{(\text{S})} = \overline{\Psi}(x_1, x_2) \left(\sum_{i=1}^{2} \frac{-1}{2m_i} \frac{\overleftarrow{\partial}}{\partial x_{i\mu}} \frac{\overrightarrow{\partial}}{\partial x_i^{\mu}} - U(x_1, x_2) \right) \Psi(x_1, x_2)$$

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (3)

where the bilocal field $\Psi(x_1, x_2)$ has the dimension of bosons [M^1] and fermions [$M^{3/2}$] for $\mathcal{L}_{\rm free}^{\rm (KG)}$ and $\mathcal{L}_{\rm free}^{\rm (S)}$, respectively, except for the dimension of internal wave functions [M^2].

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (4)

In the previous work we have found that the dimension of bilocal meson fields is bosonic for light-quark systems, while fermionic for heavy-light and heavy-heavy systems. Since the present work is concerned with light-quark systems, here we take the Klein-Gordon-like Lagrangian $\mathcal{L}^{(\mathrm{KG})}_{\mathrm{free}}$.

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (5)

- Electromagnetic currents of quark-antiquark meson systems

The interaction of quark-antiquark meson systems with an electromagnetic field can be obtained by the minimal substitutions

$$\frac{\partial}{\partial x_i^\mu} \to \frac{\partial}{\partial x_i^\mu} + ieQ_i A_\mu(x_i)$$

in the free Lagrangian $\mathcal{L}_{\rm free}^{\rm (KG)}$, in which the heuristic prescription by Feynman, Kislinger, and Ravndal (1971) is adopted as

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (6)

the following replacements

$$\frac{\overleftarrow{\partial}}{\partial x_{i\mu}}\frac{\overrightarrow{\partial}}{\partial x_i^{\mu}} \to \frac{\overleftarrow{\partial}}{\partial x_i^{\mu}}\gamma^{\mu}\gamma^{\nu}\frac{\overrightarrow{\partial}}{\partial x_i^{\nu}}$$

Here Q_i $(i=1,2)$ are the quark and antiquark charges in units of e. Then the action for the electromagnetic interaction of meson systems is obtained, up to the first order of e, as

$$S_{\rm int}^{\rm (KG)} = \int d^4x_1 \int d^4x_2 \sum_{i=1}^{2} j_i^{({\rm KG})\mu}(x_1,x_2)A_\mu(x_i) \equiv \int d^4X J^{({\rm KG})\mu}(X)A_\mu(X)$$

with the conserved currents

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (7)

$$j_i^{(\mathrm{KG})\mu}(x_1,x_2) = 2(m_1+m_2)\left\langle \overline{\Psi}(x_1,x_2)\frac{-ieQ_i}{2m_i}\left(\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}} - ig_M^{(i)}\sigma^{\mu\nu}\left(\frac{\overrightarrow{\partial}}{\partial x_i^\nu}+\frac{\overleftarrow{\partial}}{\partial x_i^\nu}\right)\right)\Psi(x_1,x_2)\right\rangle$$

where $\langle\cdots\rangle$ means taking trace concerning the Dirac indices and $g_M^{(i)}$ are the parameters related to the anomalous magnetic moments of constituent quarks. The electric charges of meson systems are given by the diagonal elements

$$Q_{\mathrm{meson}}^{(\mathrm{KG})} = \int d^3X\langle i|J^{(\mathrm{KG})0}(X)|i\rangle$$

ELECTROMAGNETIC CURRENTS FOR QUARK-ANTIQUARK MESON SYSTEMS (8)

- Features of the Klein-Gordon-like current
  - The meson charge is given by

$$Q^{(\mathrm{KG})}_{\mathrm{meson}} = (Q_1 + Q_2)e$$

    which reproduces the physical one correctly.
  - The currents $j_i^{(\mathrm{KG})\mu}(x_1, x_2)$ do not have the absolute mass scales of quarks. This would seem to be natural for light-quark systems from the viewpoint of QCD in the chiral limit.

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (1)

- Magnetic moment of the ρ(770) meson

  The above effective electromagnetic interaction $S_{\rm int}^{(\rm KG)}$ describes systematically all the electromagnetic interactions of quark-antiquark meson systems.

  For the magnetic moment of the ρ(770) meson the following expression is obtained:

$$\mu_\rho = 2g_M\left(\frac{e}{2M_\rho}\right)$$

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (2)

where M_ρ is the $\rho(770)$ meson mass and g_M is the parameter related to the anomalous magnetic moment of u and d quarks, assuming the isospin symmetry $g_M^{(u)} = g_M^{(d)}$. The $g_M = 1$ corresponds to the normal moment of quarks.

Since the $\rho(770)$ meson is unstable, its lifetime being very short, there are not ever direct measurements for its magnetic moment.

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (3)

So, we use the lattice QCD results for the ρ-meson magnetic moment to estimate reasonable values of the parameter g_M. The values of μ_ρ in the lattice QCD calculations are tabulated in the following table:

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (4)

< The ρ-meson magnetic moment in the lattice QCD calculations (in units of $e/2M_\rho$) >

<table>
<tr><th></th><th>μ_ρ</th><th>Weighted avg.</th><th>Simple avg.</th></tr>
<tr><td>1</td><td>2.25 ± 0.34</td><td rowspan="4">2.38 ± 0.01</td><td rowspan="5">2.20</td></tr>
<tr><td>2</td><td>2.39 ± 0.01</td></tr>
<tr><td>3</td><td>2.21 ± 0.08</td></tr>
<tr><td>4</td><td>2.11 ± 0.10</td></tr>
<tr><td>5</td><td>2.02</td><td>-</td></tr>
</table>

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (5)

[1] W. Andersen and W. Wilcox (1997)
[2] F. X. Lee et al. (2008)
[3] B. Owen et al. (2015)
[4] E. V. Lushevskaya et al. (2017)
[5] J. N. Hedditch et al. (2007)

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (6)

Using the weighted and simple averages of the lattice QCD results for the μ_ρ, we obtain

$$g_M = 1.19 \text{ and } 1.10$$

respectively.

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (7)

- Radiative decay of the $\rho(770)$ meson

For the radiative decays of light-quark mesons the decay width is obtained, following the usual procedure with the Klein-Gordon-like interaction $S_{\rm int}^{\rm (KG)}$, as

$$\Gamma = \frac{1}{2J_i+1}\frac{|\mathbf{q}|}{8\pi M_i^2}\sum_{\rm spin}|\mathcal{M}_{fi}|^2$$

where M_i (J_i) are the mass (spin) of the initial-state meson and $|\mathrm{q}|$ is the photon three-momentum.

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (8)

Here we restrict ourselves to a discussion on the radiative decays of isovector mesons with nonstrange quarks. Then the present radiative decay model has two parameters, Ω, the inverse of the Regge slope, and g_M. Here we take the value of $\Omega = 1.14$ GeV^2 with the slope of the leading $\rho(770)$-meson trajectory.

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (9)

For the spin wave functions of the pion we consider two ideal cases,

$$\frac{1}{2\sqrt{2}}\gamma^5(1-\gamma\cdot v) \quad \text{and} \quad \frac{\gamma^5}{2}$$

the nonrelativistic and chiral wave functions, respectively.

For the decay width of $\rho(770)^{\pm} \to \pi^{\pm}\gamma$, we tabulate the numerical results in the following table:

MAGNETIC MOMENT AND RADIATIVE DECAY OF THE ρ MESON (10)

< Numerical results of the decay widths for $\rho(770)^{\pm} \to \pi^{\pm}\gamma$ in keV >

g_M	NR case	Chiral case	Experiment
1.10	115	57	68 ± 7
1.19	134	67	

These numerical results show that the chiral spin wave function would be favored over the nonrelativistic one for the reasonable values of g_M, the $\rho(770)$-meson magnetic moment.

SUMMARY AND FUTURE WORK (1)

- The covariant spin wave function of the pion corresponding to the irreducible representation (1/2, 1/2) of the chiral $SU(2)_L \times SU(2)_R$ is adopted to take into consideration the NG boson nature of the pion in the COQM.
- The calculated results for the radiative decay and magnetic moment of the $\rho(770)$ meson show that they would be both described consistently by the chiral spin wave function of the pion.

SUMMARY AND FUTURE WORK (2)

- It is necessary to examine the validity of the chiral spin wave function of the pion by applying to other radiative decay processes, such as the $\pi\gamma$ decays of excited light-quark mesons.
- It is also intriguing to investigate a mixture of the nonrelativistic and chiral spin wave function for excited-state mesons.