Baryon Properties from Poincaré-Covariant Faddeev Equation

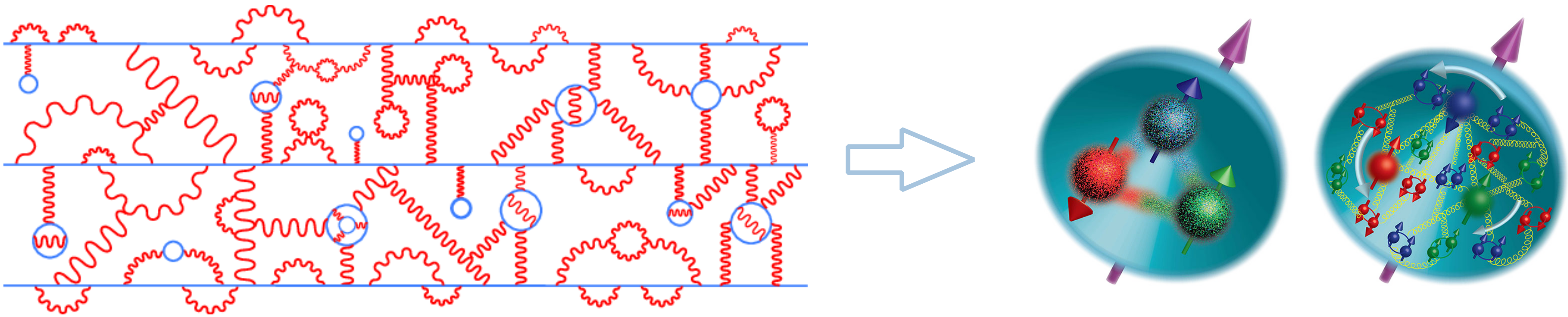
Si-xue Qin

（秦思学）

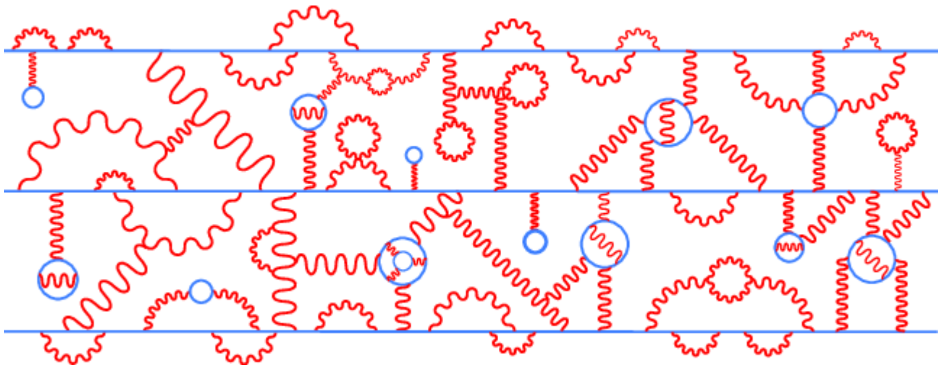
Department of Physics, Chongqing University

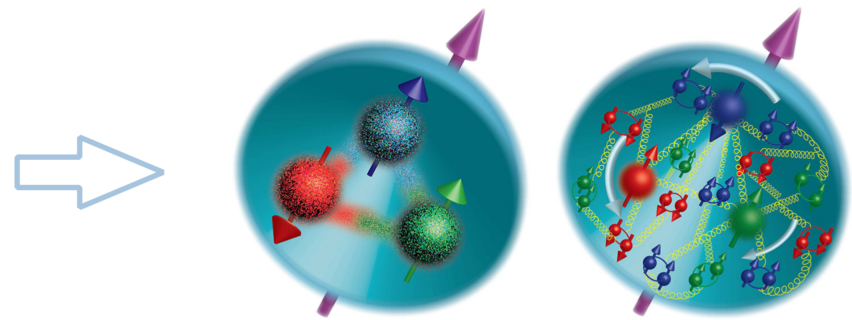
http://cqutp.org/sqin/

Introduction: Baryons of Special Importance



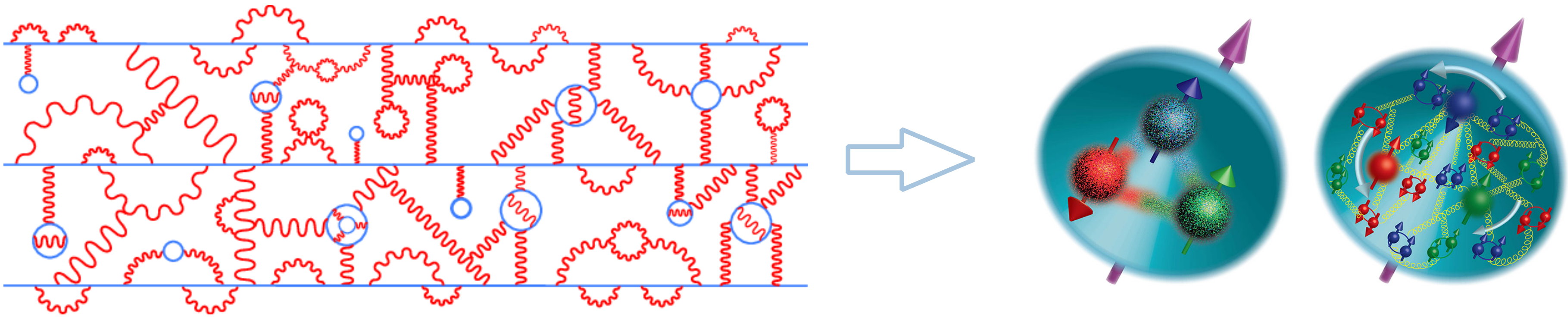
Introduction: Baryons of Special Importance





- ✓ Quarks are **complex** objects which have many intrinsic degrees of freedom, e.g, spins, colors, flavors, and etc, and are **strongly bound** by gluons.
- ✓ Baryons are **three-body** systems of quarks, whose dynamics is a well-known **difficult** problem, even in the classical level.

Introduction: Baryons of Special Importance

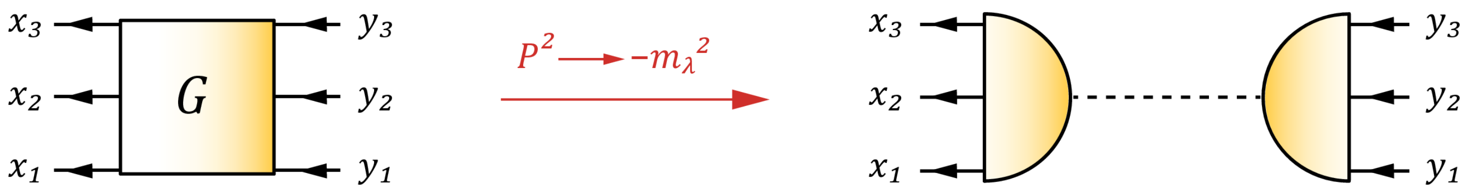


- ✓ Quarks are **complex** objects which have many intrinsic degrees of freedom, e.g, spins, colors, flavors, and etc, and are **strongly bound** by gluons.

- ✓ Baryons are **three-body** systems of quarks, whose dynamics is a well-known **difficult** problem, even in the classical level.

relativistic strongly-coupled three-body MΦNSTER

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



$$G^{(6)}(x_1, x_2, x_3, y_1, y_2, y_3) = \langle \Omega | q(x_1) q(x_2) q(x_3) q(y_1) q(y_2) q(y_3) | \Omega \rangle$$

1.1 DSE: QCD-connected Framework

Classical Mechanics	Quantum Field Theory

Degrees of freedom

Generalized coord.	Fields on spacetime

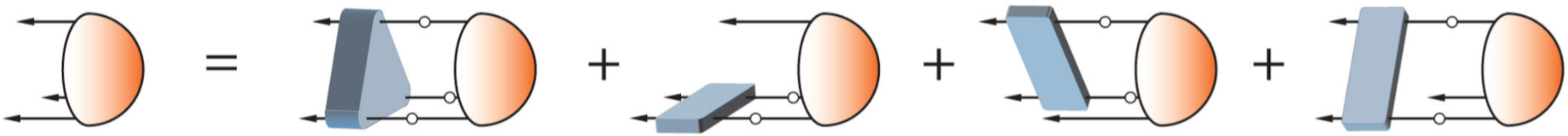
Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0$$	$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

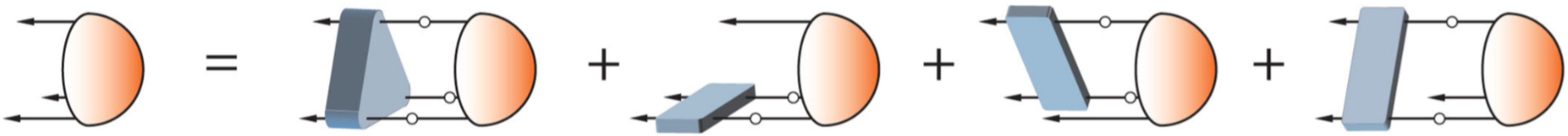
Equations of Motion (EoM)

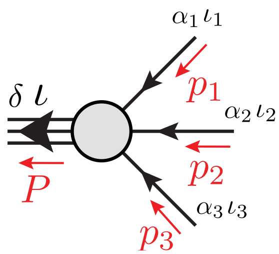
Euler-Lagrange Equation	Dyson-Schwinger Equations

1.2 DSE: Three-Body Faddeev Equation



1.2 DSE: Three-Body Faddeev Equation





$$ {}_{c_1c_2c_3}\boldsymbol{\Psi}^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\boldsymbol{\Psi}^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,, $$

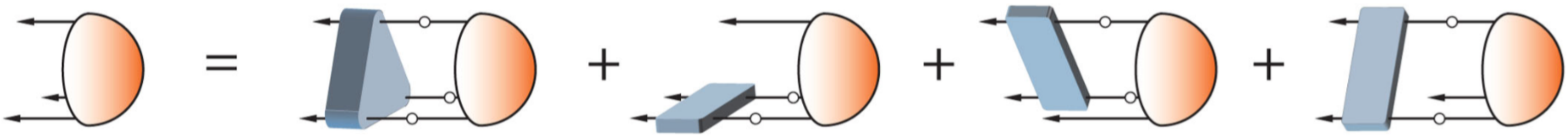
Spins: $4\times4\times4\times4=256$
Flavors: $2\times2\times2\times2=16$

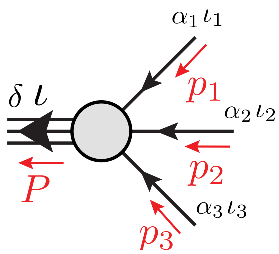
128-terms

Jacobi Coordinates: $P_\mu,\,p_\mu,\,q_\mu$
$P^2=-M_N^2;\,p^2,\,q^2,\,Pq,\,Pp,\,pq$

5-dim

1.2 DSE: Three-Body Faddeev Equation





$$ {}_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,, $$

Spins: $4\times4\times4\times4=256$

Flavors: $2\times2\times2\times2=16$

128-terms

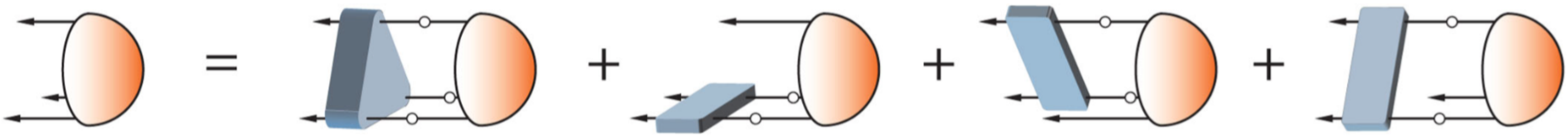
Jacobi Coordinates: P_μ, p_μ, q_μ

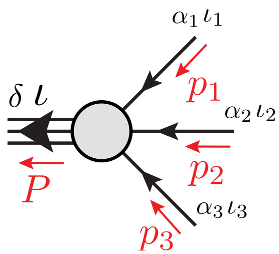
$P^2=-M_N^2; p^2, q^2, Pq, Pp, pq$

5-dim

Highly complex algebra + Cutting edge numerical technique

1.2 DSE: Three-Body Faddeev Equation





$$ {}_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,, $$

Spins: $4\times4\times4\times4=256$
Flavors: $2\times2\times2\times2=16$

128-terms

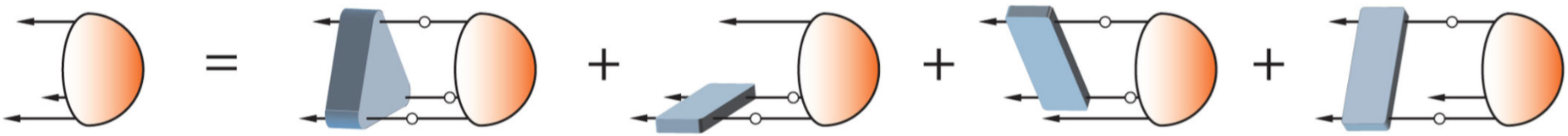
Jacobi Coordinates: P_μ, p_μ, q_μ
$P^2=-M_N^2; p^2, q^2, Pq, Pp, pq$

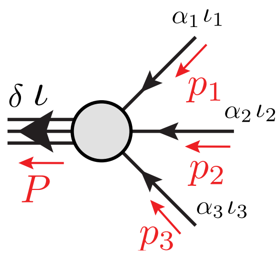
5-dim

Highly complex algebra + Cutting edge numerical technique

Diquark approximation: Reduce three-body problem to two two-body ones.

1.2 DSE: Three-Body Faddeev Equation





$${}_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P) = \frac{1}{\sqrt{6}}\varepsilon_{c_1c_2c_3}\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1,p_2,p_3;P)\,,$$

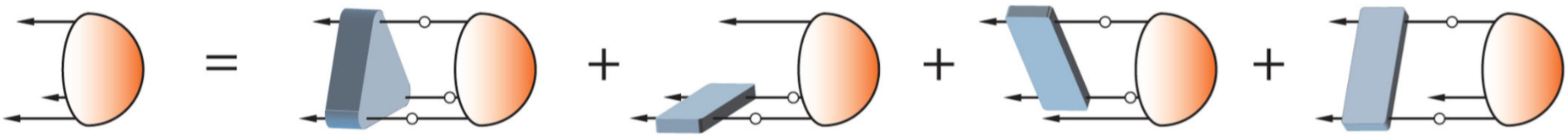
Spins: $4\times4\times4\times4=256$
Flavors: $2\times2\times2\times2=16$
128-terms

Jacobi Coordinates: P_μ, p_μ, q_μ
$P^2=-M_N^2; p^2, q^2, Pq, Pp, pq$
5-dim

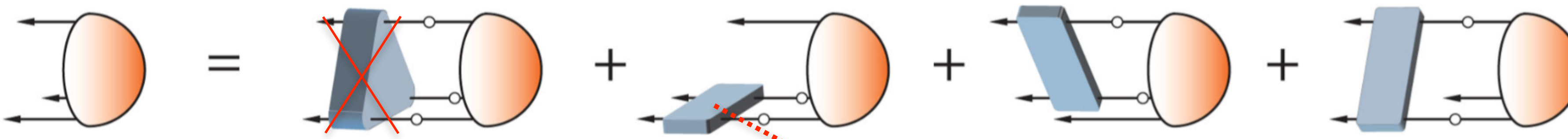
$\boxed{\text{Highly complex algebra}}$ + $\boxed{\text{Cutting edge numerical technique}}$

~~**Diquark**~~ approximation: Reduce three-body problem to two two-body ones.

1.2 DSE: Three-Body Faddeev Equation

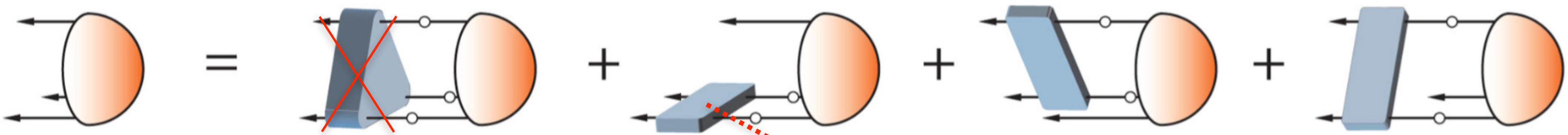


1.2 DSE: Three-Body Faddeev Equation



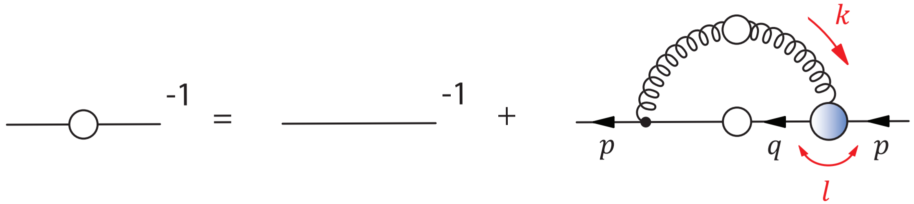
Scattering kernel

1.2 DSE: Three-Body Faddeev Equation

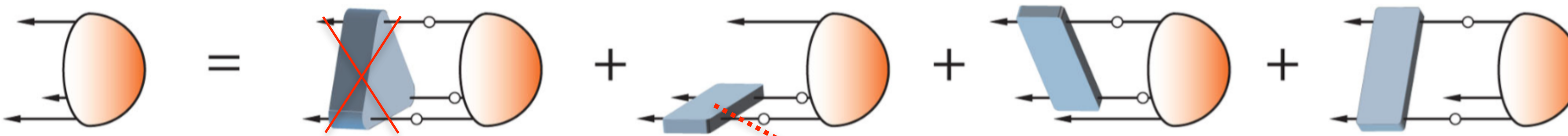


Scattering kernel

One-body gap equation

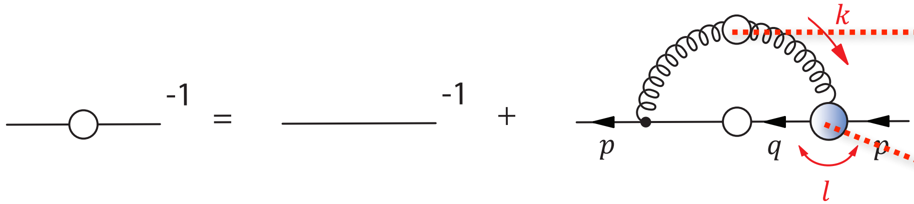


1.2 DSE: Three-Body Faddeev Equation



Scattering kernel

One-body gap equation



Gluon propagator

Quark-gluon vertex

1.3 Approximation: Rainbow-Ladder Truncation

Quark-gluon vertex

Rainbow approximation

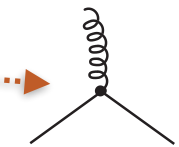
Scattering kernel

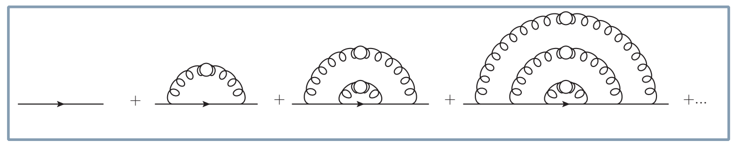
Ladder approximation

1.3 Approximation: Rainbow-Ladder Truncation

Quark-gluon vertex

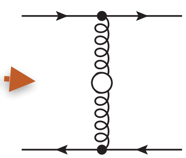
Rainbow approximation

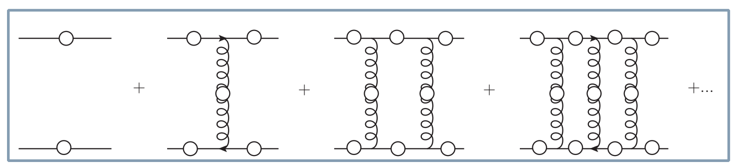




Scattering kernel

Ladder approximation

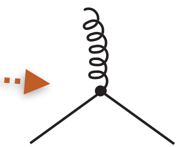


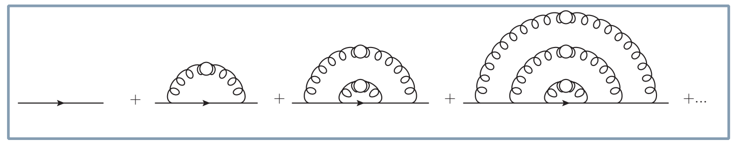


1.3 Approximation: Rainbow-Ladder Truncation

Quark-gluon vertex

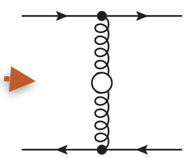
Rainbow approximation

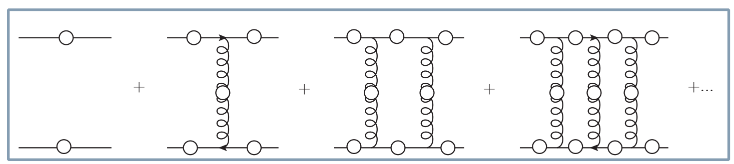


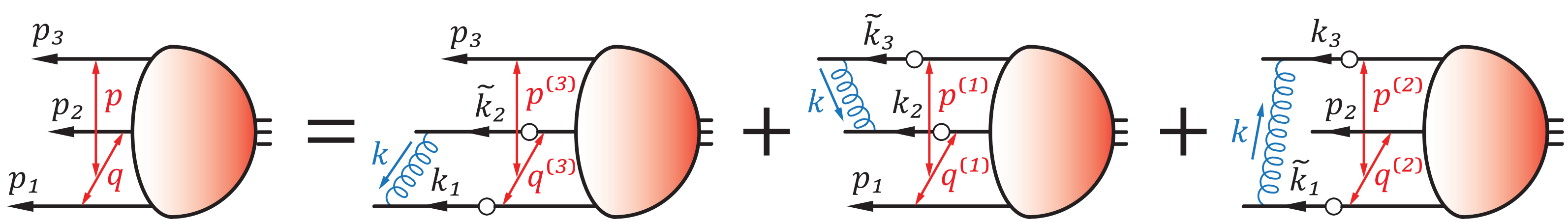


Scattering kernel

Ladder approximation



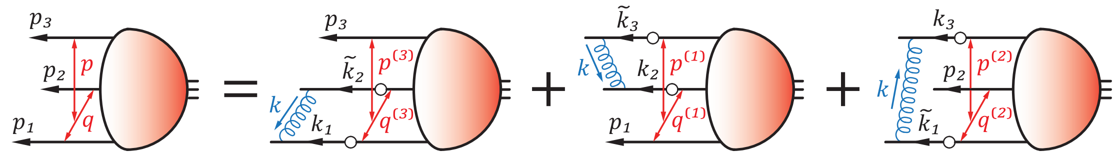




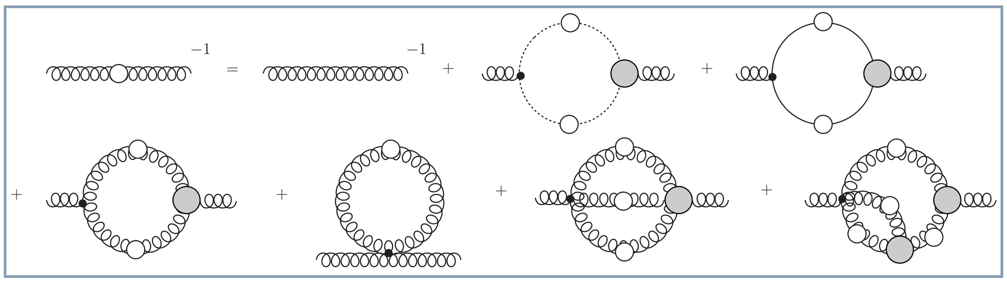
1.3 Approximation: Dynamically Massive Gluon

CHONGQING UNIVERSITY

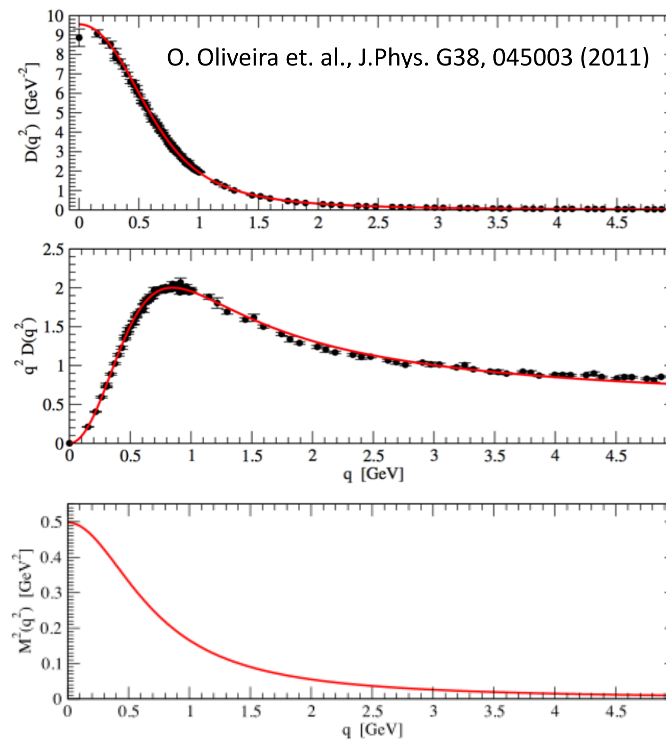
Gluon propagator



Gluon gap equation: Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero

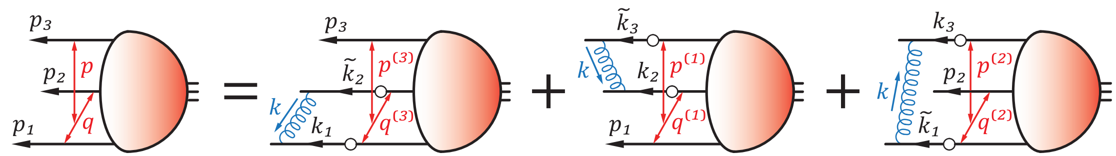


Lattice simulations:

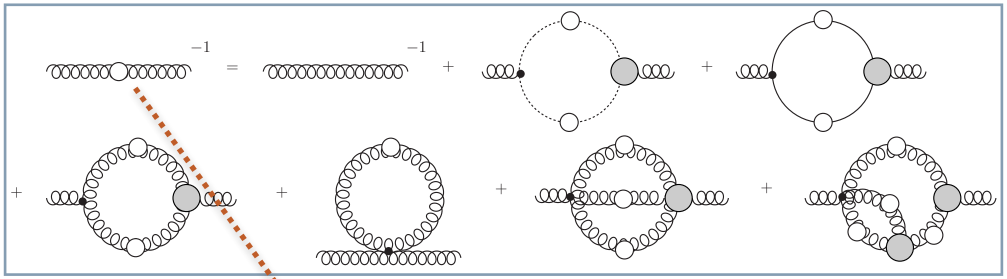


See, e.g., PRC 84. 042202 (2018)

Gluon propagator



Gluon gap equation: Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero

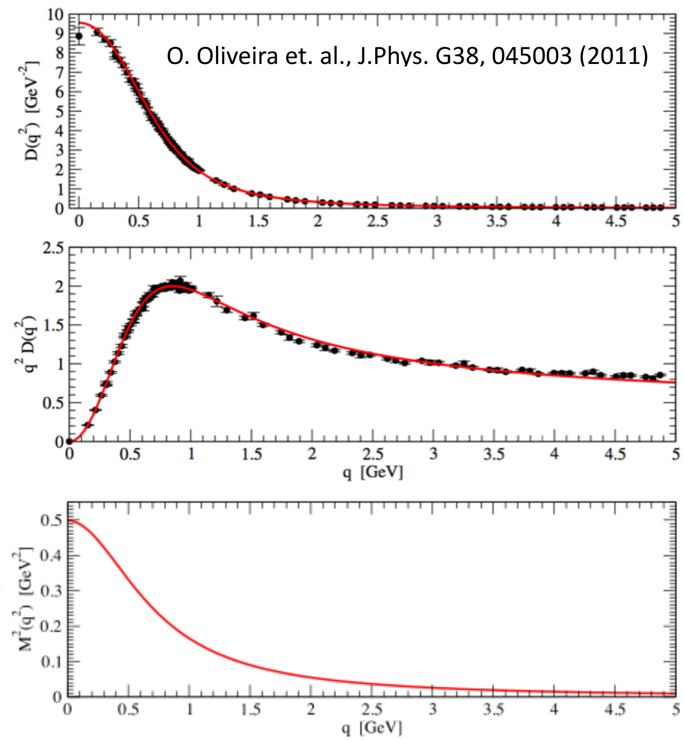


- The gluon dressing function: ***gluon running mass*** + ***effective running coupling constant***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \quad \text{(Landau gauge)}$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}, \qquad m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

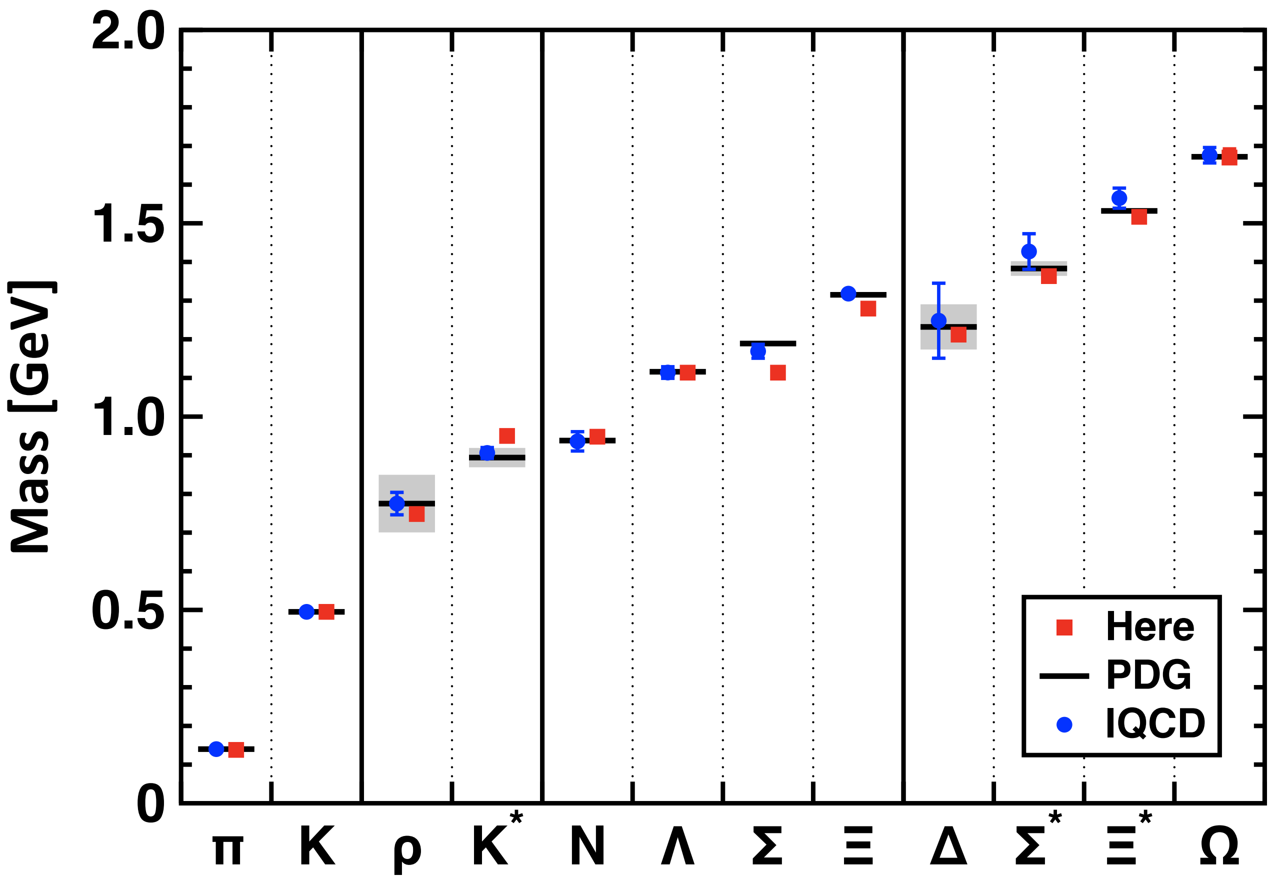
Lattice simulations:



See, e.g., PRC 84. 042202 (2018)

Baryon Spectra

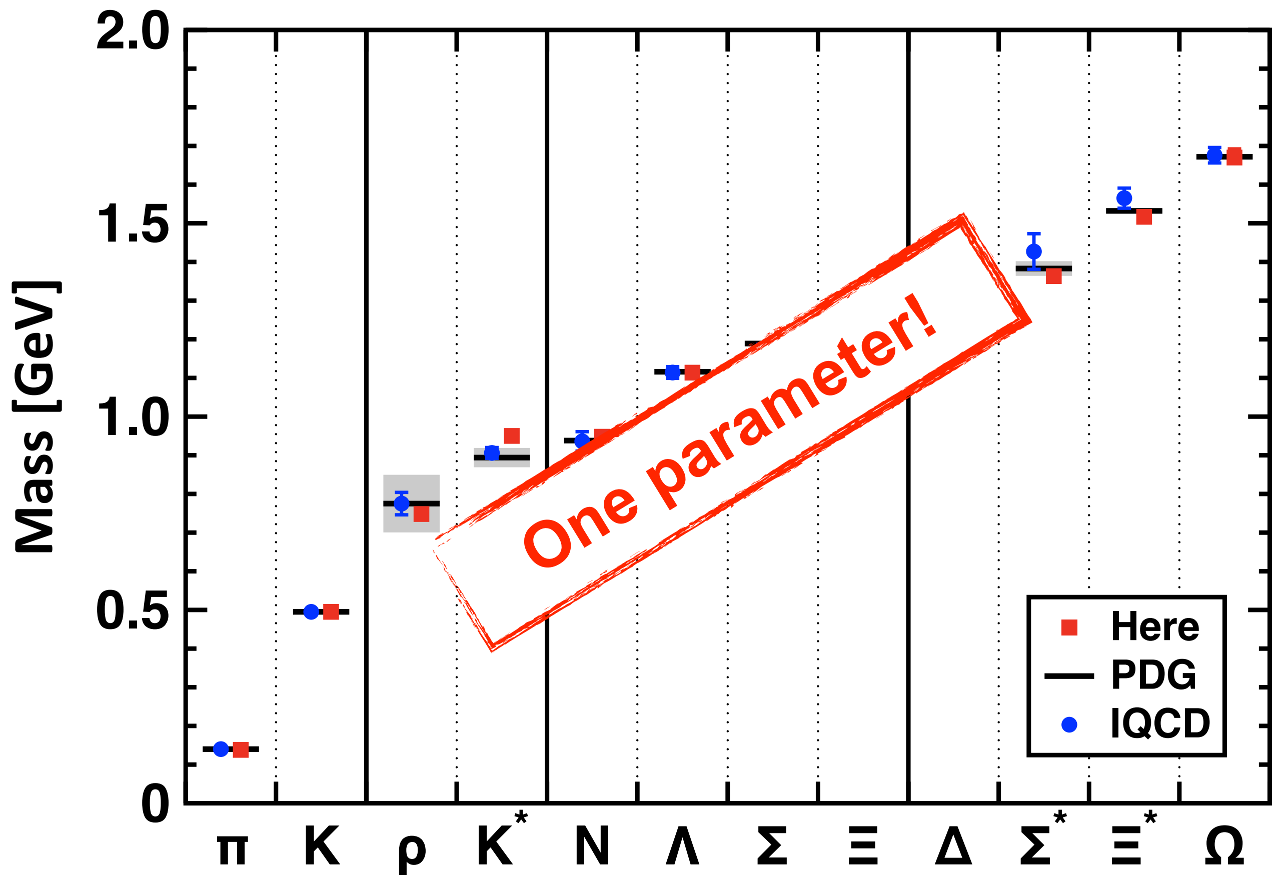
2.1 Spectra: Light & Strange flavors



The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

See, e.g., Few-Body Syst 60, 26 (2019)

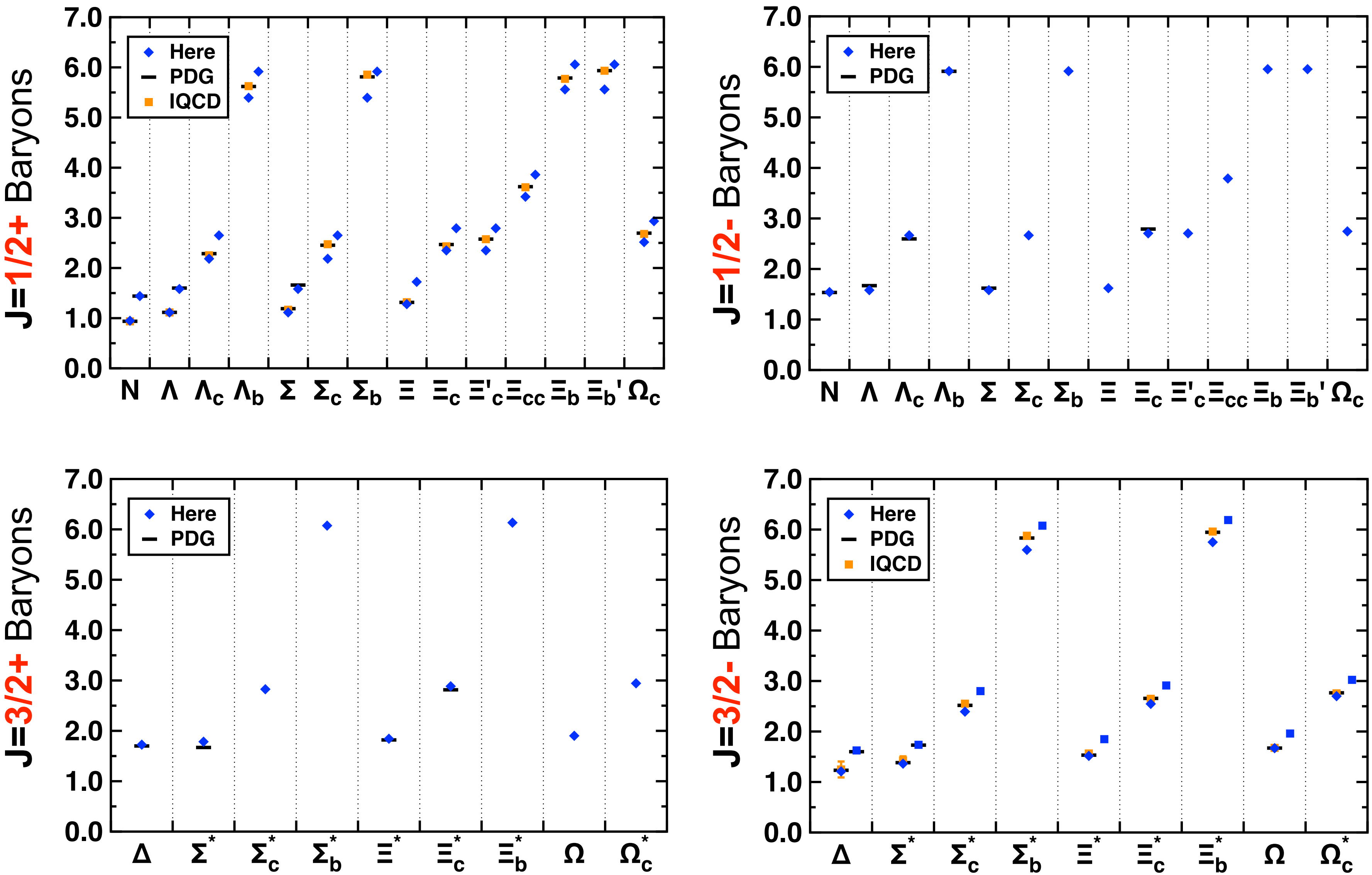
2.1 Spectra: Light & Strange flavors



The interaction strength and current quark masses are fixed by properties of pseudo-scalar mesons, e.g., pion, kaon, and etc.

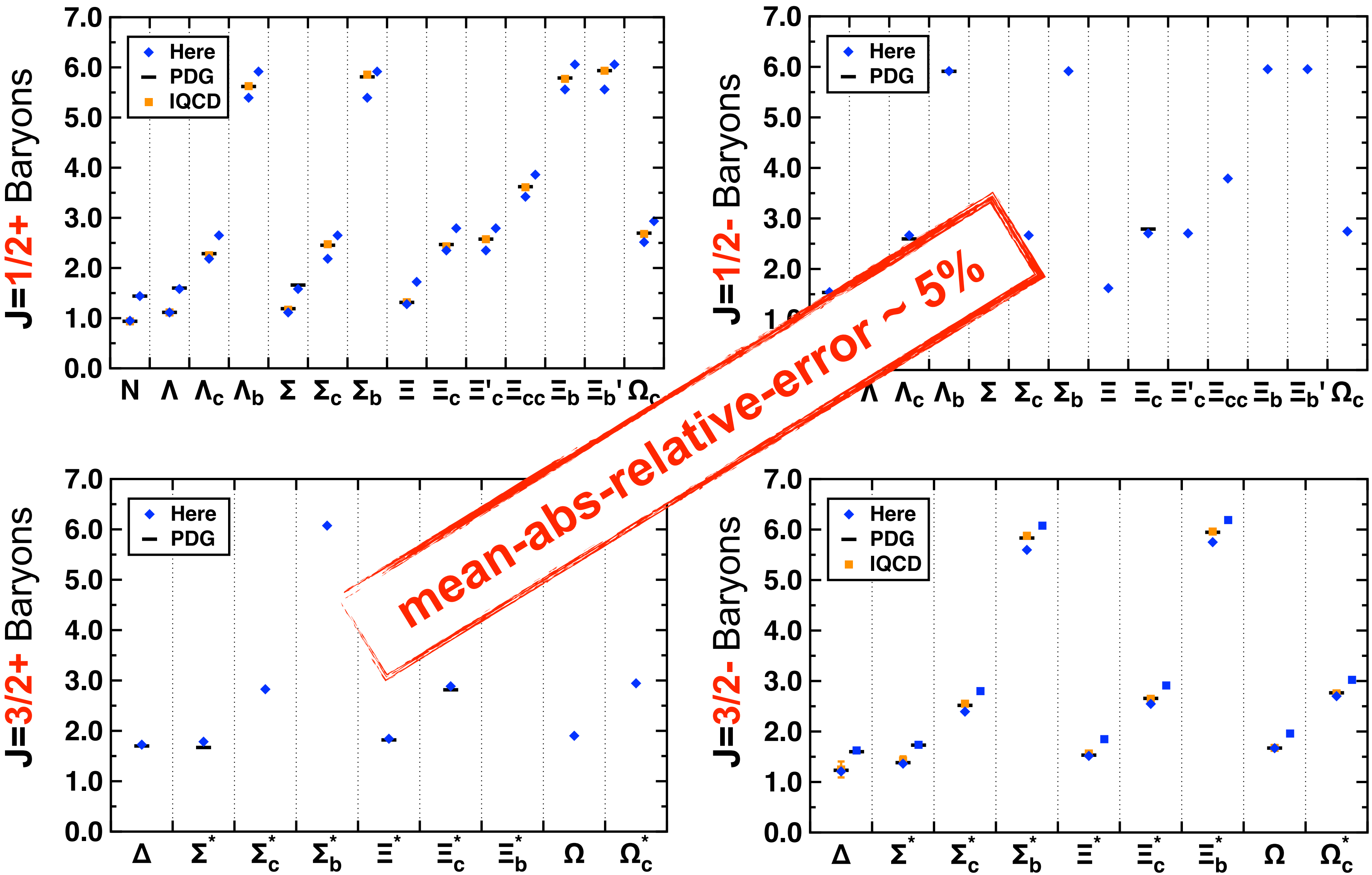
See, e.g., Few-Body Syst 60, 26 (2019)

2.2 Spectra: Charm & Bottom flavors



See, e.g., Few-Body Syst 60, 26 (2019)

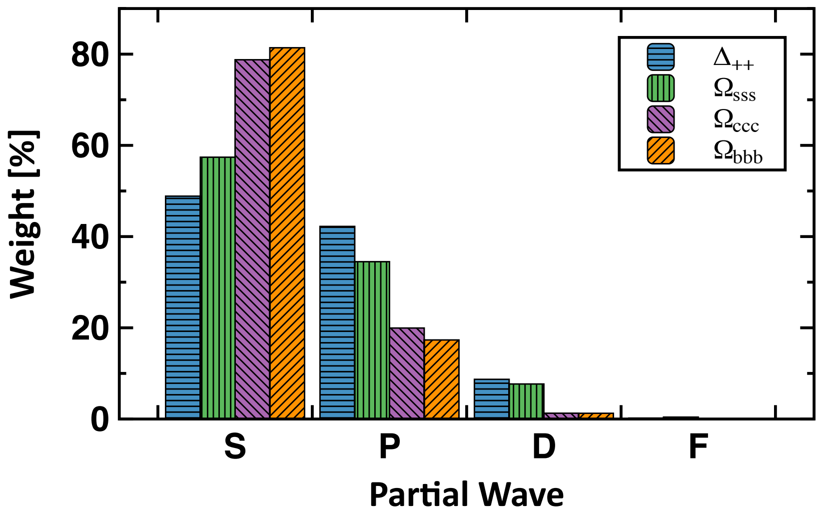
2.2 Spectra: Charm & Bottom flavors



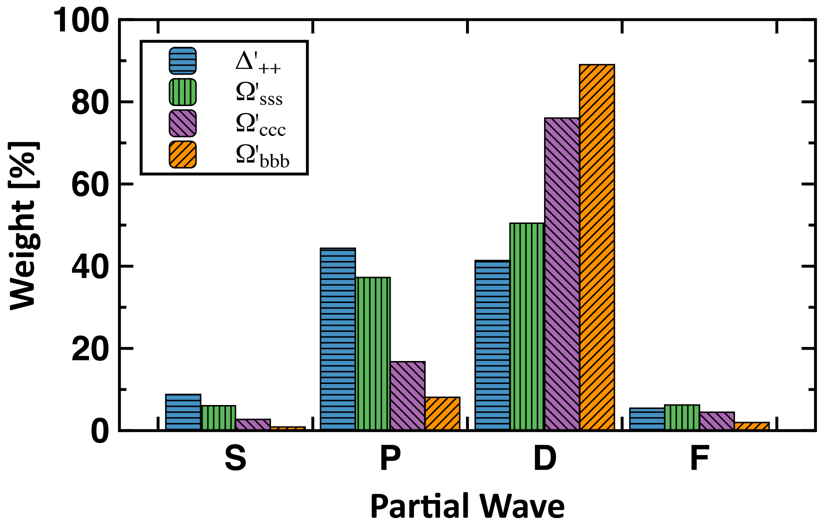
See, e.g., Few-Body Syst 60, 26 (2019)

Baryon Structures

3.1 Structures: Wave Functions



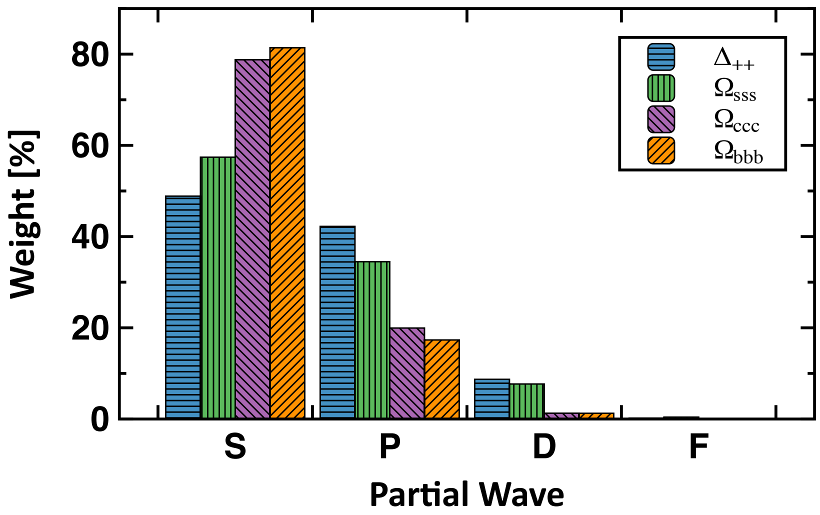
✓ S-waves dominate for ground states, but p-waves grow for light baryons.

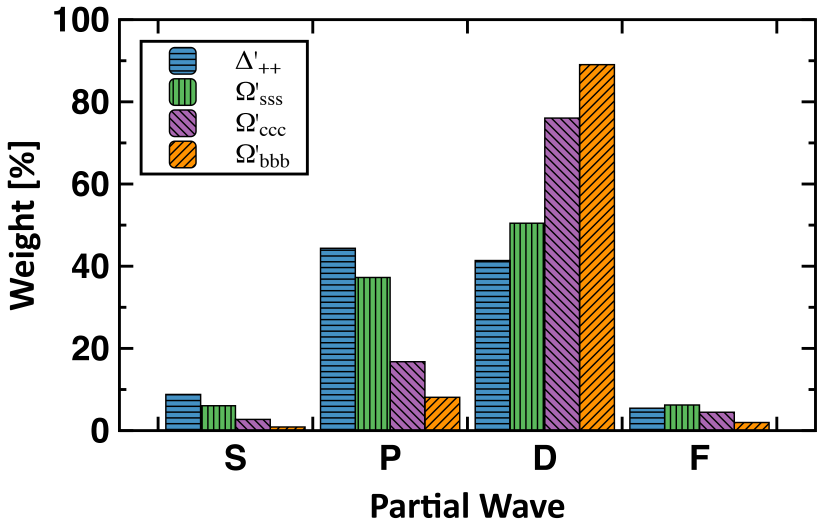


✓ D-waves dominate for excited states, but p-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

3.1 Structures: Wave Functions





✓ S-waves dominate for ground states, but p-waves grow for light baryons.

How quark potential model works

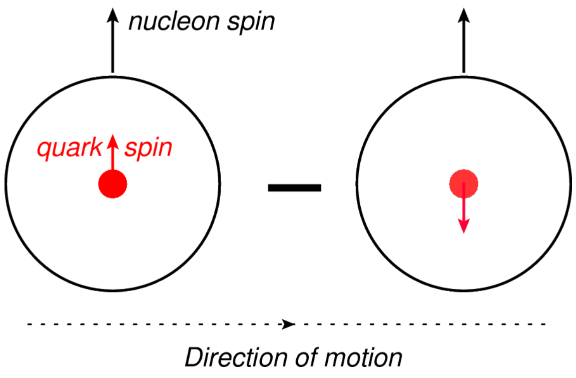
✓ D-waves dominate for excited states, but p-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

3.2 Structures: Tensor Charges

- New generation experiments aim to determine nucleon TMDs, which, in a limit, are connected with the tensor charges:

$$\delta_T q = \int_{-1}^{1} dx\, h_{1T}^{q}(x) = \int_{0}^{1} dx\, \left[h_{1T}^{q}(x) - h_{1T}^{\bar{q}}(x)\right]$$

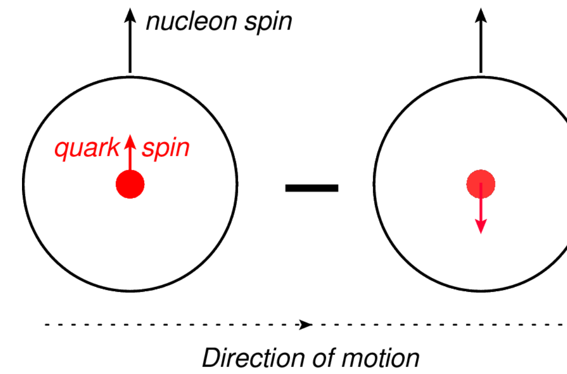


See, e.g., PRD 98, 054019 (2018)

3.2 Structures: Tensor Charges

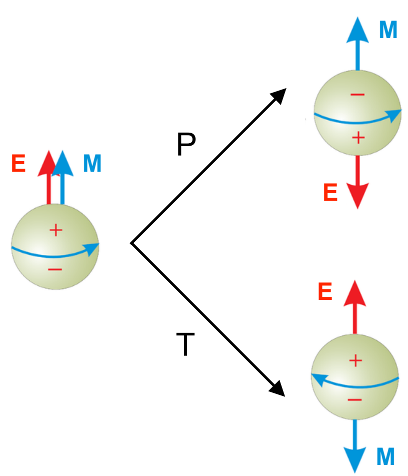
- New generation experiments aim to determine nucleon TMDs, which, in a limit, are connected with the tensor charges:

$$\delta_T q = \int_{-1}^{1} dx\, h_{1T}^{q}(x) = \int_{0}^{1} dx\, \left[h_{1T}^{q}(x) - h_{1T}^{\bar{q}}(x)\right]$$



- The quark-EDM (electric dipole moment) contribution to a nucleon's EDM is determined once its tensor charges are known:

$$\langle P(p)|\mathcal{J}_{\mu\nu}^{\mathrm{EDM}}|P(p)\rangle = \left[\mathsf{E}_u \delta_T u + \mathsf{E}_d \delta_T d\right] \bar{\mathrm{u}}(p)\gamma_5\sigma_{\mu\nu}\mathrm{u}(p) \qquad \langle P(p)|\bar{q}\sigma_{\mu\nu}q|P(p)\rangle = \delta_T q\, \bar{\mathrm{u}}(p)\sigma_{\mu\nu}\mathrm{u}(p)$$



Parity violation

CP violation

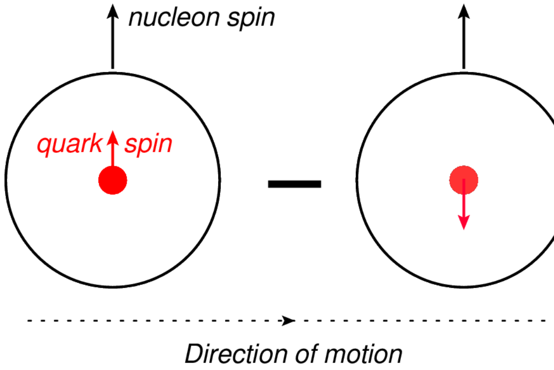
Time reversal violation

See, e.g., PRD 98, 054019 (2018)

3.2 Structures: Tensor Charges

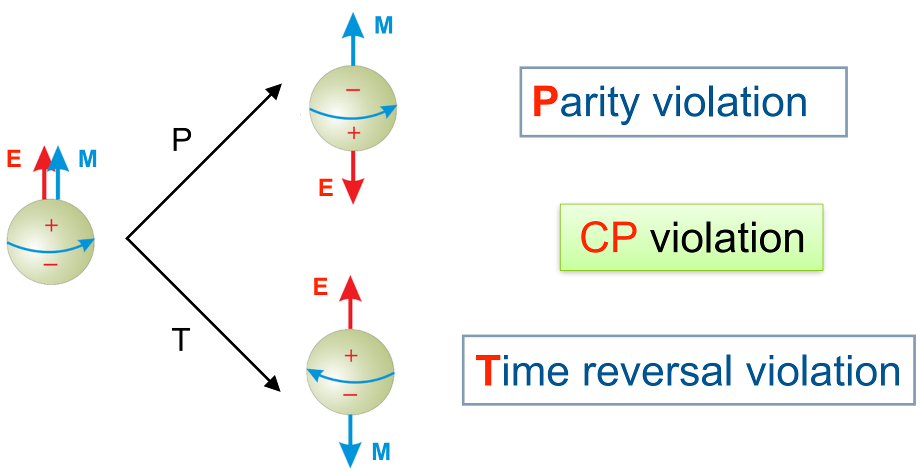
- New generation experiments aim to determine nucleon TMDs, which, in a limit, are connected with the tensor charges:

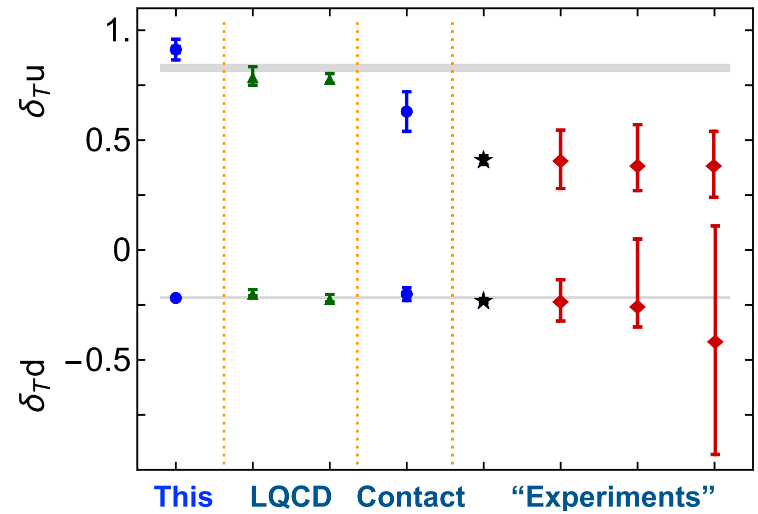
$$\delta_T q = \int_{-1}^{1} dx\, h_{1T}^{q}(x) = \int_{0}^{1} dx\, \left[h_{1T}^{q}(x) - h_{1T}^{\bar{q}}(x)\right]$$



- The quark-EDM (electric dipole moment) contribution to a nucleon's EDM is determined once its tensor charges are known:

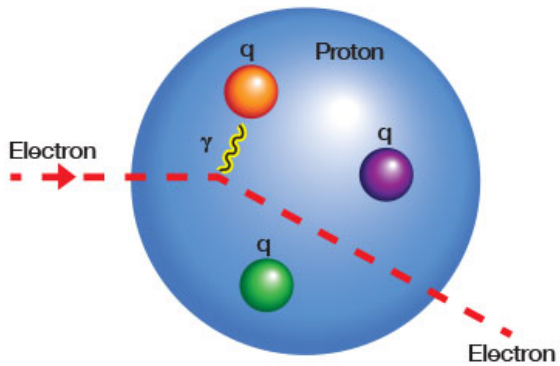
$$\langle P(p)|\mathcal{J}_{\mu\nu}^{\rm EDM}|P(p)\rangle = [\mathsf{E}_u \delta_T u + \mathsf{E}_d \delta_T d]\,\bar{\mathsf{u}}(p)\gamma_5\sigma_{\mu\nu}\mathsf{u}(p) \qquad \langle P(p)|\bar{q}\sigma_{\mu\nu}q|P(p)\rangle = \delta_T q\,\bar{\mathsf{u}}(p)\sigma_{\mu\nu}\mathsf{u}(p)$$

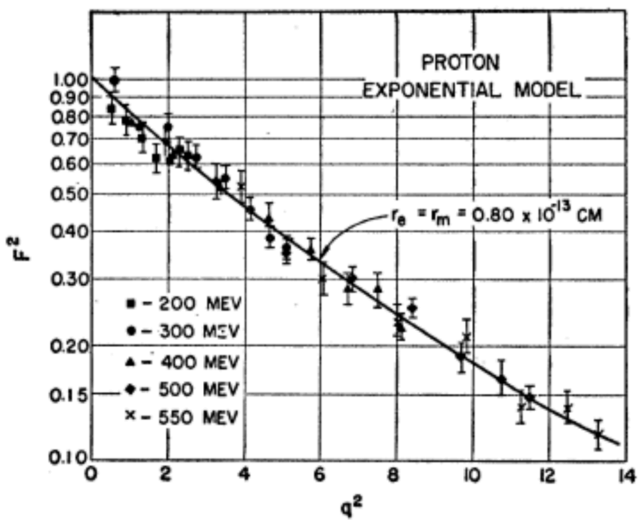




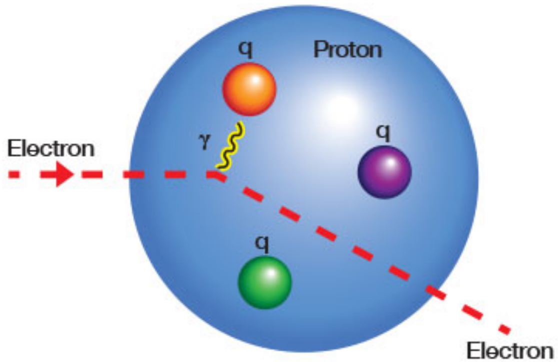
See, e.g., PRD 98, 054019 (2018)

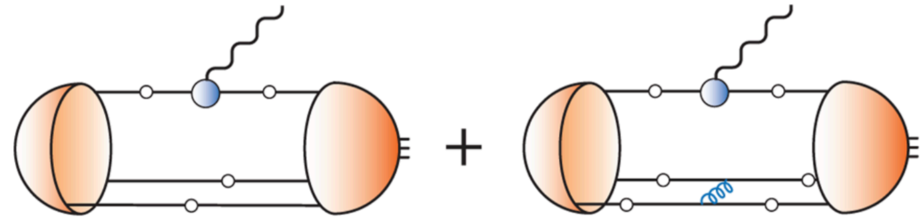
3.3 Structures: EM form factors



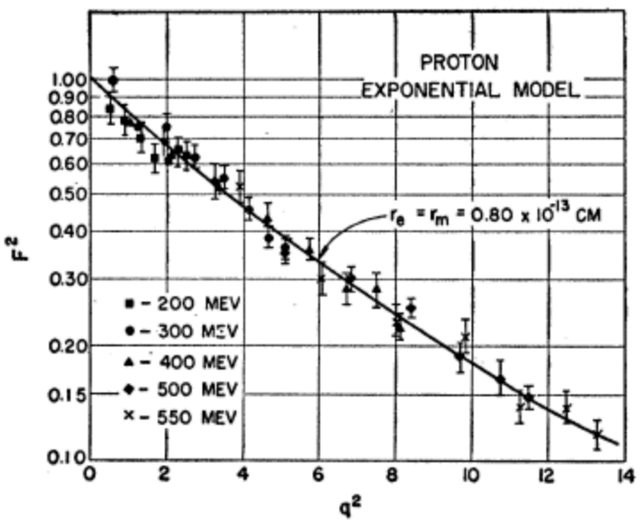


Robert Hofstadter
(Nobel prize 1961)



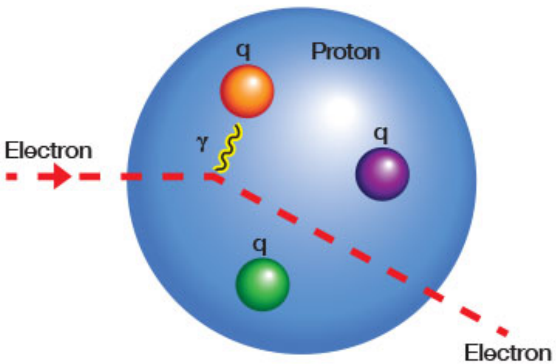


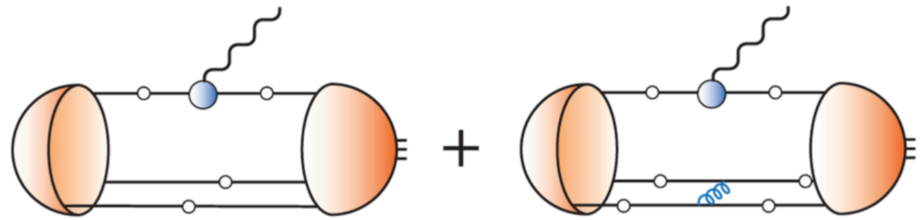
$$\langle N(P')|J^\mu(x)|N(P)\rangle \sim \Gamma^\mu(P',P)$$



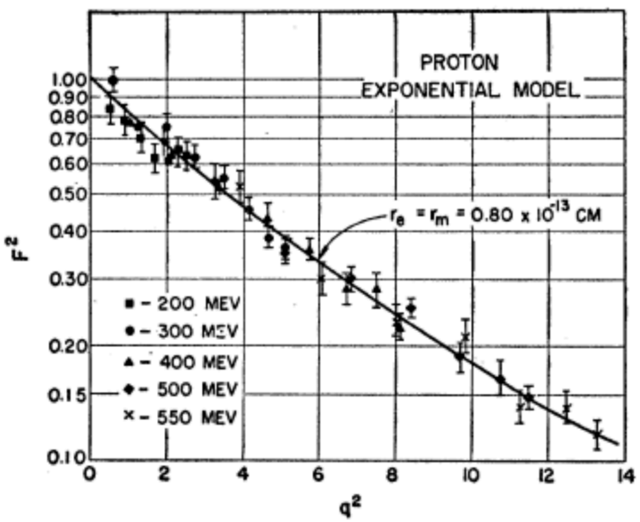
Robert Hofstadter
(Nobel prize 1961)

3.3 Structures: EM form factors





$$\langle N(P')|J^\mu(x)|N(P)\rangle \sim \Gamma^\mu(P',P)$$



Robert Hofstadter
(Nobel prize 1961)

The current can be decomposed by form factors:

$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_N} Q^\nu F_2(Q^2)$$

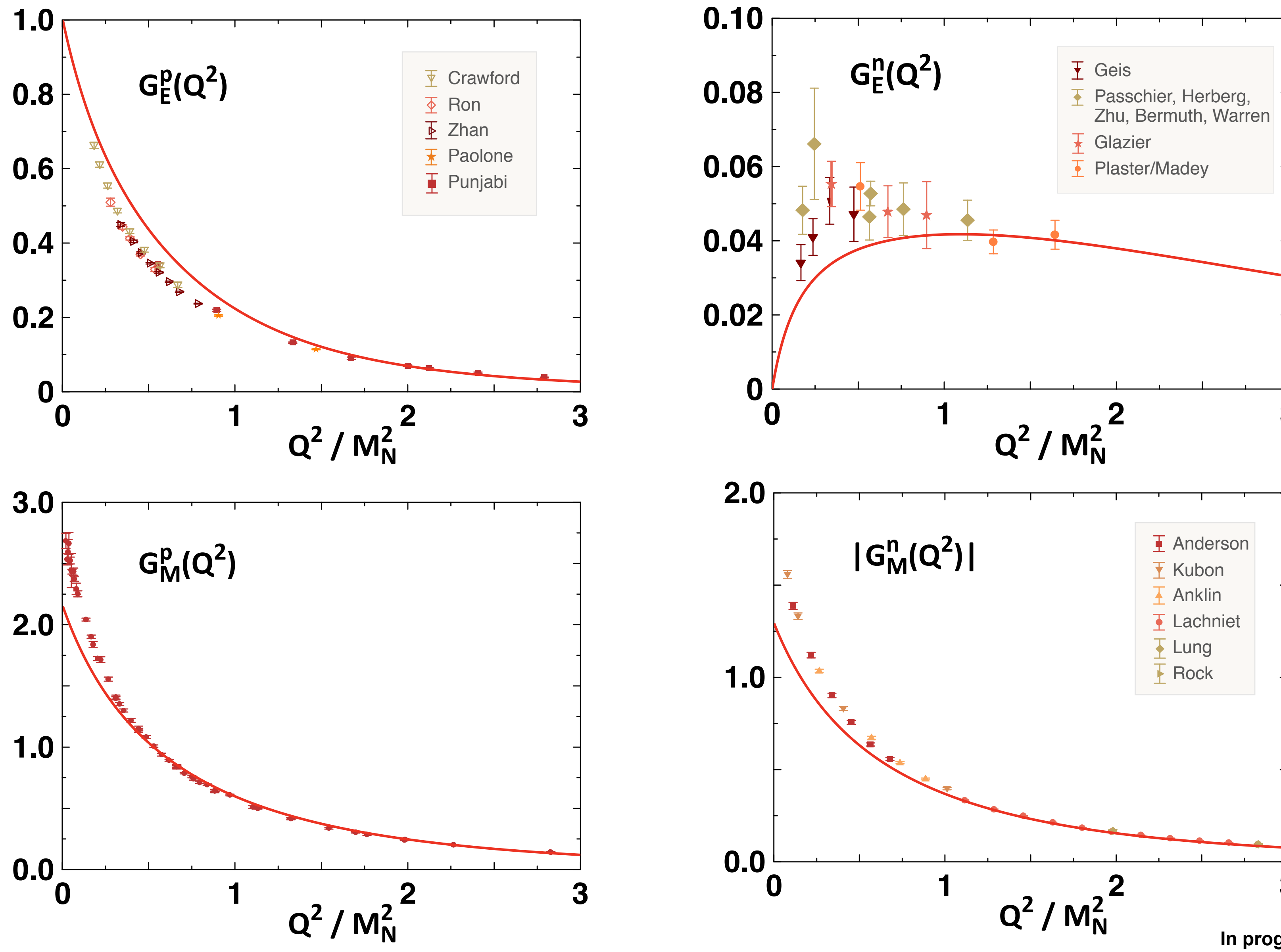
which express charge and magnetization densities.

Alternatively, the Sachs form factors read:

$$G_E(Q^2) := F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

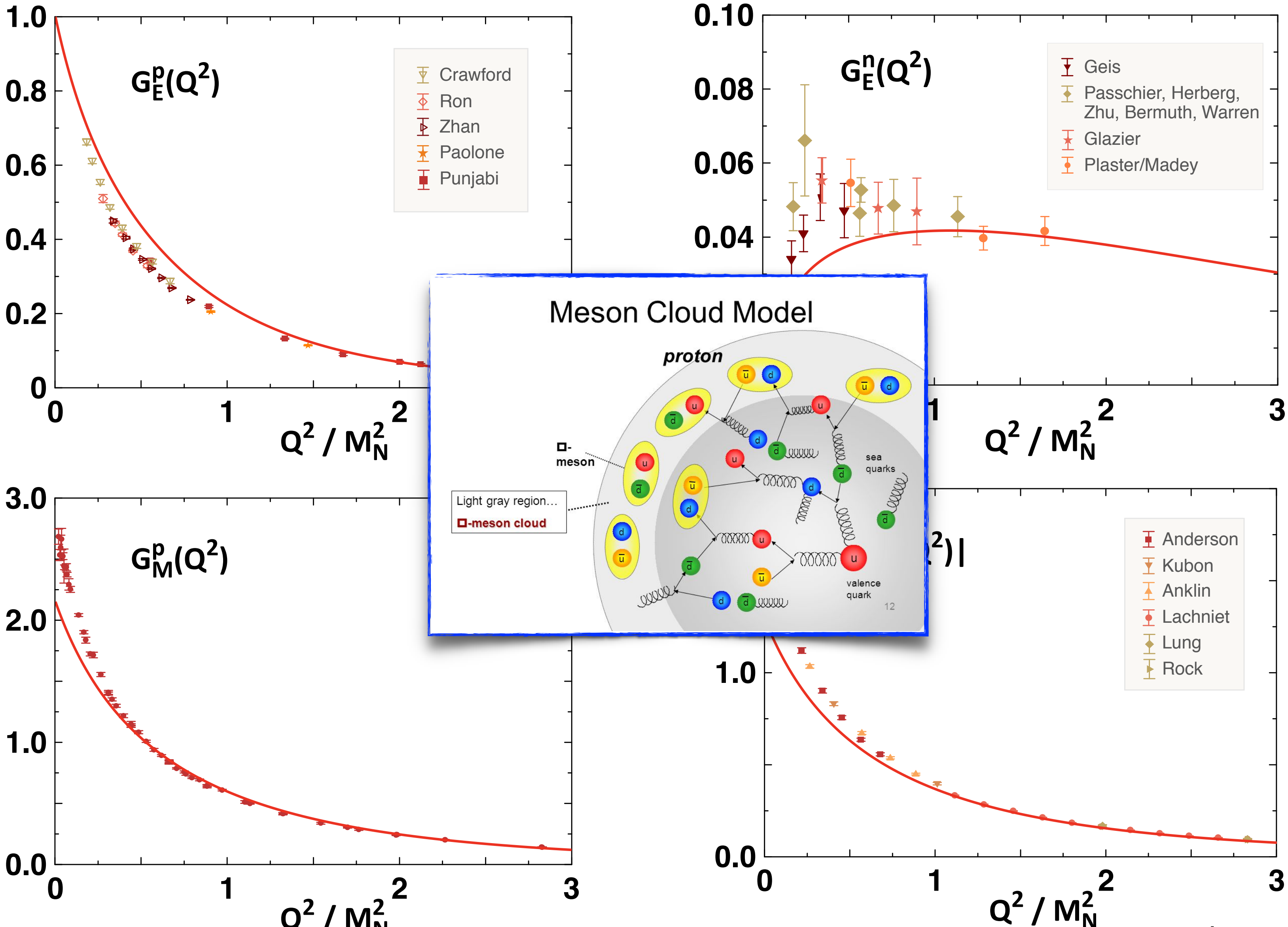
$$G_M(Q^2) := F_1(Q^2) + F_2(Q^2)$$

3.3 Structures: EM form factors



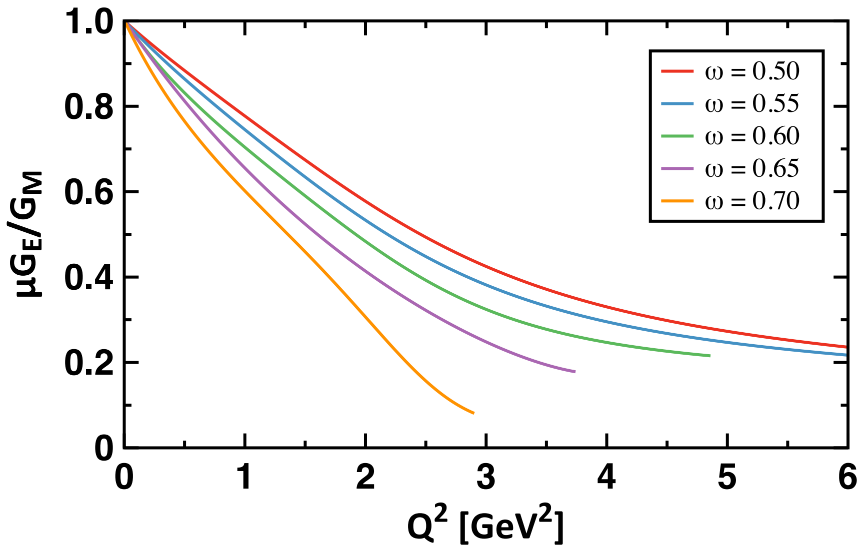
In progress (2019)

3.3 Structures: EM form factors

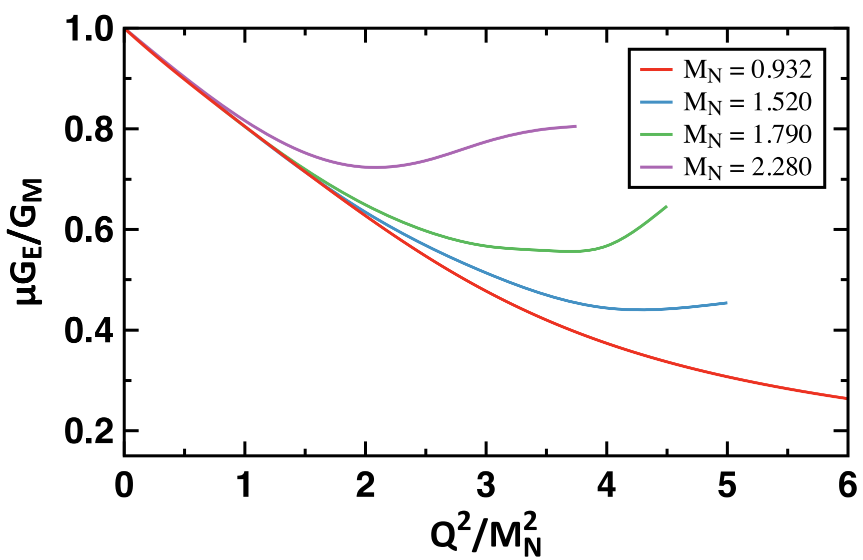


In progress (2019)

3.3 Structures: EM form factors



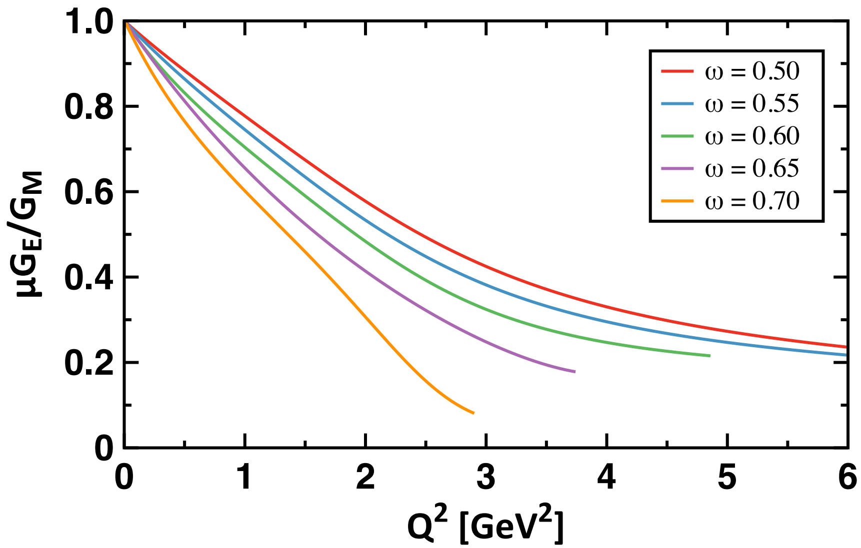
✓ With the transition speed of DCSB decreasing, the ratio suppresses and gains a zero-crossing.

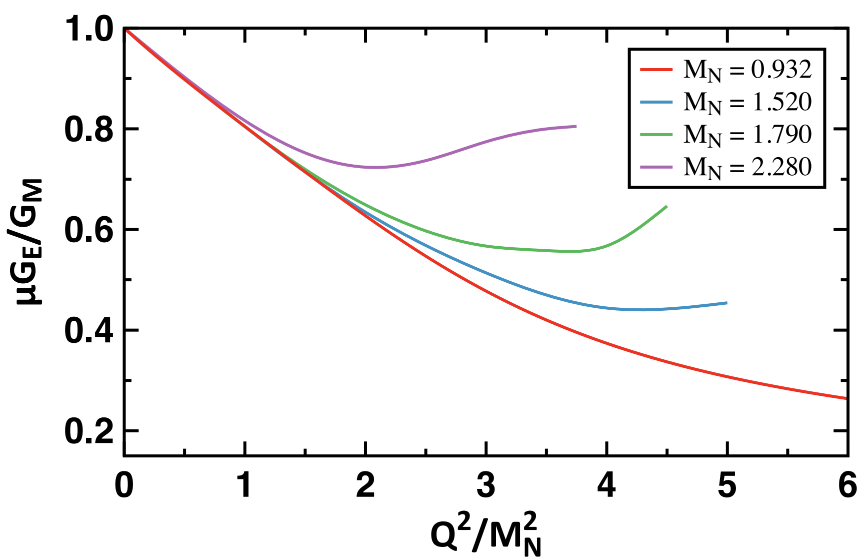


✓ With the baryon mass increasing, the ratio raises and approaches the point-particle limit.

In progress (2019)

3.3 Structures: EM form factors





✓ With the transition speed of DCSB decreasing, the ratio suppresses and gains a zero-crossing.

Theory vs Exps

✓ With the baryon mass increasing, the ratio raises and approaches the point-particle limit.

In progress (2019)

Summary

CHONGQING UNIVERSITY

✦ The framework of three-body Faddeev equation, which describes baryons in continuum QCD, and its rainbow-ladder truncation scheme was introduced.

✦ Baryon properties are studied: **a**) full mass spectrum of J=1/2 and J=3/2 baryons; **b**) delta wave functions and nucleon tensor charges; c) nucleon EM form factors.

Summary

重庆大学 CHONGQING UNIVERSITY

- The framework of three-body Faddeev equation, which describes baryons in continuum QCD, and its rainbow-ladder truncation scheme was introduced.

- Baryon properties are studied: **a**) full mass spectrum of J=1/2 and J=3/2 baryons; **b**) delta wave functions and nucleon tensor charges; c) nucleon EM form factors.

Outlook

- Use the three-body Faddeev equation to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.

- Study baryon states beyond the simple rainbow-ladder truncation, e.g., radial excitations, parity partners, their structures, and etc.