# Light Mesons Parton Distribution Functions From Basis Light Front Quantization

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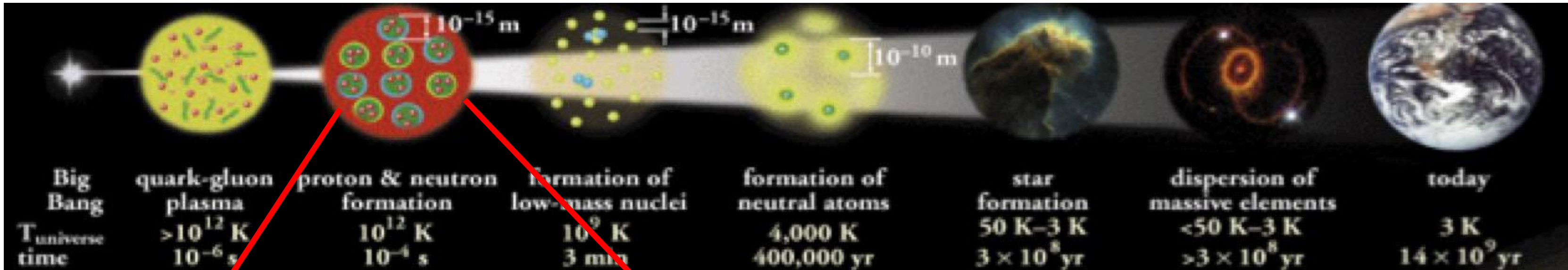
# Introduction




baryon： proton & neutron

**meson： pion & kaon**

- **The most common light mesons**
- **Easy to produce in the experiment**
- **Not only a quark and antiquark**

# Basis Light Front Quantization (BLFQ)

**Xingbo ZHAO**
20th, (Tuesday)

Approach for solving quantum field theory

- Nonperturbative for systems with strong interaction
- First-principles & effective Hamiltonian as input
- Directly access to wave function of bound states
- Light Front dynamics

  spectrum and light front Fock state wave functions are obtained from

$$\boxed{H_{LF}\,|\psi\rangle = M^2\,|\psi\rangle}$$

$$H_{LF} = P^- = \frac{m^2 + P_\perp^2}{P^+} \qquad P^\pm = P^0 \pm P^3$$

[Vary et al, 2008]

# Hamiltonian & Basis

**Shaoyang JIA**
20th, (Tuesday)

We choose $H_{LF}$ as effective Hamiltonian:

$$H_{LF} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x\left(x(1-x)\partial_x\right)}_{\text{Longitudinal}} + H^{\text{eff}}_{\text{NJL}}$$

- We only consider the valence Fock sector, such as:

$$|\Psi_{\pi^+}\rangle = |u\bar{d}\rangle + |u\bar{d}\, g\rangle + |u\bar{d}\, q\bar{q}\rangle + \ldots$$

- Discretized basis [S. Jia and J. P. Vary, PRC (2018)]
    - Transverse: 2D harmonic oscillator basis: $\phi_{nm}(\vec{p}_\perp; b)$
    - Longitudinal: the Jacobi polynomial, labeled by *l*.
    - Basis truncation:

$$|m| \le M_{\max} \qquad n \le N_{\max} \qquad l \le L_{\max}$$

# Wave function

Parameters are adjusted to reproduce the **masses** of $\pi^+$, $\rho^+$, $K^+$ , and $K^{*+}$ , as well as the experimental **charge radii** of $\pi^+$ and $K^+$.

$\pi, \rho$

| $m_l$ | $\kappa_{ll}$ | $G_\pi$ | $N_{max}$ | $M_{max}$ | $L_{max}$ |
|---|---|---|---|---|---|
| 337.01 MeV | 227.00 MeV | 18.5095 GeV$^{-2}$ | 8 | 2 | 8 |

$K, K^*$

| $m_l$ | $m_s$ | $\kappa_{ls}$ | $G_K$ | $N_{max}$ | $M_{max}$ | $L_{max}$ |
|---|---|---|---|---|---|---|
| 307.66 MeV | 445.14 MeV | 276.00 MeV | 13.6455 GeV$^{-2}$ | 8 | 2 | 8 |

| | BLFQ-NJL | PDG |
|---|---|---|
| $m_{\pi+}$ **mass** | 139.57 MeV | 139.57 MeV |
| $m_{\rho+}$ | $775.23 \pm 0.04$ MeV | $775.26 \pm 0.25$ MeV |
| $f_\pi$ **decay** | 202.10 MeV | $130.2 \pm 1.7$ MeV |
| $f_\rho$ | 100.12 MeV | $221 \pm 2$ MeV |
| $\sqrt{\langle r_c^2 \rangle}\|_{\pi+}$ **radii** | $0.68 \pm 0.05$ fm | $0.672 \pm 0.008$ fm |

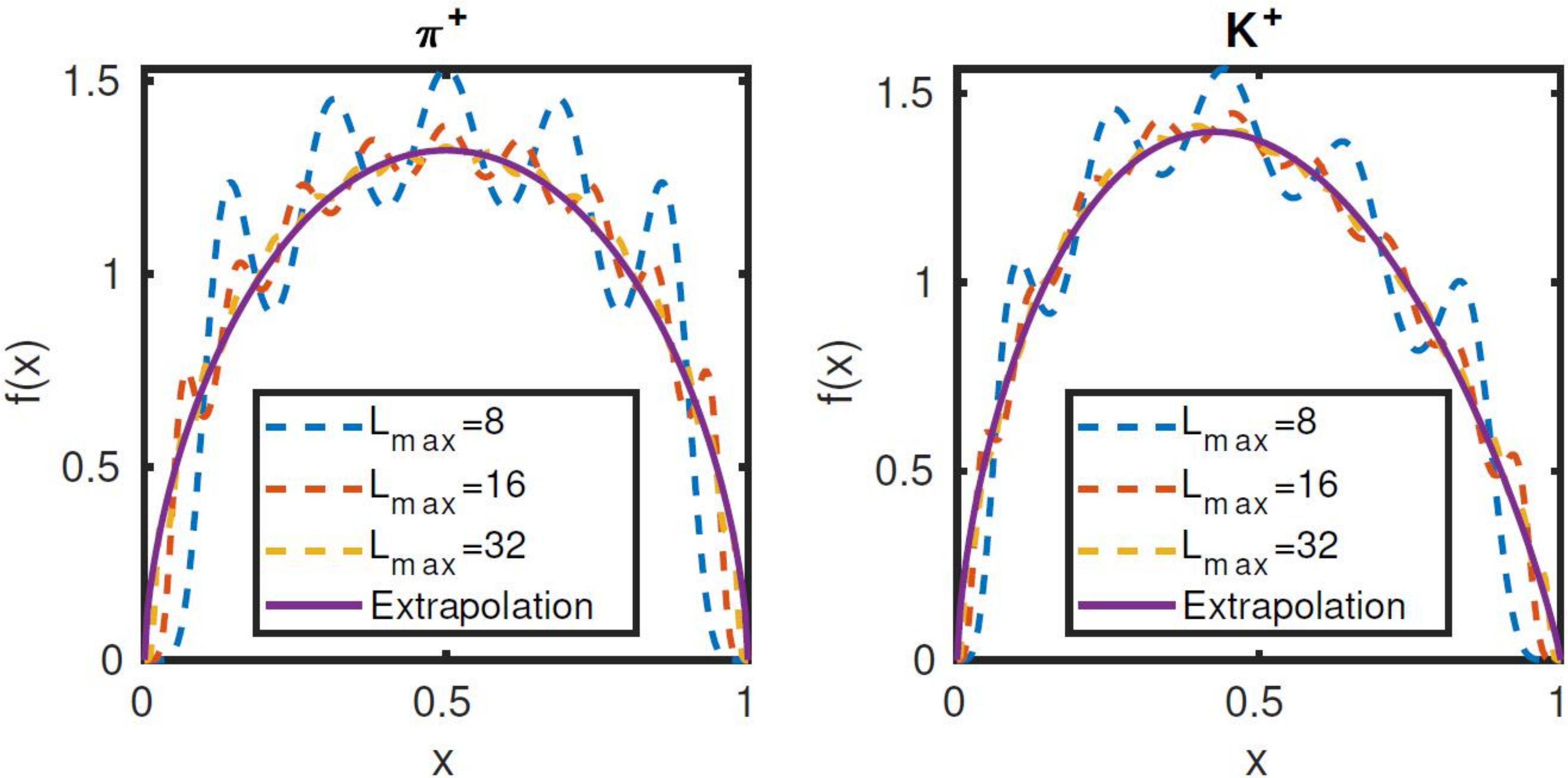
[S. Jia and J. P. Vary, PRC (2018)]

| | BLFQ-NJL | PDG |
|---|---|---|
| $m_{K+}$ | 493.68 MeV | $493.68 \pm 0.02$ MeV |
| $m_{K*+}$ | $891.82 \pm 0.06$ MeV | $891.76 \pm 0.25$ MeV |
| $m_{K_0^{*+}}$ | 858.35 MeV | $824 \pm 30$ MeV |
| $f_K$ | 235.99 MeV | $155.6 \pm 0.4$ MeV |
| $f_{K*}$ | 104.57 MeV | $224 \pm 11$ MeV |
| $\sqrt{\langle r_c^2 \rangle}\|_{K+}$ | $0.54 \pm 0.03$ fm | $0.560 \pm 0.031$ fm |

# Parton Distribution Functions (PDFs)

PDF for the valence quark result from the light front wave functions obtain by diagonalizing the effective Hamiltonian.

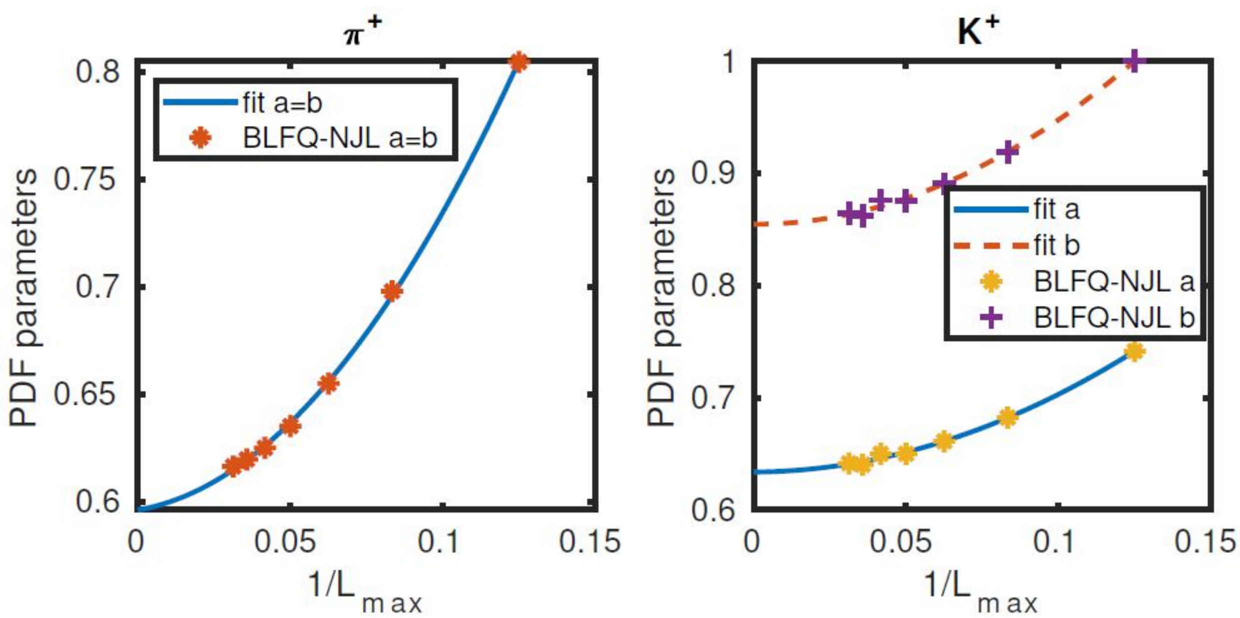
[Y. Li, P. Maris, and J. P. Vary, PRD (2017)]



# Parton Distribution Functions (PDFs)

We fit the resulting PDFs using the function

$$f(x) = x^a(1-x)^b/B(a+1,b+1),$$

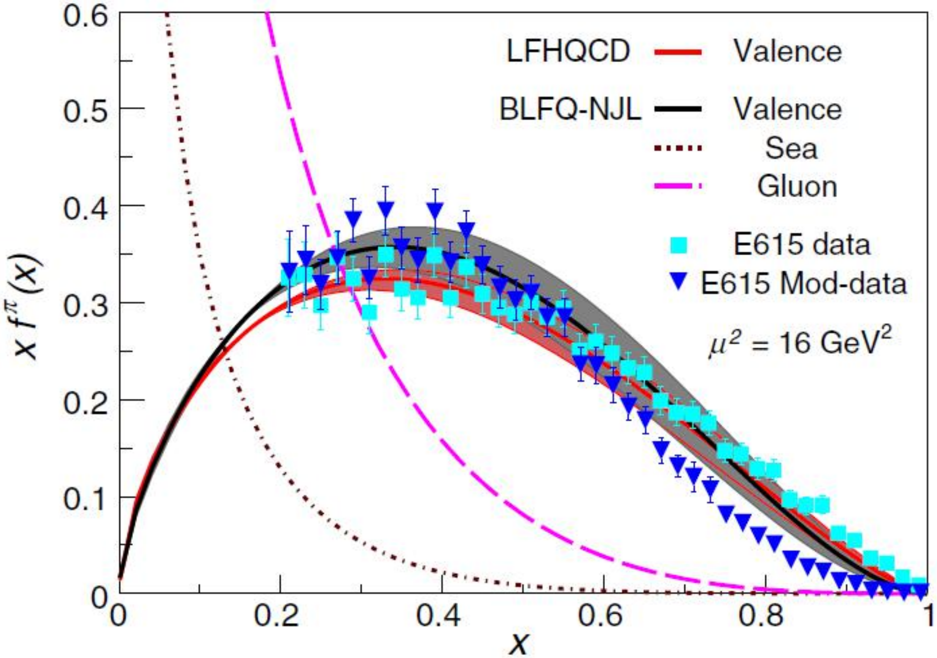


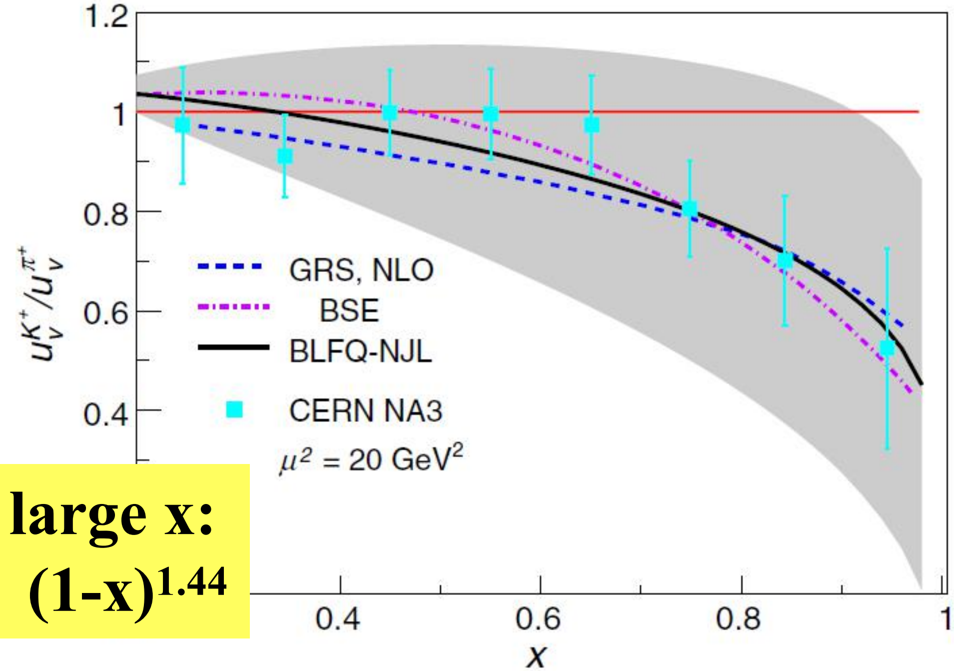

The resulting parameters are **a= b= 0.5961** for the **pion**, while **a= 0.6337** and **b = 0.8546** for the **kaon**

# PDF with QCD Evolution

[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

We use NNLO DGLAP to evolve, and fix initial scale by fitting <x>



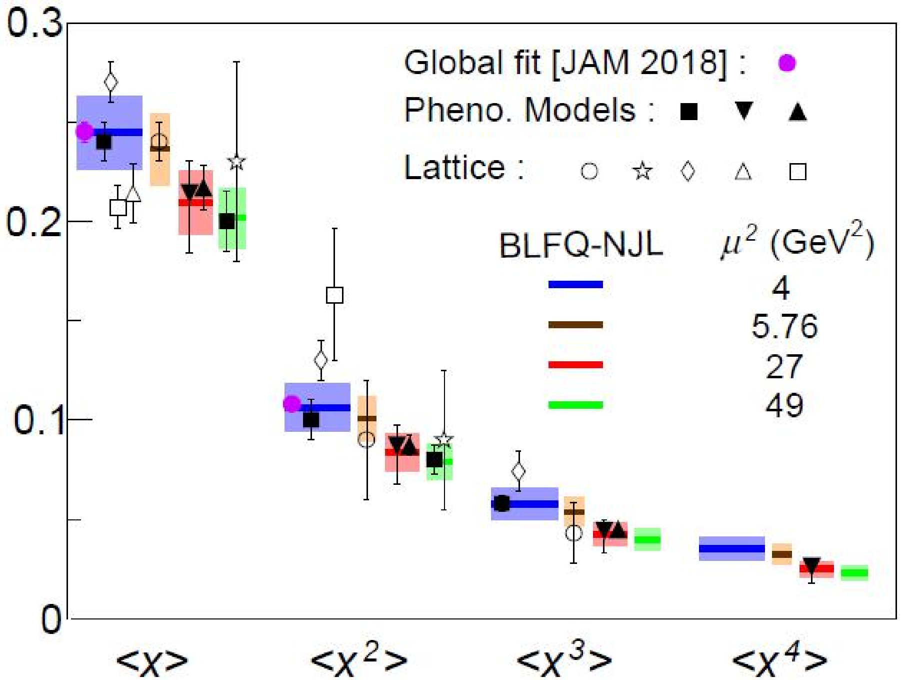

**large x: $(1-x)^{1.44}$**

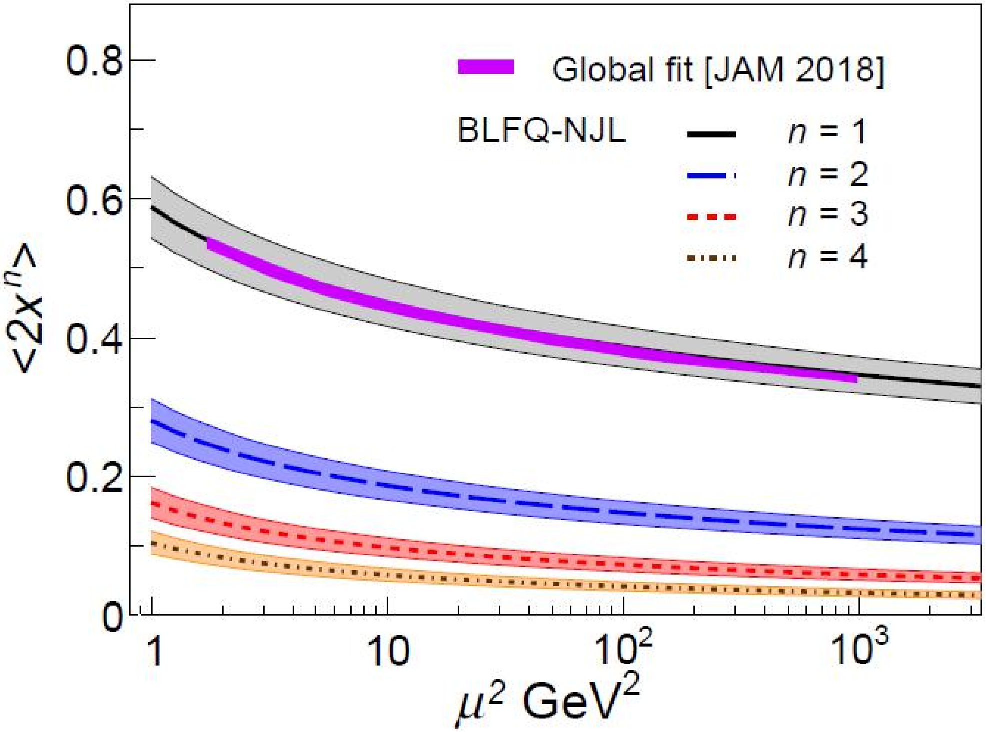
| Order | Initial scale of $\pi$ | Initial scale of $K$ | E615 $\chi^2_{/dof}$ | NA3 $\chi^2_{/dof}$ |
|---|---|---|---|---|
| LO | $0.120 \pm 0.012$ GeV$^2$ | $0.133 \pm 0.013$ GeV$^2$ | 6.71 | 0.88 |
| NLO | $0.205 \pm 0.020$ GeV$^2$ | $0.210 \pm 0.021$ GeV$^2$ | 4.67 | 0.56 |
| NNLO | $0.240 \pm 0.024$ GeV$^2$ | $0.246 \pm 0.024$ GeV$^2$ | 3.64 | 0.50 |

# PDF with QCD Evolution [Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

The moments of **pion** valence quark PDF:

$$\langle x^n \rangle = \int_0^1 dx\; x^n f_v^{\pi/K}(x,\mu^2),\;\; n = 1,2,3,4.$$




| $<x>$ @4 GeV$^2$ | Valence | Gluon | Sea |
|---|---|---|---|
| **BLFQ-NJL** | **0.489** | **0.398** | **0.113** |
| [Aguilar *et. al.,* Pion and Kaon Structure at the Electron-Ion Collider] | 0.48(3) | 0.41(2) | 0.11(2) |

# PDF & QCD evolution

[Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

For **Kaon** the moment of valence quark PDFs

| $\langle x \rangle$ @27 GeV$^2$ | u | \bar{s} | u/\bar{s} |
|---|---|---|---|
| **BLFQ-NJL** | **0.201** | **0.228** | **0.882** |
| [Chen, et al, dressed quark model (2016)] | 0.28 | 0.36 | 0.78 |
| [Watanabe, et al, meson cloud model (2018)] | 0.23 | 0.24 | 0.96 |

# Drell-Yan cross section

The cross section for the Drell-Yan process $\pi^- \ \mathrm{Nucleus} \to \mu^+ \ \mu^- \ \mathrm{X}$ is related to the pion and the nuclear PDFs by

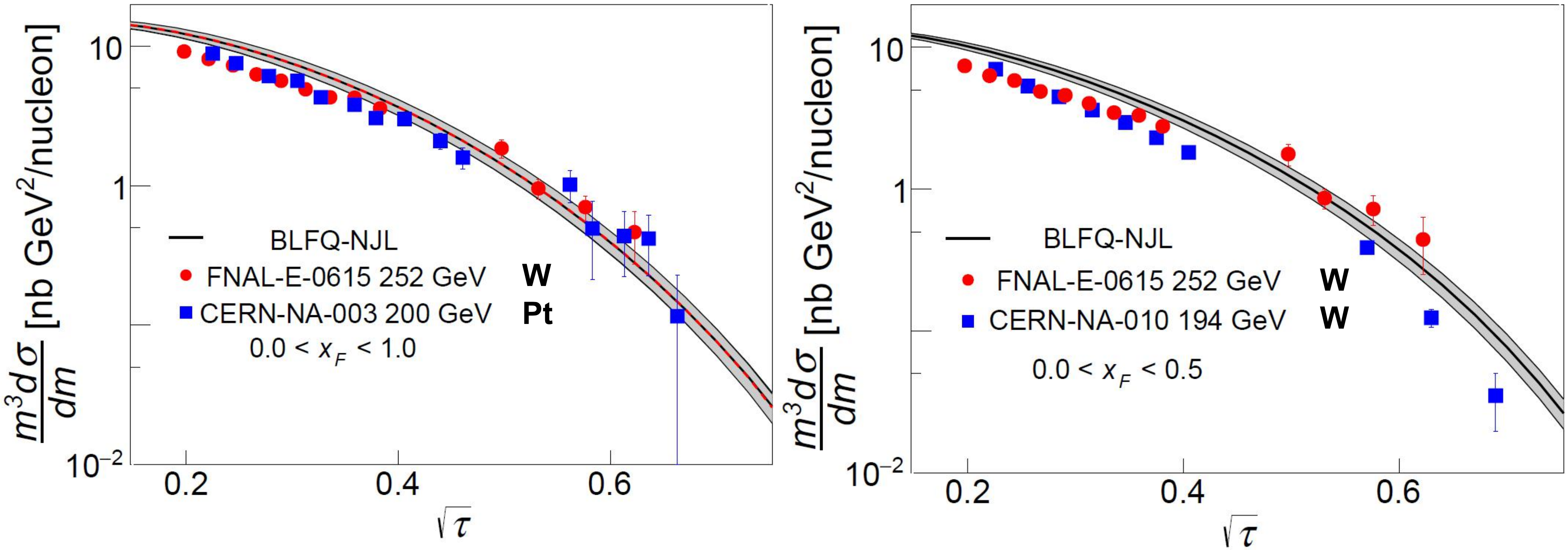
$$\frac{m^3 d^2\sigma}{dm\, dx_{\mathrm{F}}} = \frac{8\pi\alpha^2}{9} \frac{x_1 x_2}{x_1 + x_2} \sum_a e_a^2 f_a^{\pi^-}(x_1) f_{\bar{a}}^N(x_2)$$

[S. D. Drell and T.-M. Yan, PRL (1970)]
[McGaughey ea al, Drell-Yan experiment FNAL-E-0772, PRD (1994)]

# Drell-Yan cross section

[Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

We use our Pion PDFs and the target nuclear PDFs from "nCTEQ 2015"




W: tungsten; Pt: platinum; Be: beryllium; C: carbon.

# Drell-Yan cross section

[Lan, Mondal, Jia, Zhao, Vary, arxiv: 1907.01509]

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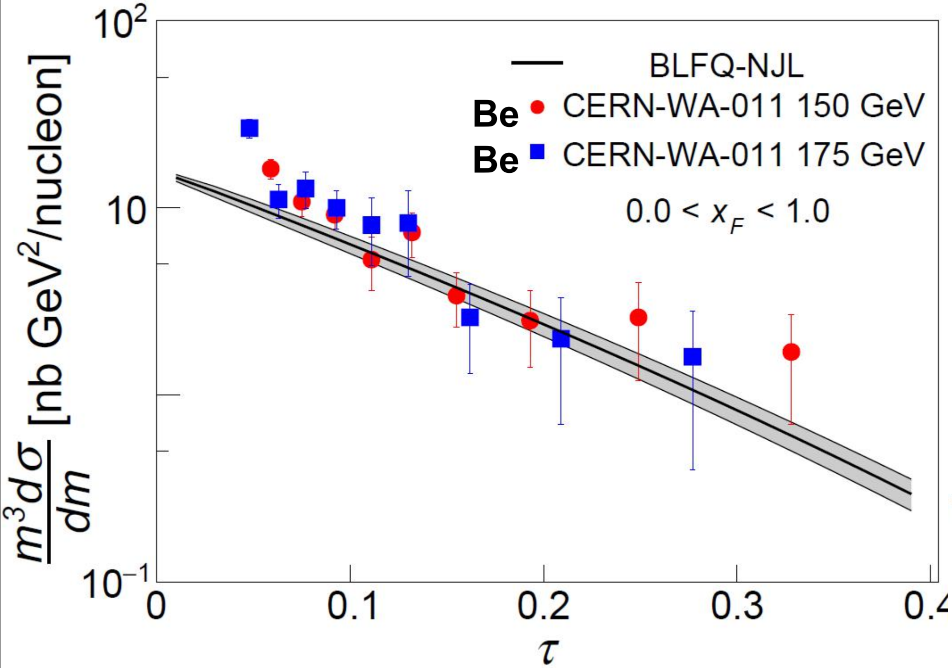


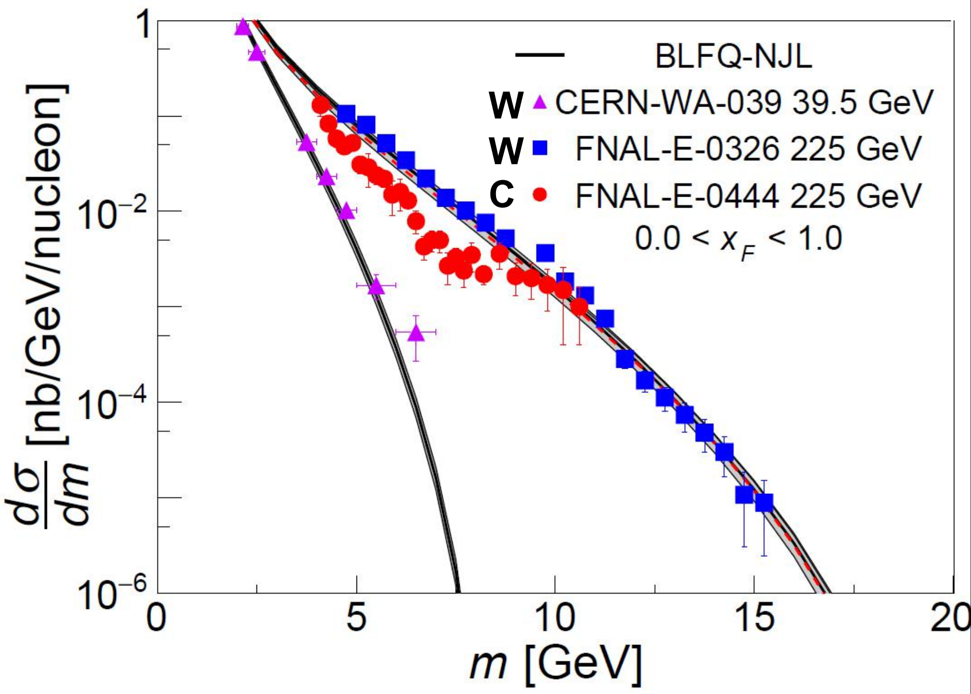


Results are in acceptable agreement with data from widely different experimental conditions (**FNAL E615**, **326**, **444**, and **CERN NA3, NA10**, **WA-011**, **WA-039**).

# Conclusions

We used the wave functions of light mesons from BLFQ to calculate the initial PDFs and investigate QCD evolution.

- ✓ The PDFs of pion
  agree with FNAL E615
- ✓ The ratio of up quark PDF in kaon to that in pion
  agree with CERN NA3
- ✓ The moment of pion
  agree with JAM & Lattice & others phenom. results
- ✓ The moment of kaon
- ✓ The pion-nucleus-induced DY process Cross Section
  agree with Exp. data
  (FNAL E615, 326, 444, and CERN NA3, NA10, WA-011, WA-039)

# Outlook

- For light mesons, we will study the GPD, TMD, GTMD, Wignar distribution...
- Investigate the heavy quarkonium
- Consider the higher Fock sectors for directly calculating gluon and sea quark PDFs

Hopefully the BLFQ approach will be useful in unstanding the structure of hadrons in the future!

*Thank you for your attention!*