Three Dimension Imaging of Proton in Basis Light-Front Quantization

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With

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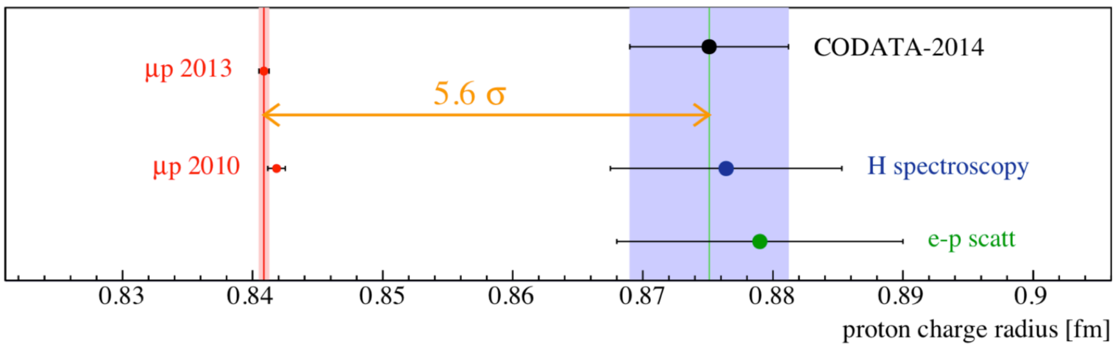
Outline

- Background
- **B**asis **L**ight-**F**ront **Q**uantization (**BLFQ**)
- Nucleon Properties in BLFQ
- Conclusion and Outlook

Background

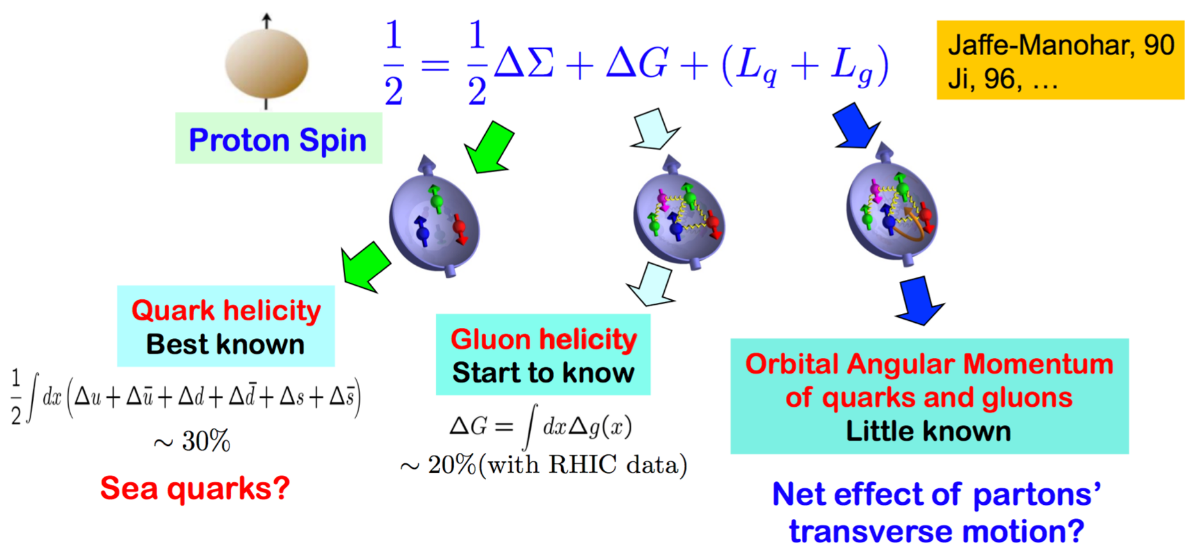
➢ Proton radius puzzle



- ❖ Elastic electron scattering established the extended nature of the proton, [*R. Hofstadter, Nobel Prize 1961*]
- ❖ Different experiments give the different ***radius***.

➢ Spin crisis

- ❖ **In 1988s, EMC(European Muon Collaboration) found the contribution of *spin of quark* is smaller than *expected*.**

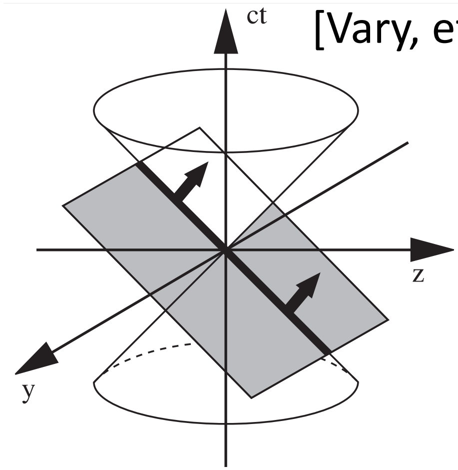


Basis Light-Front Quantization

➢ Solve the time-independent Schrödinger Equation:

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

[Vary, et.al, Phys.Rev.C '10]



- $\boldsymbol{P^-}$: Light-Front Hamiltonian;
- $|\boldsymbol{\beta}\rangle$: Eigenstates;
- $\boldsymbol{P^-_\beta}$: Eigenvalues for eigenstates.

➢ Quantum numbers of basis states in BLFQ:

I. Longitudinal direction

— discrete longitudinal momentum (labeled by **k**): $p^+ = \frac{2\pi}{L}k$

II. Transverse direction

— 2-dimensional harmonic oscillator (labeled by **n** , **m**)

Truncation : $\begin{cases}\text{Nmax}\\ \text{Kmax}\end{cases}$

$$\Phi^b_{n,m}(p_\perp) = \frac{1}{b\sqrt{\pi}}\sqrt{\frac{n!}{(n+|m|)!}}\, e^{-\frac{p^2}{2b^2}} e^{-im\phi}\left(\frac{p}{b}\right)^{|m|} L_n^{|m|}\left(\frac{p^2}{b^2}\right)$$

Prof. Xingbo Zhao's Talk
In the afternoon, Session 6

Light-Front Hamiltonian

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

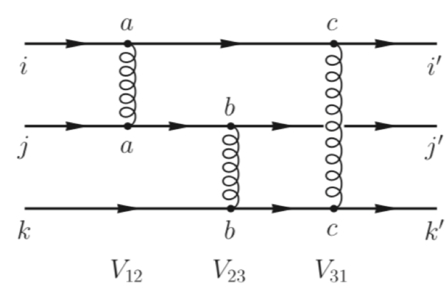
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$H_{OGE} = -\frac{C_F 4\pi\alpha_s}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$ Color factor : $C_F = -\frac{2}{3}$

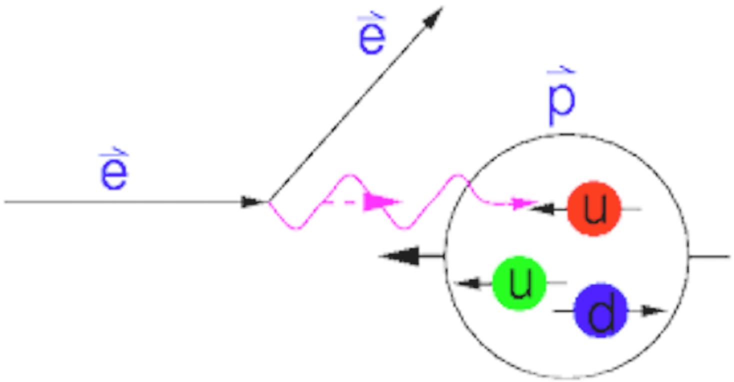
$|P_{baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$



Three active-quark approach

Electromagnetic Form Factor

- **Elastic scattering of proton**



[*R. Hofstadter, Nobel Prize 1961*]

$$e(p) + h(P) \rightarrow e(p') + h(P')$$

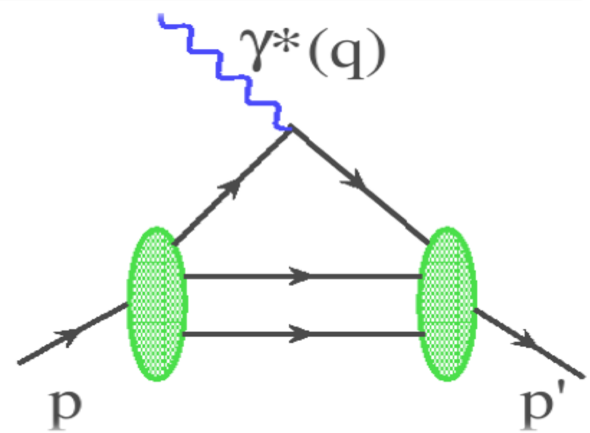
- Elastic electron scattering established the extended nature of the proton (proton radius).

The Fourier transformation of these **form factors provide spatial distributions** (***charge and magnetization distributions***).

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle =$$

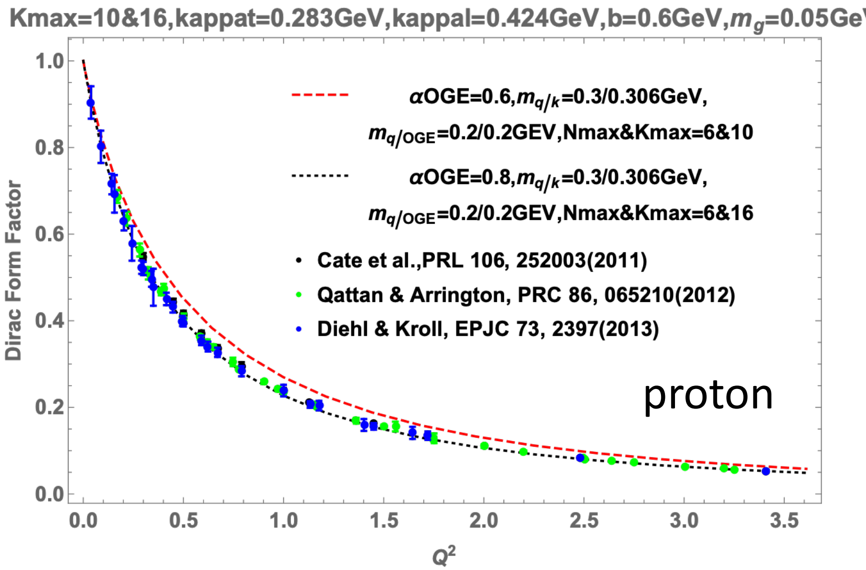
$$\bar{u}(p')\left[\gamma^{\mu} \underbrace{F_1(q^2)}_{\text{Dirac Form Factor}} + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} \underbrace{F_2(q^2)}_{\text{Pauli Form Factor}}\right] u(p)$$

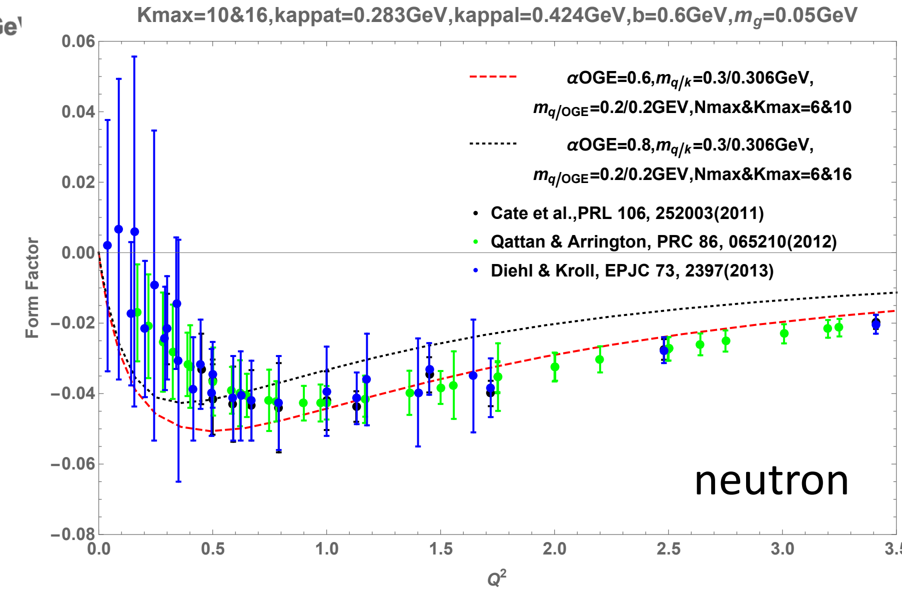
Dirac Form Factor   Pauli Form Factor

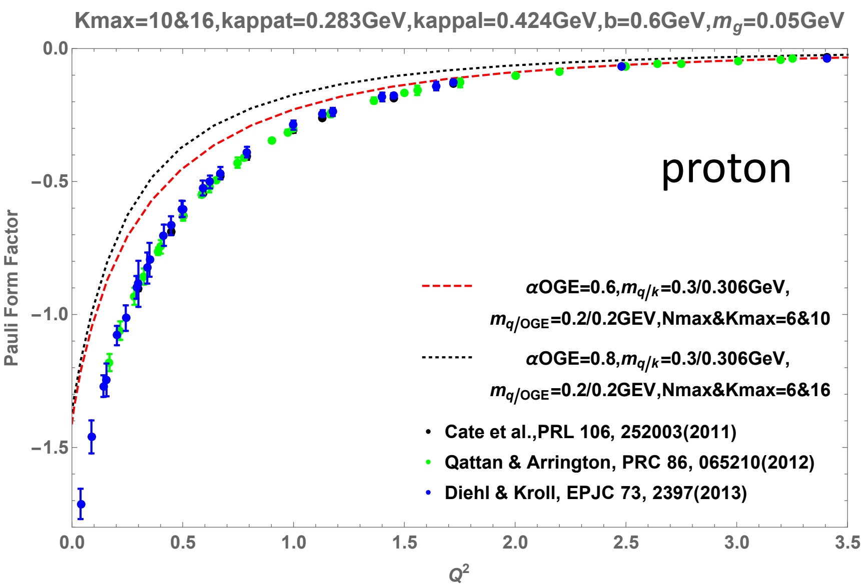


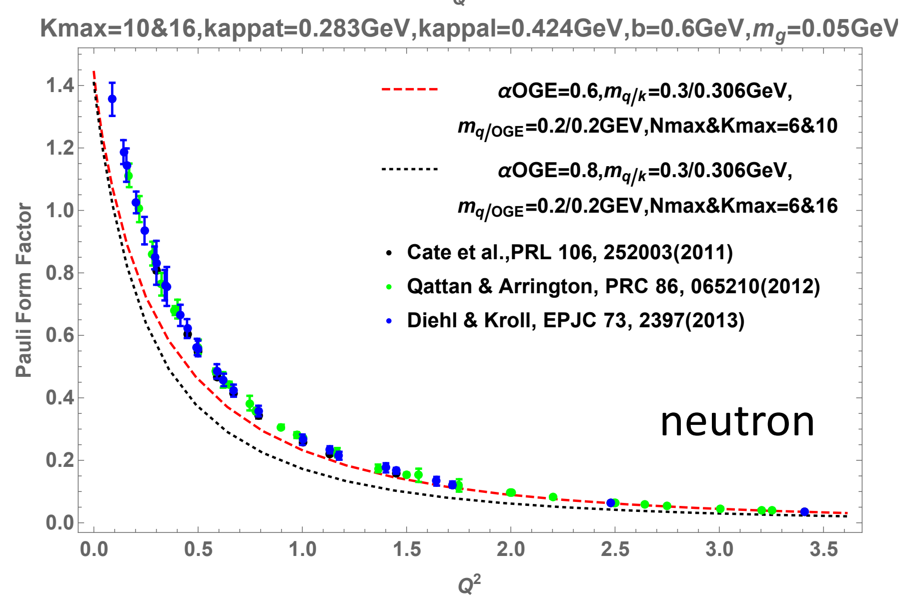
Nucleon Form Factor

[Work in progress, C. Mondal, Siqi Xu, *et.al*]





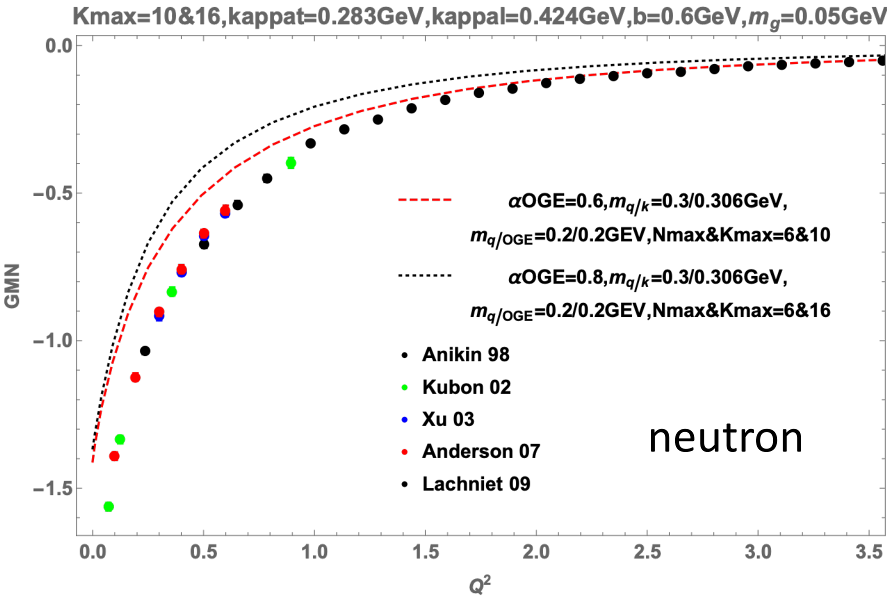


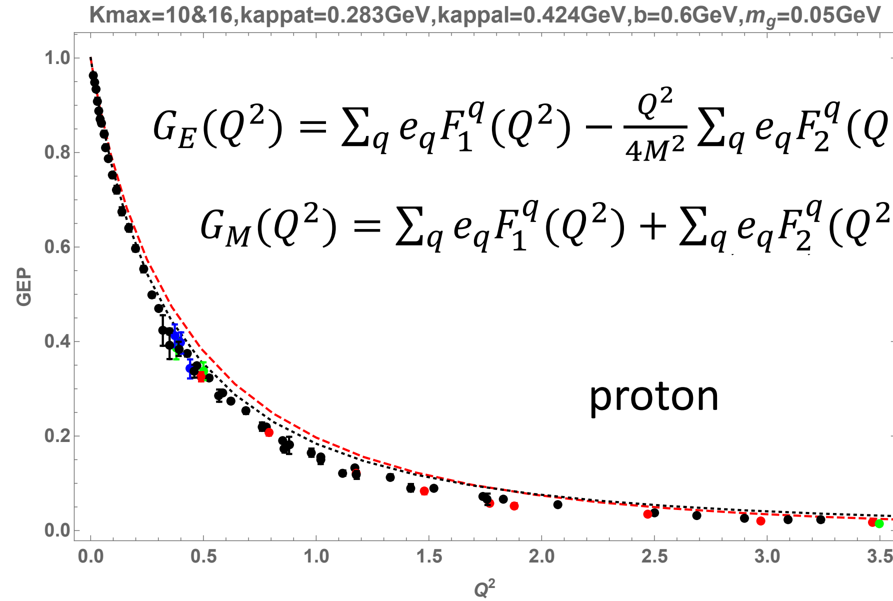


With ***increasing the basis size***, the nucleon form factors ***approach the data***

Nucleon Form Factor

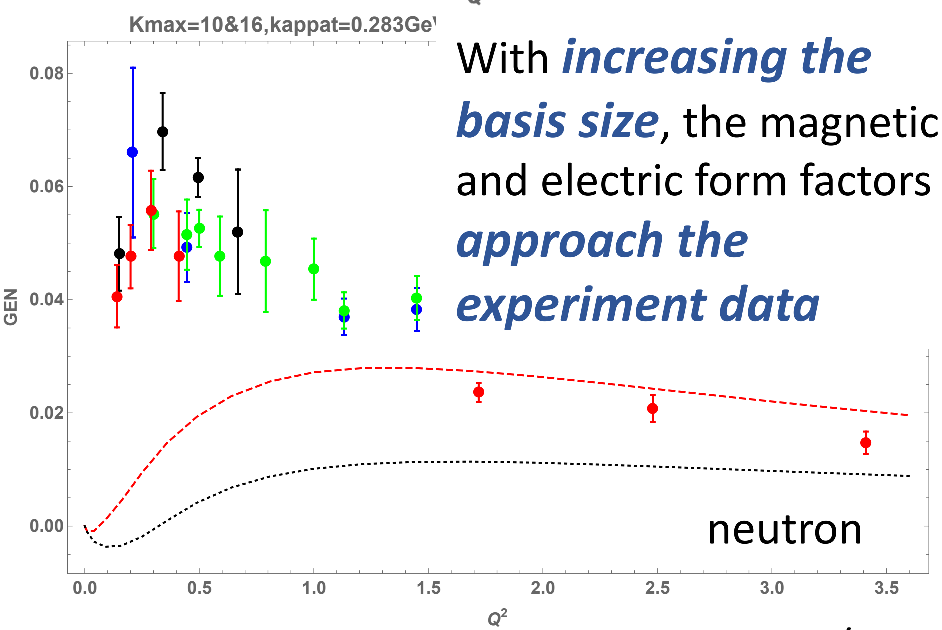
[Work in progress, C. Mondal, Siqi Xu, *et.al*]



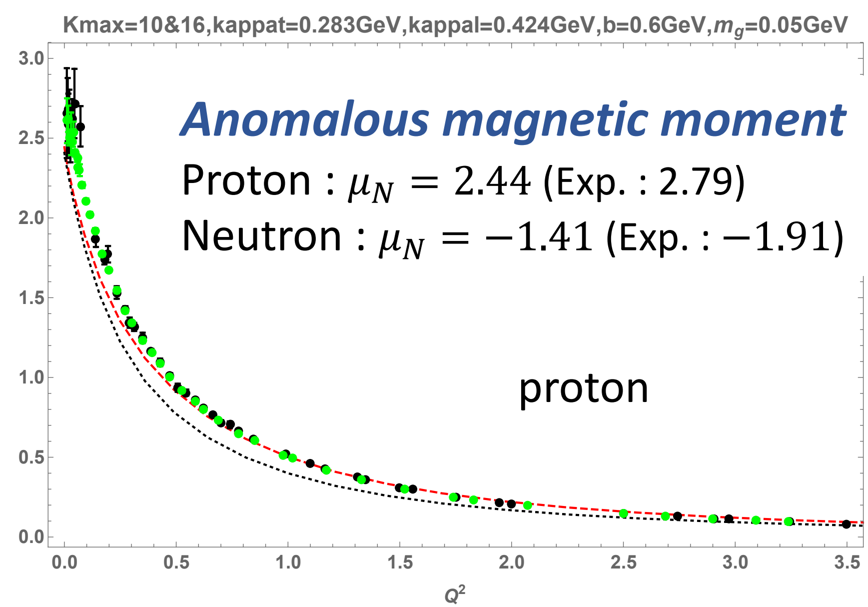


$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M^2} \sum_q e_q F_2^q(Q^2)\,,$$

$$G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2).$$



With ***increasing the basis size***, the magnetic and electric form factors ***approach the experiment data***



Anomalous magnetic moment

Proton : $\mu_N = 2.44$ (Exp. : 2.79)

Neutron : $\mu_N = -1.41$ (Exp. : -1.91)

Nucleon Form Factor

[Work in progress, C. Mondal, Siqi Xu, *et.al*]

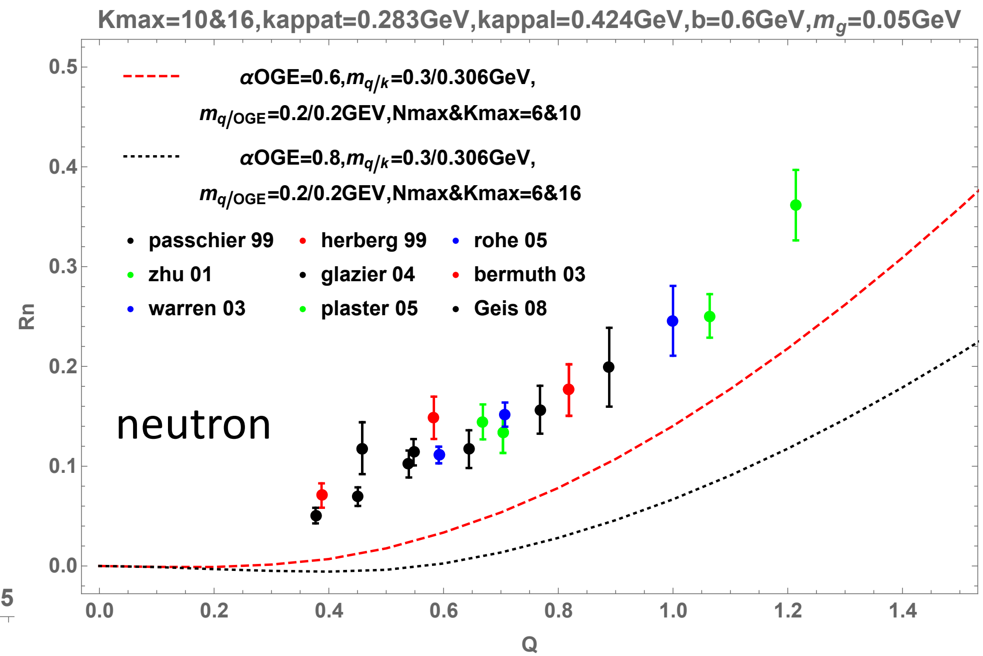
- The ratio of nucleon form factor

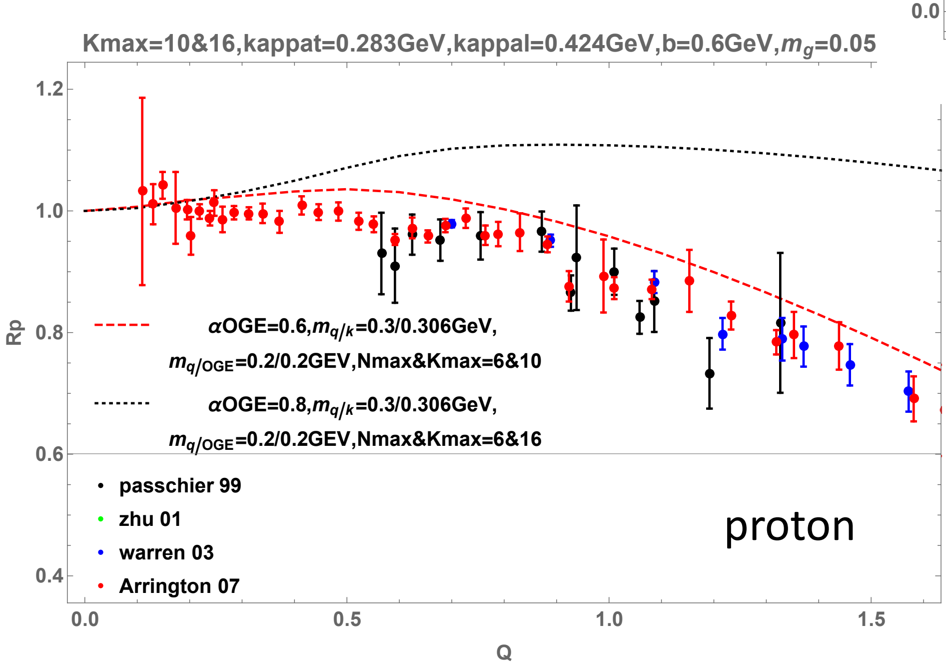
$$R_{n/p} = \mu_{n/p}\, G_E(Q^2)/G_M(Q^2)$$

Anomalous magnetic moment

Proton : $\mu_p = 2.44$ (Exp. : 2.79)

Neutron : $\mu_n = -1.41$ (Exp. : -1.91)



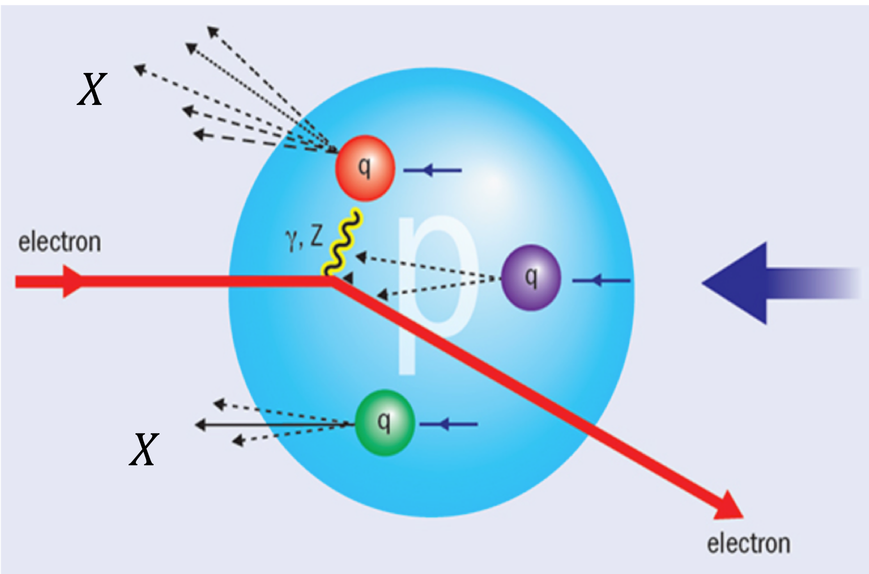


With ***increasing the basis size***, the ratio of proton and neutron ***approach the experiment data***

Parton Distribution Functions (PDF)

➢ **Deep Inelastic Scattering (SLAC 1968)**

$$e(p) + h(P) = e'(p') + X(P')$$

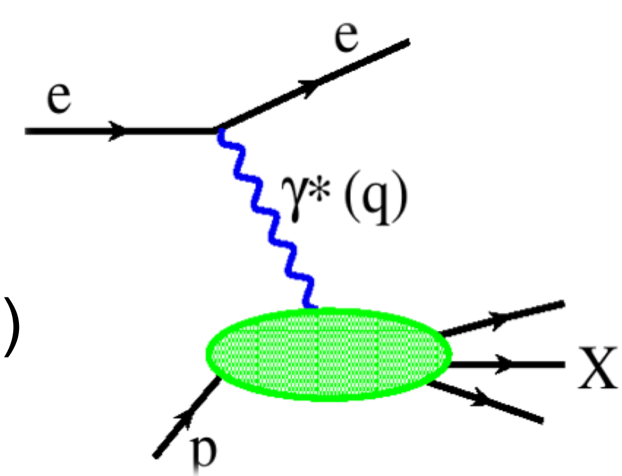


✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1 \text{ fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1 \text{ fm}$$

Discovery of spin ½ quarks and partonic structure

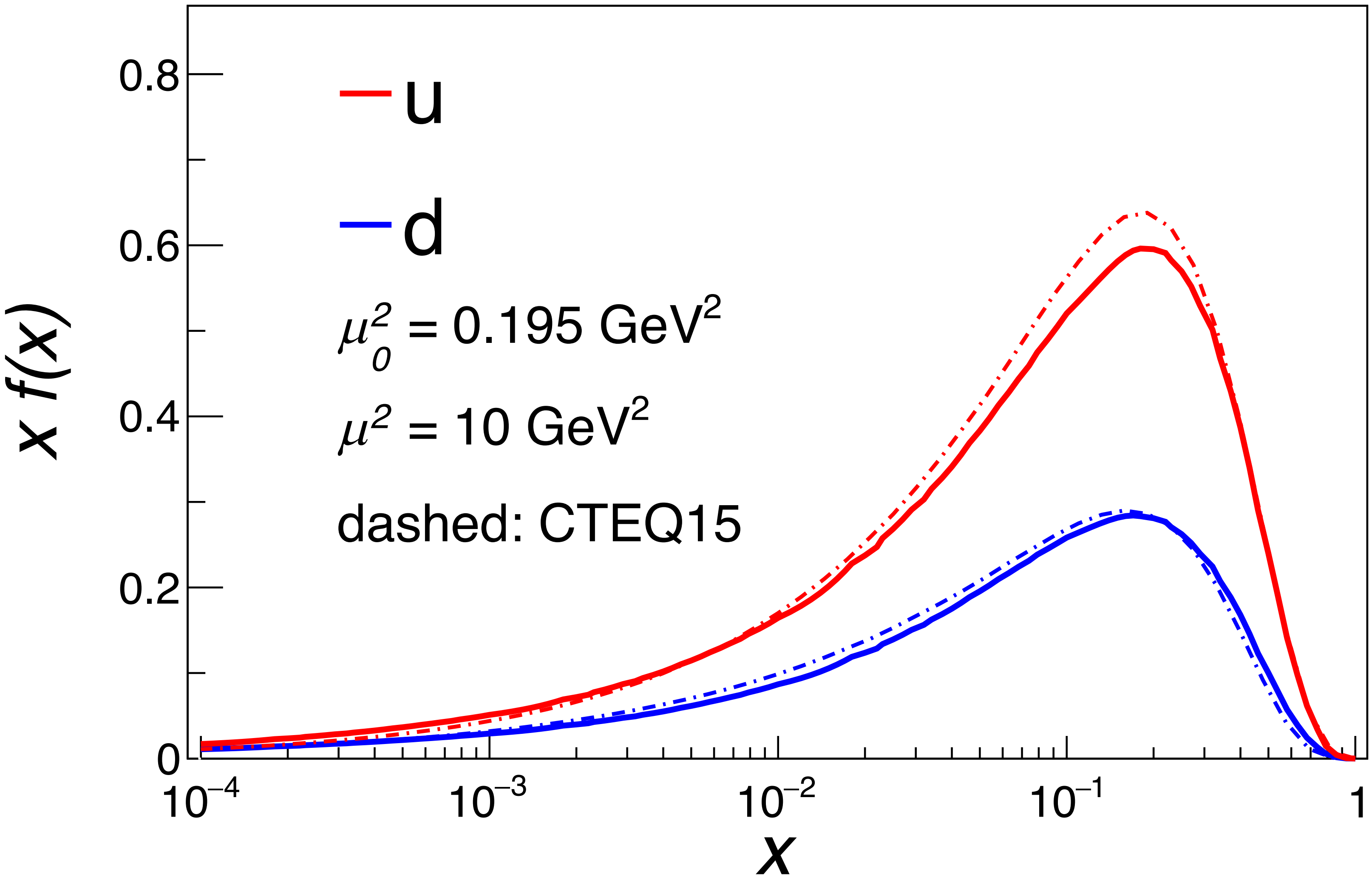


➢ **Parton distribution functions** (PDFs) are extracted from **DIS** processes.

PDFs encode the *distribution of longitudinal momentum and polarization* carried by the constituents

Parton Distribution Functions (PDF)

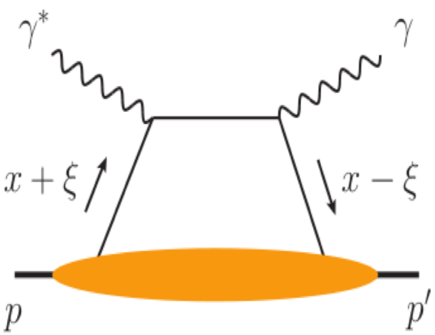
[Work in progress, C. Mondal, Siqi Xu, Jiangshan Lan, *et.al*]

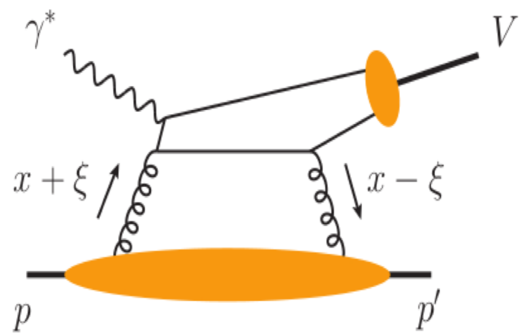


Use the NNLO DGLAP to evolve the PDF. Qualitative behavior is consistent with **the CTEQ 15 PDF**.

Generalized Parton Distribution Functions (GPDs)

- ***Deeply Virtual Compton Scattering* (*DVCS*)/ vector meson productions *experiment:***

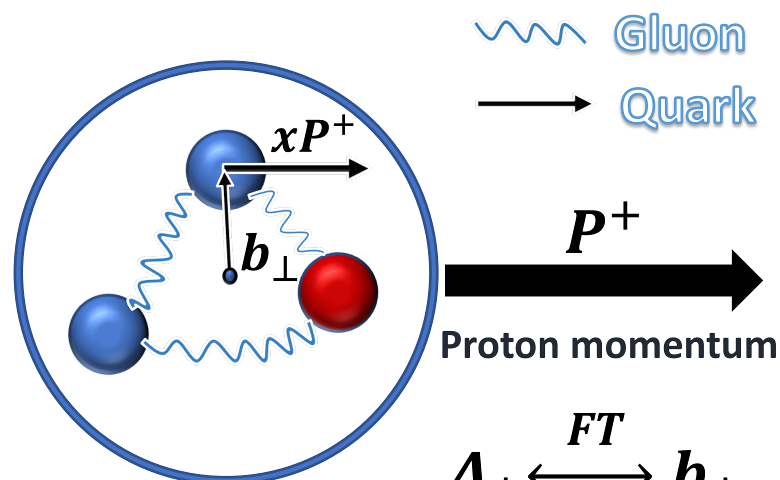




Encode the information about
three dimensional spatial structure
the spin and orbital angular momentum

- **GPDs** appear in **DVCS** processes.

- GPDs are functions of three variables :
  - ***Longitudinal momentum fraction*** $x = \frac{k^+}{P^+}$
  - ***Longitudinal momentum transfer --> skewness*** $\xi = \frac{\Delta^+}{P^+} = 0$
  - ***Square of total mom transfer***
    $$t = \Delta^2 = (P' - P)^2$$

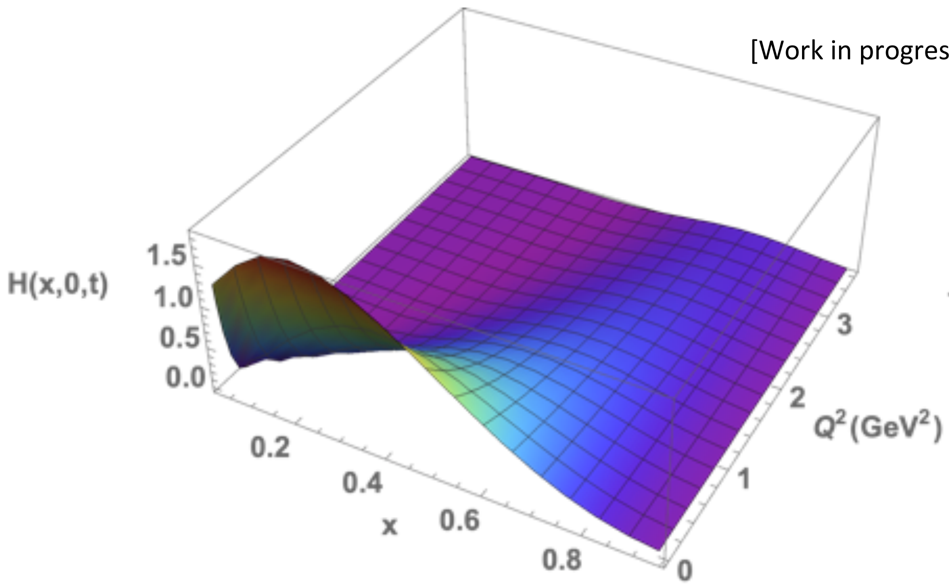


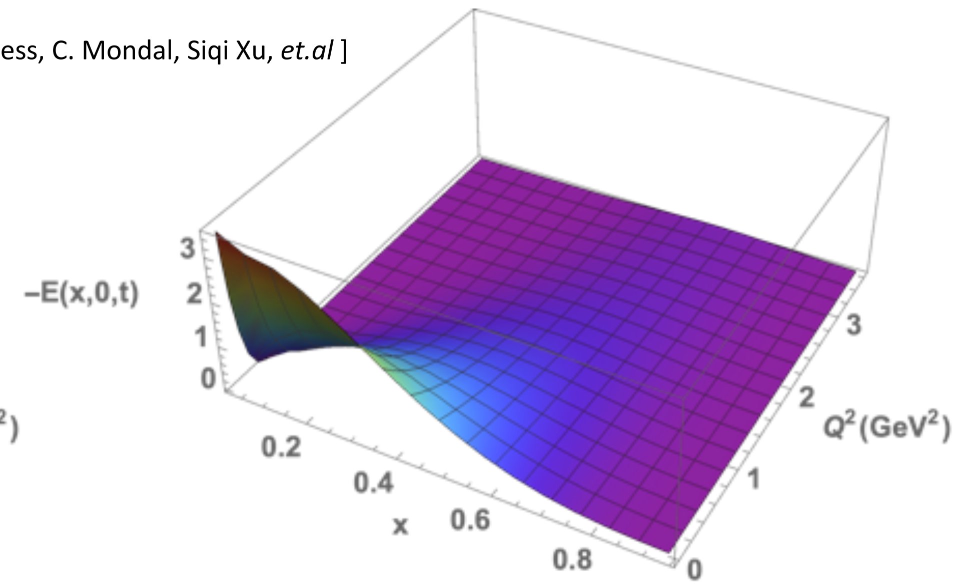
$$\Delta_\perp \overset{FT}{\longleftrightarrow} b_\perp$$

where the $b_\perp$ is transverse position of parton

Generalized Parton Distribution Functions (GPD)

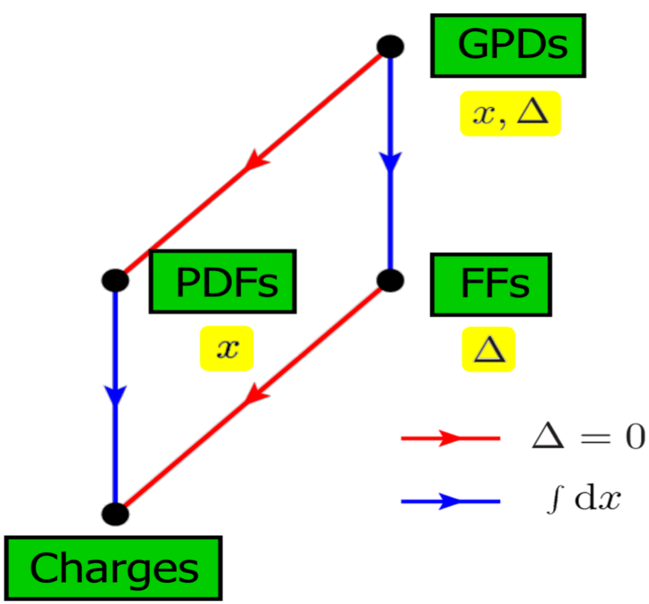
[Work in progress, C. Mondal, Siqi Xu, *et.al*]





With **increasing momentum transfer (t)**, the **peaks of distributions** shift to **larger x**;

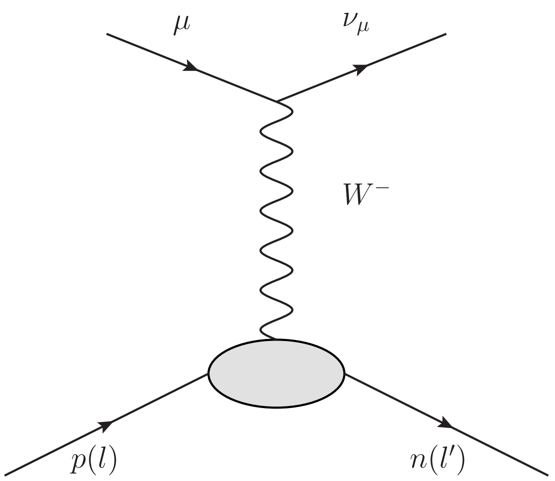
$$t = \Delta^2, x = \frac{k^+}{P^+}, \zeta = \frac{\Delta^+}{P^+} = 0$$



Axial Form Factor of The Proton

Axial form factor of the proton can be measured by ordinary muon capture (OMC)

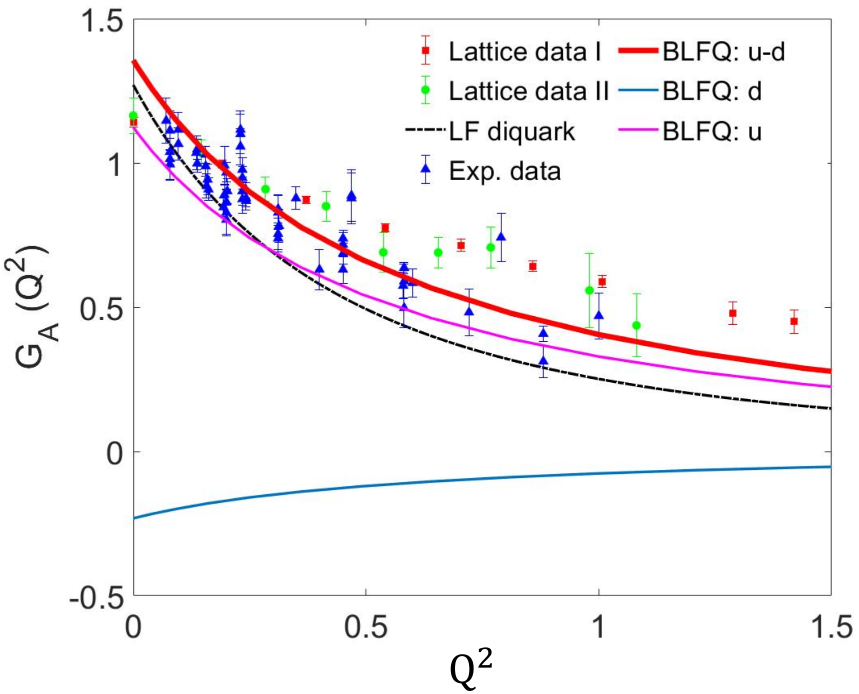
$$\mu^-(l) + p(r) \to \nu_\mu(l') + n(r')$$



Provide information on spin-isospin distributions

$$A_\mu^a = \bar{q}\gamma_\mu\gamma_5 T^a q$$

$$\langle N(p')|A_\mu^a|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m}G_P(t)\right]\gamma_5\frac{\tau^a}{2}u(p)$$



$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$

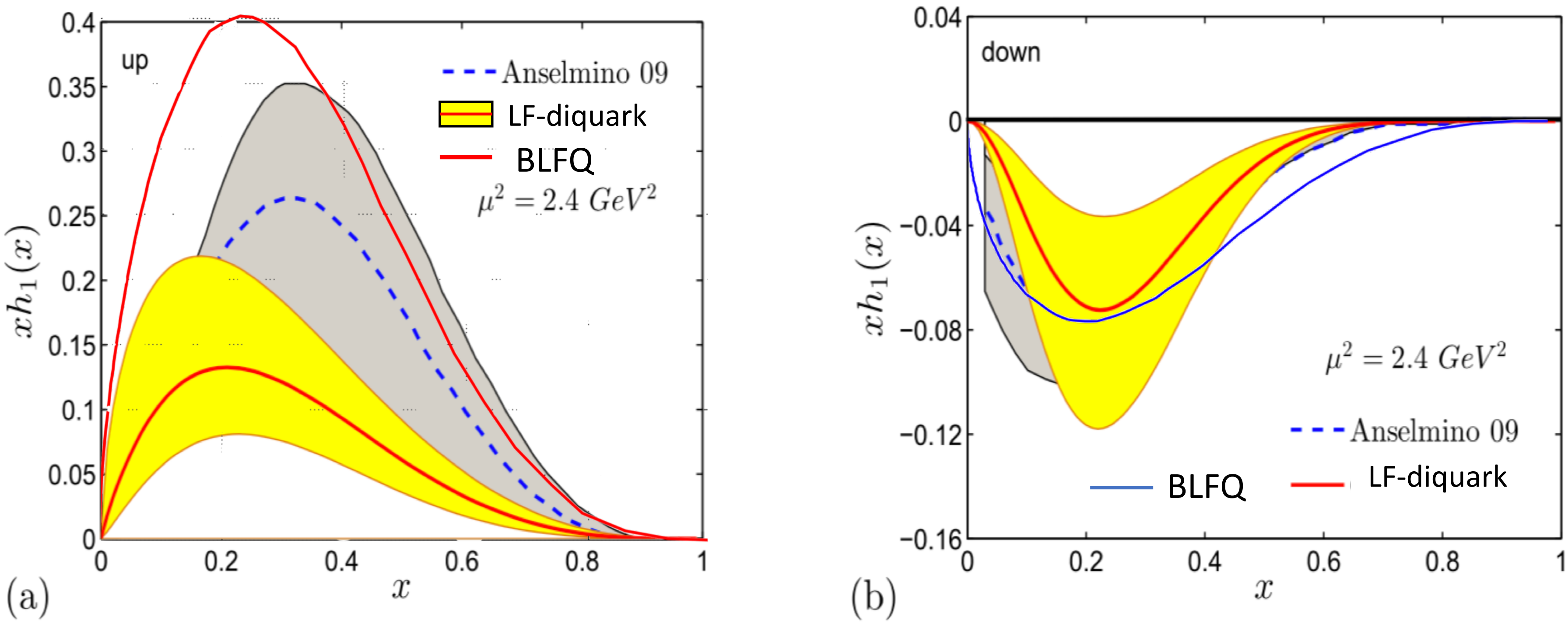
In our work, we consider only **the valence quark** contribution, And neglect **the sea quark** and **gluon** contributions.

[Work in progress, C. Mondal, Siqi Xu, *et.al*]
[Chandan Mondal, EPJC 2017]

Transversity PDF h_1

- Definition of Transversity PDF

$$h_1(x) \sim \int d^2p_\perp(\psi^+_+(x,p_\perp)\psi^-_-(x,p_\perp) + \psi^+_-(x,p_\perp)\psi^-_+(x,p_\perp))$$



[Tammy Maji, Dipankar Chakrabarti,PRD94,094020]

[Work in progress, C. Mondal, Siqi Xu, *et.al*]

Conclusion

- In the effective Hamiltonian, we include confining potential in both transverse and longitudinal directions and a One-Gluon-Exchange interaction.
- Consider the leading Fock sector (three active quark).
- We have calculated various observables - Form Factors, Axial Form Factors, PDFs, Transversity PDFs and GPDs:
  - Our results more or less agree with the experiment data & Global fits.
  - For the spin dependent distribution functions, the effects of sea quark and gluon need to be accounted.

Outlook

- Calculating other distribution functions : spin asymmetries ,GTMD, DPD…
- We will include the higher Fock sectors : $|qqqg\rangle, |qqq\bar{q}q\rangle$
- Investigate the spin structure of proton – spin decomposition.
- Mechanical properties, mass decomposition…