Is the $Y(2175)$ a strangeonium hybrid meson?

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Based on: PRD98(2018),096020; PRD100(2019),034012.
Collaborators: J. Ho, R. Berg, T. G. Steele, D. Harnett

Outline

1. Background of the $\phi(2170)/Y(2175)$ and hybrid mesons
2. Two methods of QCD Sum Rules: LSRs and GSRs
3. LSRs and GSRs investigations for the vector $\bar{s}gs$ hybrid mesons
4. Summary

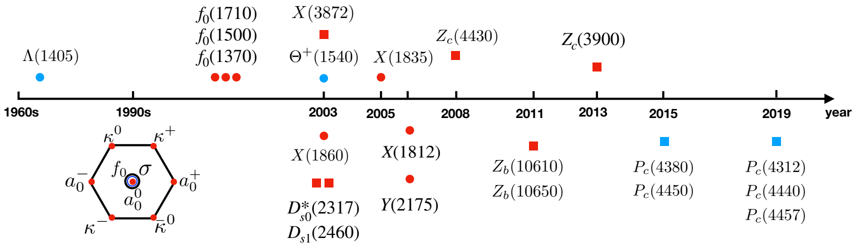
Searching for exotica

Light hadron sector:

- **Dibaryon**: Deuteron, H states, $d^*(2380)$.
- **Hybrid** candidates: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ (dispute).
- **Glueball** candidates: $a_0(980)$ and $f_0(980)$.
- **Tetraquark** candidates: light scalar mesons.

Heavy hadron sector: **breakthough in multiquarks!**

- $P_c(4380)$, $P_c(4312)$, $P_c(4440)$, $P_c(4457)$: hidden-charm pentaquark states.
- Plenty of **XYZ states**: candidates of molecules, tetraquarks, hybrids...



Y. R. Liu *et. al.*, Prog. Part. Nucl. Phys. 107 (2019) 237-320

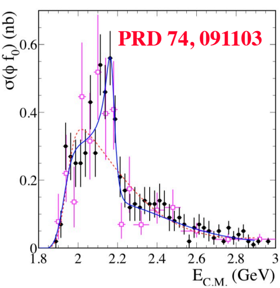
Searching for exotica

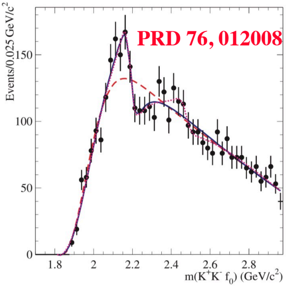
These XYZ and P_c states have inspired vigorous theoretical activity:

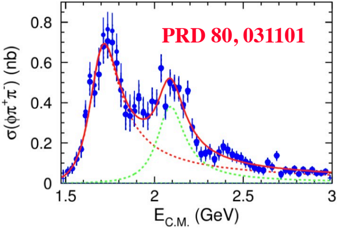
- Y. R. Liu, H. X. Chen, W. Chen, X. Liu, S. L. Zhu, Prog. Part. Nucl. Phys. 107 (2019) 237-320.
- H. X. Chen, W. Chen, X. Liu, S. L. Zhu, Phys. Rep. 639 (2016) 1-121.
- F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, Rev. Modern Phys. 90(2018) 015004.
- A. Esposito, A. Pilloni, A.D. Polosa, Phys. Rep. 668 (2016) 1-97.
- S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Modern Phys. 90 (1) (2018) 015003.
- **Multiquark (tetraquark, pentaquark, hadron molecule) configurations** were extensively investigated to understand the structures of the XYZ and P_c states.
- **Hybrid meson configuration** was much less studied. However, hybrid meson deserves more investigations due to the explicit gluonic degree of freedom in it!

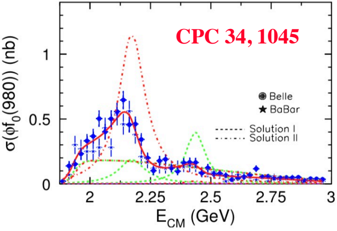
$\phi(2170)/Y(2175)$

Observed by BaBar in 2006 in the ISR process $e^+e^- \to \phi f_0(980)\gamma$:

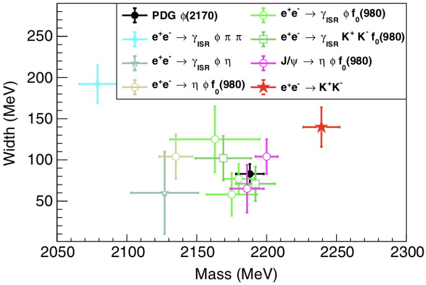








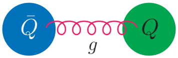
Theoretical interpretations of $\phi(2170)$

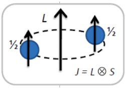


The present data can not determine the nature of $\phi(2170)$:

- $2\,^3D_1$ or $3\,^3S_1$ $s\bar{s}$ state: PRD85,074024; PRD99,074015;PLB657,49.
- Tetraquark $ss\bar{s}\bar{s}$ state: NPA791,106; PRD98, 014011;D78,034012;D99,036014.
- Molecular $\Lambda\bar{\Lambda}$ state: PRD96,074027; PRD87,054034.
- $\phi f_0(980)$ resonance with FSI: PRD80,054011.
- Three body ϕKK system: PRD78, 074031; PRD83, 116002.
- **Hybrid meson $\bar{s}gs$:** PLB650,390;PRD59, 034016;PRD100(2019),034012.

Hybrid meson: $[q\bar{q}]_{\mathbf{8_c}}$ + one excited gluonic field





$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10}^* \oplus \mathbf{27}$$

Model	$J^{PC}_{q\bar{q}'}$	J^{PC}_{g}	J^{PC}	Mass (GeV/c^2)
Bag [2, 3]	0^{-+}	1^{+-} (TE)	1^{--}	~ 1.7
	1^{--}	1^{+-} (TE)	$(0,1,2)^{-+}$	$\sim 1.3, 1.5, 1.9$
	0^{-+}	1^{--} (TM)	1^{+-}	heavier
	1^{--}	1^{--} (TM)	$(0,1,2)^{++}$	heavier
Flux tube [4, 5]	0^{-+}	1^{+-}	1^{--}	1.7-1.9
	1^{--}	1^{+-}	$(0,1,2)^{-+}$	1.7-1.9
	0^{-+}	1^{-+}	1^{++}	1.7-1.9
	1^{--}	1^{-+}	$(0,1,2)^{+-}$	1.7-1.9
Constituent gluon [6]/[7]	0^{-+}	1^{--}	1^{+-}	1.3-1.8 / 2.1
	1^{--}	1^{--}	$(0,1,2)^{++}$	1.3-1.8 / 2.2
	1^{+-}	1^{--}	$(0,1,2)^{-+}$	1.8-2.2 / 2.2
	$(0,1,2)^{++}$	1^{--}	$1^{--}, (0,1,2)^{--}, (1,2,3)^{--}$	1.8-2.2 / 2.3
Constituent gluon / LQCD [8, 9]	0^{-+}	1^{+-}	1^{--}	(2.3)
	1^{--}	1^{+-}	$(0,1,2)^{-+}$	(2.1, 2.0, 2.4)
	1^{+-}	1^{+-}	$(0,1,2)^{++}$	(>2.4)
	$(0,1,2)^{++}$	1^{+-}	$1^{+-}, (0,1,2)^{+-}, (1,2,3)^{+-}$	(>2.4)

arXiv:1208.5125

Hybrid meson: $[q\bar{q}]_{\mathbf{8_c}}$ + one excited gluonic field

Lightest hybrid supermultiplet: $1^{--}, (0, 1, 2)^{-+}$

J^{PC}	1^{--}	0^{-+}	1^{-+}	2^{-+}
$q\bar{q}$ operator	$\bar{q}\gamma_\mu q$	$\bar{q}\gamma_5 q$		$\bar{q}\gamma_5[\overleftrightarrow{D}_\mu, \overleftrightarrow{D}_\nu]q$
$n\,^{2S+1}L_J$	$n\,^3S_1$ $[q\bar{q}]$ **spin-triplet**	$n\,^1S_0$ $[q\bar{q}]$ **spin-singlet**		$n\,^1D_2$ $[q\bar{q}]$ **spin-singlet**
state	ρ, ω	π, η		
$\bar{q}gq$ operator	$\bar{q}\frac{\lambda^a}{2}\gamma^\nu\gamma_5\tilde{G}^a_{\mu\nu}q$ $[q\bar{q}]$ **spin-singlet**	$\bar{q}\frac{\lambda^a}{2}\gamma^\nu\tilde{G}^a_{\mu\nu}q$ $[q\bar{q}]$ **spin-triplet**	$\bar{q}\frac{\lambda^a}{2}\gamma^\nu\gamma_5 G^a_{\mu\nu}q$ $[q\bar{q}]$ **spin-triplet**	$\bar{q}\frac{\lambda^a}{2}\sigma^\alpha_\mu\gamma_5\tilde{G}^a_{\alpha\nu}q$ $[q\bar{q}]$ **spin-triplet**

For the same quantum numbers, the spins of the $[q\bar{q}]$ pair in the $q\bar{q}$ and $\bar{q}gq$ operators are different! Such difference results in distinct decay properties of the conventional and hybrid mesons!

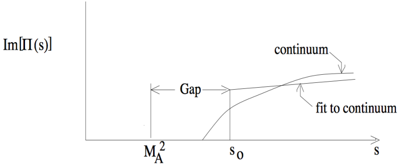
QCD Sum Rules

- Study two-point correlation function of current $J_\mu(x)$ with the same quantum numbers with hadron state:

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq\cdot x} \langle \Omega | T[J_\mu(x) J_\nu^\dagger(0)] | \Omega \rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle \Omega | J_\mu(x) | X \rangle \neq 0$
- Hadron level: described by the dispersion relation

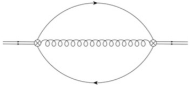
$$\begin{aligned} \Pi(q^2) &= \frac{(q^2)^N}{\pi} \int \frac{\mathrm{Im}\Pi(s)}{s^N(s - q^2 - i\epsilon)} ds + \sum_{n=0}^{N-1} b_n (q^2)^n, \\ \rho(s) &= \frac{1}{\pi} \mathrm{Im}\Pi(s) = \sum_n \delta(s - m_n^2) \langle 0 | J | n \rangle \langle n | J^\dagger | 0 \rangle \end{aligned}$$



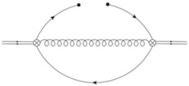
Hybrid Sum Rules

The hybrid interpolating currents:

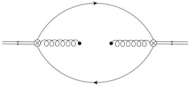
$$
\begin{aligned}
J_\mu &= g_s \bar{s} \frac{\lambda^a}{2} \gamma^\nu G^a_{\mu\nu} s, \quad & J^{PC} &= 1^{-+}, 0^{++}, \\
J_\mu &= g_s \bar{s} \frac{\lambda^a}{2} \gamma^\nu \gamma_5 G^a_{\mu\nu} s, \quad & J^{PC} &= 1^{+-}, 0^{--}, \\
J_{\mu\nu} &= g_s \bar{s} \frac{\lambda^a}{2} \sigma^\alpha_\mu \gamma_5 G^a_{\alpha\nu} s, \quad & J^{PC} &= 2^{-+}, 1^{++}, 1^{-+}, 0^{-+}, \\
\tilde{J}_\mu &= g_s \bar{s} \frac{\lambda^a}{2} \gamma^\nu \tilde{G}^a_{\mu\nu} s, \quad & J^{PC} &= 1^{++}, 0^{-+}, \\
\tilde{J}_\mu &= g_s \bar{s} \frac{\lambda^a}{2} \gamma^\nu \gamma_5 \tilde{G}^a_{\mu\nu} s, \quad & J^{PC} &= 1^{--}, 0^{+-}, \\
\tilde{J}_{\mu\nu} &= g_s \bar{s} \frac{\lambda^a}{2} \sigma^\alpha_\mu \gamma_5 \tilde{G}^a_{\alpha\nu} s, \quad & J^{PC} &= 2^{++}, 1^{-+}, 1^{++}, 0^{++}.
\end{aligned}
$$



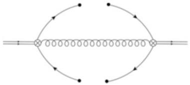
(a) Diagram I (LO perturbation theory)



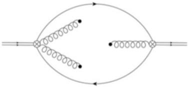
(b) Diagram II (dimension-three)



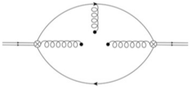
(c) Diagram III (dimension-four)



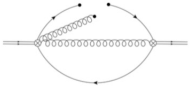
(d) Diagram IV (dimension-six)



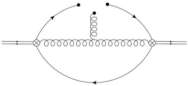
(e) Diagram V (dimension-six)



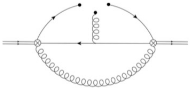
(f) Diagram VI (dimension-six)



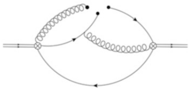
(g) Diagram VII (dimension-five)



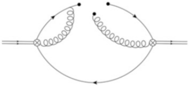
(h) Diagram VIII (dimension-five)



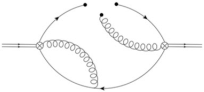
(i) Diagram IX (dimension-five)



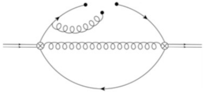
(j) Diagram X (dimension-five)



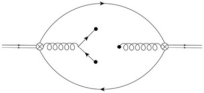
(k) Diagram XI (dimension-five)



(l) Diagram XII (dimension-five)



(m) Diagram XIII (dimension-five)



(n) Diagram XIV (dimension-five)

Correlation functions for all $\bar{s}gs$ hybrids

Two-point correlation functions for all channels:

$$\Pi\left(q^2\right) = \alpha_s\left(A_1 q^6 + A_2 m^2 q^4\right)\log\left(\frac{-q^2}{\mu^2}\right) + \left(A_4 q^2\langle\alpha G^2\rangle + \alpha_s(A_3 q^2 m\langle\bar{q}q\rangle + A_5\langle\bar{q}q\rangle^2 + A_6\langle g^3 G^3\rangle + A_7 m\langle g\bar{q}\sigma Gq\rangle)\right)\log\left(\frac{-q^2}{\mu^2}\right) + \alpha_s\left(B_1 q^6 + B_2 m^2 q^4 + B_3 q^2 m\langle\bar{q}q\rangle + B_4 q^2\langle\alpha G^2\rangle + B_5\langle\bar{q}q\rangle^2 + B_6\langle g^3 G^3\rangle + B_7 m\langle g\bar{q}\sigma Gq\rangle\right)$$

	0^{++}	1^{-+}	0^{--}	1^{+-}	0^{-+}	1^{++}	0^{+-}	1^{--}
A_1	$-\frac{1}{480\pi^3}$	$-\frac{1}{240\pi^3}$	$-\frac{1}{480\pi^3}$	$-\frac{1}{240\pi^3}$	$-\frac{1}{480\pi^3}$	$-\frac{1}{240\pi^3}$	$-\frac{1}{480\pi^3}$	$-\frac{1}{240\pi^3}$
A_2	0	$\frac{1}{12\pi^3}$	$\frac{1}{16\pi^3}$	$\frac{5}{48\pi^3}$	0	$\frac{1}{12\pi^3}$	$\frac{1}{16\pi^3}$	$\frac{5}{48\pi^3}$
A_3	$\frac{1}{3\pi}$	$-\frac{2}{9\pi}$	$-\frac{1}{3\pi}$	$-\frac{4}{9\pi}$	$\frac{1}{3\pi}$	$-\frac{2}{9\pi}$	$-\frac{1}{3\pi}$	$-\frac{4}{9\pi}$
A_4	$\frac{1}{24\pi}$	$-\frac{1}{36\pi}$	$\frac{1}{24\pi}$	$-\frac{1}{36\pi}$	$-\frac{1}{24\pi}$	$\frac{1}{36\pi}$	$-\frac{1}{24\pi}$	$\frac{1}{36\pi}$
A_5	0	0	0	0	0	0	0	0
A_6	0	0	0	0	0	0	0	0
A_7	$\frac{1}{9\pi}$	0	$\frac{11}{72\pi}$	$-\frac{19}{72\pi}$	$-\frac{1}{9\pi}$	0	$-\frac{11}{72\pi}$	$\frac{19}{72\pi}$

Correlation functions for all $\bar{s}gs$ hybrids

Coefficients of the finite terms:

	0^{++}	1^{-+}	0^{--}	1^{+-}	0^{-+}	1^{++}	0^{+-}	1^{--}
B_1	$\frac{97}{19200\pi^3}$	$\frac{39}{3200\pi^3}$	$\frac{97}{19200\pi^3}$	$\frac{39}{3200\pi^3}$	$\frac{19}{6400\pi^3}$	$\frac{77}{9600\pi^3}$	$\frac{19}{6400\pi^3}$	$\frac{77}{9600\pi^3}$
B_2	$\frac{1}{32\pi^3}$	$-\frac{7}{32\pi^3}$	$-\frac{55}{384\pi^3}$	$-\frac{109}{384\pi^3}$	$\frac{1}{32\pi^3}$	$-\frac{13}{96\pi^3}$	$-\frac{31}{384\pi^3}$	$-\frac{23}{128\pi^3}$
B_3	$-\frac{1}{2\pi}$	$\frac{7}{27\pi}$	$\frac{1}{6\pi}$	$\frac{17}{27\pi}$	$\frac{1}{6\pi}$	$-\frac{5}{27\pi}$	$-\frac{1}{2\pi}$	$-\frac{7}{27\pi}$
B_4	$-\frac{13}{144\pi}$	$\frac{11}{216\pi}$	$-\frac{13}{144\pi}$	$\frac{11}{216\pi}$	$-\frac{5}{144\pi}$	$\frac{7}{216\pi}$	$-\frac{5}{144\pi}$	$\frac{7}{216\pi}$
B_5	$-\frac{4\pi}{3}$	$\frac{4\pi}{9}$	$\frac{4\pi}{3}$	$-\frac{4\pi}{9}$	0	$-\frac{8\pi}{9}$	0	$-\frac{8\pi}{9}$
B_6	$-\frac{1}{192\pi^2}$	$\frac{1}{192\pi^2}$	$-\frac{1}{192\pi^2}$	$\frac{1}{192\pi^2}$	$\frac{5}{192\pi^2}$	$-\frac{5}{192\pi^2}$	$\frac{5}{192\pi^2}$	$-\frac{5}{192\pi^2}$
B_7	$-\frac{461}{1728\pi}$	$-\frac{83}{1728\pi}$	$-\frac{731}{1728\pi}$	$\frac{1019}{1728\pi}$	$-\frac{217}{1728\pi}$	$\frac{265}{1728\pi}$	$\frac{41}{1728\pi}$	$\frac{71}{1728\pi}$

Method I: Laplace Sum Rules Analyses (LSRs)

- Borel transform: suppress the contributions of continuum and higher excited states in correlation functions:

$$\Pi_k(M_B^2) = M_B^2 \lim_{\substack{N,Q^2\to\infty \\ M_B^2=Q^2/N}} \frac{\left(-Q^2\right)^N}{\Gamma(N)} \left(\frac{d}{dQ^2}\right)^N \left\{(-Q^2)^k \Pi(Q^2)\right\},$$

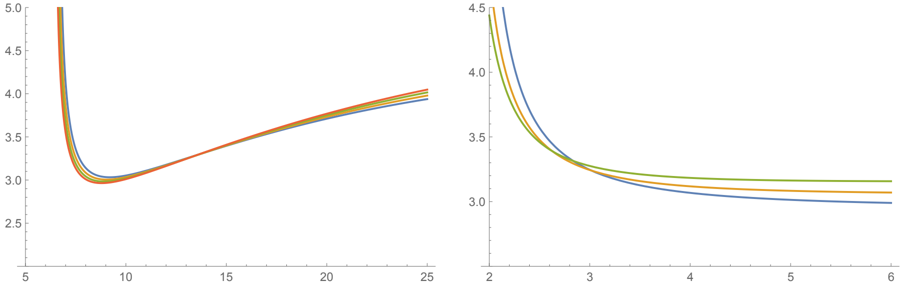
- Quark-hadron duality: Laplace Sum Rules with QCD spectral function

$$\mathcal{L}_k\left(s_0, M_B^2\right) = \int_{4m_s^2}^{s_0} ds e^{-s/M_B^2} \rho(s) s^k = f_X^2 m_X^{2k} e^{-m_X^2/M_B^2}.$$

where s_0 is continuum threshold and M_B is the Borel parameter.

Method I: Laplace Sum Rules Analyses

For the vector $\bar{s}Gs$ hybrid meson with $J^{PC}=1^{--}$: $M_B^2 \in [3.2, 4.2]\text{GeV}^2$



The extracted mass $\mathbf{m_X \sim 3.1 - 3.3}$ **GeV** is much higher than the mass of $\phi(2170)/Y(2175)$!

Method II: Gaussian sum rules analyses (GSRs)

- The unsubtracted GSRs of integer weight k are defined as

$$
\begin{aligned}
G_k(\hat{s},\,\tau) =& \sqrt{\frac{\tau}{\pi}} \lim_{\substack{N,\Delta^2\to\infty \\ \tau=\Delta^2/(4N)}} \frac{\left(-\Delta^2\right)^N}{\Gamma(N)} \times \\
& \left(\frac{d}{d\Delta^2}\right)^N \left\{ \frac{(\hat{s}+i\Delta)^k \Pi(-\hat{s}-i\Delta) - (\hat{s}-i\Delta)^k \Pi(-\hat{s}+i\Delta)}{i\Delta} \right\} \\
=& \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty t^k e^{-\frac{(\hat{s}-t)^2}{4\tau}} \frac{1}{\pi} \mathrm{Im}\Pi^{\mathrm{QCD}}(t)\,\mathrm{d}t\,,
\end{aligned}
$$

  providing a fundamentally different weighting of the spectral function that makes them well-suited to analyzing distributed resonance strength hadron models.

- QCD Gaussian sum-rules:

$$
G^{\mathrm{QCD}}(\hat{s},\,\tau,\,s_0) \equiv \frac{1}{\sqrt{4\pi\tau}} \int_0^{s_0} e^{-\frac{(\hat{s}-t)^2}{4\tau}} \frac{1}{\pi} \mathrm{Im}\Pi^{\mathrm{QCD}}(t) dt\,,
$$

  where the GSRs have a duality interval of width $\sim \sqrt{2\tau}$ near $\hat{s}$. By varying the free parameter $\hat{s}$, GSRs can probe a wide region of the hadronic spectral function with the same sensitivity as the ground state region.

Gaussian sum rules analyses

The normalized GSRs (NGSRs) are defined as:

$$N^{\text{QCD}}(\hat{s},\,\tau,\,s_0)=\frac{\frac{1}{\sqrt{4\pi\tau}}\int_0^{s_0}e^{-\frac{(\hat{s}-t)^2}{4\tau}}\rho^{\text{had}}(t)\,\mathrm{d}t}{\int_0^{s_0}\rho^{\text{had}}(t)\,\mathrm{d}t}.$$

In a double-narrow resonance model

$$\rho^{\text{had}}(t)=f_1^2\delta\left(t-m_1^2\right)+f_2^2\delta\left(t-m_2^2\right)\,.$$

The normalized GSRs

$$N^{\text{had}}\left(\hat{s},\,\tau\right)=\frac{1}{\sqrt{4\pi\tau}}\left(re^{-\frac{(\hat{s}-m_1^2)^2}{4\tau}}+(1-r)e^{-\frac{(\hat{s}-m_2^2)^2}{4\tau}}\right),$$

where the normalized couplings are defined as

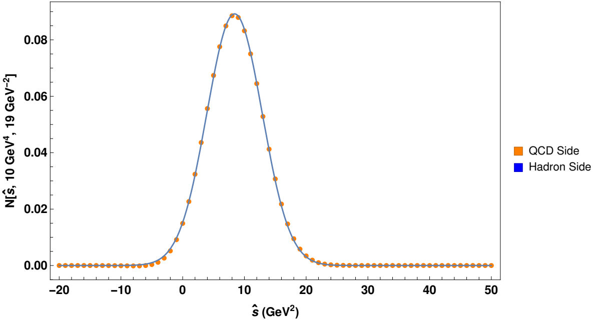
$$r=\frac{f_1^2}{f_1^2+f_2^2}\,,1-r=\frac{f_2^2}{f_1^2+f_2^2}\,,0\leq r\leq 1.$$

Gaussian sum-rules analyses

Fitting the $\hat{s}$ dependence of the QCD prediction and hadronic model

$$\chi^2\left(r, m_2, s_0\right) = \sum_{\hat{s}_{min}}^{\hat{s}_{max}} \left[N^{\text{had}}\left(\hat{s},\, \tau\right) - N^{\text{QCD}}\left(\hat{s},\, \tau\, s_0\right)\right]^2 ,$$

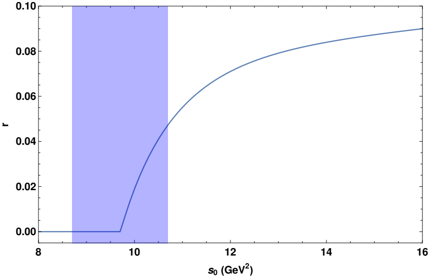
with the modelled resonance $m_1 = m_{Y(2175)} = 2.188$ GeV.

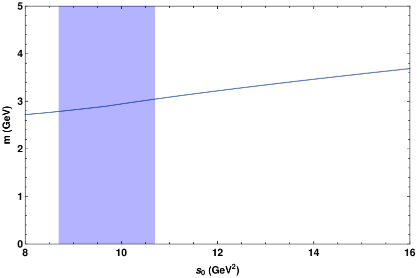


Gaussian sum-rules analyses

The resulting predictions:

$$s_0^{\rm opt} = 9.7 \pm 1.0\,\text{GeV}^2$$
$$m_2 = m_{\rm fit} = 2.90 \pm 0.16\,\text{GeV}$$
$$r \le 0.033\,.$$





No evidence for a significant strangeonium hybrid component of the $\phi(2170)$, which is consistent with the LQCD calculation(PRD84(2011),074023).

Hybrid decay properties

Hybrid meson decay selection Rules:

- Kinematically allowed.
- Conversation of $I^G J^{PC}$.
- $[s\bar{s}]$ quark spin symmetry: the spin of $[s\bar{s}]$ pair favors to be preserved in process $\bar{s}gs \to \phi + X$.
- S+P-wave selection rule: hybrids cannot decay into two identical mesons (S+S-wave, P+P-wave).

For $\phi(2170)$, the following decay modes are important:

- Branching fractions of $\phi(2170) \to \phi + f_0(980)$, $\phi(2170) \to \phi + \eta$.
- Open-strange channel: $\phi(2170) \to K^+ + K^-$.

BESIII's new measurment in $e^+e^- \to K^+K^-$

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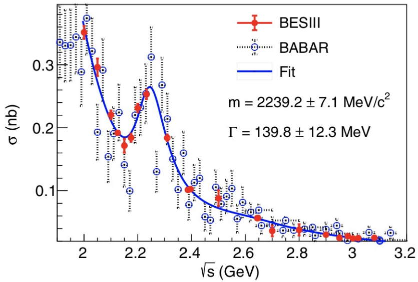

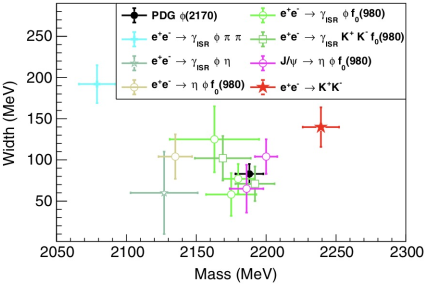

- **If this structure can be identified with $\phi(2170)$**, the observed K^+K^- decay mode would **disfavour** the $3\,^3S_1$ $s\bar{s}$ meson, **strangeonium hybrid**, and $ss\bar{s}\bar{s}$ tetraquark interpretations.
- **If this structure can not be identified with $\phi(2170)$**, the lack of K^+K^- decay mode would **disfavour** the $2\,^3D_1$ strangeonium meson and $\Lambda\bar{\Lambda}$ interpretations.
- Further experimental and theoretical studies are needed.

Summary

- We have calculated the two-point correlation functions of $\bar{s}Gs$ hybrid mesons in various channels.
- The LSRs analyses show that the vector $\bar{s}Gs$ hybrid is much heavier than the $\phi(2170)/Y(2175)$.
- In a double-narrow resonance model, the GSRs are used to probe the distributed resonance strength in the hadronic spectral function.
- We fit NGSRs of the QCD prediction and hadronic model to predict (m_2, r, s_0) and find the relative coupling strength $r < 5\%$.
- Our investigations don't support the $\bar{s}gs$ hybrid interpretation for $\phi(2170)/Y(2175)$.
- $\phi(2170) \to K^+K^-$ decay mode is important to distinguish its nature!

Thank you very much!