Spectroscopy of a_1 mesons from lattice QCD with the truncated overlap fermions

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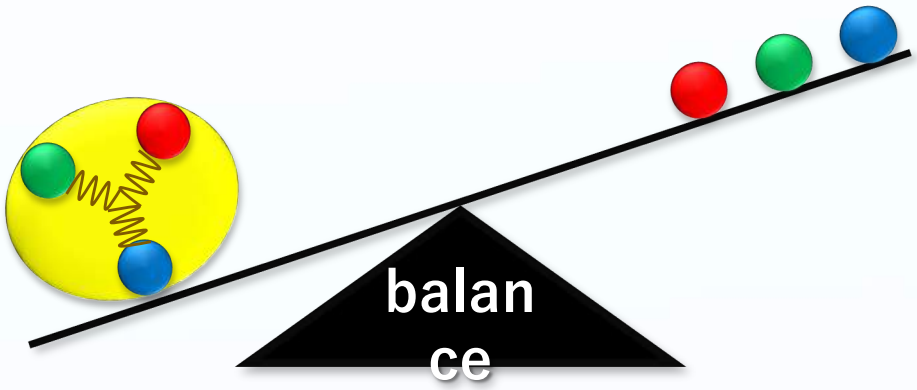
Mass of Origin

Nucleon is made from three current quarks.
But …



Nucleon mass
~ 1000 MeV

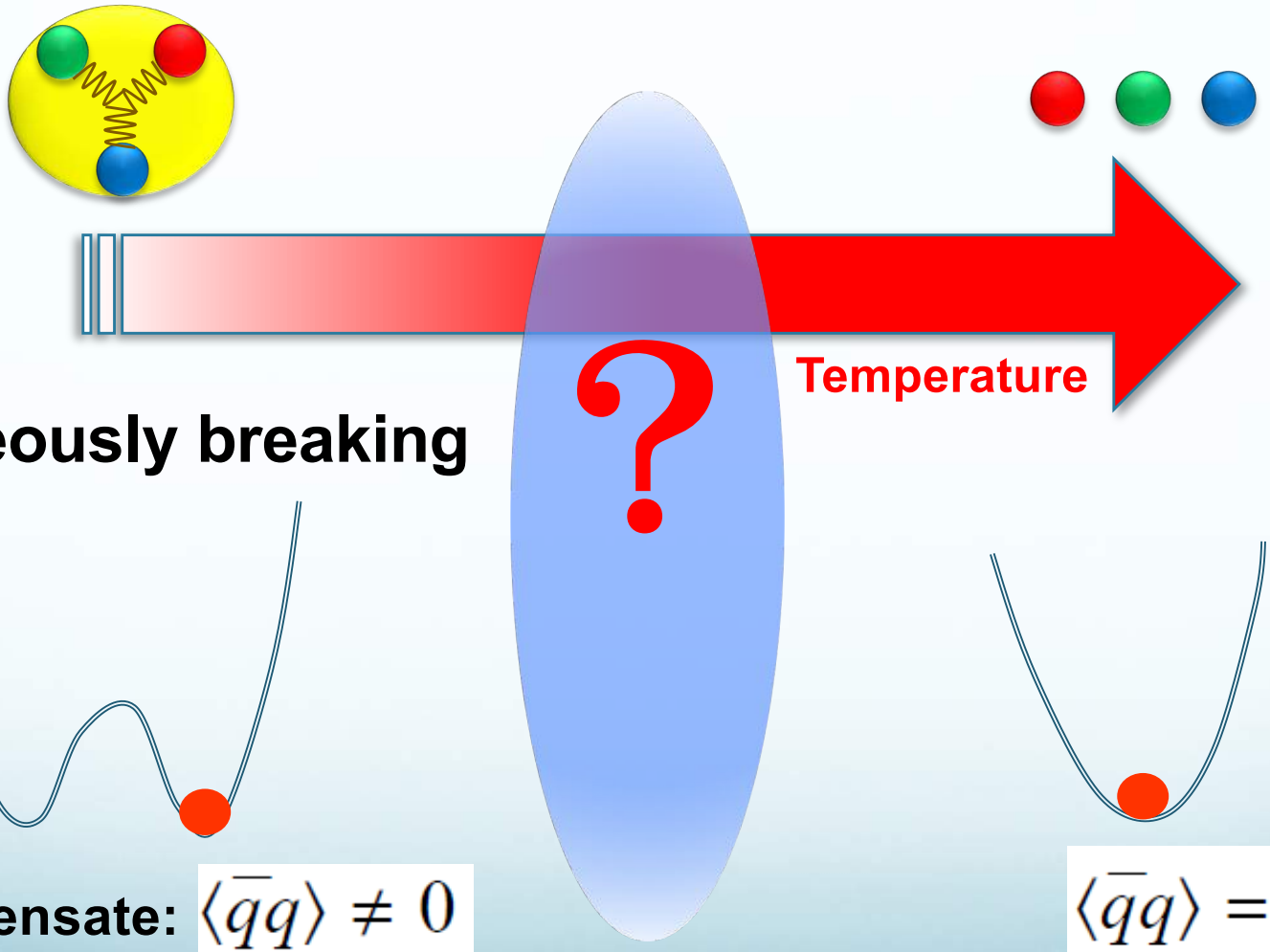
(Quark mass) × 3
~ 10 MeV

Keyword : Spontaneously chiral symmetry breaking in QCD

Chiral Symmetry

Hadron phase (Our World)

Quark-Gluon Plasma phase



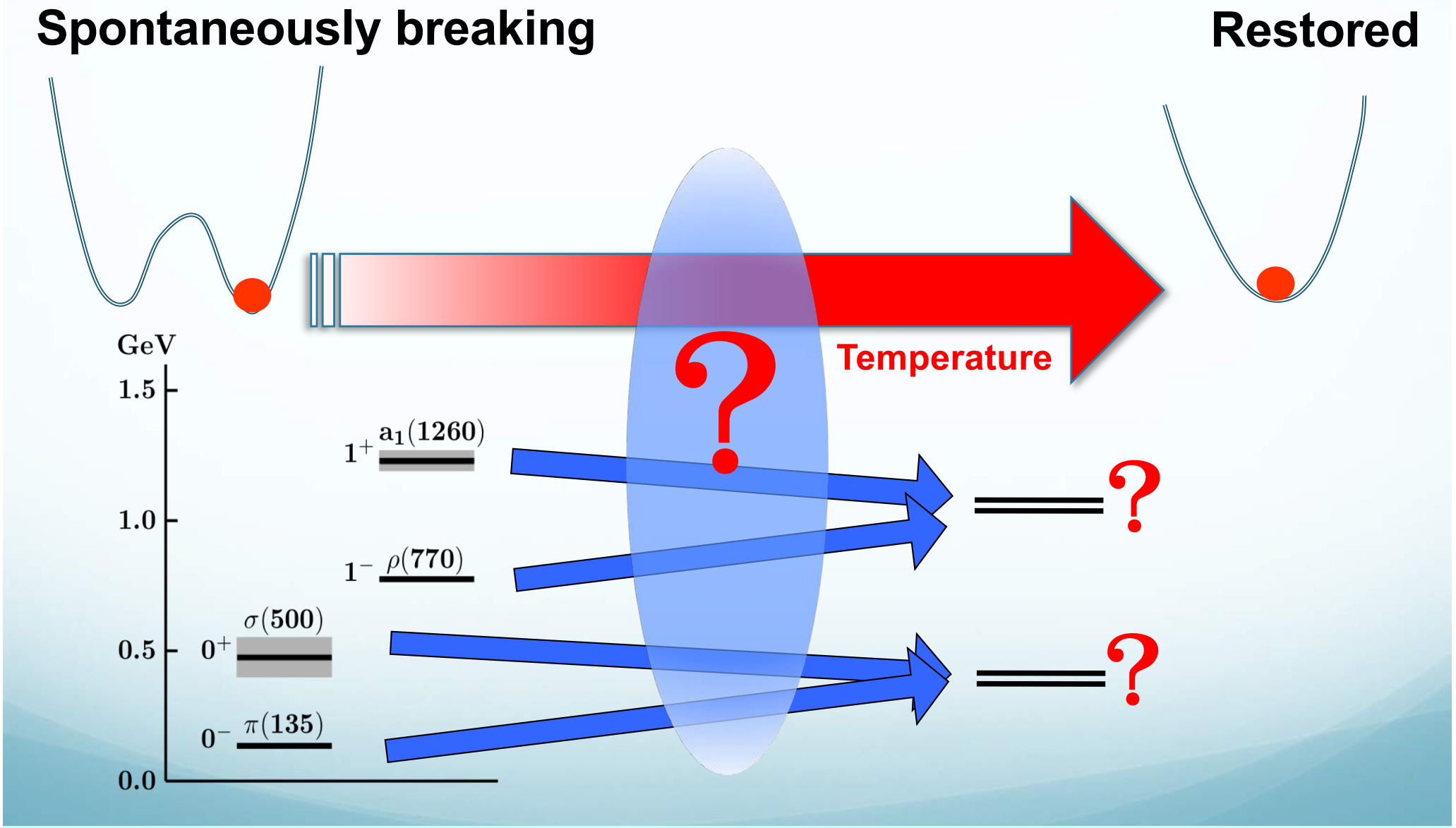
Spontaneously breaking

Restored

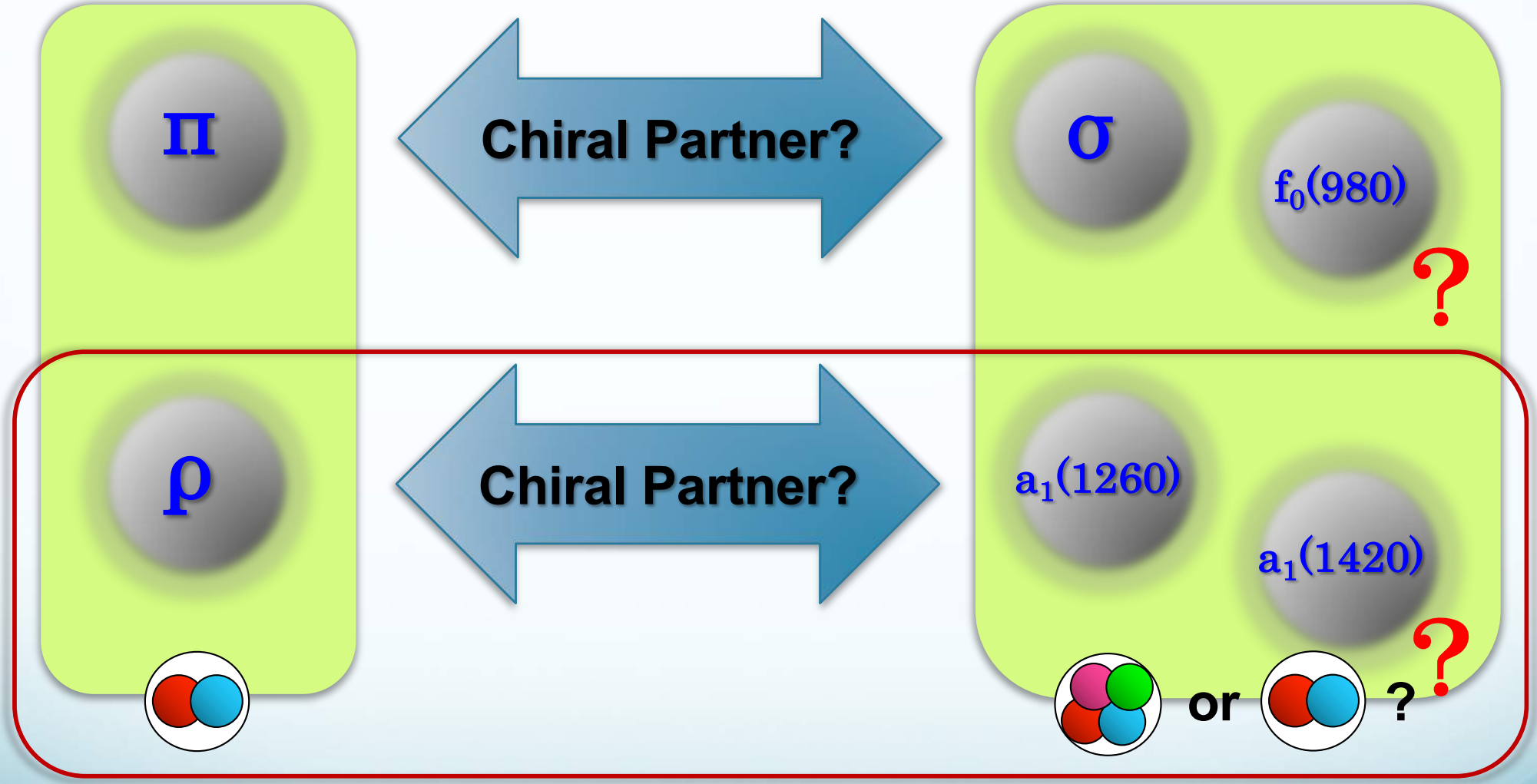
Quark condensate: $\langle \bar{q}q \rangle \neq 0$

$\langle \bar{q}q \rangle = 0$

Chiral Partners



Really Chiral Partners?



This Talk

a_1 mesons from effective theories

- **NJL model**
  Nambu & Jona-Lasinio, PR124, 246(1961)
  Takizawa, Kubodera & Myhrer, PLB261, 221(1991)

- **Holographic QCD**
  Sakai & Sugimoto, PTP113, 843(2005); PTP114, 1083(2005)
  Nawa, Suganuma & Kojo, PRD75, 086003(2007)

- **Hidden Local symmetry**
  Harada, Sasaki & Weise, PRD78, 114003(2008)

a_1 mesons from experiments

Particle Data Group (2018)

$a_1(1260)$: Mass = 1230(40) MeV (Summary Table)

$a_1(1640)$: Mass = 1654(19) MeV (Particle Listings)

Report in HADRON2013

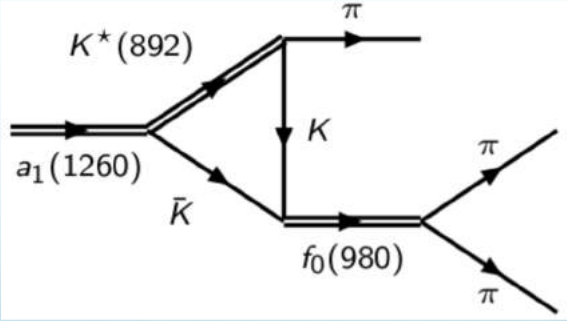
COMPASS, PRL115, 082001(2015)

$a_1(1420)$: Mass = 1414(15) MeV (Particle Listings from PDG2016)

$\bar{q}\bar{q}qq$ **?** H-X. Chen. et al., PRD91, 094022(2015)

Report in HADRON2017

COMPASS, "The $a_1(1420)$ – evidence for three-body effects"



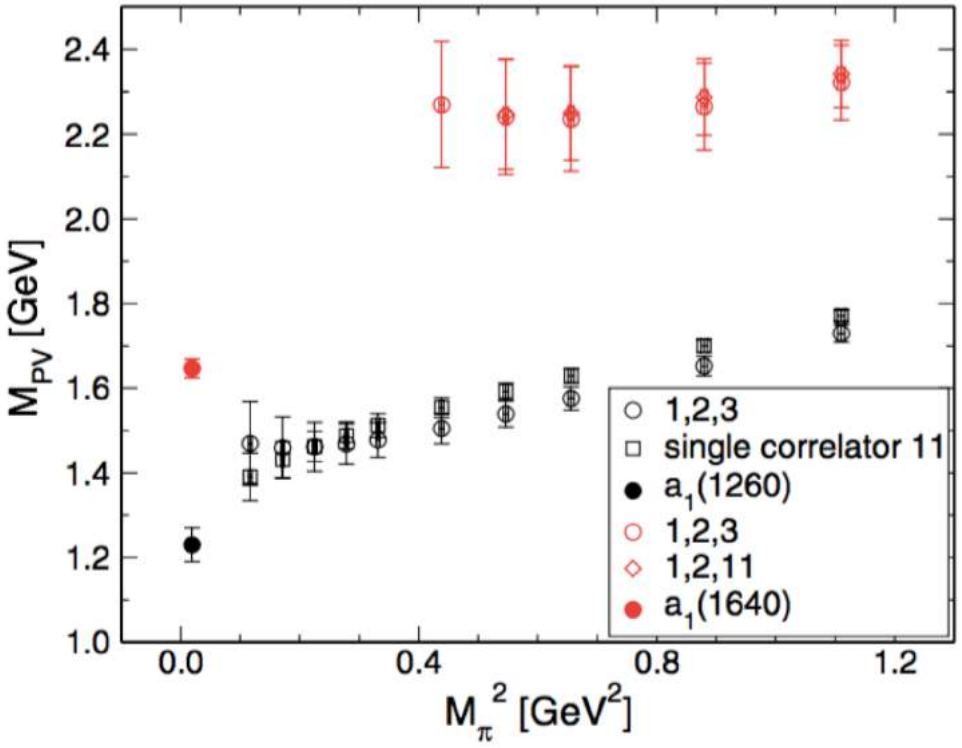
$a_1(1420)$ is resonance of three-body decay for $a_1(1260)$?

$a_1(1930)$, $a_1(2095)$, $a_1(2270)$ maybe exist ? (Further States in Particle Listings)

a_1 mesons from lattice QCD

- Wingate, DeGrand, Collins & Heller, PRL74, 23(1995)
  Dynamical configuration with staggered fermions
  Calculation of propagator: Wilson fermions
  a_1 meson's mass: 1250(80) MeV
- Gattringer, Glozman, Lang, Mohler & Prelovsek, PRD78, 034501(2008)
  Quenched
  Lüscher-Weisz gauge action
  Chirally Improved Dirac operator
  $\bar{q}q$ operator

Approach to the a_1 meson with lattice QCD is still developing !



Lattice Chiral Symmetry

In lattice QCD, the chiral symmetry cannot be respected due to the fermion-doubling problem.

$$\gamma_5 D + D\gamma_5 = 0$$

- **(improved) Wilson fermion : explicitly break the symmetry**

But there is the "lattice" chiral symmetry so-called the Ginsparg-Wilson relation.

P.H. Ginsparg & K.G. Wilson,
Phys. Rev. D25 (1982)

$$\gamma_5 D + D\gamma_5 = D\gamma_5 D$$

- **Truncated overlap fermion : satisfied with the GW relation (Additional fifth dimension)**

Truncated overlap fermions

$$D_{TOF} = \epsilon_s P_{st} \left(D_{PV}^{-1}\right)_{tu} \left(D_{DWF}\right)_{uv} P_{vw} \epsilon_w$$

A. Boriçi, Nucl. Phys. Proc. Suppl. 83, 771 (2000)

※ Indexes represent fifth dimensional sites.

$$D_{DWF} = \begin{pmatrix} D_{WF}+1 & -P_L & 0 & \cdots & 0 & m_f P_R \\ -P_R & D_{WF}+1 & -P_L & \ddots & \vdots & 0 \\ 0 & -P_R & D_{WF}+1 & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & -P_L & 0 \\ 0 & \vdots & \ddots & -P_R & D_{WF}+1 & -P_L \\ m_f P_L & 0 & \cdots & 0 & -P_R & D_{WF}+1 \end{pmatrix}$$

fifth dimension

Wilson Fermion:

$$D_{WF}(x,y) = (4 - M_5)\,\delta_{x,y} - \frac{1}{2}\sum_{\mu=1}^{4}\left[(1-\gamma_\mu)\,U_\mu(x)\delta_{y,x+\hat{\mu}} + (1+\gamma_\mu)\,U_\mu^\dagger(y)\delta_{y,x-\hat{\mu}}\right]$$

Projection to four-dimensional space

$$\begin{aligned} D_{PV}(x,y) &= D_{DWF}(m_f = 1)(x,y)\ , \\ P_{st} &= P_L\delta_{s,t} + P_R\delta_{s+1,t} + P_R\delta_{s,N_5}\delta_{t,1}\ , \quad \epsilon_s = \delta_{1,s} \end{aligned}$$

Truncated overlap fermions

$$D_{TOF} = \epsilon_s P_{st} \left(D_{PV}^{-1}\right)_{tu} \left(D_{DWF}\right)_{uv} P_{vw} \epsilon_w$$

A. Boriçi, Nucl. Phys. Proc. Suppl. 83, 771 (2000)

$$H_w = \gamma_5 \frac{D_{WF}}{D_{WF} + 2}$$

$$D_{TOF} = \frac{1 + m_f a}{2} + \frac{1 - m_f a}{2} \gamma_5 \frac{(1 + H_w)^{N_5} - (1 - H_w)^{N_5}}{(1 + H_w)^{N_5} + (1 - H_w)^{N_5}}$$

$$V = \gamma_5 \, \mathrm{sgn}\left(H_w\right)$$

Lattice size of fifth dimension : $N_5 \rightarrow$ infinity
quark mass : $m_f \rightarrow$ zero

$$D_{TOF} = \frac{1}{Ra} \left[1 + V\right] \ , \quad \gamma_5 V \gamma_5 = V^\dagger \ , \quad V V^\dagger = 1$$

GW relation : $\gamma_5 D + D \gamma_5 = D \gamma_5 D$

COST (Truncated overlap fermions)
~ COST (Wilson fermions) × Order(100) !

Simulation Setup

For Gauge configuration

Two-flavor quenched Lattice QCD

- Plaquette gauge action
  - Lattice size : 8*8*8*24
  - Lattice spacing : a = 0.189(2) fm
  - β = 5.7

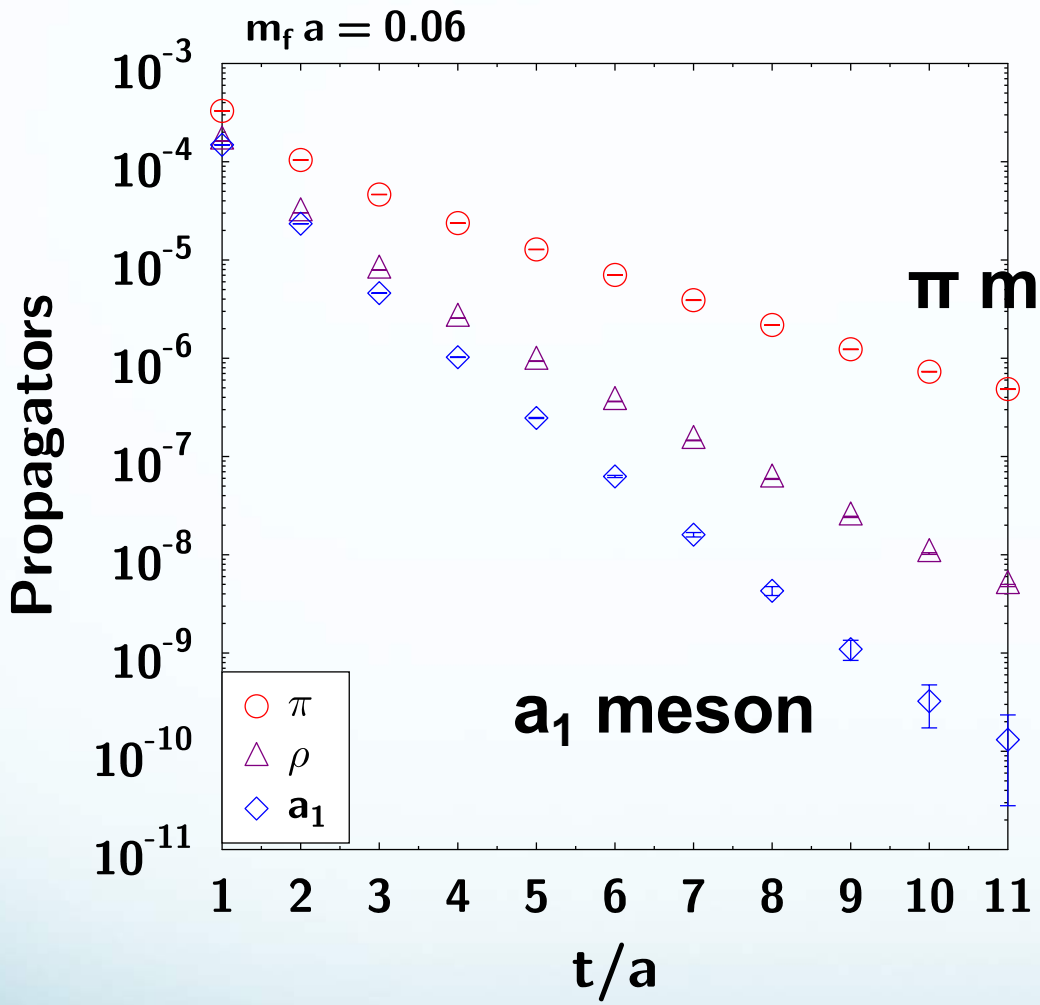
For quark propagator

- **Truncated overlap fermion action**

T. Blum *et al.*,
Phys. Rev. D69 (2004)

  - Fifth lattice size : $N_5 = 32$
  - Domain wall height parameter : $M_5 = 1.65$
  - Quark mass : $m_f = 0.08, 0.07, 0.06, 0.05, 0.04$
  - # of configurations : 3000, 3000, 3000, 3600, 7986 conf.
  - Source and sink are used $\bar{q}q$ operator

Propagators of π, ρ & a_1



π meson

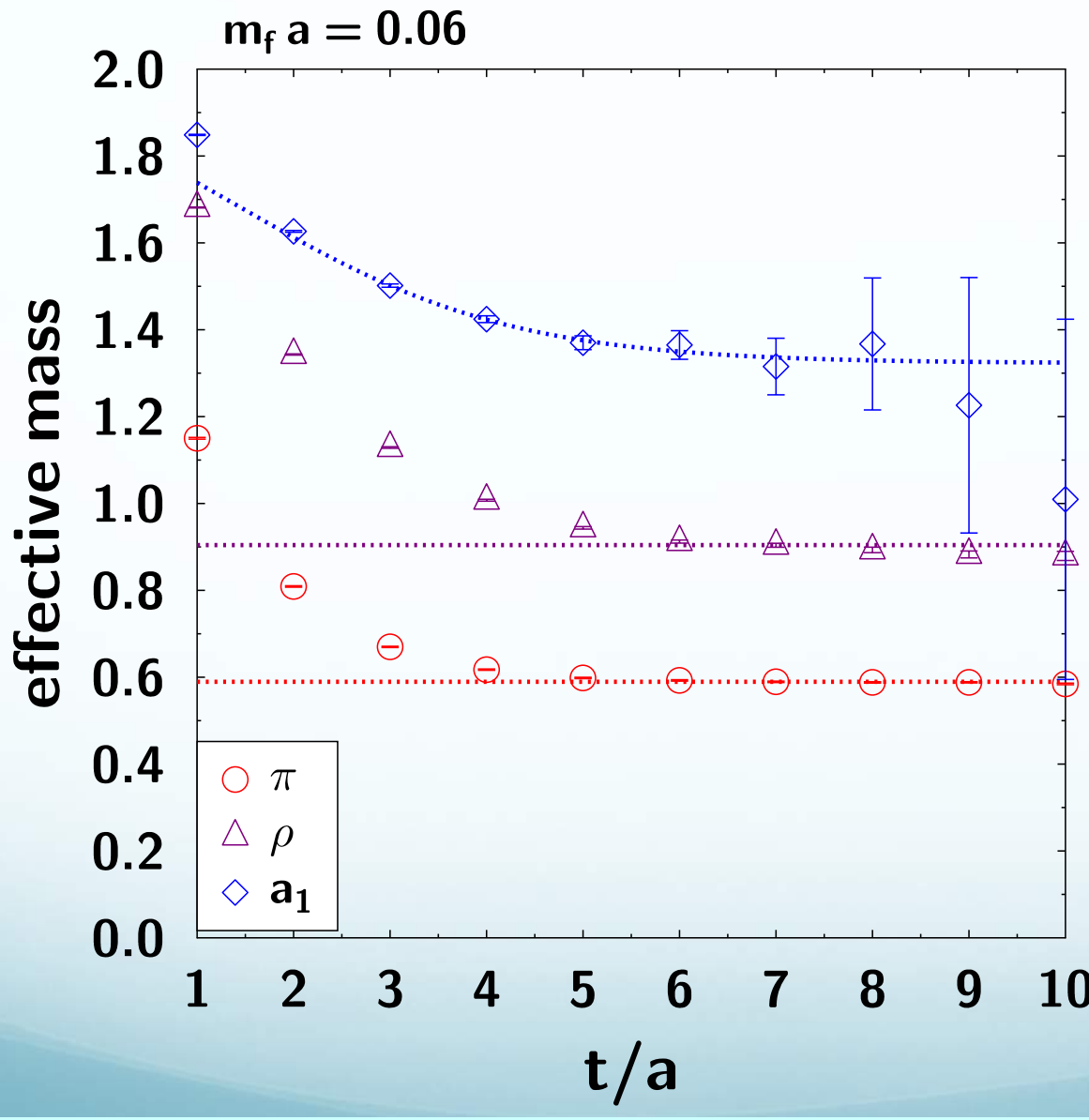
ρ meson

a_1 meson

$$P(t) = Z_0 e^{-m_0 t} + Z_1 e^{-m_1 t} + \ldots$$

By fitting the propagator with a one- or two-pole function at the large time region, we can get the meson masses.

Effective masses of π, ρ & a_1

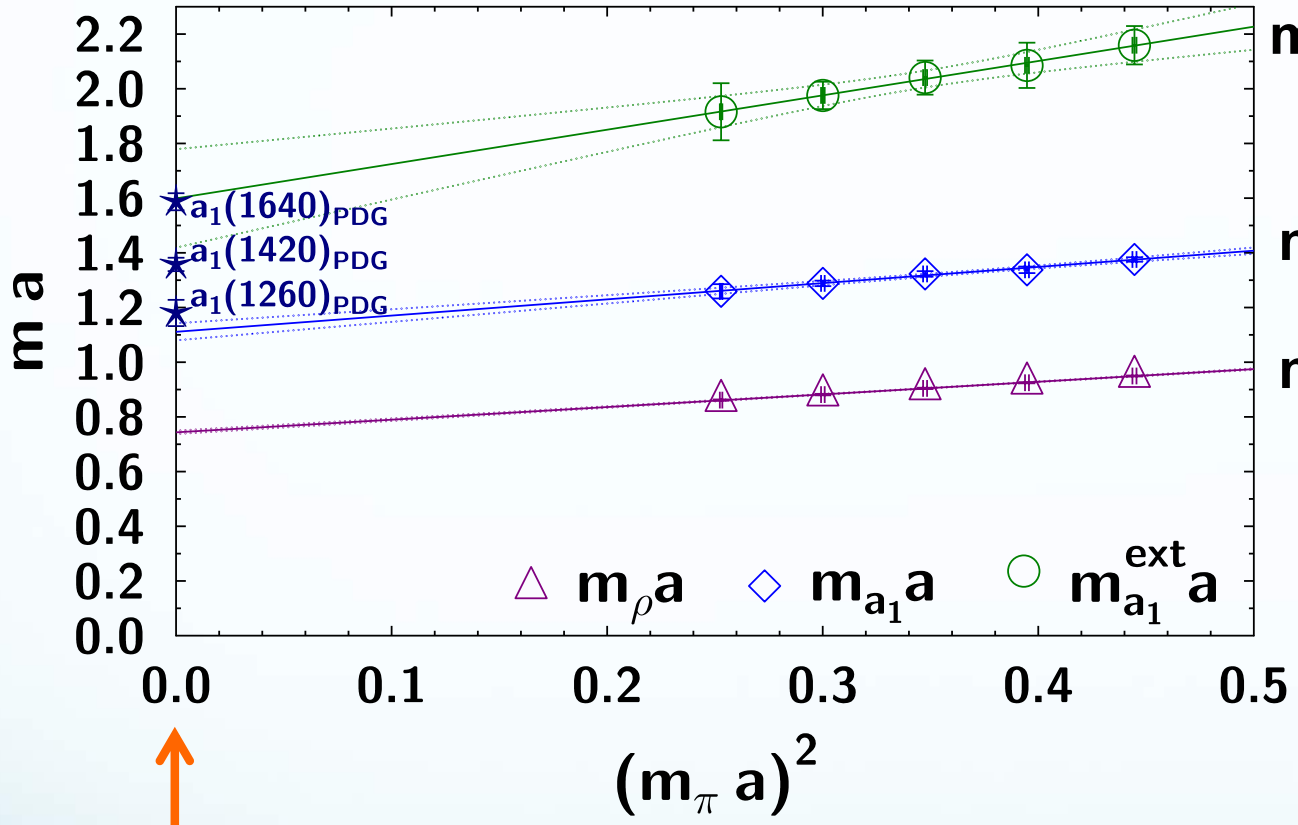


a_1 meson
(double-pole fit)

ρ meson
(single-pole fit)

π meson
(single-pole fit)

Mass dependence of π, ρ & a_1



mass of exited a_1 meson

mass of lowest a_1 meson

mass of ρ meson

Experimental values:
$a_1(1260)$: 1230(40) MeV
$a_1(1420)$: 1414(15) MeV
$a_1(1640)$: 1654(19) MeV

In the chiral limit, we tune the ρ meson's mass to 775 MeV.

lowest a_1 meson's mass: 1158(42) MeV
exited a_1 meson's mass: 1667(202) MeV

Summary & Future work

- **The experimental data for a_1 mesons sector are reported**
  $a_1(1260)$, $a_1(1420)$, $a_1(1640)$, $a_1(1930)$, $a_1(2095)$, $a_1(2270)$
- **As the first step, we calculate the mass of the a_1 meson with the truncated overlap fermions.**
- **The results support that**
  $a_1(1260)$ and $a_1(1640)$ is the $\bar{q}q$ states
  $a_1(1420)$ is the peculiar state
  because we perform the quenched approximation and use the $\bar{q}q$ operator for sink and source.
- **Dynamical calculations with lighter quark masses**
- **Calculations at finite temperature**