RUHR UNIVERSITÄT BOCHUM

XVIII International conference on Hadron Spectroscopy and Structure

16-21 Aug. 2019, Guilin, China

Baryon-baryon scattering in manifestly Lorentz-invariant formulation of ChPT

Xiu-Lei Ren（任修磊）

Institute of theoretical physics II, Ruhr-Universität Bochum

In collaboration with:

Vadim Baru (Bonn U.), Evgeny Epelbaum (RUB), and Jambul Gegelia(RUB)

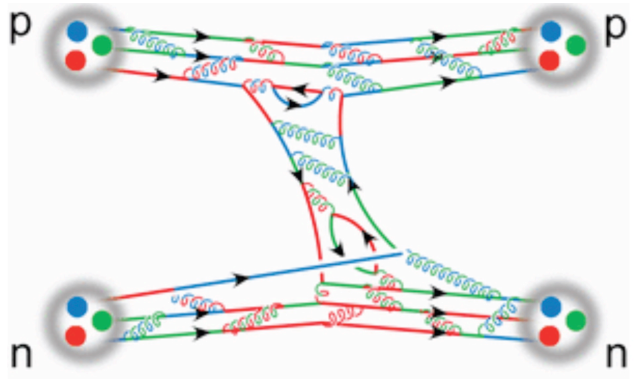
Outline

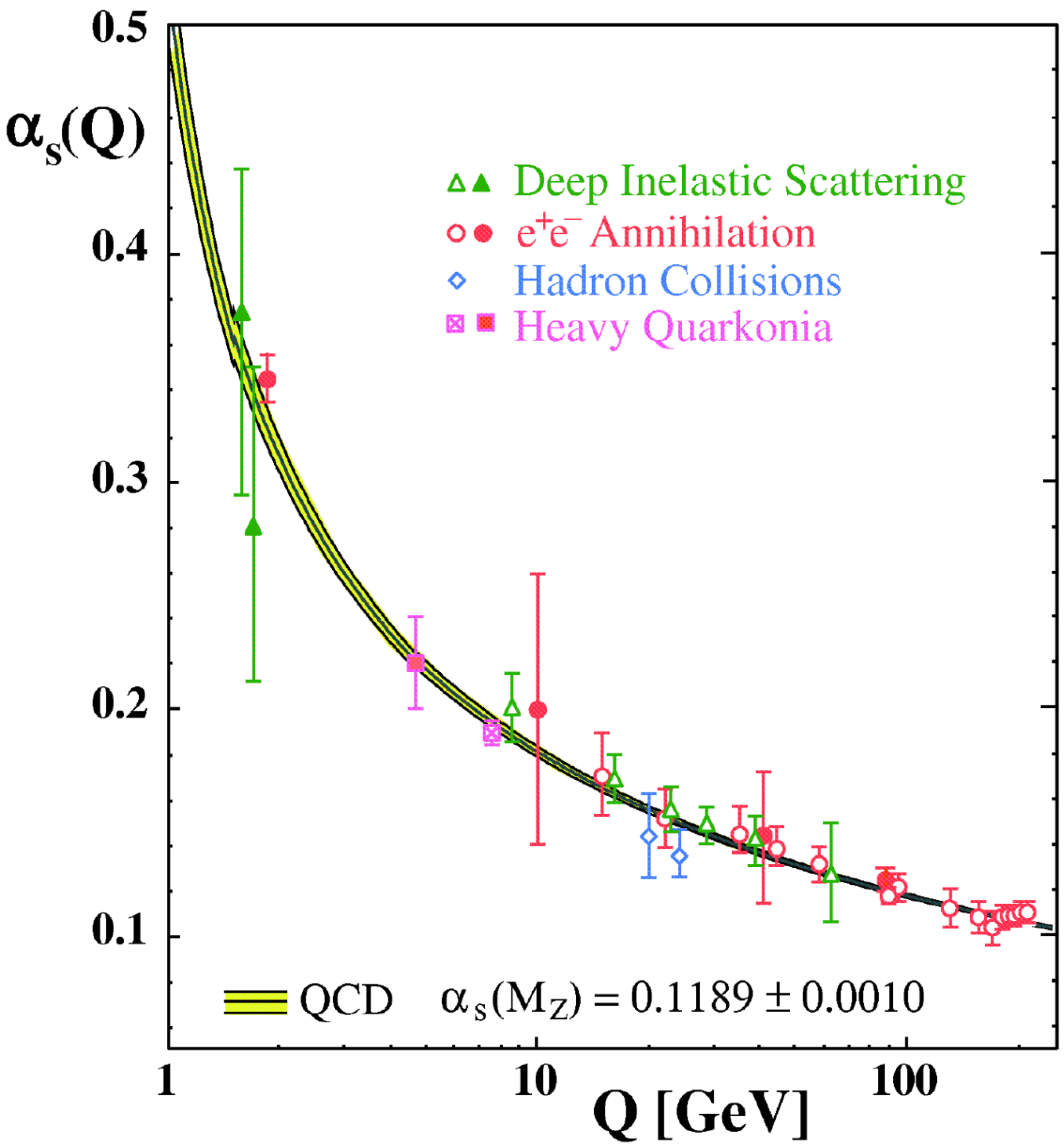
RUB

- Introduction
- Theoretical Framework
- Results and discussion
- Summary

Baryon–baryon interactions from QCD

- Residual quark-gluon strong interaction
- Understood from Quantum Chromo-Dynamics





S.Bethke, PPNP(2013)

At low-energy region

- Running coupling constant $\alpha_s > 1$
- Nonperturbative QCD -- unsolvable

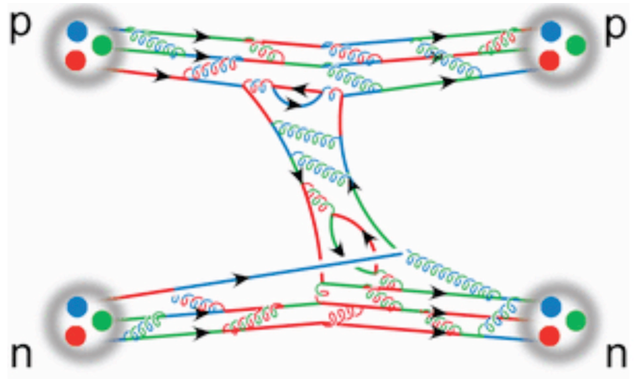
Studies of BB interactions

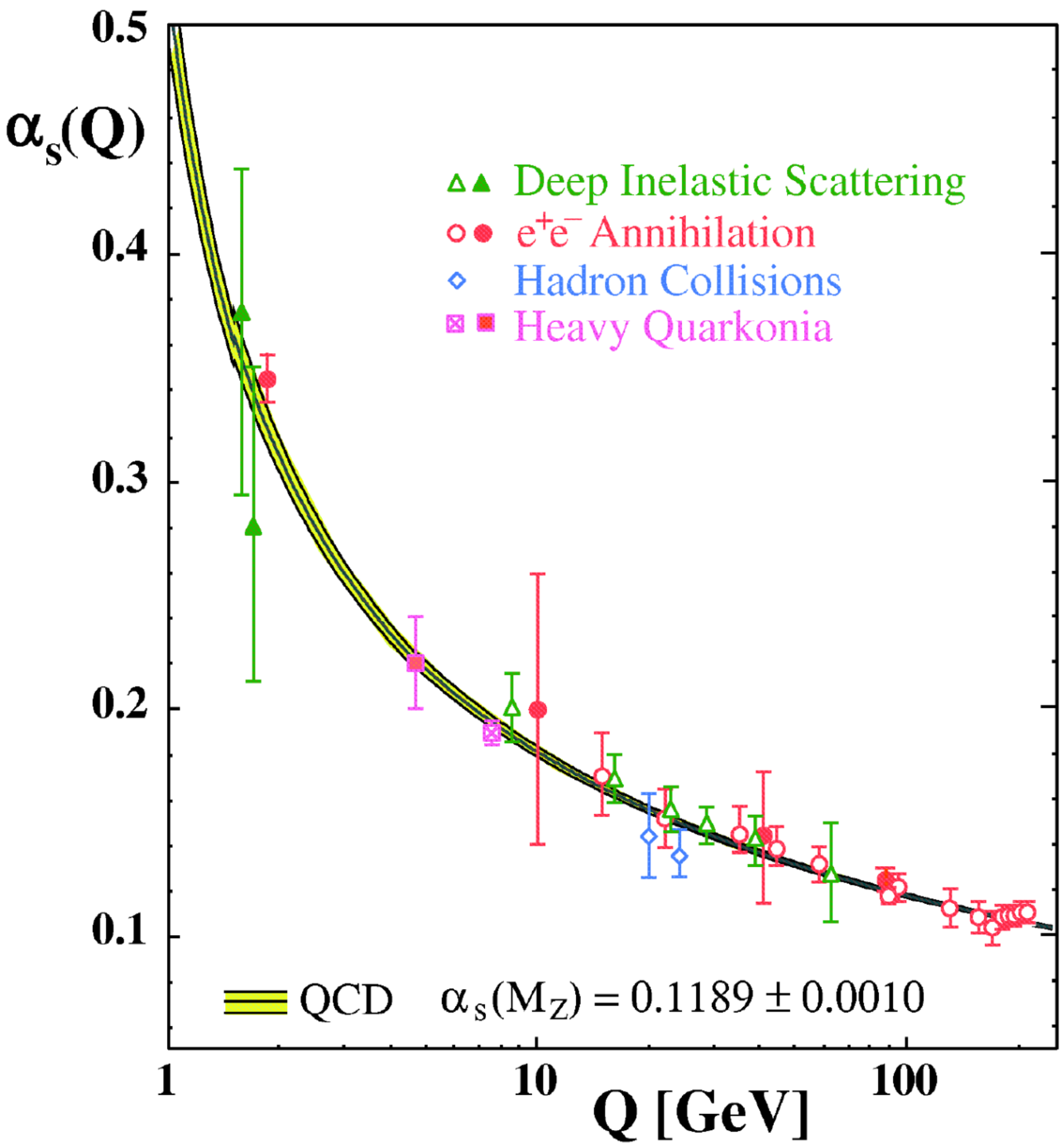
- Phenomenological models (since 1935~)
- Lattice QCD simulation (since 2010~)
- Chiral effective field theory (since 1990~)

Baryon–baryon interactions from QCD

RUB

- Residual quark-gluon strong interaction
- Understood from Quantum Chromo-Dynamics





S.Bethke, PPNP(2013)

At low-energy region

- Running coupling constant $\alpha_s > 1$
- Nonperturbative QCD -- unsolvable

Studies of BB interactions

- Phenomenological models (since 1935~)
- Lattice QCD simulation (since 2010~)
- Chiral effective field theory (since 1990~)

BB interaction in ChEFT

RUB

Weinberg's proposal

S. Weinberg, PLB251(1990)288-292; NPB363(1991)3-18

- NN potential calculated in chiral perturbation theory order by order

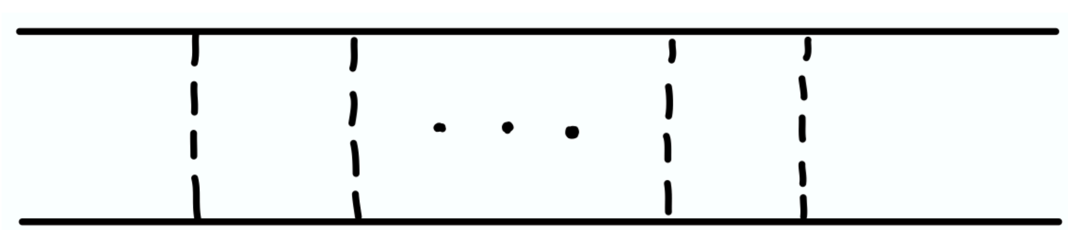
$$V(p',p) = V_{\rm LO} + V_{\rm NLO} + V_{\rm NNLO} + \cdots$$

Weinberg's Power Counting: $\mathcal{O}(Q^0)$ $\mathcal{O}(Q^2)$ $\mathcal{O}(Q^3)$ Q: small external momenta

- Scattering amplitude obtained by solving the Schrödinger or Lippmann-Schwinger equations

$$T(p',p) = V(p',p) + \int_0^\infty \frac{k^2 dk}{(2\pi)^3} V(p',k) \frac{m}{p^2 - k^2 + i\epsilon} T(k,p)\,.$$

- BUT, e.g., a series of ladder diagrams



M. Savage_arXiv:nucl-th/9804034

$\xrightarrow{k \to \infty}$ **Ultraviolet Divergence!!!**

cannot be absorbed by contact terms

WPC is inconsistent with renormalization, even at leading order (LO)!

Deal with the renormalization issue

RUB

Possible solutions (still controversial…)

- Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
  - ✓ WPC is consistent *G.P. Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*
  - ✓ Achieve great successes

	2NF	3NF	4NF
LO (Q^0)	*S. Weinberg, PLB 1990, NPB1990*	—	—
NLO (Q^2)	*U. van Kolck et al, PLB1992, PRL1994* *N. Kaiser et al., NPA1997*	—	—
N^2LO (Q^3)	*U. van Kolck et al., PRC1994* *E. Epelbaum et al., NPA1998, 2000*	*U. van Kolck et al., PRC1994*	—
N^3LO (Q^4)	… *R. Machleidt et al., PRC2003* *E. Epelbaum et al., NPA2005*	… *S. Ishikwas et al., PRC2007* *V. Bernard et al., PRC2007*	… *E. Epelbaum, PLB2006, EPJA2007*
N^4LO (Q^5)	… *R. Machleidt et al., PRC2015* *E. Epelbaum et al., PRL2015*	… *H. Krebs et al, PRC2012,2013* *Short-range loop contrib. still missing*	… *Not yet…*

1990	LO
2003	N^3LO: 2N
2007	N^3LO: 3N
2015	N^4LO: 2N
In future	N^4LO: 3N & 4N

P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339
E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

Deal with the renormalization issue

RUB

Possible solutions (still controversial…)

- Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
  - ✓ WPC is consistent — G.P. Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161
  - ✓ Achieve great successes — E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773; R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

- Kaplan, Savage, and Wise (KSW) power counting
  - ✓ Treat the exchange of pions perturbatively — D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390
  - ✓ Fail to converge in certain spin-triplet channels — S. Fleming, et al., Nucl.Phys. A677 (2000) 313
  - ✓ Recently, some improvements of KSW proposed by Kaplan — D.B. Kaplan, arXiv: 1905.07485

Deal with the renormalization issue

RUB

- Possible solutions (still controversial…)
  - Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
    - ✓ WPC is consistent *G.P. Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*
    - ✓ Achieve great successes *E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773; R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1*
  - Kaplan, Savage, and Wise (KSW) power counting
    - ✓ Treat the exchange of pions perturbatively *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*
    - ✓ Fail to converge in certain spin-triplet channels *S. Fleming, et al., Nucl.Phys. A677 (2000) 313*
    - ✓ Recently, some improvements of KSW proposed by Kaplan *D.B. Kaplan, arXiv: 1905.07485*
  - Modified WPC with renormalization group invariance (RGI)
    - ✓ Rearrange the higher order contact terms to the lower chiral order
      A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002. B. Long and C.-J. Yang, PRC84(2011)057001 …

Deal with the renormalization issue

RUB

- Possible solutions (still controversial…)
  - Keep cutoff lower than hard scale: $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
    - ✓ WPC is consistent G.P. Lepage, nucl-th/9706029. E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161
    - ✓ Achieve great successes E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773; R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1
  - Kaplan, Savage, and Wise (KSW) power counting
    - ✓ Treat the exchange of pions perturbatively D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390
    - ✓ Fail to converge in certain spin-triplet channels S. Fleming, et al., Nucl.Phys. A677 (2000) 313
    - ✓ Recently, some improvements of KSW proposed by Kaplan D.B. Kaplan, arXiv: 1905.07485
  - Modified WPC with renormalization group invariance (RGI)
    - ✓ Rearrange the higher order contact terms to the lower chiral order
      A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002. B. Long and C.-J. Yang, PRC84(2011)057001 …
  - Lorentz invariant framework to reformulate chiral force
    - ✓ The fundamental symmetry of our nature

Relativistic studies of chiral force

RUB

Talk by Prof. Li-Sheng Geng @ Session4, 8/20

- Relativistic chiral force (covariant form)

  XLR, et al., CPC42(2018)014103; K.-W. Li, et al., CPC42(2018)014105; K.-W. Li, et al., PRC98(2018)065203 …

Here, we focus on the renormalization issue of chiral force

- Modified Weinberg approach *E. Epelbaum and J. Gegelia_PLB716(2012)338-344*
  - Based on the Lorentz invariant chiral Lagrangians
  - Adopt WPC to expand the NN potential and the relativistic corrections are perturbatively included

$$V(p',p) = \bar{u}_1\bar{u}_2\,\mathscr{A}\,u_1u_2 \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

  - Use the Kadyshevsky equation to calculate the scattering T-matrix

$$T(p',p) = V(p',p) + \int \frac{k^2dk}{(2\pi)^3}\,\frac{m^2}{2\sqrt{k^2+m^2}}\,\frac{1}{\sqrt{p^2+m^2}-\sqrt{k^2+m^2}+i\epsilon}\,T(k,p)\,.$$

V. Kadyshevsky, NPB (1968)

    ✓ Milder ultraviolet behavior than in LS equation

  - Result in **a renormalizable LO potential !**

    ✓ All divergences absorbed in parameters of the LO potential

In this work

- Based on the idea of modified Weinberg approach, we proposed a systematic framework within the old-fashioned (time-ordered) perturbation theory using the Lorentz invariant chiral Lagrangians
  - Derive the rules of time-ordered diagrams, especially for the rules with spin-1/2 fermion *(as far as we know, there was no such rules in the literatures)*
  - Extend the framework from the nucleon-nucleon scattering to the baryon-baryon sector with different strangeness
    - ✓ Calculate the NN and YN scatterings up to leading order
    - ✓ Discuss the renormalization issue of our obtained potentials

V.Baru, E.Epelbaum, J. Gegelia, XLR, arXiv:1905.02116

XLR, E.Epelbaum, J.Gegelia, Hyperon-nucleon scattering, in preparation

Theoretical framework

RUB

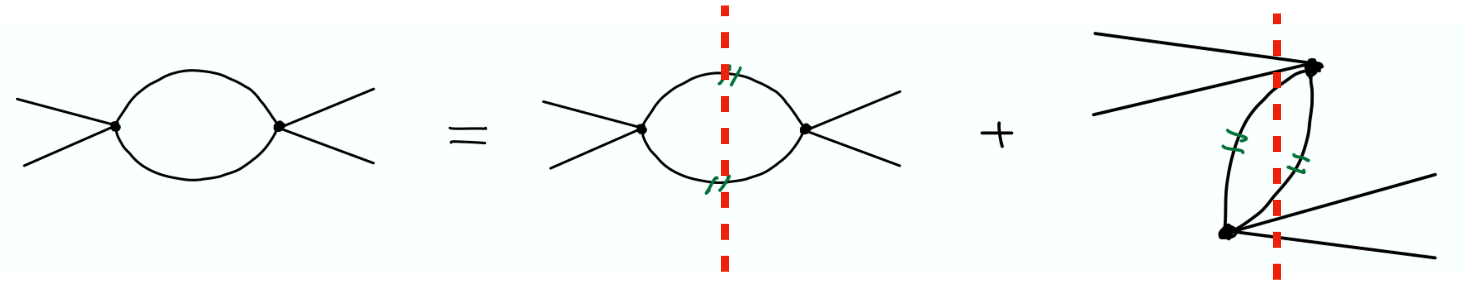
Time-ordered perturbation theory (TOPT)

- Definition

  S. Weinberg, Phys.Rev.150(1966)1313
  G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

  ✓ Re-express the Feynman integral in a form that makes the connection with on-shell state explicit. This form is called **TOPT or old-fashioned PT**

  ✓ (In short) Instead the propagators for internal lines as the energy denominators for intermediate states



- Advantages

  ✓ Explicitly show the unitarity

  ✓ One-to-one relation between internal lines and intermediate states

  ✓ Easily to tell the contributions of a particular diagram

- Derive the rules for time-ordered diagrams

  ✓ Perform Feynman integrations over the zeroth components of the loop momenta

  ✓ Decompose Feynman diagram into sums of time-ordered diagrams

  ✓ Match to the rules of time-ordered diagrams

Diagram rules in TOPT

RUB

External lines

- Incoming (outgoing) baryon lines: $u(p)\ [\bar{u}(p')]$ *Dirac spinors*

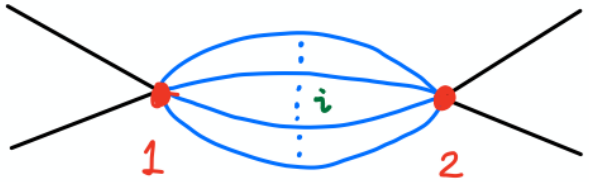
Internal lines

- Pseudo-scalar meson lines: $\dfrac{1}{2\omega(q_i, M_i)}$ $\quad \omega(q,M) = \sqrt{q^2 + M^2}$
- Baryon lines: $\dfrac{m_i}{\omega(p_i, m)} \sum u(p_i)\bar{u}(p_i)$
- Anti-baryon lines: $\dfrac{m_i}{\omega(p_i, m)} \sum u(p_i)\bar{u}(p_i) - \gamma_0$ $\quad \gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$

Interaction vertices

- Follow the standard Feynman rules
- Take care of zeroth components of momenta p^0
  - ✓ Replaced as $\omega(p, m)$ for particle
  - ✓ Replaced as $-\omega(p, m)$ for antiparticle

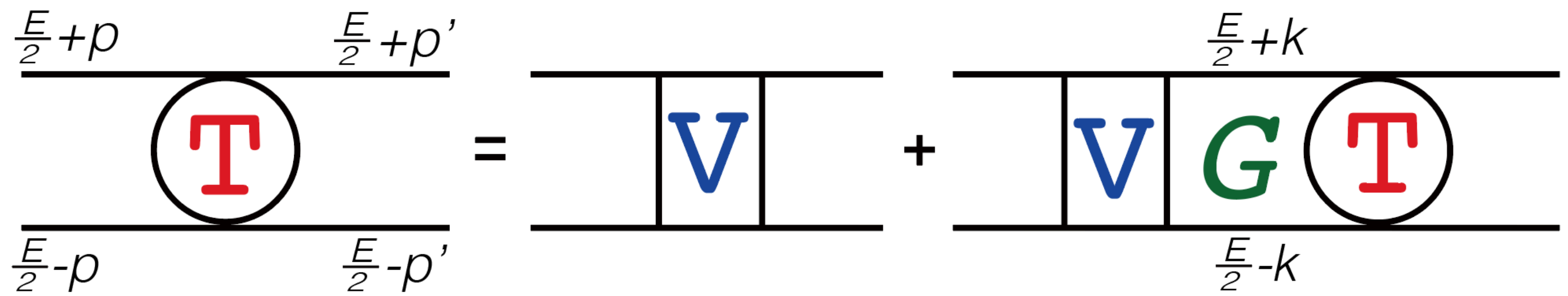
Intermediate state: a set of lines between any two vertices



$$[E - \sum_i \omega(p_i, m_i) + i\epsilon]^{-1}$$

E is the total energy of the system

Scattering amplitude T



- Potential V: sum up the two-particle **irreducible time-ordered diagrams**
  - ✓ Employ the Weinberg power counting to perturbatively calculate potential

$$
V = \begin{pmatrix}
V_{NN,NN} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & V_{\Lambda N,\Lambda N} & V_{\Lambda N,\Sigma N} & 0 & 0 & 0 & 0 \\
0 & V_{\Sigma N,\Lambda N} & V_{\Sigma N,\Sigma N} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & V_{\Lambda\Lambda,\Lambda\Lambda} & V_{\Lambda\Lambda,\Xi N} & V_{\Lambda\Lambda,\Sigma\Sigma} & V_{\Lambda\Lambda,\Sigma\Lambda} \\
0 & 0 & 0 & V_{\Xi N,\Lambda\Lambda} & V_{\Xi N,\Xi N} & V_{\Xi N,\Sigma\Sigma} & V_{\Xi N,\Sigma\Lambda} \\
0 & 0 & 0 & V_{\Sigma\Sigma,\Lambda\Lambda} & V_{\Sigma\Sigma,\Xi N} & V_{\Sigma\Sigma,\Sigma\Sigma} & V_{\Sigma\Sigma,\Sigma\Lambda} \\
0 & 0 & 0 & V_{\Sigma\Lambda,\Lambda\Lambda} & V_{\Sigma\Lambda,\Xi N} & V_{\Sigma\Lambda,\Sigma\Sigma} & V_{\Sigma\Lambda,\Sigma\Lambda}
\end{pmatrix}
$$

S=0

S=-1

S=-2

- Two-body Green functions *G*:

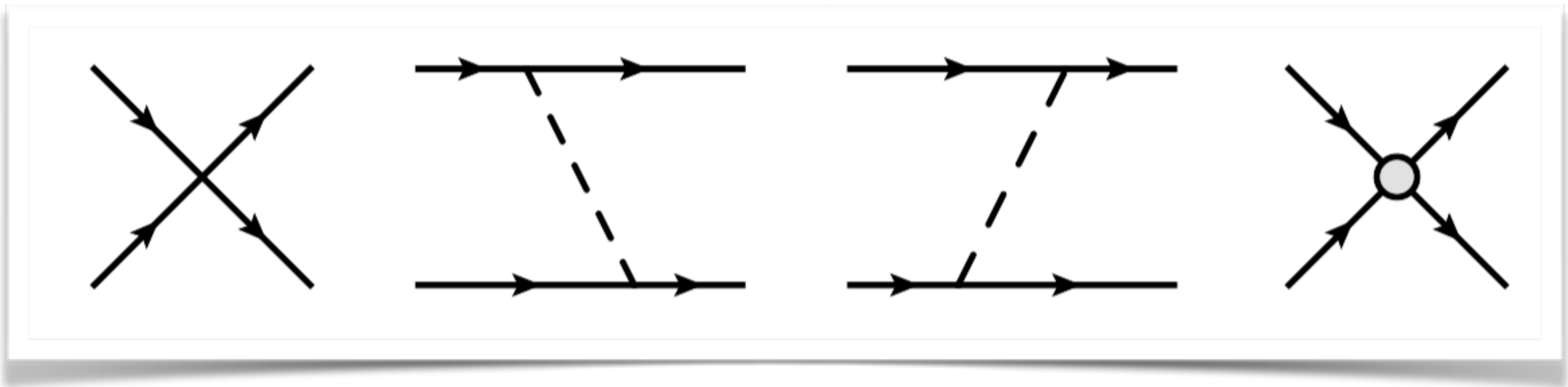
$$
G_{ij}(E) = \frac{1}{\omega(k,m_i)\,\omega(k,m_j)}\,\frac{m_i m_j}{E - \omega(k,m_i) - \omega(k,m_j) + i\epsilon}
$$

V. Kadyshevsky, NPB (1968)

  - ✓ This is the generalized Kadyshevsky propagator of NN scattering
  - ✓ **SELF-CONSISTENTLY obtained** in TOPT!

Leading order contributions in TOPT

- Time-ordered diagrams at LO



- Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\phi} + \mathcal{L}_{\phi\text{B}} + \mathcal{L}_{\text{BB}} + \cdots$$

  - Mesonic Lagrangian *J. Gasser and H. Leutwyler, Ann. Phys.158,142(1984)*

$$\mathcal{L}_{\phi}^{(2)} = \frac{F_0^2}{4}\text{Tr}\left\{u_{\mu}u^{\mu} + \chi_{+}\right\}$$

  - Meson-baryon Lagrangian *A. Krause, Helv. Phys. Acta 63 (1990) 3-70*

$$\mathcal{L}_{\phi\text{B}}^{(1)} = \text{Tr}\left\{\bar{\text{B}}\left(i\gamma_{\mu}D^{\mu} - m\right)\text{B}\right\} + \frac{D/F}{2}\text{Tr}\left\{\bar{\text{B}}\gamma_{\mu}\gamma_5[u^{\mu},\text{B}]_{\pm}\right\}$$

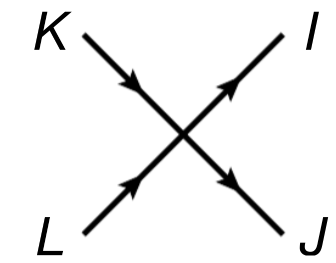
  - Baryon-baryon Lagrangian *H.Polinder, J. Haidenbauer, Ulf-G. Meißner, NPA779(2006)244-266*

$$\begin{aligned}\mathcal{L}_{\text{BB}}^{(0)} \;=\; & C_i^1\,\text{Tr}\left\{\bar{\text{B}}_{\alpha}\bar{\text{B}}_{\beta}\left(\Gamma_i\text{B}\right)_{\beta}\left(\Gamma_i\text{B}\right)_{\alpha}\right\} + C_i^2\,\text{Tr}\left\{\bar{\text{B}}_{\alpha}\left(\Gamma_i\text{B}\right)_{\alpha}\bar{\text{B}}_{\beta}\left(\Gamma_i\text{B}\right)_{\beta}\right\} \\ & + C_i^3\,\text{Tr}\left\{\bar{\text{B}}_{\alpha}\left(\Gamma_i\text{B}\right)_{\alpha}\right\}\text{Tr}\left\{\bar{\text{B}}_{\beta}\left(\Gamma_i\text{B}\right)_{\beta}\right\},\end{aligned}$$

$$\Gamma_1 = 1,\quad \Gamma_2 = \gamma^{\mu},\quad \Gamma_3 = \sigma^{\mu\nu},\quad \Gamma_4 = \gamma^{\mu}\gamma_5,\quad \Gamma_5 = \gamma_5.$$

Leading order potentials

Contact baryon-baryon interaction



- S = 0, *NN* single channel

$$V_{0,C}^{IJ,KL} = C_S(\bar{u}_K u_I)(\bar{u}_L u_J) + C_A(\bar{u}_K \gamma_5 u_I)(\bar{u}_L \gamma_5 u_J) + C_V(\bar{u}_K \gamma_\mu u_I)(\bar{u}_L \gamma^\mu u_J)$$
$$+ C_{AV}(\bar{u}_K \gamma_\mu \gamma_5 u_I)(\bar{u}_L \gamma^\mu \gamma_5 u_J) + C_T(\bar{u}_K \sigma_{\mu\nu} u_I)(\bar{u}_L \sigma^{\mu\nu} u_J)$$

XLR, K.-W. Li, L.-S. Geng, B. Long, P. Ring, J. Meng, CPC 42 (2018) 014103

✓ Contains higher order contributions according to WPC

✓ Perform the expansion for the baryon energies $\sqrt{\omega(p,m)+m} = \sqrt{2m} + \mathcal{O}(p^2)$

$$\Rightarrow \quad \boxed{V_{LO,C}^{IJ,KL} = (C_S + C_V) - (C_{AV} - 2C_T)\vec{\sigma}_1 \cdot \vec{\sigma}_2}$$

Same as the non-relativistic potential *S. Weinberg, PLB251(1990)288-292*

- S = -1, ΛN-ΣN two coupled channels

✓ Expressions are the same as the non-relativistic potential

H.Polinder, J. Haidenbauer, Ulf-G. Meißner, NPA779(2006)244-266

- S = -2, $\Lambda\Lambda$, ΞN, $\Sigma\Sigma$, $\Sigma\Lambda$ four coupled channels

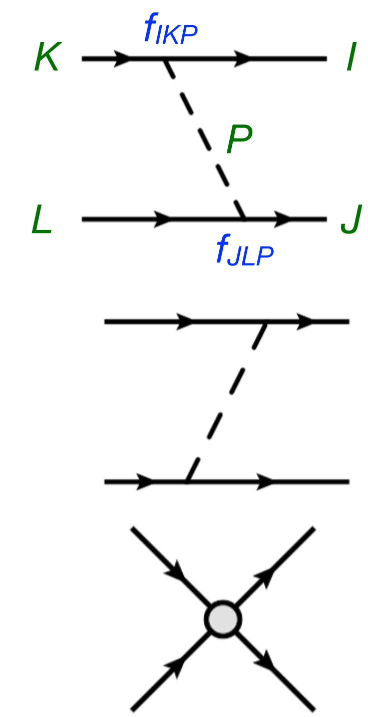
✓ Expressions are the same as the non-relativistic potential

H.Polinder, J. Haidenbauer, Ulf-G. Meißner, PLB 653 (2007) 29–37

Leading order potentials

RUB

One-meson-exchange contribution



$$V^{IJ,KL}_{0,M_P} = \frac{f_{IKP} f_{JLP} \mathcal{I}_{IJ,KL}}{2\,\omega(q,M_P)} \left[\frac{(\bar{u}_I \gamma_\mu \gamma_5 q^\mu u_K)(\bar{u}_J \gamma_\nu \gamma_5 q^\nu u_L)}{\omega(q,M_P) + \omega(p_K,m_K) + \omega(p_J,m_J) - E - i\,\epsilon} + \frac{(\bar{u}_I \gamma_\mu \gamma_5 q^\mu u_K)(\bar{u}_J \gamma_\nu \gamma_5 q^\nu u_L)}{\omega(q,M_P) + \omega(p_L,m_L) + \omega(p_I,m_I) - E - i\,\epsilon} \right]$$

- Contains higher order contributions according to WPC
- Perform the expansion for the baryon energies

$$\sqrt{\omega(p,m)+m} = \sqrt{2m} + \mathcal{O}(p^2)$$

✓ Keep the baryon energies in denominator (consistent with Kadysevsky eq.)

$$V^{IJ,KL}_{\mathrm{LO},M_P} = -\frac{f_{IKP} f_{JLP} \mathcal{I}_{IJ,KL}}{2\,\omega(q,M_P)} \left[\frac{1}{\omega(q,M_P) + \omega(p_K,m_K) + \omega(p_J,m_J) - E - i\,\epsilon} + \frac{1}{\omega(q,M_P) + \omega(p_L,m_L) + \omega(p_I,m_I) - E - i\,\epsilon} \right] \frac{(m_I + m_K)\,(m_J + m_L)}{\sqrt{m_I m_J m_K m_L}} \times \frac{(m_K \vec{\sigma}_1 \cdot \vec{p}_I - m_I \vec{\sigma}_1 \cdot \vec{p}_K)\,(m_L \vec{\sigma}_2 \cdot \vec{p}_J - m_J \vec{\sigma}_2 \cdot \vec{p}_L)}{\sqrt{\omega(p_I,m_I) + m_I}\,\sqrt{\omega(p_J,m_J) + m_J}\,\sqrt{\omega(p_K,m_K) + m_K}\,\sqrt{\omega(p_L,m_L) + m_L}}.$$

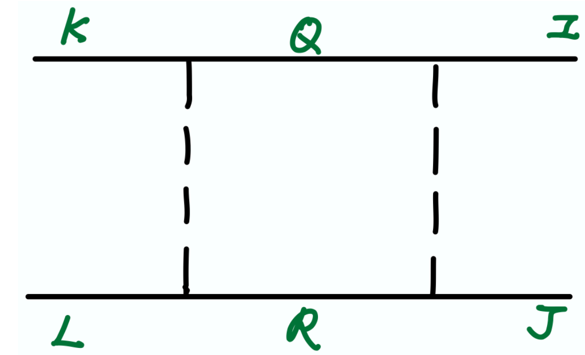
It has a milder ultraviolet behaviour than the non-relativisitc OMEP

Behavior of long-range potential

RUB

One-loop integral $V_M G V_M$

$$I_{VGV} = \sum_{Q,R} \int \frac{d^3 \vec{k}}{(2\pi)^3} V_{\mathrm{LO},M_P}^{IJ,QR} \, G^{QR}(E) \, V_{\mathrm{LO},M_P}^{QR,KL}$$

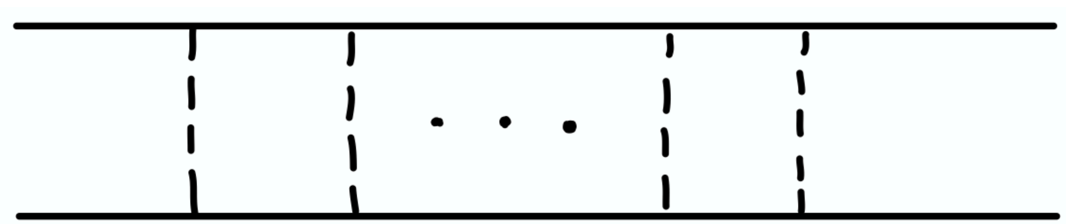


$$\xrightarrow{k \equiv |\vec{k}| \to \infty} \begin{cases} \textbf{Our:} \quad I_{VGV}^{Our} \to \int dk^3 \, \dfrac{1}{k^5} \\ \textbf{NR:} \quad I_{VGV}^{NR} \to \int dk^3 \, \dfrac{1}{k^2} \end{cases}$$

Ultraviolet convergent!

Ultraviolet divergent!

Iteration of our OMEP



$$\xrightarrow{k \to \infty}$$

Finite diagram!

- Scattering amplitude from OMEP is cutoff independent

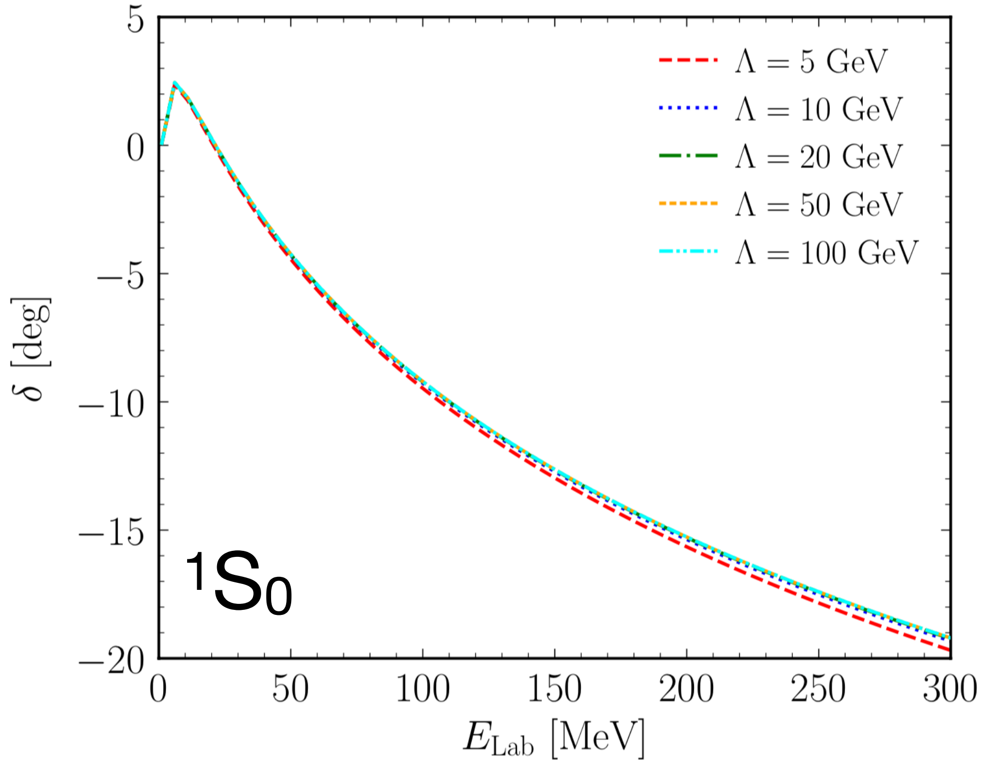
$$T_M = V_M + V_M \, G \, T_M$$

Renormalizable!

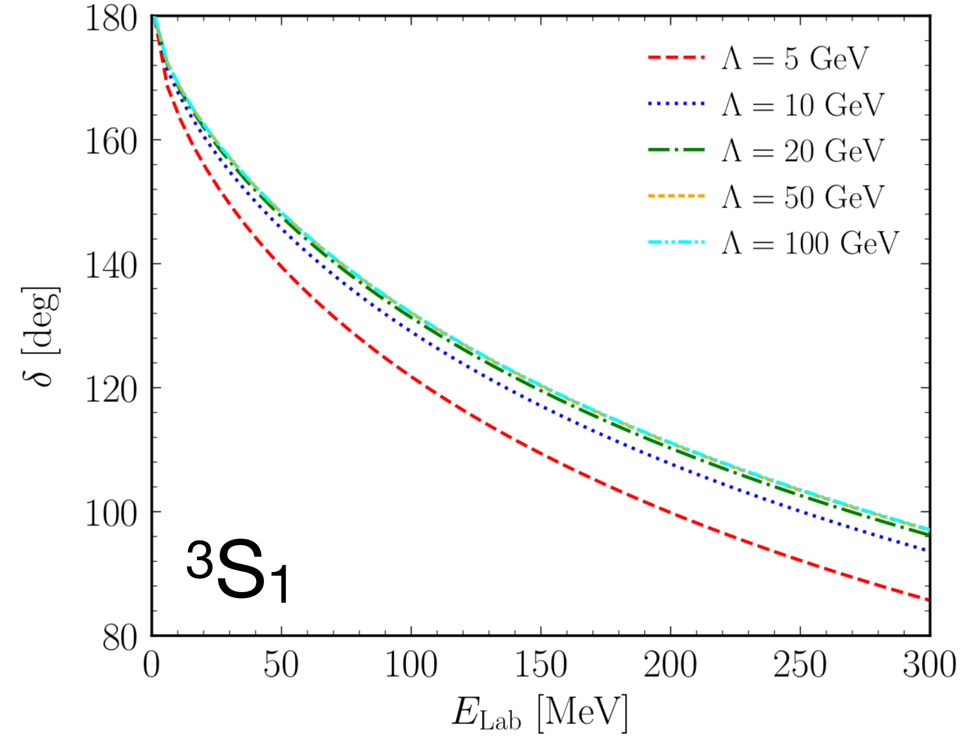
Phase shifts: cutoff-independent

RUB

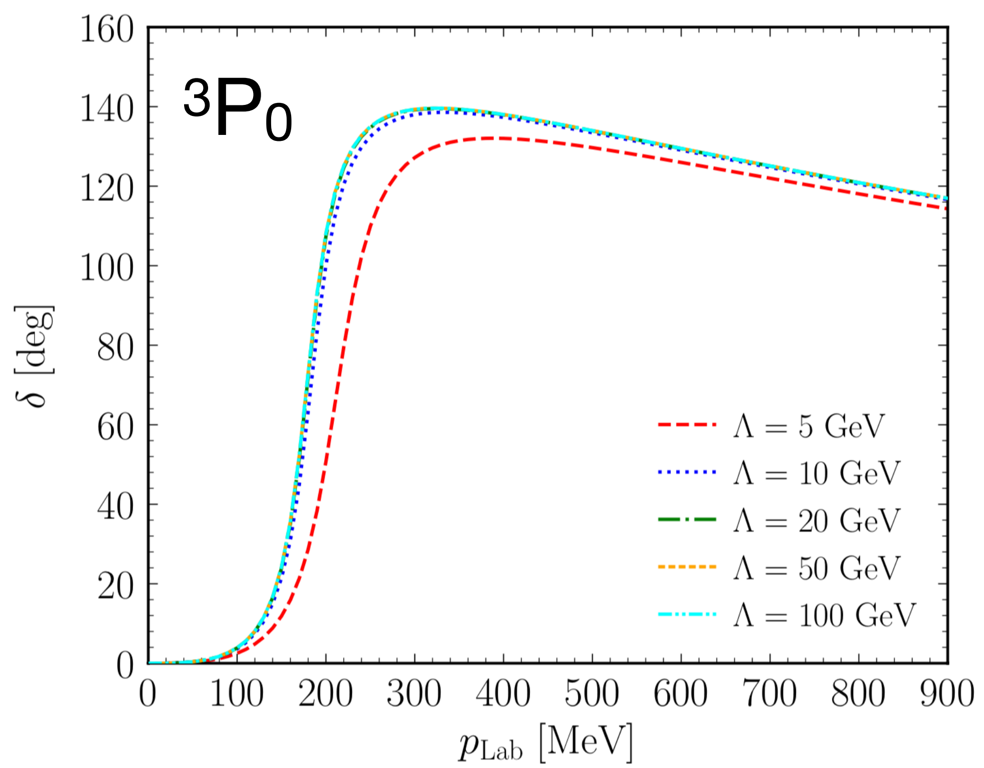
- NN single channel



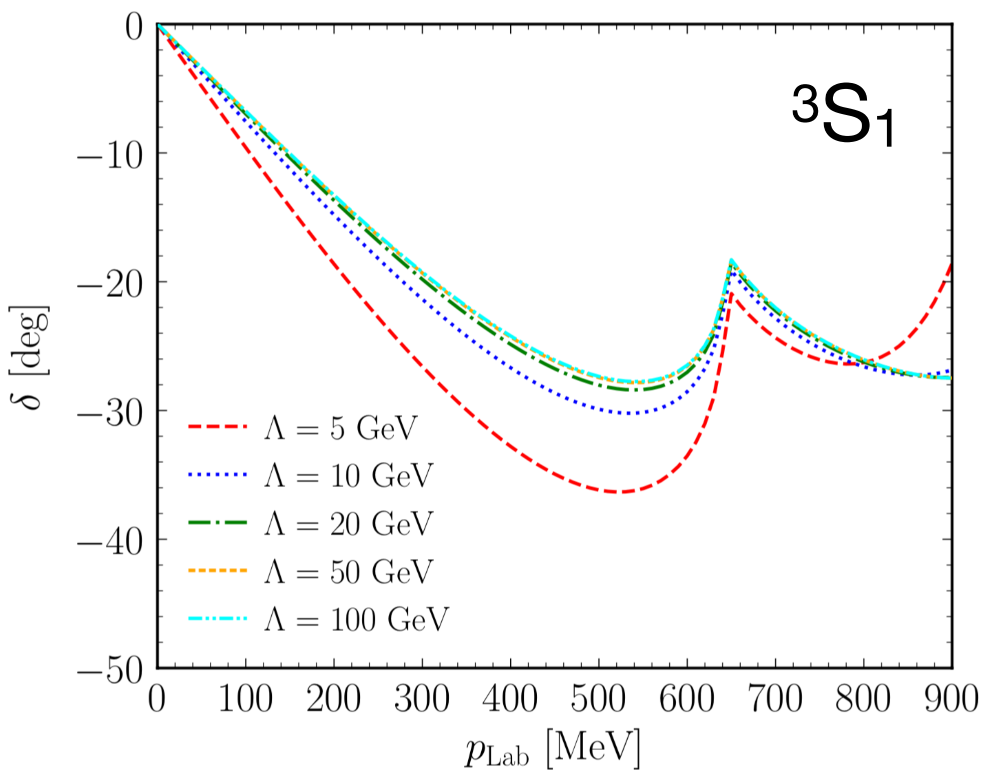
- NN couple channels



- ΛN-ΣN single channel



- ΛN-ΣN couple channels



Phase shifts of NN scattering

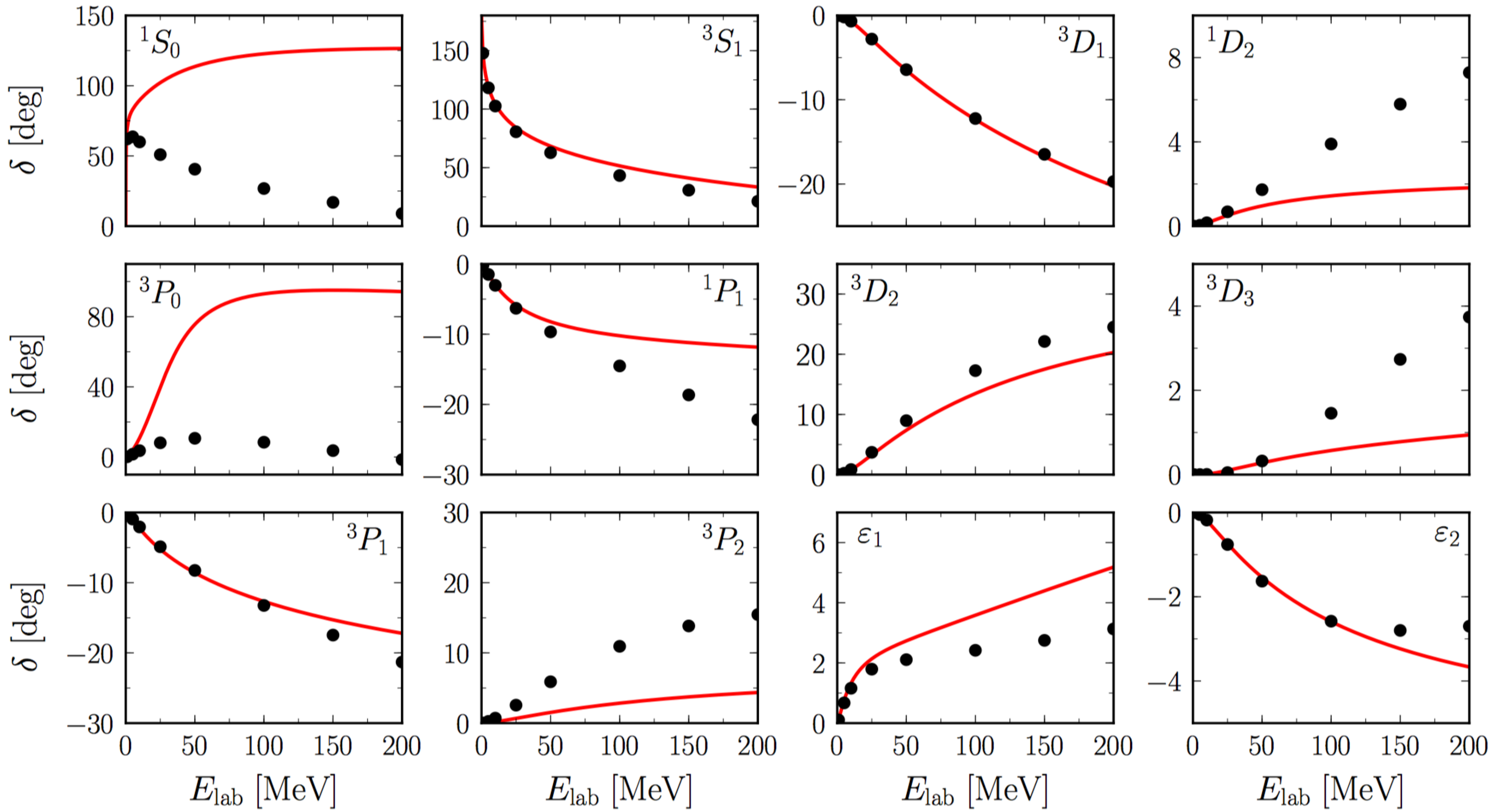
RUB

- Leading order chiral NN potential

$$V_C = C_S - C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_\pi(\vec{p}\,', \vec{p}) = -\frac{g_A^2}{4F_0^2} \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{\sqrt{(\vec{p} - \vec{p}\,')^2 + M_\pi^2}} \frac{4m_N^2}{(m_N + \sqrt{\vec{p}^{\,2} + m_N^2})(m_N + \sqrt{\vec{p}\,'^2 + m_N^2})} \times \frac{[\vec{\sigma}_1 \cdot (\vec{p} - \vec{p}\,')]\,[\vec{\sigma}_2 \cdot (\vec{p} - \vec{p}\,')]}{\sqrt{(\vec{p} - \vec{p}\,')^2 + M_\pi^2} + \sqrt{\vec{p}^{\,2} + m_N^2} + \sqrt{\vec{p}\,'^2 + m_N^2} - E - i\epsilon},$$

- Phase shifts at LO with cutoff → ∞



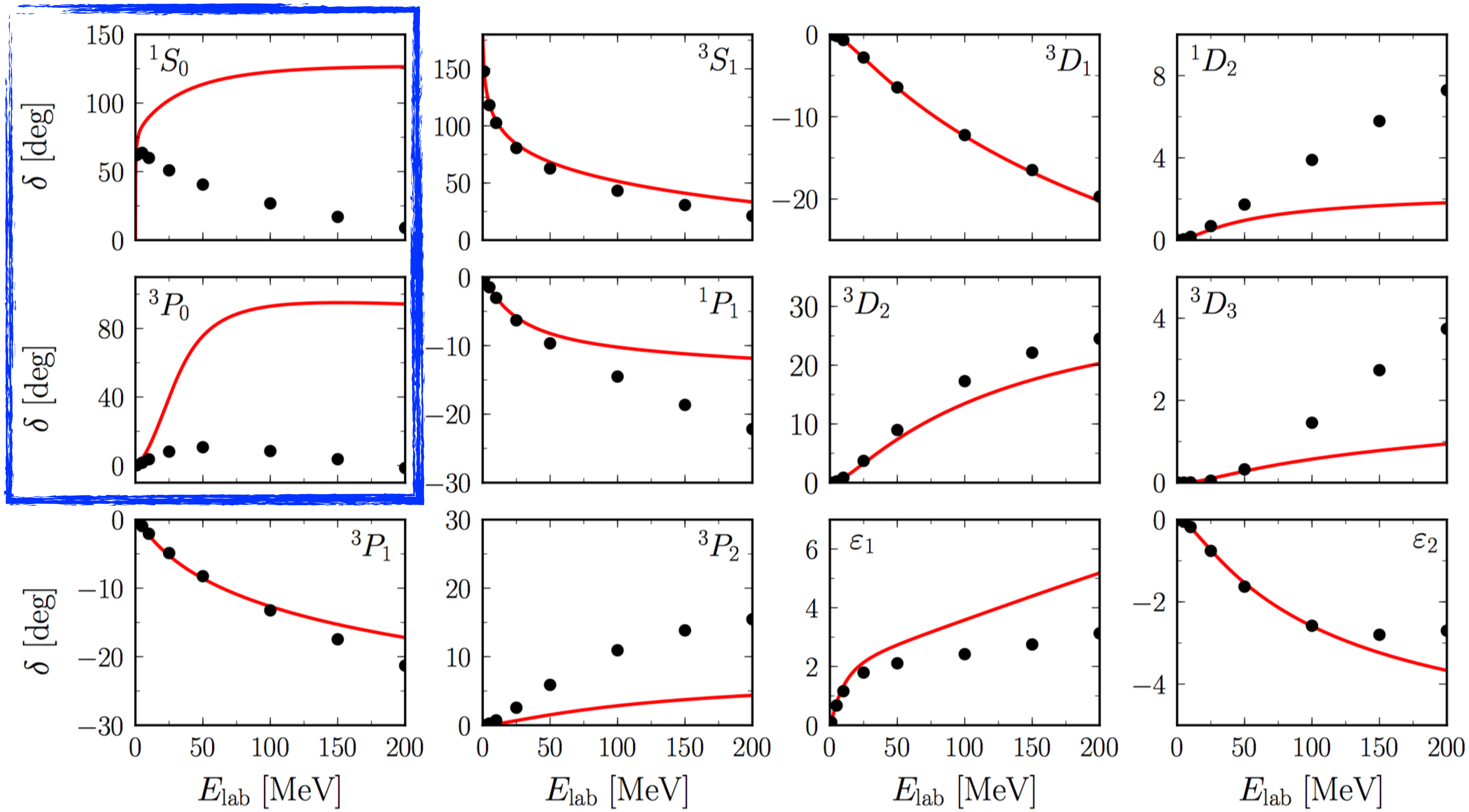
Phases shift of NN scattering

- Leading order chiral NN potential

$$V_C = C_S - C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_\pi(\vec{p}\,', \vec{p}) = -\frac{g_A^2}{4F_0^2} \frac{\vec{\tau}_1 \cdot \vec{\tau}_2}{\sqrt{(\vec{p}-\vec{p}\,')^2 + M_\pi^2}} \frac{4m_N^2}{(m_N + \sqrt{\vec{p}^{\,2}+m_N^2})(m_N + \sqrt{\vec{p}\,'^2 + m_N^2})} \times \frac{[\vec{\sigma}_1 \cdot (\vec{p}-\vec{p}\,')]\,[\vec{\sigma}_2 \cdot (\vec{p}-\vec{p}\,')]}{\sqrt{(\vec{p}-\vec{p}\,')^2 + M_\pi^2} + \sqrt{\vec{p}^{\,2}+m_N^2} + \sqrt{\vec{p}\,'^2+m_N^2} - E - i\epsilon},$$

- Phase shifts at LO with cutoff → ∞



$^1S_0$ and $^3P_0$: Large differences

- At least a part of the subleading corrections must be treated non-perturbatively
- For simplicity, we choose to promote the NLO contact terms up to LO.

Role of NLO contact terms in renormalization

RUB

Take $^3P_0$ partial wave for example

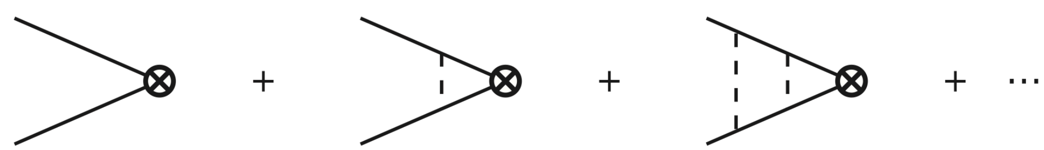
- Promote the NLO contact term to the lowest order
- Potential: $V_{3P0}(p',p) = C\,p'p + V_\pi$

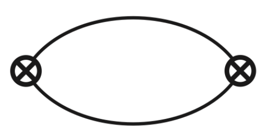
Renormalizable or not?

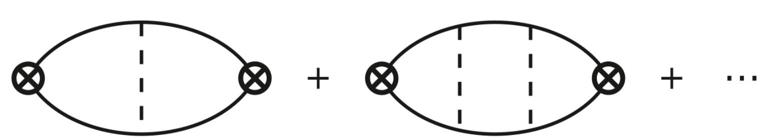
- Amplitude: *E.Epelbaum, A.M. Gasparyan, J. Gegelia, and H. Krebs., EPJA51(2015)71*

$$T_{3P0} = T_\pi + \frac{[(1+T_\pi G)p'][p(1+GT_\pi)]}{C^{-1} - pGp' - pGT_\pi Gp'}$$

✓ $T_\pi = V_\pi + V_\pi G\, T_\pi :\ \xrightarrow{k\to\infty}$ **Finite term**

✓ $(1+T_\pi G)p' :$  $\xrightarrow{k\to\infty}$ **Finite term**

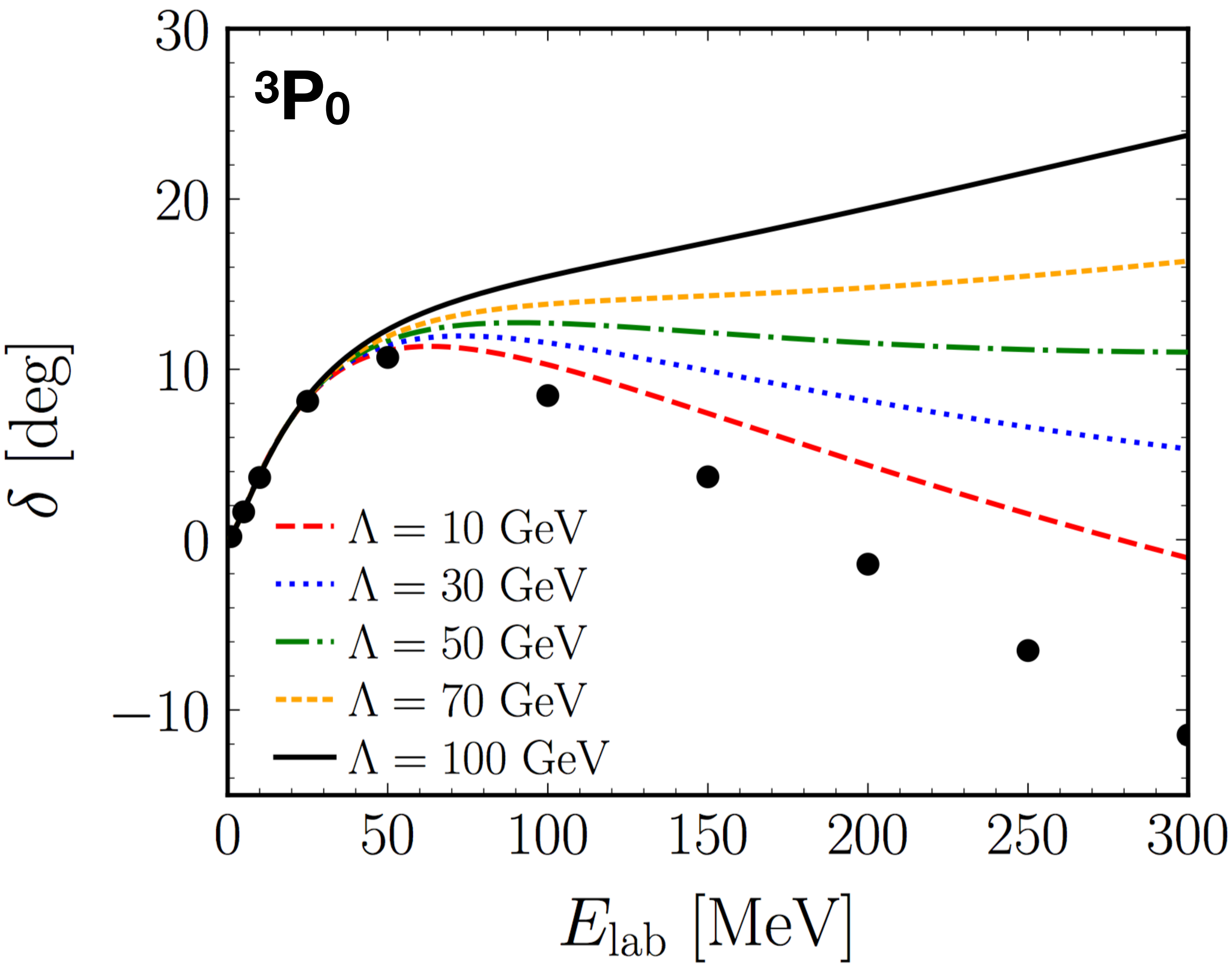
✓ $pGp' :$  $\xrightarrow{k\to\infty}$ $\propto \Lambda^2 + E\Lambda + (3m^2 - E^2/2)\log\frac{\Lambda}{m}$

✓ $pGT_\pi Gp' :$  $\xrightarrow{k\to\infty}$ **Logarithmic divergence**

It is cutoff dependent, not renormalizable!

Cutoff dependence!

RUB



From the point view of Renormalization Group Invariance *C.-J. Yang_arXiv:1905.12510*

- The $^3P_0$ potential is not singular, and therefore does not require a contact term to achieve RG-invariance.
- Promoting a contact term to LO in the non-perturbative treatment will destroy the RG, **unless extra care is taken to further subtract the divergences**.

Amplitude renormalization

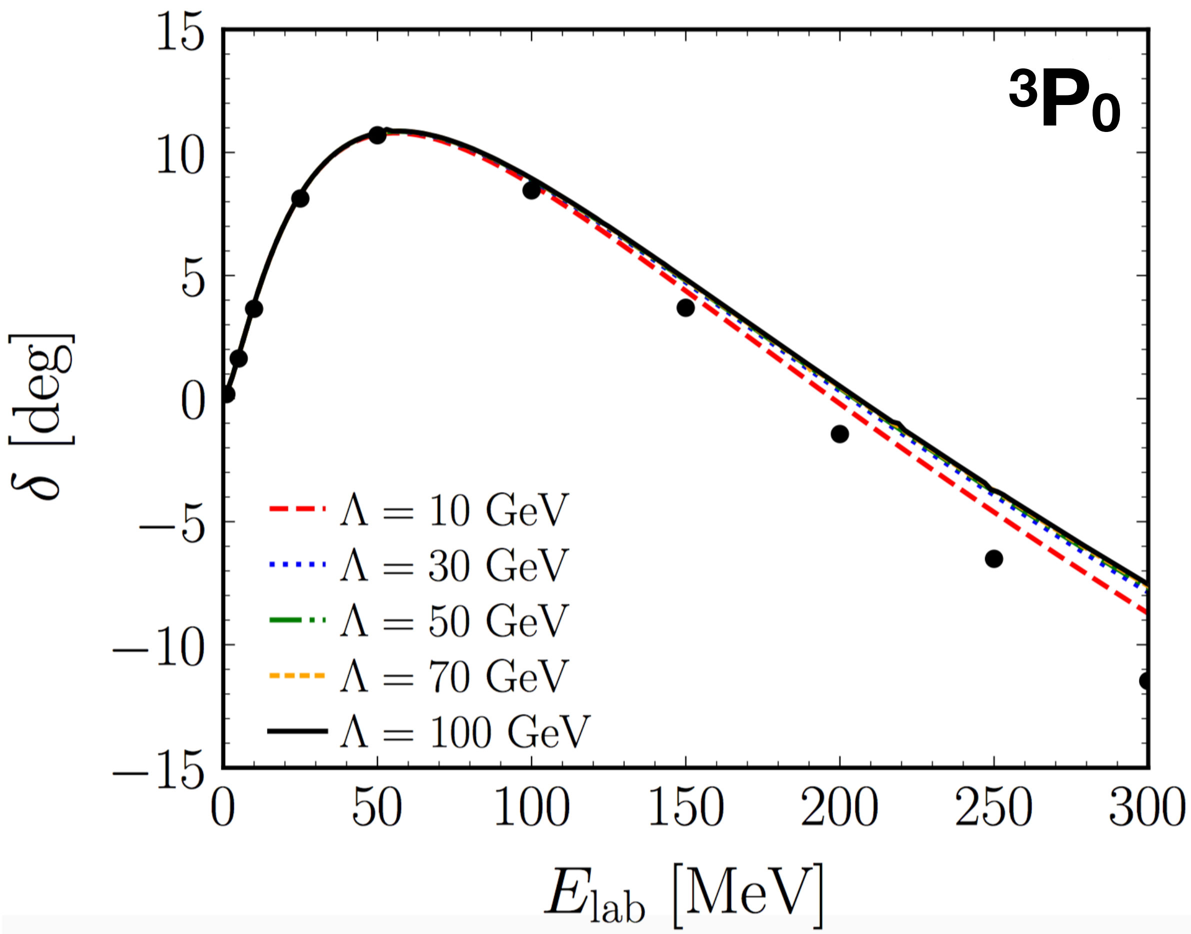
- Use subtractive renormalization to subtract all the divergences in loop diagrams

J. Collins, Renormalization, Cambridge Uni. Press
E. Epelbaum, et al., EPJA51(2015)71

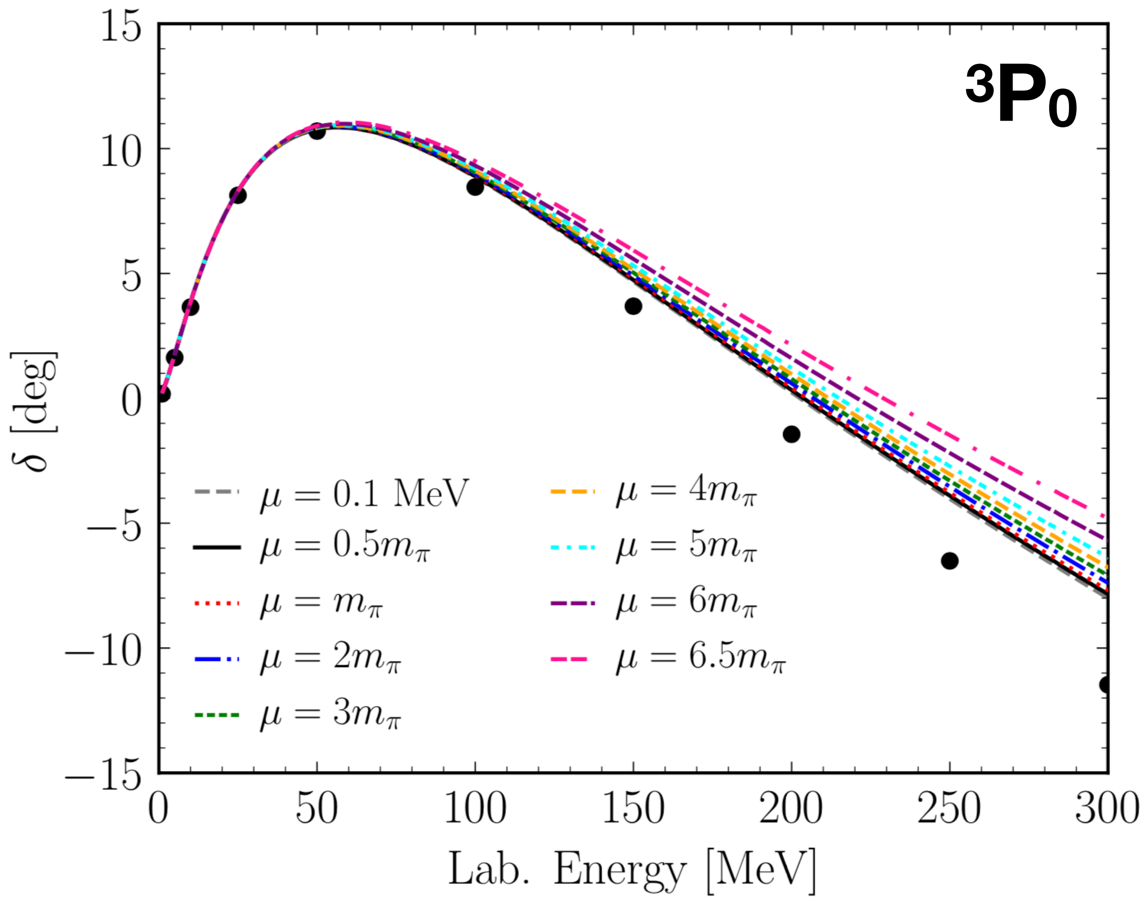
$$(pGp')^R = \frac{m^2p^2}{4\pi^2 E}\left[2p\left(\sinh^{-1}\frac{p}{m} - i\pi\right) - \pi m\right) + \frac{m^2p^2}{8\pi^2\sqrt{m^2-\mu^2}}\left(2\mu\left(\sin^{-1}\frac{\mu}{m} - \pi\right) + \pi m\right].$$

- Renormalized amplitude: $T^R_{3P0} = T_\pi + \frac{[(1+T_\pi G)p'][p(1+GT_\pi)]}{(C^R)^{-1} - (pGp')^R - pGT_\pi Gp'}$

Cutoff independent!

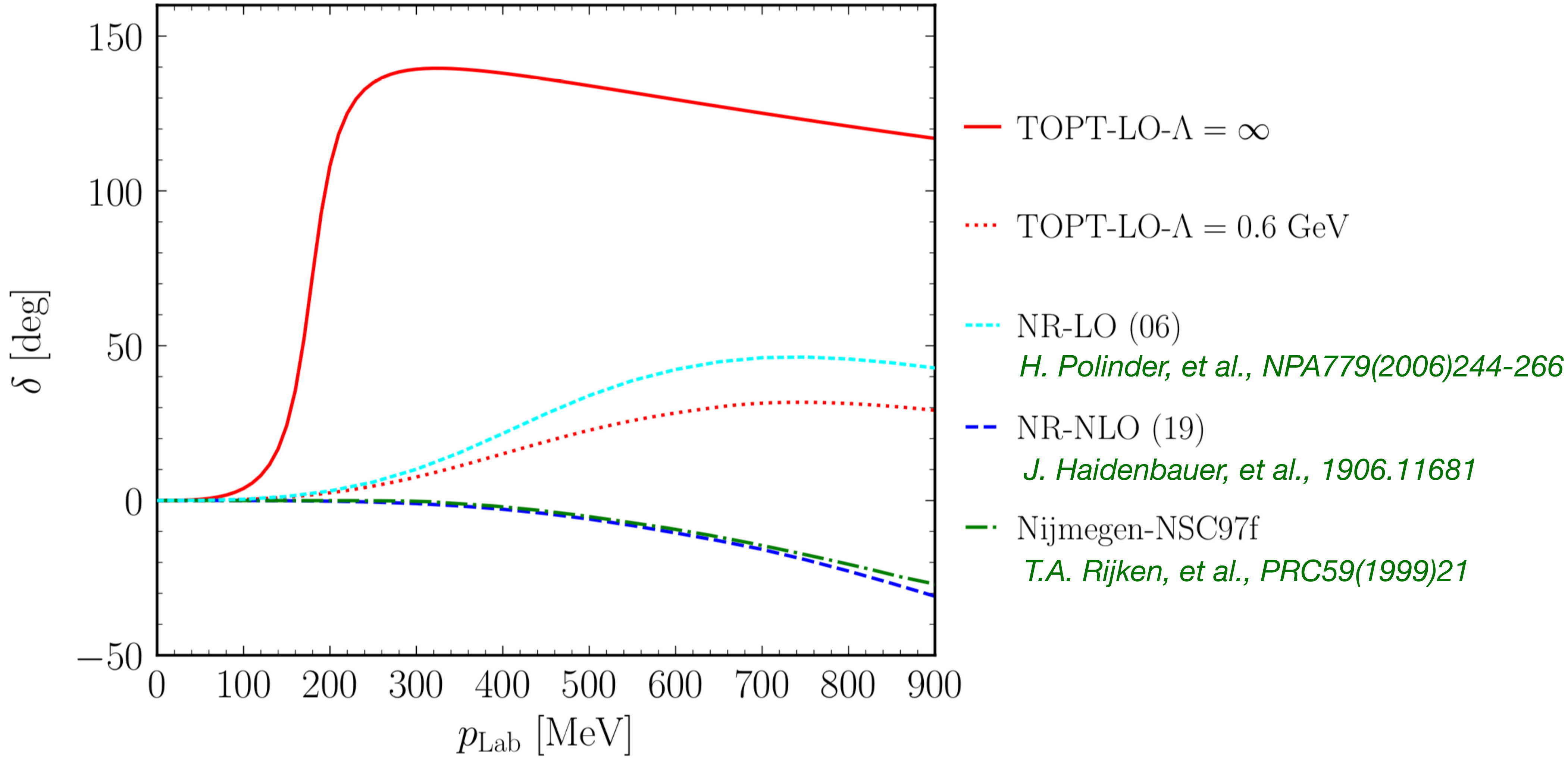


Scale dependence is weak!



ΛN–ΣN scattering

- $^3P_0$ phase shifts up to LO



- In order to improve the description of $^3P_0$ phases
  - NLO contact terms are promoted to LO

$$V^{^3P_0}_{\mathrm{LO}}(p_1', p_2'; p_1, p_2) = V_C(p_1', p_2'; p_1, p_2) + V^{IJ,KL}_{\mathrm{LO},M_P}$$

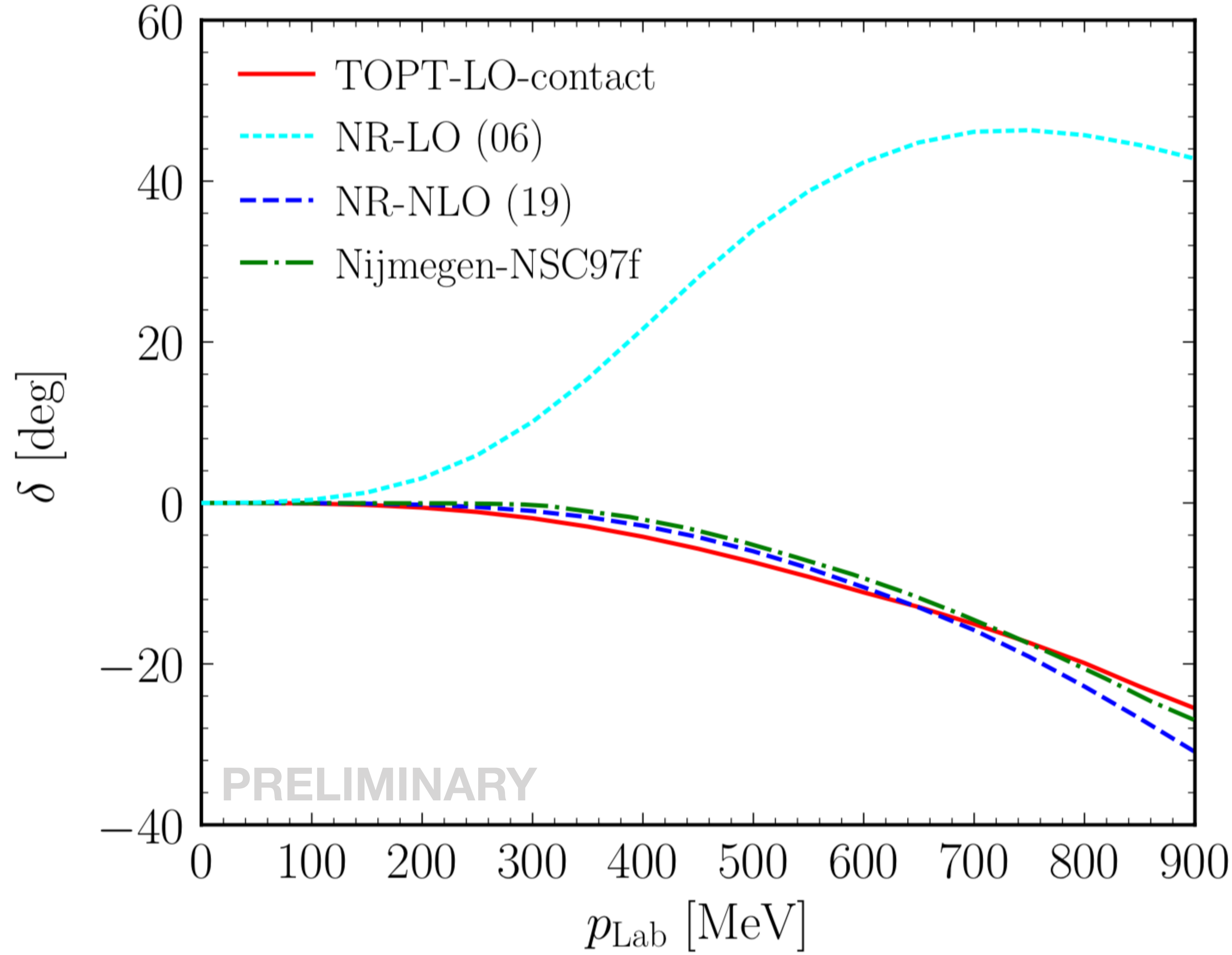
$$V_C = \xi(p_1', p_2')\,\mathcal{C}\,\xi(p_1, p_2), \quad \mathcal{C} = \begin{pmatrix} C_{\Lambda N,\Lambda N} & C_{\Lambda N,\Sigma N} \\ C_{\Sigma N,\Lambda N} & C_{\Sigma N,\Sigma N} \end{pmatrix}, \quad \xi(p_1, p_2) = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}.$$

  - Use the subtractive renormalization to achieve a renormalizable potential

ΛN–ΣN scattering

RUB

- $^3P_0$ phase shifts



XLR, E.Epelbaum, J.Gegelia, Hyperon-nucleon scattering, in preparation

Summary and perspectives

RUB

- We proposed a systematic framework to formulate the baryon-baryon interactions based on the time-ordered perturbation theory using the Lorentz invariant effective Lagrangian
  - Obtained the rules of time-ordered diagrams with spin-1/2 fermions
  - Derived the generalized Kadyshevsky equation self-consistently
  - Calculated the baryon-baryon interactions up to leading order, which is renormalizable
  - Achieved a rather good description of $^3P_0$ phases by promoting the NLO contact terms in a renormalizable way
- Higher order studies are in progress
  - Perturbatively/Non-perturbatively include NLO/NNLO contributions
  - Keep the momentum cutoff $\Lambda < \Lambda_{\chi PT}$ or $\Lambda \sim \infty$

THANK YOU FOR YOUR ATTENTION!

Backup slides

1/r^2 potential

RUB

- Whether 1/r^2 potential has a unique solution depends on the strengths

$$V(r) = \frac{\hbar^2}{m}\frac{c}{r^2} \qquad \text{with} \quad r \equiv |\boldsymbol{r}| \quad c \equiv -\tfrac{1}{4} - \nu^2$$

  - For the couplings larger than critical value (-1/4) equations do not have unique solutions.
  - For the couplings smaller than critical value (-1/4) equations have unique solutions.

- For potentials more singular than 1/r^2, the equations do not have unique solutions.

Subtractive renormalization

- Subtractive renormalization of the considered problem corresponds to the inclusion of contributions of an infinite number of counter terms generated by bare parameters of the effective Lagrangian.