Fate of Heavy Quark Bound States inside Quark-Gluon Plasma

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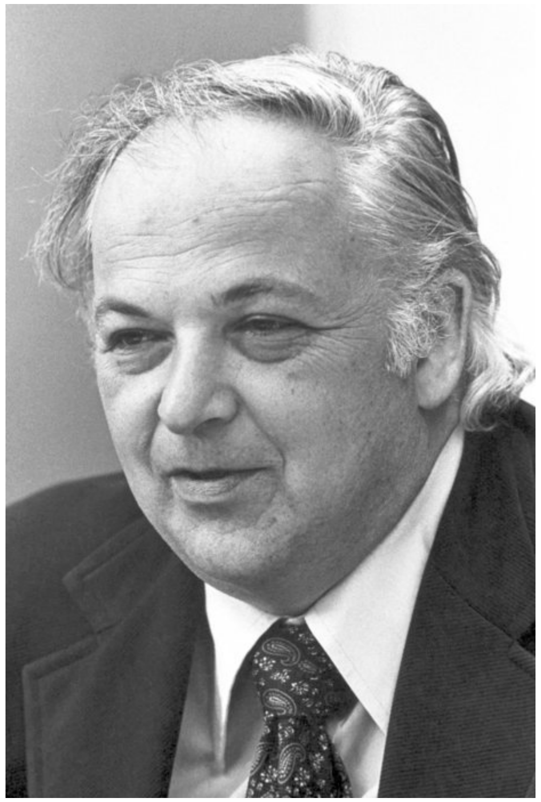
Introduction: Quarkonium

- 1974 discovery of J/Ψ at BNL and SLAC: bound state of charm anticharm —> Nobel prize in 1976



- Ground and lower excited states spectrum can be understood from potential models:

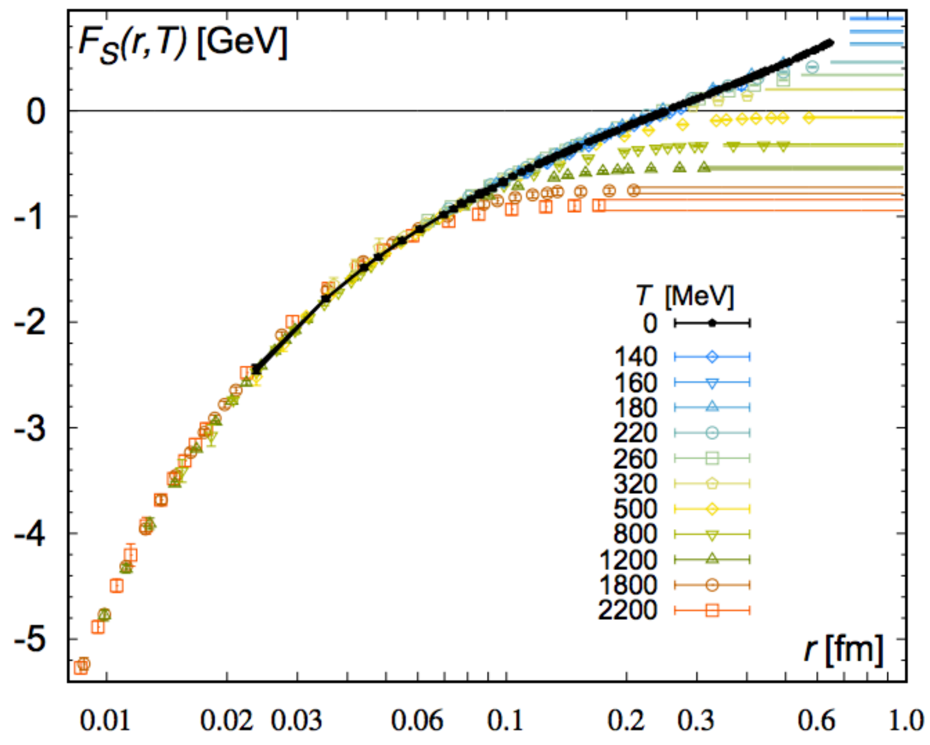
Cornell potential $$V(r) = -\frac{A}{r} + Br$$

Quarkonium inside Plasma

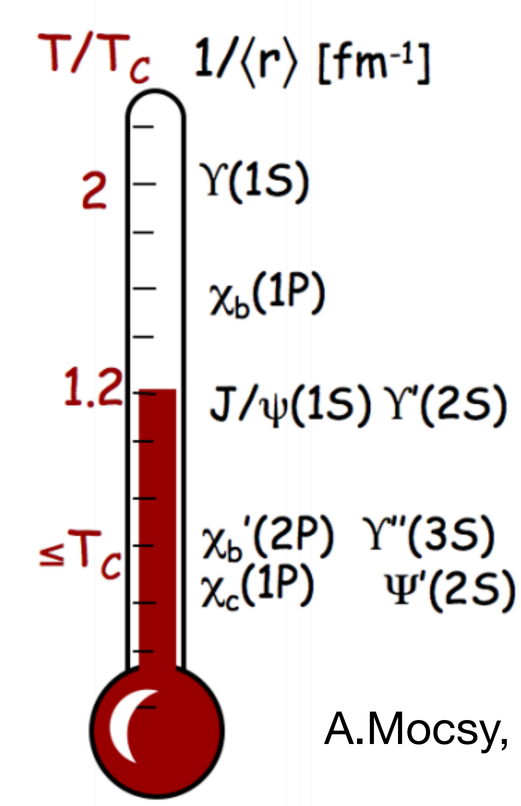
- 1986, Matsui & Satz studied Debye screening effect on heavy quark bound state inside quark-gluon plasma (QGP, deconfined phase of QCD)

- Static plasma screening: real part of attractive potential suppressed —> melting of quarkonium at high temperature —> probe QGP

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



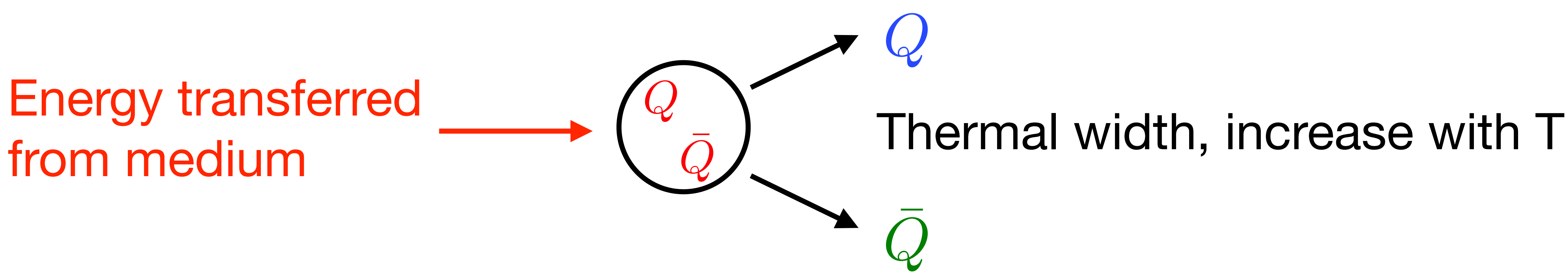
arXiv:1804.10600, A.Bazavov, N.Brambilla P.Petreczky, A.Vairo, J.H.Weber

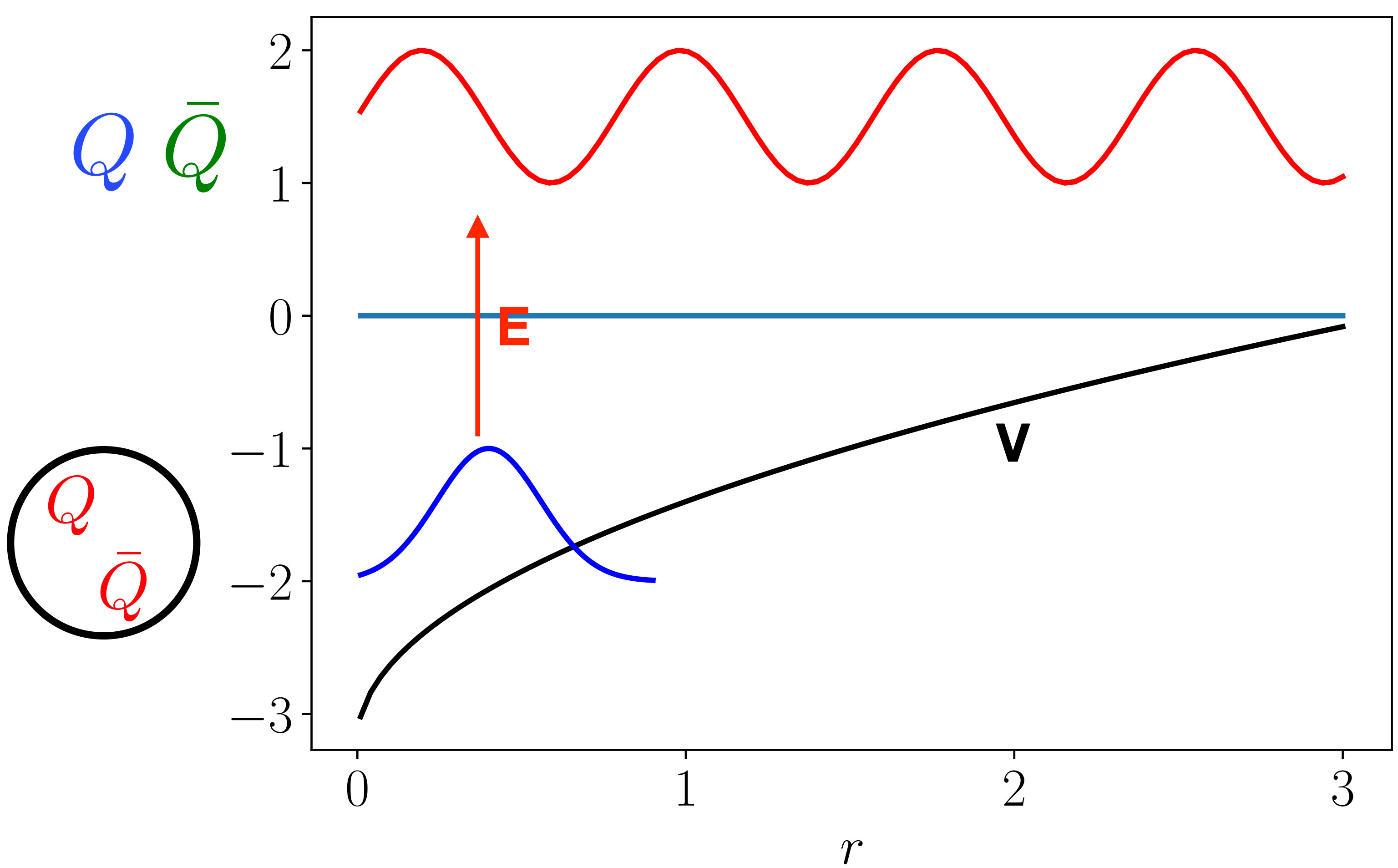


A.Mocsy, arXiv:0811.0337

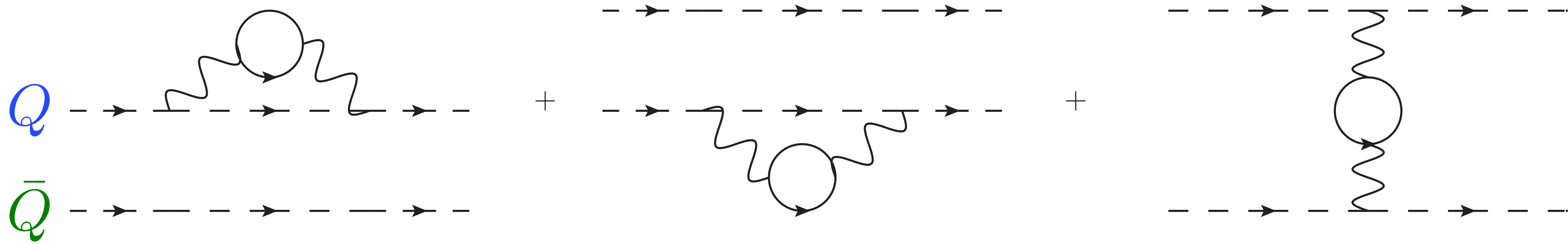
Quarkonium inside Plasma

- Dynamical plasma screening: dissociation induced by scattering (imaginary part of potential)



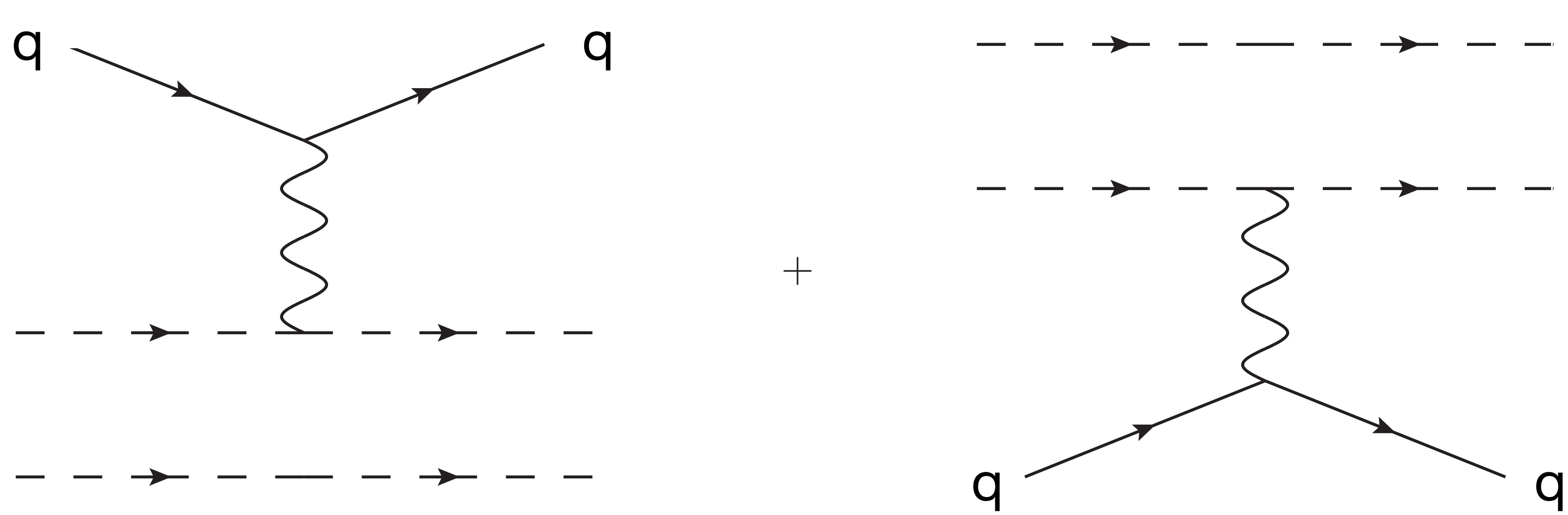


Imaginary Potential from Perturbation



Hard Thermal Loop

$$D_{00}(0,\boldsymbol{k}) = \frac{i}{\boldsymbol{k}^2 + m_D^2} + \pi \frac{T}{|\boldsymbol{k}|} \frac{m_D^2}{(\boldsymbol{k}^2 + m_D^2)^2}$$



Mass Shift and Width from Lattice at Finite T

- **Spectral function**

$$D_H^{>}(t,\vec{x}) = \langle J_H(t,\vec{x}) J_H(0,\vec{0})\rangle \,,$$

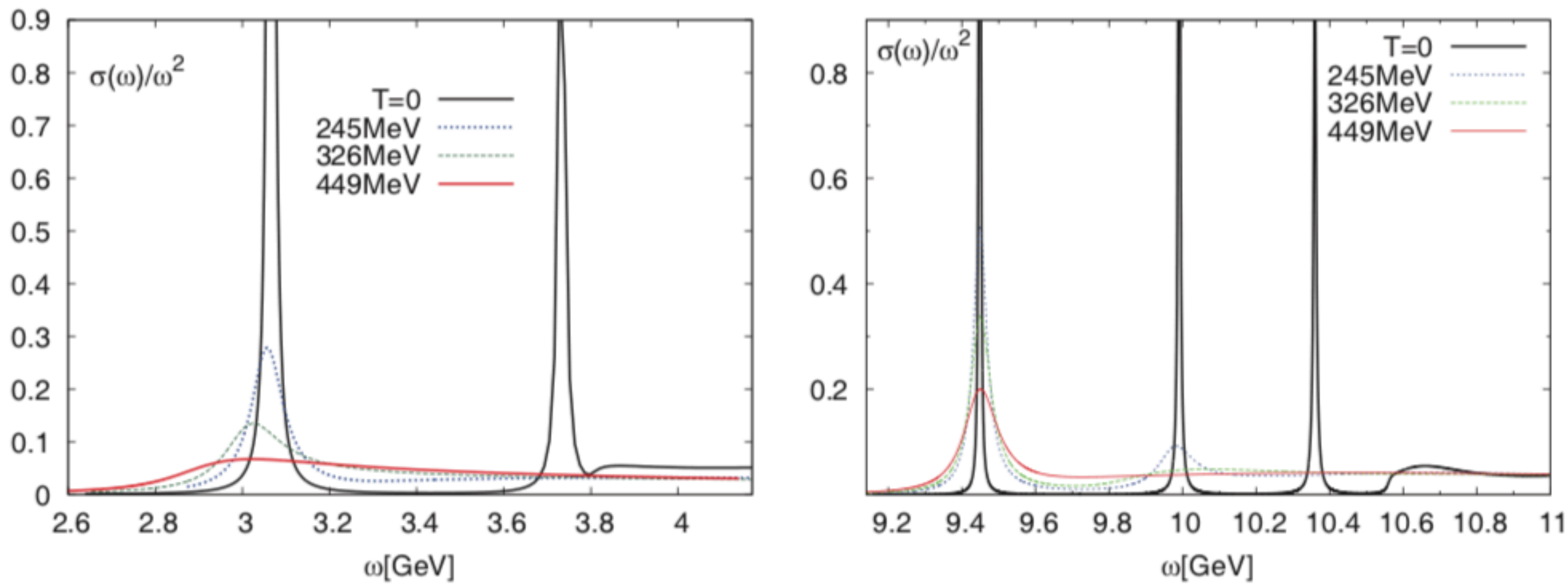
$$D_H^{<}(t,\vec{x}) = \langle J_H(0,\vec{0}) J_H(t,\vec{x})\rangle \,, \quad t>0$$

$$\sigma(\omega,\vec{p}) = \frac{1}{2\pi}\Big(D_H^{>}(\omega,\vec{p}) - D_H^{<}(\omega,\vec{p})\Big) = \frac{1}{\pi}\,\mathrm{Im}\, D_H^R(\omega,\vec{p})$$

$$J_H(t,x) = \bar{q}(t,\vec{x})\Gamma_H q(t,\vec{x})$$

$$G(\tau,\vec{p}) = \int_0^\infty d\omega\, \sigma(\omega,\vec{p}) K(\omega,\tau)\,, \quad K(\omega,\tau) = \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

A.Mocsy, P.Petreczky
M.Strickland, arXiv:1302.2180



- **Define potential from Wilson loops**

$$\frac{i\partial_t W_\square(r,t)}{W_\square(r,t)} = \frac{\int d\omega\, \omega\, e^{-i\omega t}\rho_\square(r,\omega)}{\int d\omega\, e^{-i\omega t}\rho_\square(r,\omega)} \equiv V_\square(r,t).$$

A.Rothkopf, T.Hatsuda
S.Sasaki arXiv:1108.1579

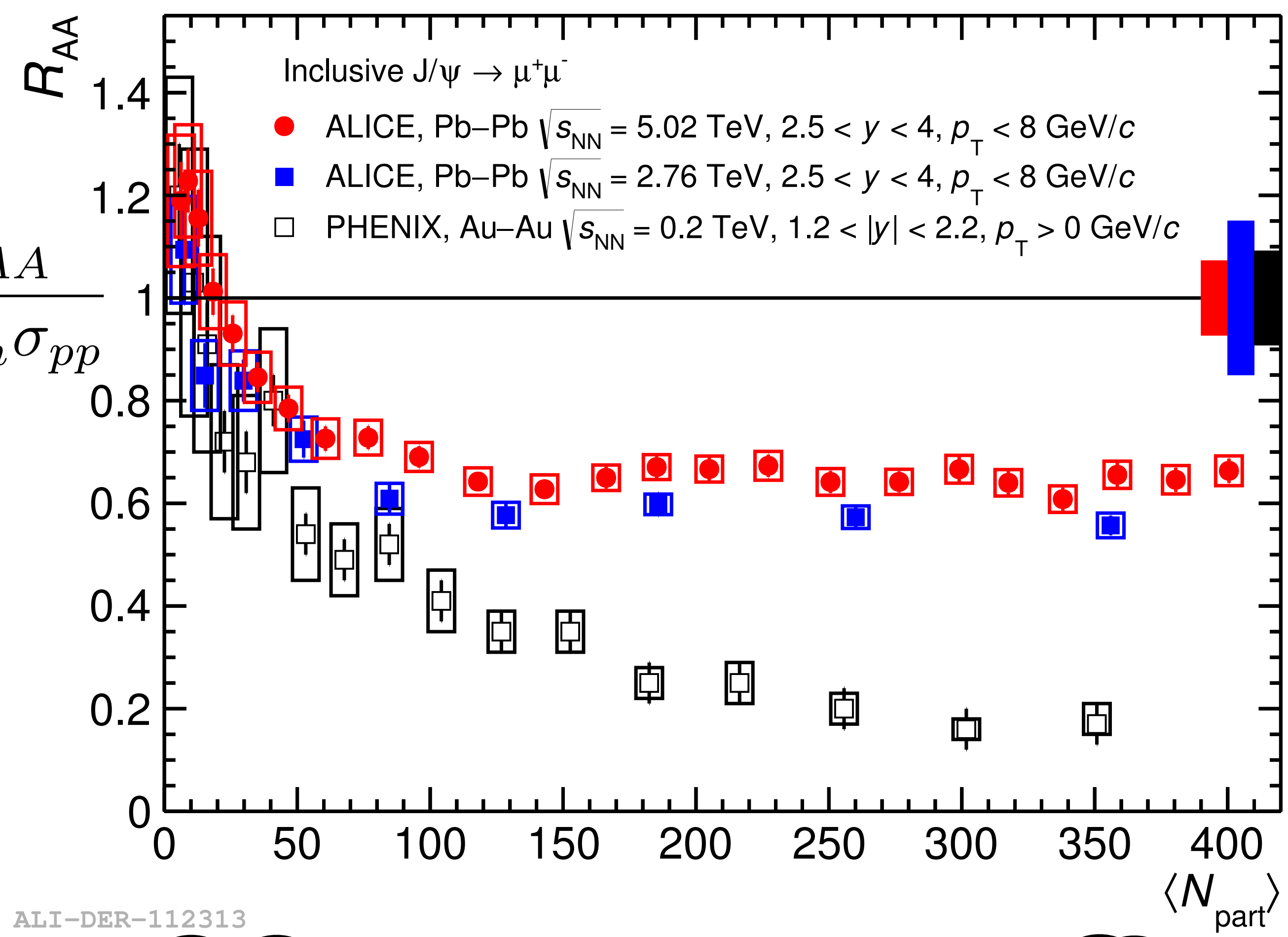
Quarkonium in Heavy Ion Collisions

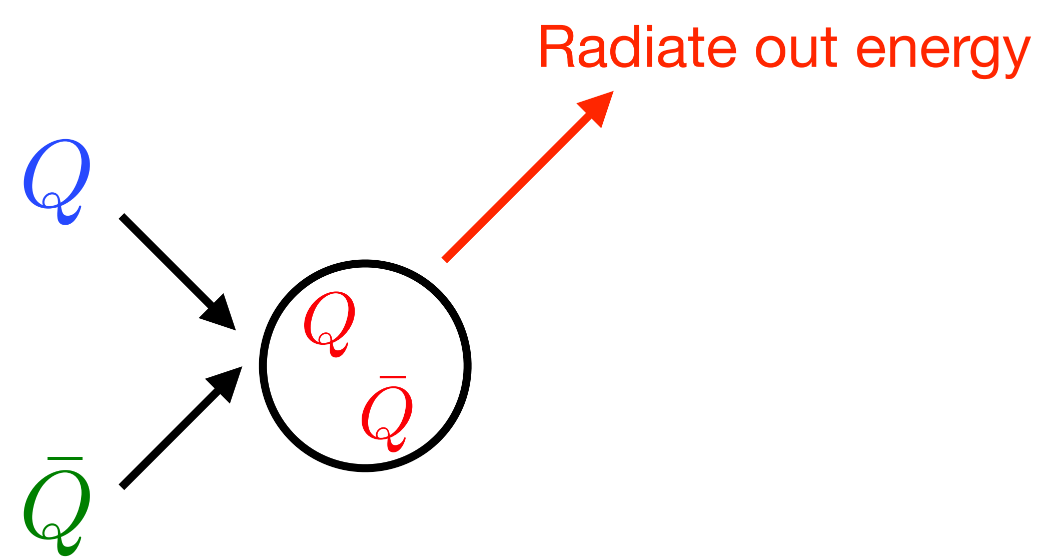
- Heavy ion collisions: quark-gluon plasma, deconfined phase of quarks and gluons, strongly-coupled fluid with small shear viscosity

- Higher collision energy —> higher QGP temperature —> quarkonium more suppressed?

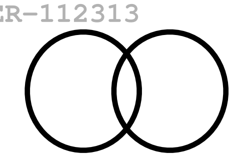
- **Recombination**: J/psi less suppressed at LHC than RHIC, can happen inside QGP below melting temperature

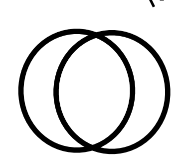
$$R_{AA} \equiv \frac{\sigma_{AA}}{N_{bin}\sigma_{pp}}$$





RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)





Quarkonium Transport inside QGP

- Transport equations: $$\frac{dN}{d\tau} = -\Gamma(T)N + \alpha(\tau)\Gamma(T)N^{\mathrm{eq}}(T)$$

  Dissociation rate calculated from QCD

  Recombination modeled: detailed balance, phenomenological factor

- Phenomenological success, but need improvement:

  Dissociation and recombination not same theoretical framework

  Cannot explain approach to equilibrium

  Recombination should depend on real-time distributions of open heavy flavors

  Relation to the underlying quantum evolution?

Open Quantum System

$$H = H_S + H_E + H_I$$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

System & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

System & environment

Trace out (integrate out) environment

$$\rho_S(t=0)$$

System(heavy quark pairs)

Non-unitary

Time irreversible

$$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$$

Evolution equation not von Neumann, but Lindblad

From Open Quantum System to Transport Equation

Lindblad equation:

$$H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

Trace perserving

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\Big)$$

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1,t_2) \langle a|O_\beta^{(S)}(t_2)|b\rangle \langle c|O_\alpha^{(S)}(t_1)|d\rangle^*$$

$$\sigma_{ab}(t) \equiv \frac{-i}{2} \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1,t_2) \mathrm{sign}(t_1 - t_2) \langle a|O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2)|b\rangle$$

$$C_{\alpha\beta}(t_1,t_2) \equiv \mathrm{Tr}_E(O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2)\rho_E)$$

$|a\rangle$ Eigenstates of H_S

$$L_{ab} = |a\rangle\langle b|$$

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}_{nls}^{(+)}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}_{nls}^{(-)}(\boldsymbol{x},\boldsymbol{k},t)$$

From Open Quantum System to Transport Equation

Lindblad equation: **Correction to Hamiltonian**

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}\Big)$$

Markovian approximation

Wigner transform

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

Recombination **Dissociation**

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t)$$

XY T.Mehen, arXiv:1811.07027

Two Key Assumptions

1. **System interacts weakly with environment ?**

2. **Markovian assumption (no memory effect) ?**

Effective field theory and separation of scales

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$ $\quad v^2 \sim 0.3$ charmonium

Bound state exists $\quad v^2 \sim 0.1$ bottomonium

QCD

M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Mv — perturbative matching / non-perturbative matching

Potential NRQCD

$Mv^2 \sim 500$ MeV — hep-ph/9907240, N.Brambilla A.Pineda, J.Soto, A.Vairo

T

m_D

NR & multipole expansions up to linear order of r $\quad \mathrm{S}(\boldsymbol{R},\boldsymbol{r},t) \quad \mathrm{O}(\boldsymbol{R},\boldsymbol{r},t)$

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\,\mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

$$H_s = \frac{(i\boldsymbol{\nabla}_{\mathrm{cm}})^2}{4M} + \frac{(i\boldsymbol{\nabla}_{\mathrm{rel}})^2}{M} + V_s^{(0)} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \cdots \longrightarrow H_{s,o} = \frac{(i\boldsymbol{\nabla}_{\mathrm{rel}})^2}{M} + V_{s,o}^{(0)}$$

- **In quarkonium c.m. frame, c.m. energy suppressed by at least one power of v when** $v_{\mathrm{med}} \lesssim \sqrt{1-v}$

- **Virial theorem**
- **No hyperfine splitting**

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + \boxed{V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.})} + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Dipole interaction $r \sim \frac{1}{Mv}$

Weak coupling between quarkonium and QGP: quarkonium small in size

$rMv^2 \sim rT \sim rm_D \sim v$ suppressed

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r \, \mathrm{Tr}\Big(\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger (iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots \Big)$$

Dipole interaction $r \sim \frac{1}{Mv}$

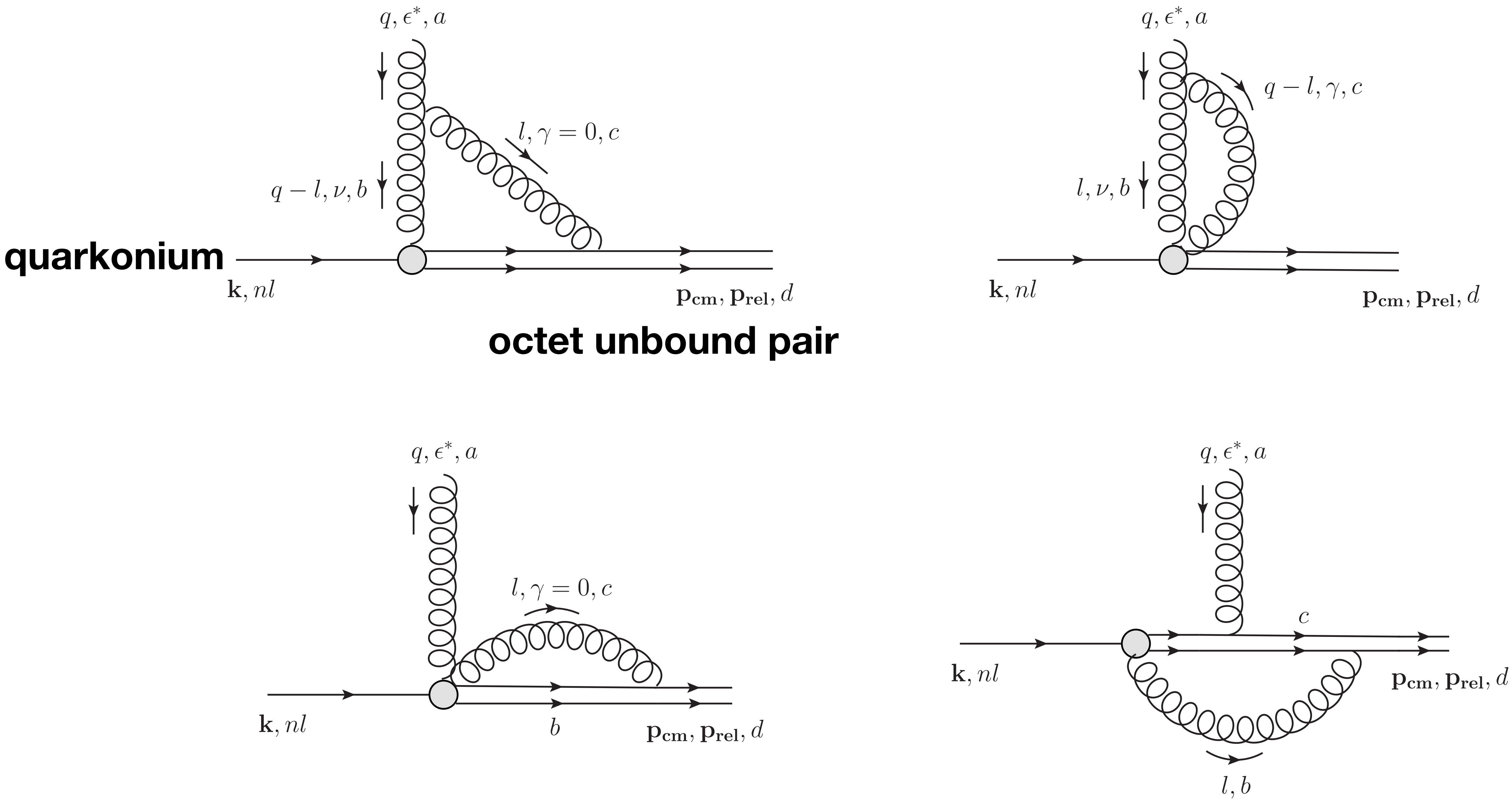
Weak coupling between quarkonium and QGP: quarkonium small in size

$rMv^2 \sim rT \sim rm_D \sim v$ suppressed

Perturbative matching gives $V_A(\mu = Mv) = 1$

Running? large log? no at one loop

Running of Dipole Interaction



$$\frac{0}{\epsilon} + \cdots \qquad \frac{d}{d\mu} V_A(\mu) = 0$$

A. Pineda and J. Soto, Phys. Lett. B 495, 323 (2000)

XY, B.Müller arXiv:1811.09644

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}$

For 1st term: $|d\rangle = |\boldsymbol{k}_1, n_1 l_1, 1\rangle$ $\quad |a\rangle = |c\rangle = |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$ $\quad |b\rangle = |\boldsymbol{k}_3, n_3 l_3, 1\rangle$

Linear order in r : transition between bound singlet & unbound octet

$$\gamma_{ab,cd} = \int d^3R_1 \int d^3R_2 \sum_{i_1,i_2,b_1,b_2} \int_0^t dt_1 \int_0^t dt_2\, C_{\boldsymbol{R}_1 i_1 b_1, \boldsymbol{R}_2 i_2 b_2}(t_1, t_2)$$
$$\langle \boldsymbol{k}_1, n_1 l_1, 1 | \langle S(\boldsymbol{R}_1, t_1) | r_{i_1} | O^{b_1}(\boldsymbol{R}_1, t_1)\rangle | \boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$$
$$\langle \boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1 | \langle O^{b_2}(\boldsymbol{R}_2, t_2) | r_{i_2} | S(\boldsymbol{R}_2, t_2)\rangle | \boldsymbol{k}_3, n_3 l_3, 1\rangle$$

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^\dagger_{cd}L_{ab},\rho_S(0)\}$

For 1st term: $|d\rangle = |\boldsymbol{k}_1, n_1 l_1, 1\rangle \qquad |a\rangle = |c\rangle = |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle \qquad |b\rangle = |\boldsymbol{k}_3, n_3 l_3, 1\rangle$

Linear order in r : transition between bound singlet & unbound octet

$$\gamma_{ab,cd} = \int d^3R_1 \int d^3R_2 \sum_{i_1,i_2,b_1,b_2} \int_0^t dt_1 \int_0^t dt_2\, C_{\boldsymbol{R}_1 i_1 b_1, \boldsymbol{R}_2 i_2 b_2}(t_1,t_2)$$

$$\langle \boldsymbol{k}_1, n_1 l_1, 1| \langle S(\boldsymbol{R}_1,t_1)| r_{i_1} | O^{b_1}(\boldsymbol{R}_1,t_1)\rangle |\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1\rangle$$

$$\langle \boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a_1| \langle O^{b_2}(\boldsymbol{R}_2,t_2)| r_{i_2} | S(\boldsymbol{R}_2,t_2)\rangle |\boldsymbol{k}_3, n_3 l_3, 1\rangle$$

$$\langle \Psi_{\boldsymbol{p}_{\rm rel}} | r_{i_2} | \psi_{n_3 l_3}\rangle \delta^{a_1 b_2} e^{-i(E_{\boldsymbol{k}_3} t_2 - \boldsymbol{k}_3\cdot \boldsymbol{R}_2)} e^{i(E_{\boldsymbol{p}} t_2 - \boldsymbol{p}_{\rm cm}\cdot \boldsymbol{R}_2)}$$

$$\frac{T_F}{N_C} g^2 \langle E^{b_1}_{i_1}(\boldsymbol{R}_1,t_1) E^{b_2}_{i_2}(\boldsymbol{R}_2,t_2)\rangle_T$$

$$= \frac{T_F}{N_C} g^2 \delta^{b_1 b_2} \int \frac{d^4q}{(2\pi)^4} e^{iq_0(t_1-t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) n_B(q_0)(2\pi)\mathrm{sign}(q_0)\delta(q_0^2 - \boldsymbol{q}^2)$$

Used < thermal correlator, can also use > because of $1 + n_B(q_0) + n_B(-q_0) = 0$

Sign fixed by energy conservation

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^{\dagger}_{cd}L_{ab},\rho_S(0)\}$

Markovian approximation: $t \longrightarrow \infty$ when doing time integral

Valid when environment correlation time << system relaxation time

T^{-1} dissociation frequency

correlation scale T rate $(grT)^2T \sim T\frac{\alpha_s T^2}{(Mv)^2} \lesssim \alpha_s v^2 T$

Putting everything together, make Wigner transform:

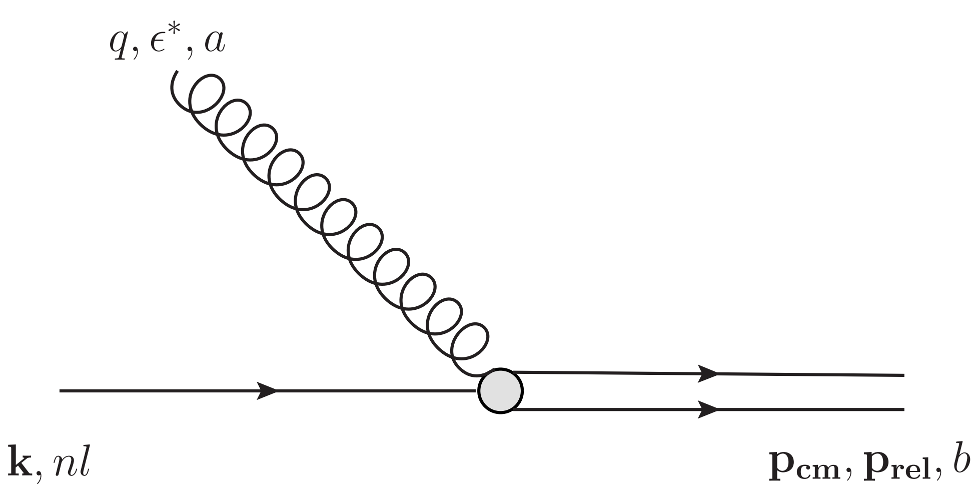
Spatial & time integrals give delta functions (E&p conservation)

E&p conservation

$$t\int \frac{d^3p_{\rm cm}}{(2\pi)^3}\frac{d^3p_{\rm rel}}{(2\pi)^3}\frac{d^3q}{(2\pi)^3 2q} n_B(q)(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\rm cm}+\boldsymbol{q})\delta(E_k-E_p+q)$$

Phase space measure

$$\frac{2}{3}C_F q^2 g^2 |\langle\psi_{nl}|\boldsymbol{r}|\Psi_{\boldsymbol{p}_{\rm rel}}\rangle|^2 f_{nl}(\boldsymbol{x},\boldsymbol{k},t=0)$$



Amplitude squared

For Coulomb potential and neglect octet repulsive potential, get Peskin-Bhanot result

Boltzmann Transport Equation

Phase space free streaming

$$\rho_S(t) = \rho_S(0) - it(H_{eff}\rho_S(0) - \rho_S(0)H_{eff}) + \cdots$$

↓ Wigner transform

$$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = f_{nl}(\boldsymbol{x},\boldsymbol{k},0) - it\int\frac{d^3k'}{(2\pi)^3}e^{i\boldsymbol{k}'\cdot\boldsymbol{x}}(E_{\boldsymbol{k}+\frac{\boldsymbol{k}'}{2}} - E_{\boldsymbol{k}-\frac{\boldsymbol{k}'}{2}})\langle\boldsymbol{k}+\frac{\boldsymbol{k}'}{2},nl,1|\rho_S(0)|\boldsymbol{k}-\frac{\boldsymbol{k}'}{2},nl,1\rangle + t\mathcal{C}_{nl}^{(+)} - t\mathcal{C}_{nl}^{(-)}$$

$$E_{\boldsymbol{k}\pm\frac{\boldsymbol{k}'}{2}} = -|E_{nl}| + \frac{(\boldsymbol{k}\pm\frac{\boldsymbol{k}'}{2})^2}{4M}$$

Add spin dependence —> transport equation:

$$\frac{\partial}{\partial t}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}_{nls}^{(+)}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}_{nls}^{(-)}(\boldsymbol{x},\boldsymbol{k},t)$$

Open quantum system
Effective field theory: separation of scales

weak coupling between quarkonium & QGP
Markovian approximation

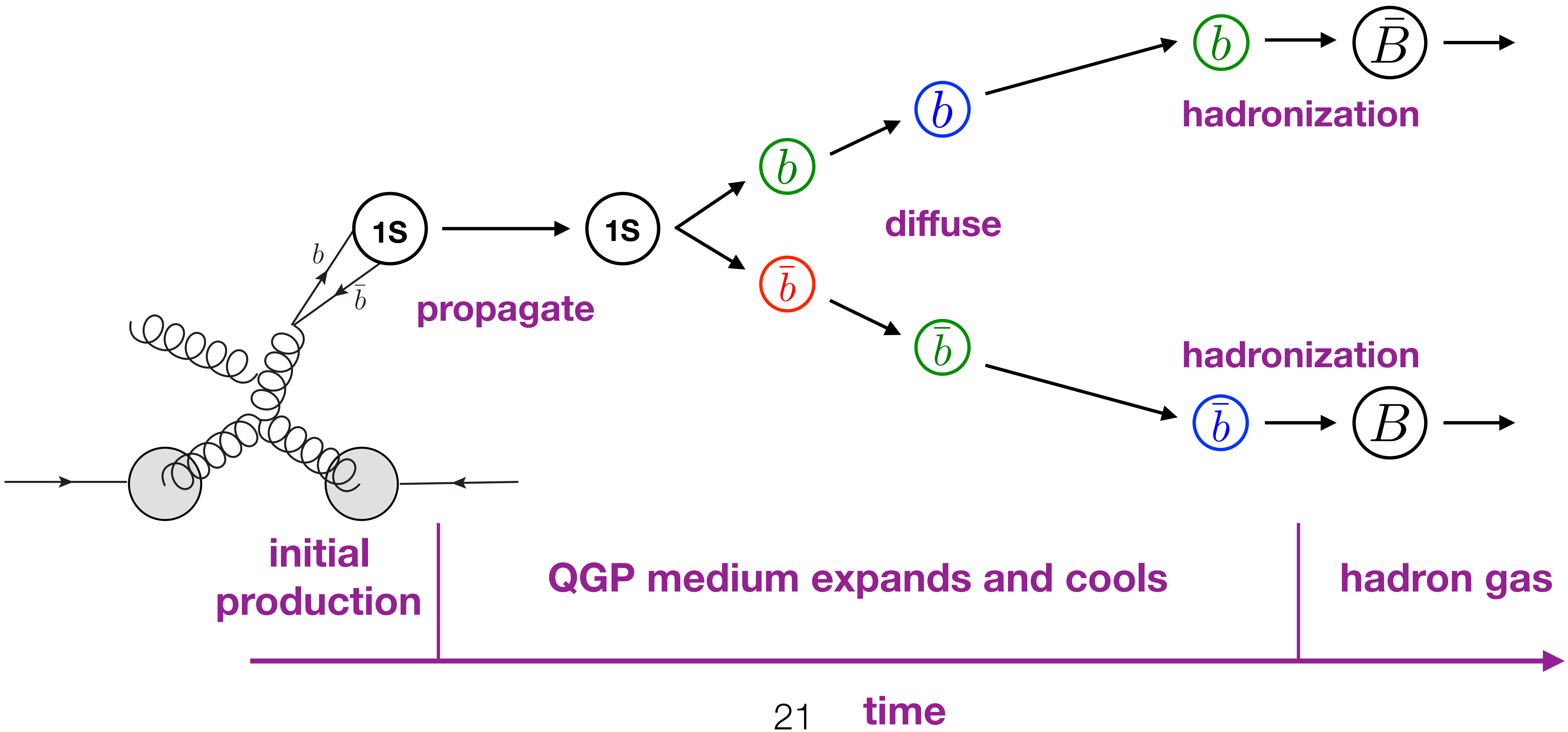
XY T.Mehen, arXiv:1811.07027

Coupled with Transport of Open Heavy Flavor

heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

anti-heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S,1P etc. $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



Coupled with Transport of Open Heavy Flavor

heavy quark
$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^{+} + \mathcal{C}_Q^{-}$$

anti-heavy quark
$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^{+} + \mathcal{C}_{\bar{Q}}^{-}$$

each quarkonium state
nl = 1S, 2S,1P etc.
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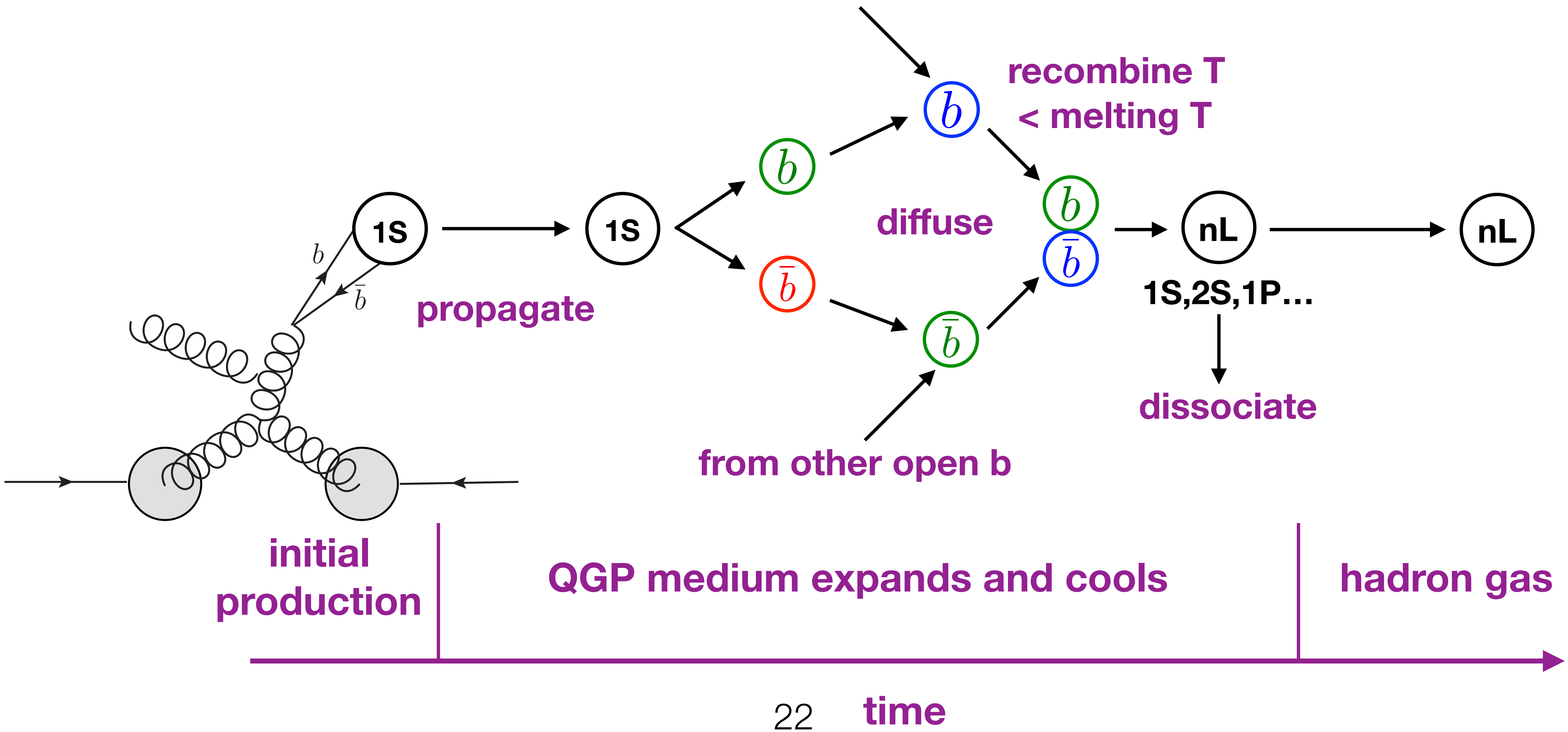
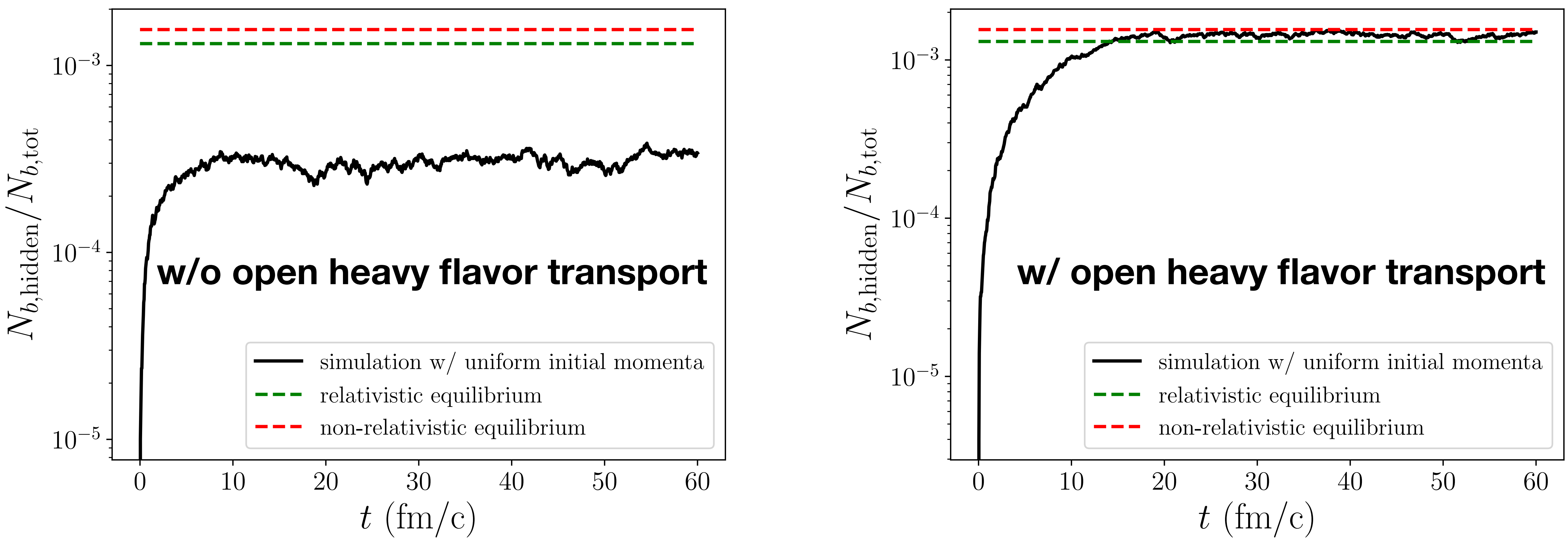


Detailed Balance and Thermalization

Setup:
- QGP box w/ const T=300 MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport

XY, B.Müller arXiv:1709.03529



Dissociation-recombination interplay drives to detailed balance

Heavy quark energy gain/loss necessary to drive kinetic equilibrium of quarkonium

Collision Event Simulation

- Initial production:

  PYTHIA 8.2: NRQCD factorization — Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159; Bodwin, Braaten, Lepage Phys. Rev. D 51, 1125 (1995)

  Nuclear PDF: EPS09 (cold nuclear matter effect) — Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

  Trento, sample position, hydro. initial condition — Moreland, Bernhard, Bass, Phys. Rev. C 92, no. 1, 011901 (2015)

- Medium background: 2+1D viscous hydrodynamics (calibrated)

  Song, Heinz, Phys.Rev.C77,064901(2008)
  Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)
  Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94,no.2,024907(2016)

- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

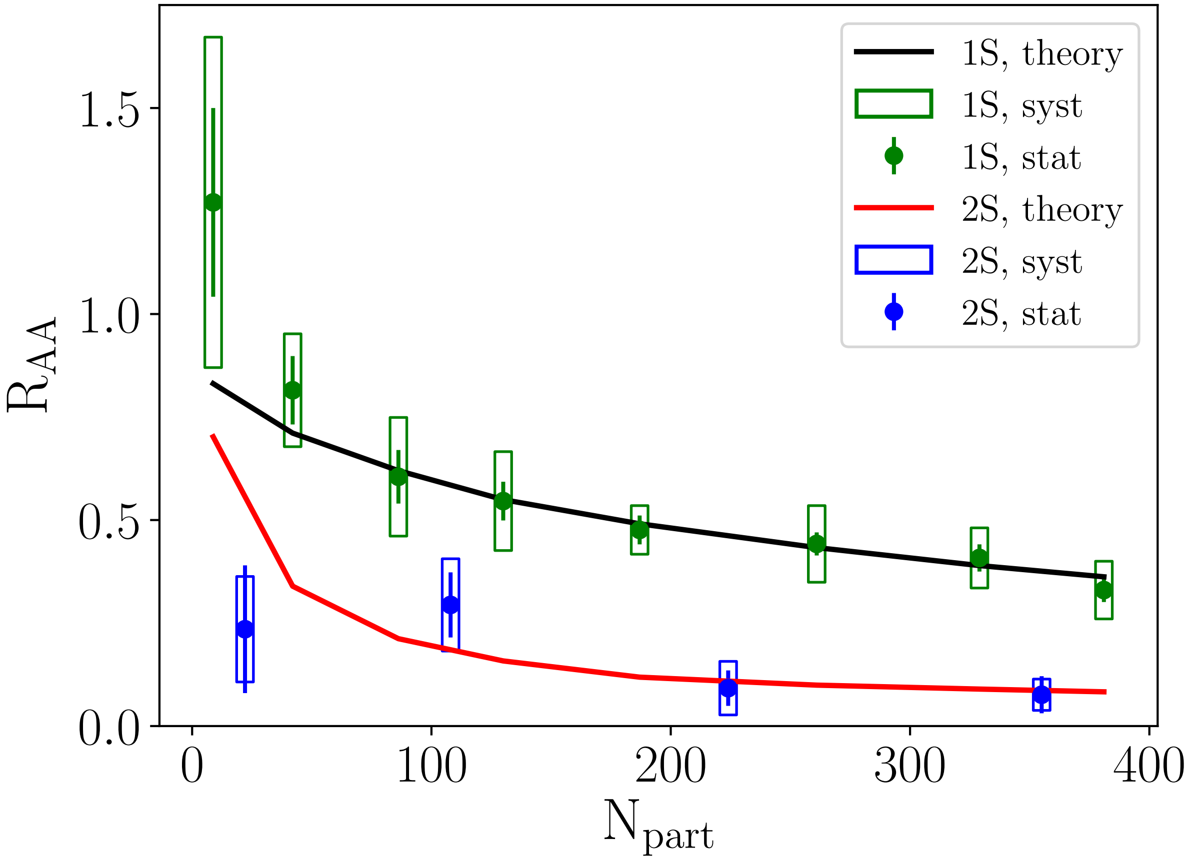
Upsilon in 2760 GeV PbPb Collision

Fix $\alpha_s = 0.3$

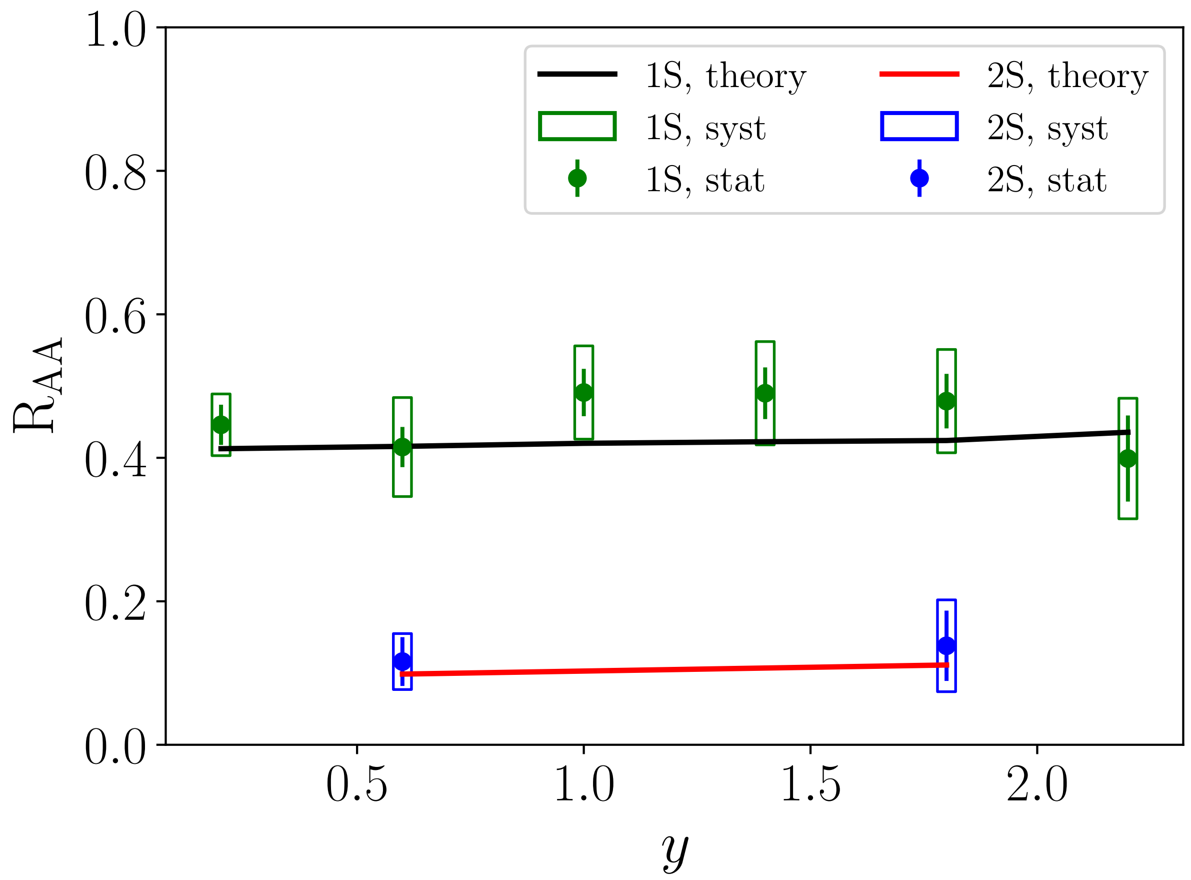
Tune $T_{\mathrm{melt}}(2S) = 210\ \mathrm{MeV}$

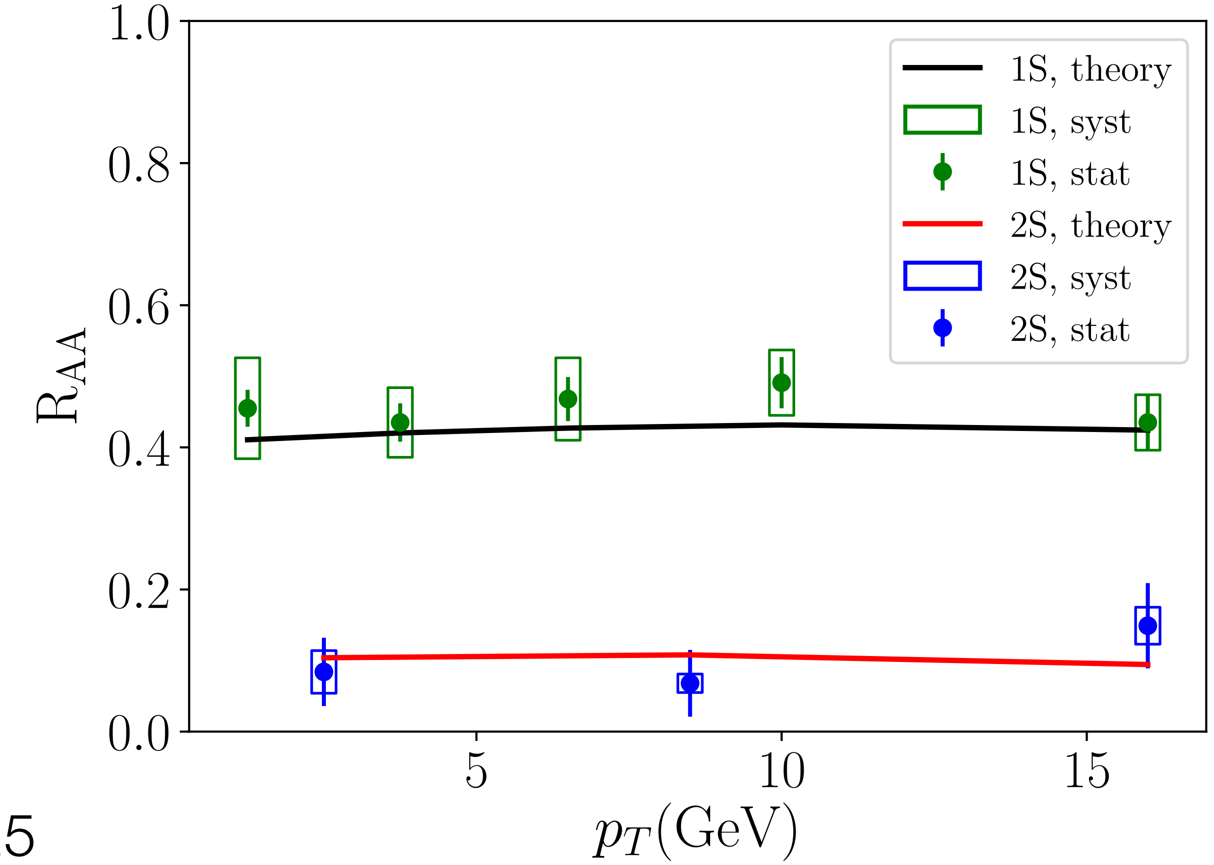
Tune $V_s = -C_F \dfrac{0.42}{r}$

Cold nuclear matter effect ~ 0.87 (PYTHIA + nPDF)



CMS Phys.Lett. B
770 (2017) 357-379

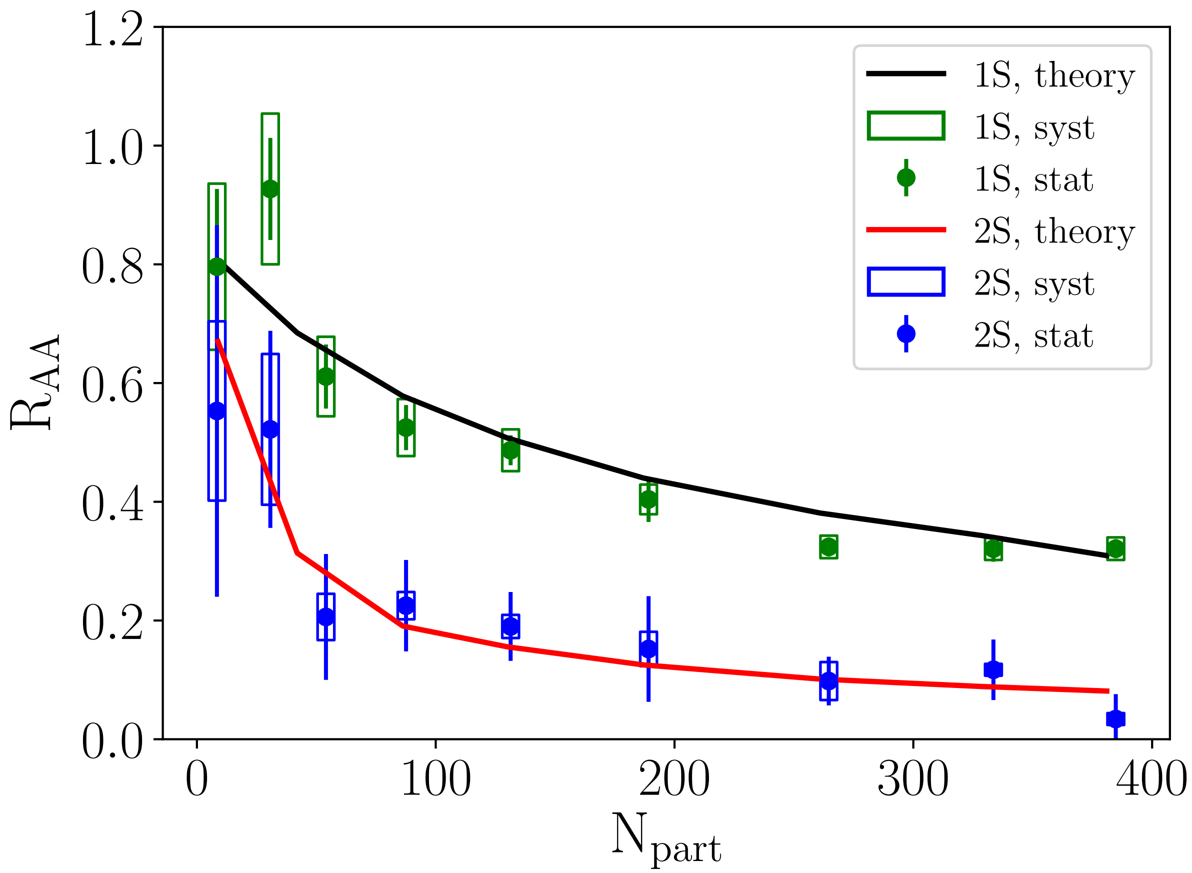




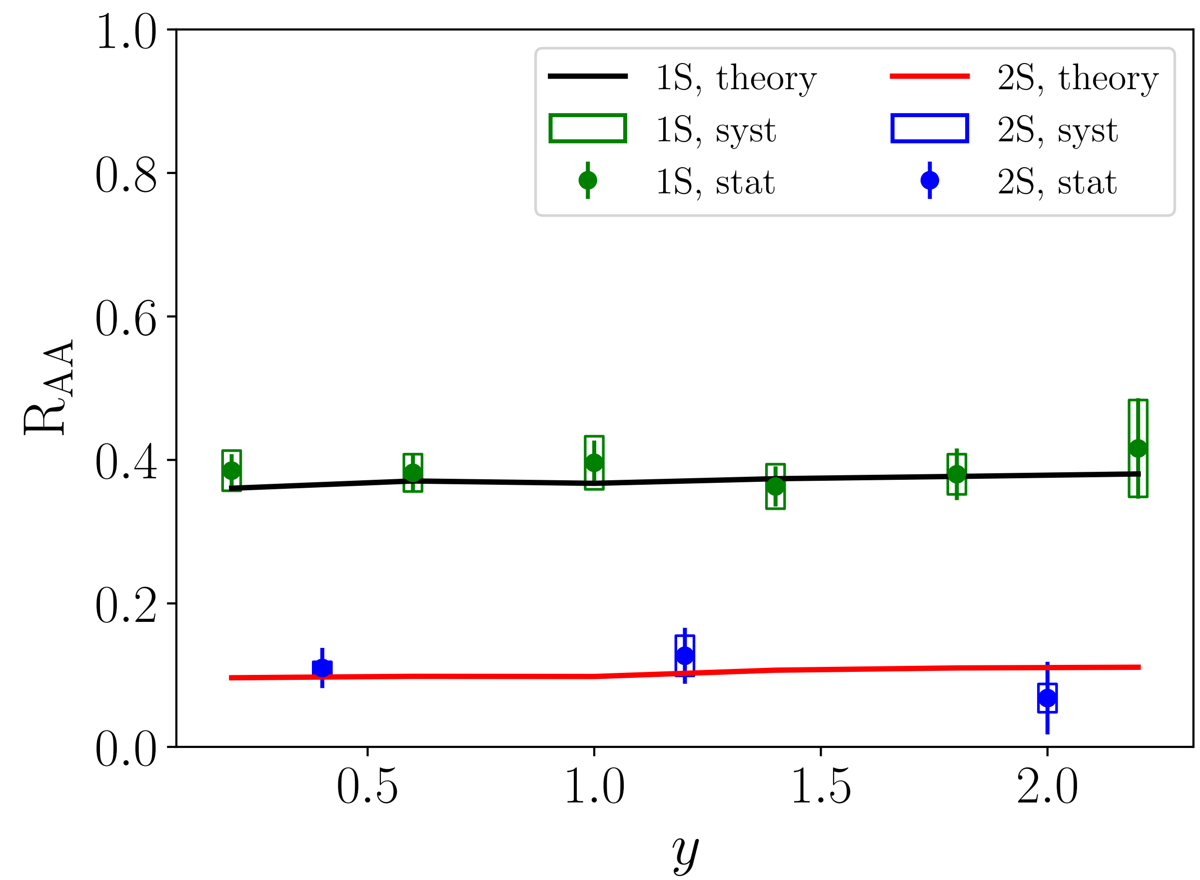
Upsilon in 5020 GeV PbPb Collision

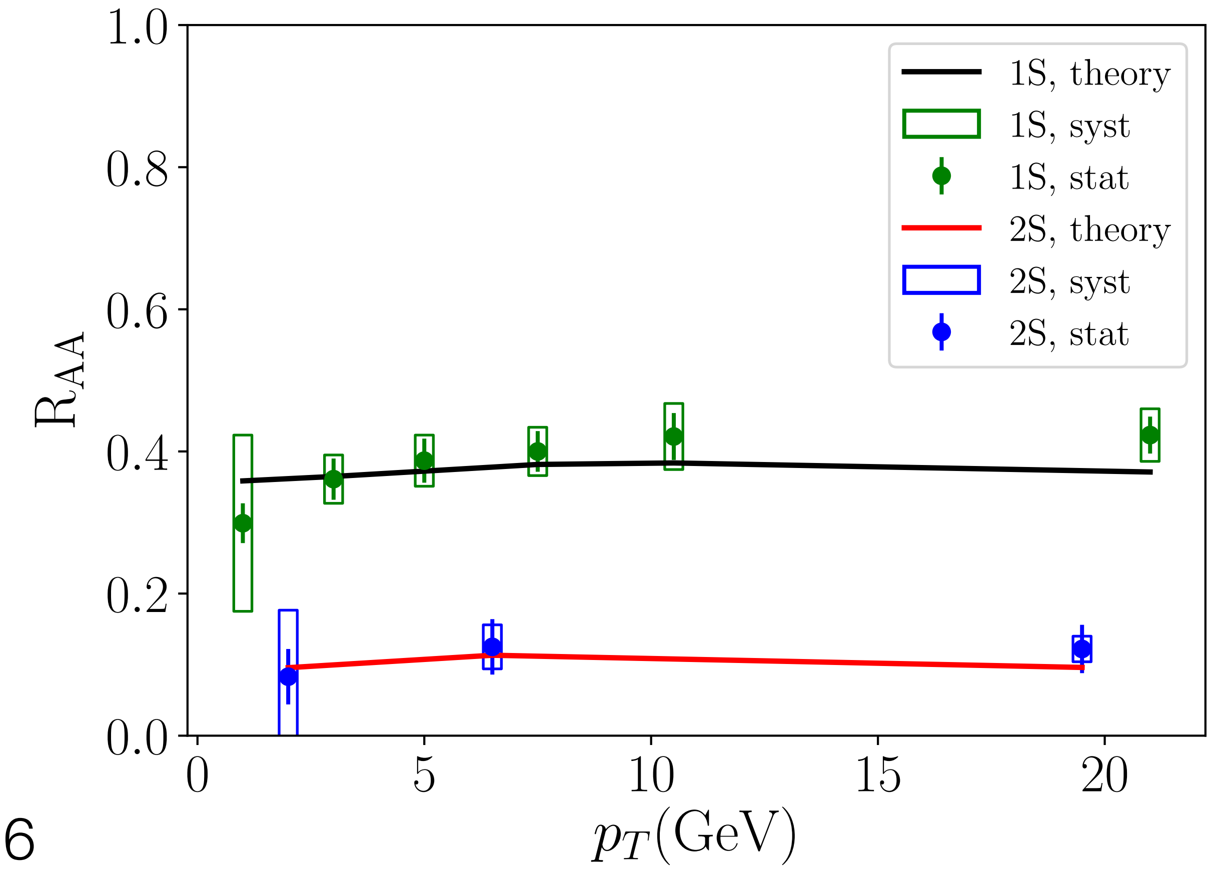
Use same set of parameters

Cold nuclear matter effect ~ 0.85 (PYTHIA + nPDF)



CMS arXiv:1805.09215

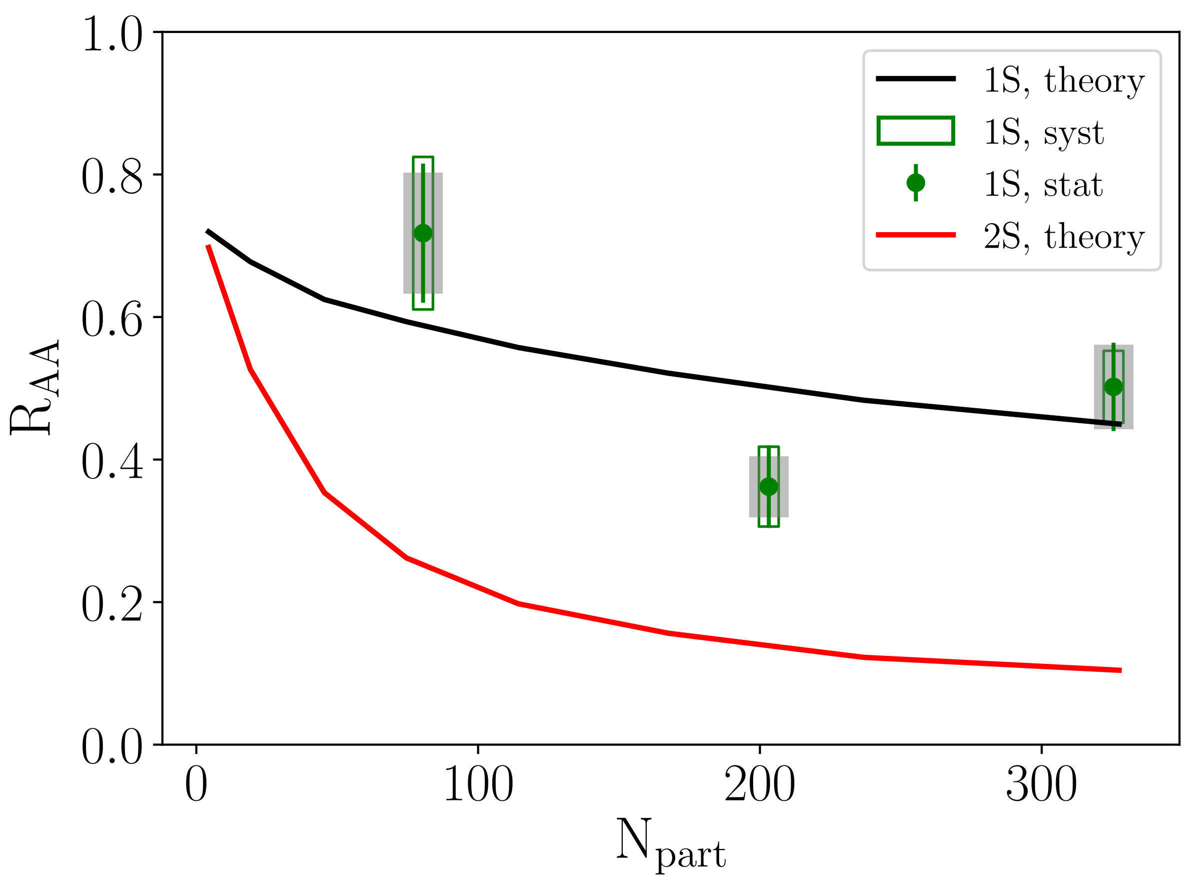


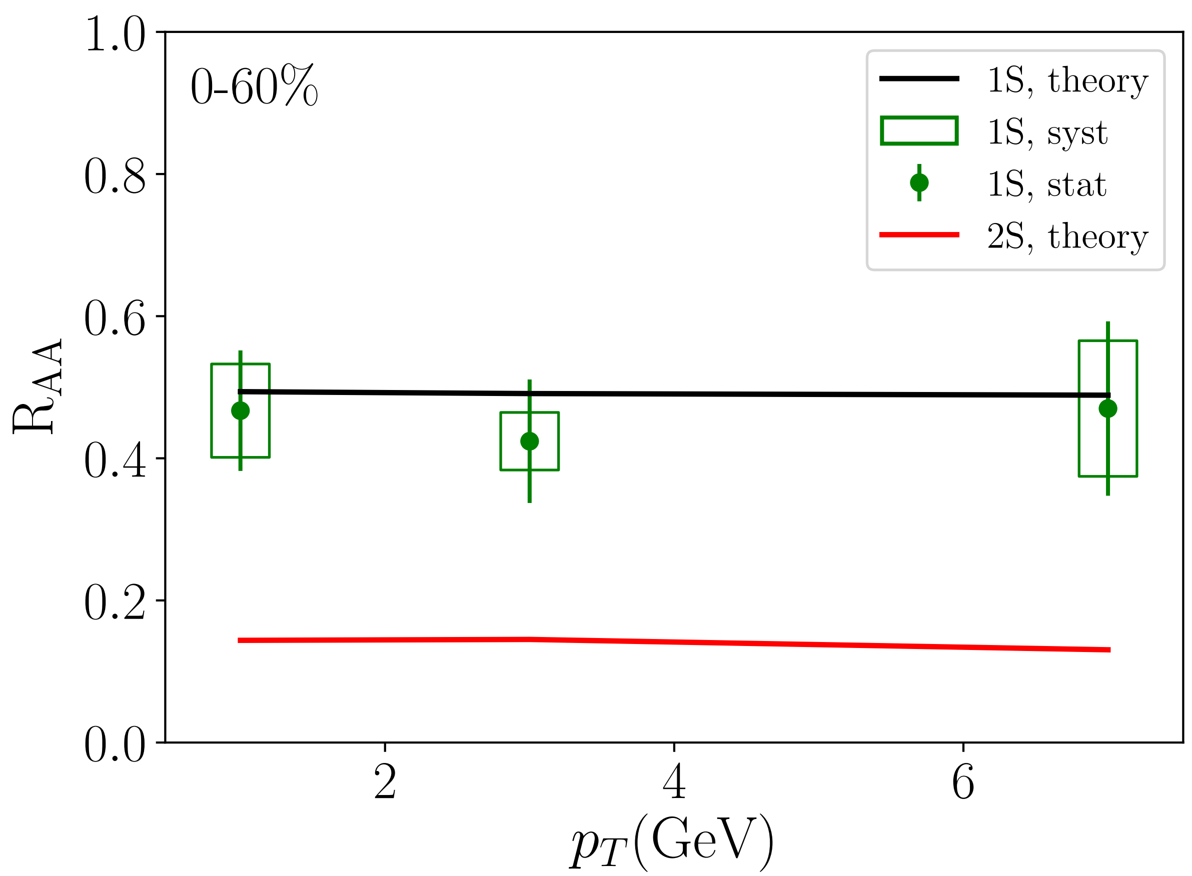


Upsilon in 200 GeV AuAu Collision

Use same set of parameters

Cold nuclear matter effect ~ 0.72 (use p-Au data of STAR)



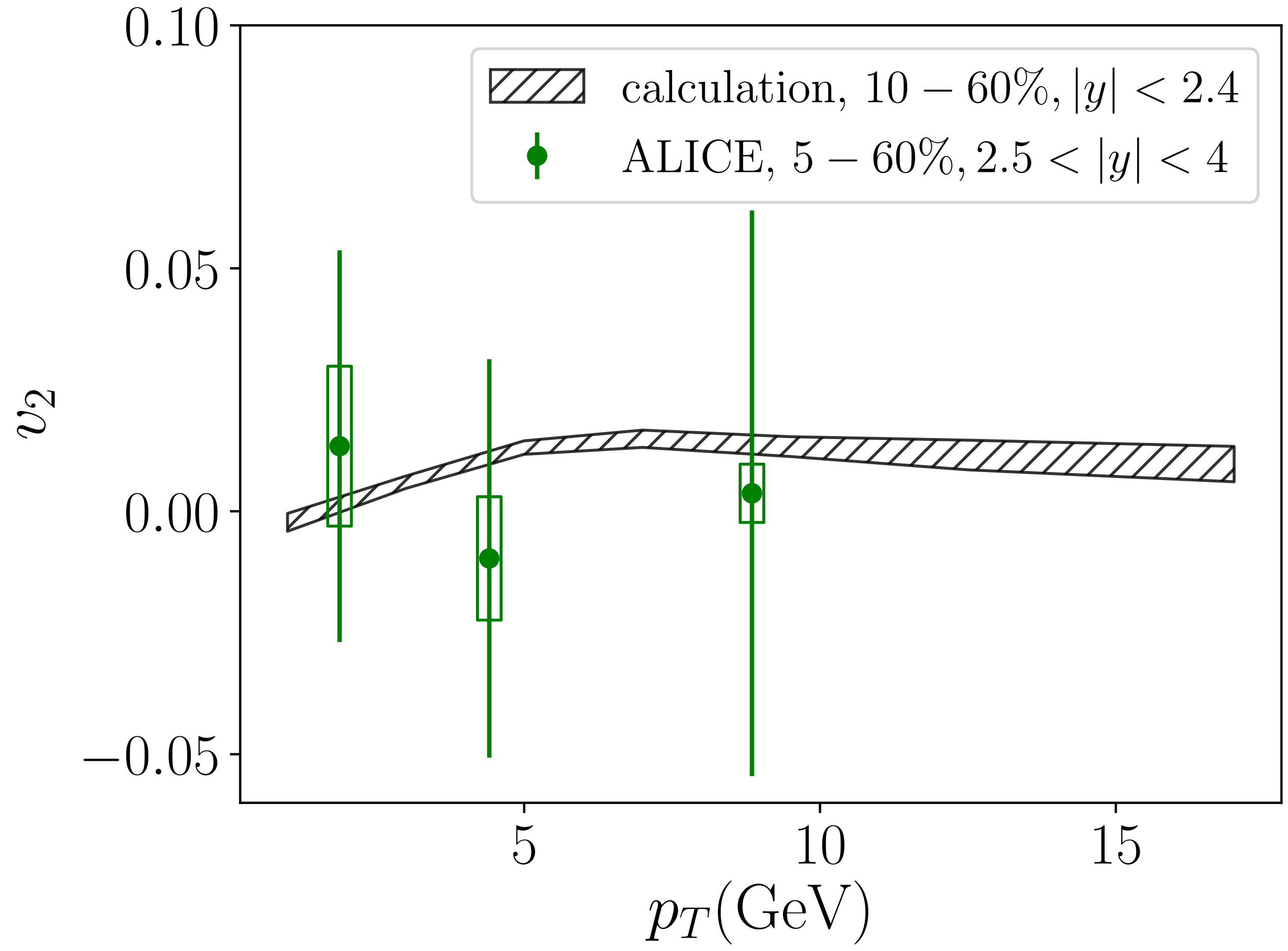


STAR measures 2S+3S

STAR Talks at QM 17&18

Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2v_2\cos(2\phi) + \cdots\right)$$



preliminary measurements by ALICE

Conclusion

- Derivation of transport equations from open quantum system and effective field theory
- **Assumptions used to derive semi-classical transport equations are justified from the separation of scales**
- **Coupled transport equations: detailed balance and thermalization**
- Phenomenological results on bottomonium production in heavy ion collisions

Backup: Mapping Operators

In general theory

$$H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)} \qquad L_{ab} \equiv |a\rangle\langle b|$$

$$\rho_S(t) = \rho_S(0) - i\Big[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2}\{L_{cd}^\dagger L_{ab}, \rho_S(0)\}\Big)$$

In pNRQCD

$$O_\alpha^{(S)} \to \langle S(\boldsymbol{R},t)|r_i|O^a(\boldsymbol{R},t)\rangle + \langle O^a(\boldsymbol{R},t)|r_i|S(\boldsymbol{R},t)\rangle$$

$$O_\alpha^{(E)} \to \sqrt{\frac{T_F}{N_C}} g E_i^a(\boldsymbol{R},t) \qquad \sum_\alpha \to \int d^3R \sum_i \sum_a$$

Complete set of states $|a\rangle$

$|\boldsymbol{k}, nl, 1\rangle = a_{nl}^\dagger(\boldsymbol{k})|0\rangle$ — **Bound singlet**

$|\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, 1\rangle = b_{\boldsymbol{p}_{\rm rel}}^\dagger(\boldsymbol{p}_{\rm cm})|0\rangle$ — **Unbound singlet**

$|\boldsymbol{p}_{\rm cm}, \boldsymbol{p}_{\rm rel}, a\rangle = c_{\boldsymbol{p}_{\rm rel}}^{a\dagger}(\boldsymbol{p}_{\rm cm})|0\rangle$ — **Unbound octet**

Wigner transform —> formulation in phase space, focus on bound state

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

Need to calculate $\langle \boldsymbol{k}_1, n_1 l_1, 1|\rho_S(t)|\boldsymbol{k}_2, n_2 l_2, 1\rangle$

Recombination $\gamma_{ab,cd} L_{ab} \rho_S(0) L_{cd}^\dagger$

Phase space measure **E&p conservation**

$$t \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) \sum_{a,i} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(-|E_{nl}| + q - \frac{\boldsymbol{p}_{\rm rel}^2}{M})$$

$$\frac{2T_F}{3N_C} q^2 g^2 \langle \psi_{nl} | r_i | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \int d^3 r \, \psi_{nl}(\boldsymbol{r}) r_i \Psi^*_{\boldsymbol{p}_{\rm rel}}(\boldsymbol{r}) f_{Q\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}_{\rm cm}, \boldsymbol{r}, \boldsymbol{p}_{\rm rel}, a, t=0)$$

$p_{\rm rel} \sim a_B^{-1} \sim Mv$

Amplitude squared, mixed with distribution function hard to implement numerically

When can we take distribution function out?

Uniformly distributed when r < Bohr radius a_B $\sqrt{Dt} \gg a_B$

$$D \sim \frac{1}{\alpha_s^2 T} \qquad t \sim \frac{a_B}{v_{\rm rel}} \sim \frac{1}{p_{\rm rel} v} \qquad p_{\rm rel} \ll \frac{Mv}{\alpha_s^2 v^2}$$

Molecular chaos assumption $f_{Q\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}_{\rm cm}, \boldsymbol{r}, \boldsymbol{p}_{\rm rel}, a, t) = \frac{1}{9} f_Q(\boldsymbol{x}_1, \boldsymbol{p}_1, t) f_{\bar{Q}}(\boldsymbol{x}_2, \boldsymbol{p}_2, t)$

$$t \frac{1}{9} \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) f_Q(\boldsymbol{x}_1, \boldsymbol{p}_1, t) f_{\bar{Q}}(\boldsymbol{x}_2, \boldsymbol{p}_2, t)$$

Spin! $g_s = \frac{3}{4}, \ \frac{1}{4}$

$$(2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(-|E_{nl}| + q - \frac{\boldsymbol{p}_{\rm rel}^2}{M}) \frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle|^2$$

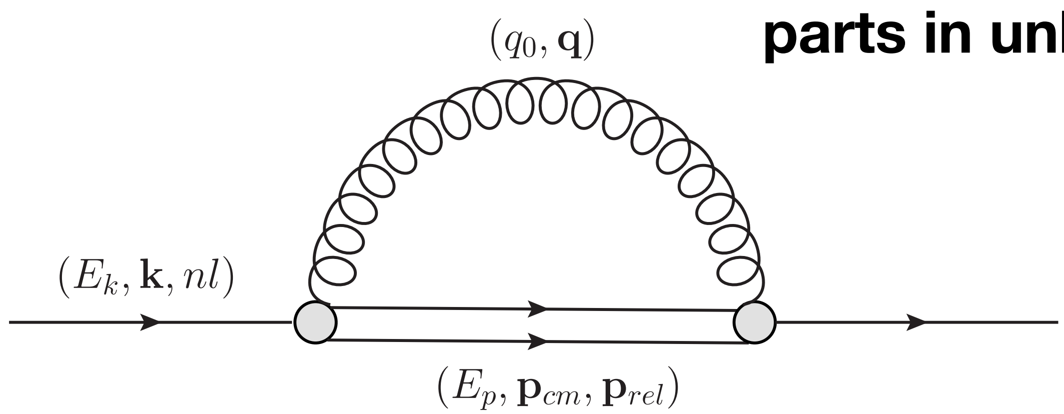
Numerical implementation when |x1-x2| large

Correction of Potential $-i\sum_{ab}\sigma_{ab}(t)[L_{ab},\rho_S(0)]$

$$\sum_{a,b}\sigma_{ab}L_{ab}\to t\sum_{n,l}\int\frac{d^3k}{(2\pi)^3}\mathrm{Re}\Big\{-ig^2C_F\sum_{i_1,i_2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4p_{\mathrm{cm}}}{(2\pi)^4}\int\frac{d^3p_{\mathrm{rel}}}{(2\pi)^3}$$
$$(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}-\boldsymbol{q})\delta(E_k-p^0_{\mathrm{cm}}-q^0)$$
$$(q_0^2\delta_{i_1i_2}-q_{i_1}q_{i_2})\Big(\frac{i}{q_0^2-\boldsymbol{q}^2+i\epsilon}+n_B(|q_0|)(2\pi)\delta(q_0^2-\boldsymbol{q}^2)\Big)$$
$$\langle\psi_{nl}|r_{i_1}\frac{i|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle\langle\Psi_{\boldsymbol{p}_{\mathrm{rel}}}|}{p^0_{\mathrm{cm}}-E_p+i\epsilon}r_{i_2}|\psi_{nl}\rangle\Big\}L_{|\boldsymbol{k},nl,1\rangle\langle\boldsymbol{k},nl,1|}$$

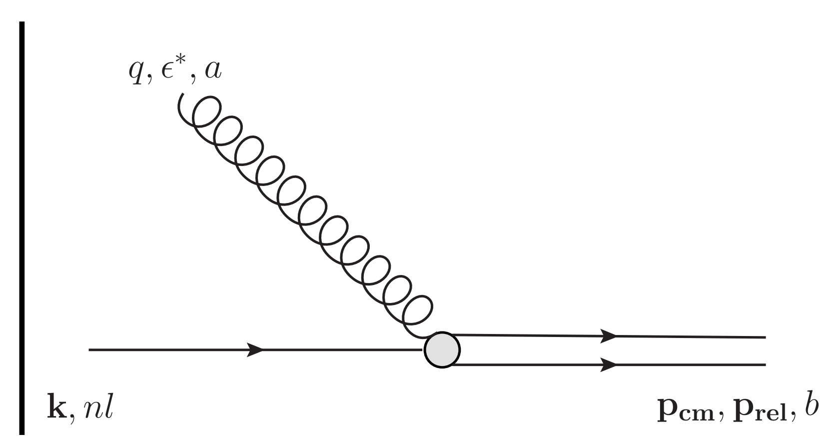
parts in unbound singlet / octet neglected

= Re



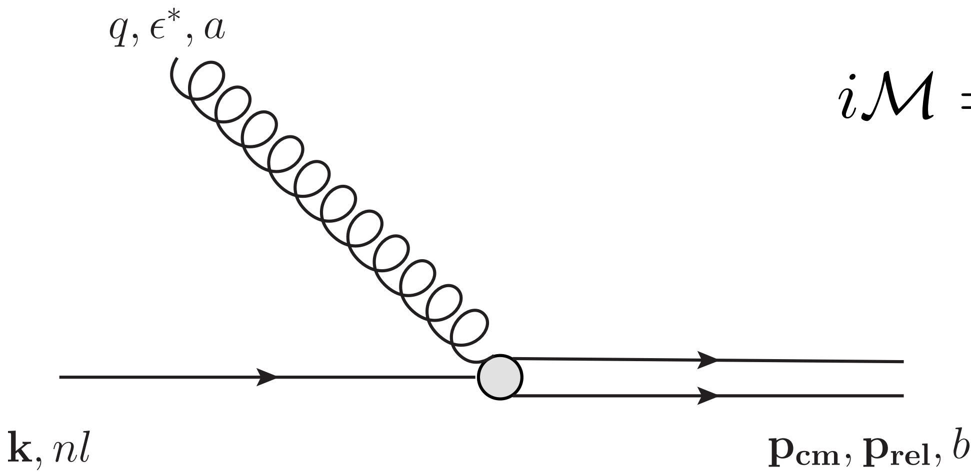
$\theta(t)=(1+\mathrm{sign}(t))/2$

= Im —> dissociation rate



2

Backup: Scattering Amplitudes



$$i\mathcal{M} = g\sqrt{\frac{T_F}{N_c}}(q^0\epsilon^{*i} - q^i\epsilon^{*0})\langle\Psi_{\boldsymbol{p}_{\rm rel}}|r^i|\psi_{nl}\rangle\delta^{ab} \equiv i\epsilon^{*\mu}(\mathcal{M})_\mu$$

Ward identity $\quad q^\mu(\mathcal{M})_\mu = 0$

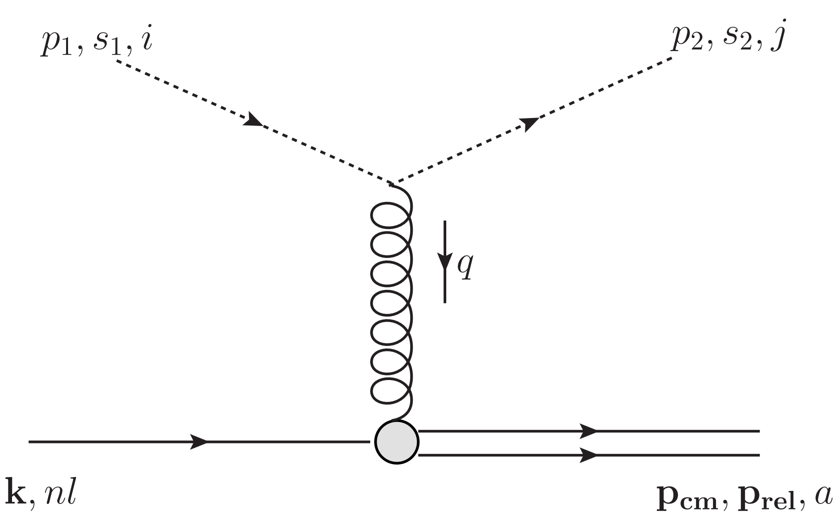
Higher order corrections of

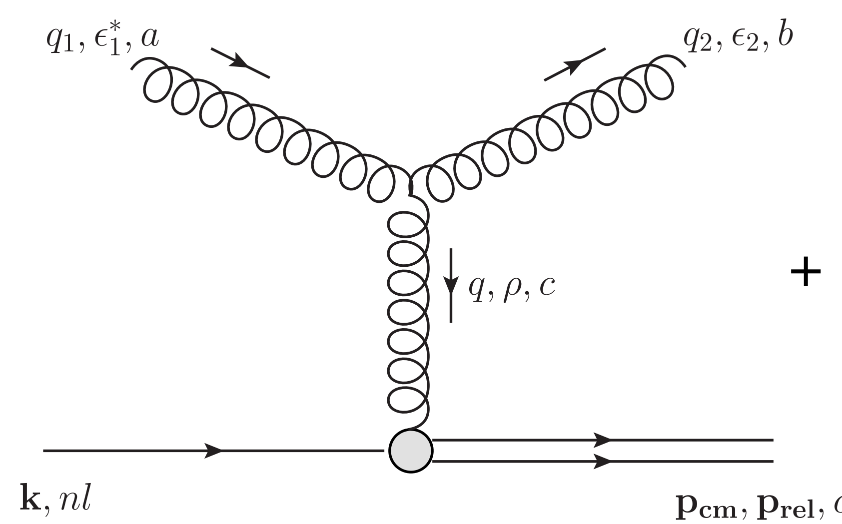
$$\frac{T_F}{N_C}g^2\langle E^{b_1}_{i_1}(\boldsymbol{R}_1, t_1)E^{b_2}_{i_2}(\boldsymbol{R}_2, t_2)\rangle_T$$

⟶ **Inelastic scattering in t-channel**

Ward identity

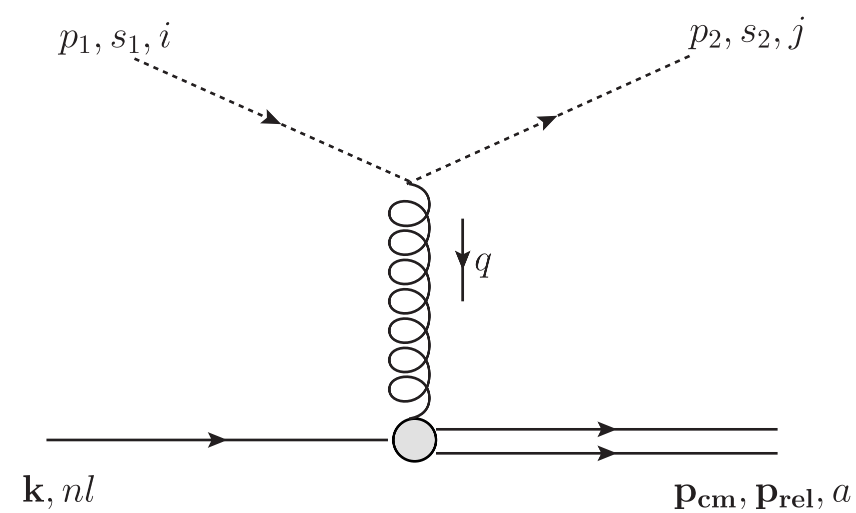
Infrared safe





+ more diagrams

Backup: NLO Amplitudes: Light Quark



Gauge independence: Dirac equation + LO Ward identity

$$i\mathcal{M}_{(b)} = g^2 V_A \sqrt{\frac{T_F}{N_c}} \langle \Psi_{\boldsymbol{p}_{\rm rel}} | r^k | \psi_{nl} \rangle \Big[\frac{-q^0(\delta^{kl} - \hat{q}^k \hat{q}^l)}{(q^0)^2 - \boldsymbol{q}^2 + i\epsilon} \bar{u}_{s_2}(p_2) \gamma^l T^a u_{s_1}(p_1) + \frac{q^k}{\boldsymbol{q}^2} \bar{u}_{s_2}(p_2) \gamma^0 T^a u_{s_1}(p_1) \Big]$$

$$\sum |\mathcal{M}_{(b)}|^2 \equiv \sum_{a,i,j} \sum_{s_1,s_2} \sum_{u,\bar{u},d,\bar{d}} |\mathcal{M}_{(b)}|^2 = \frac{16}{3} g^4 V_A^2 T_F C_F |\langle \Psi_{\boldsymbol{p}_{\rm rel}} | \boldsymbol{r} | \psi_{nl} \rangle|^2$$

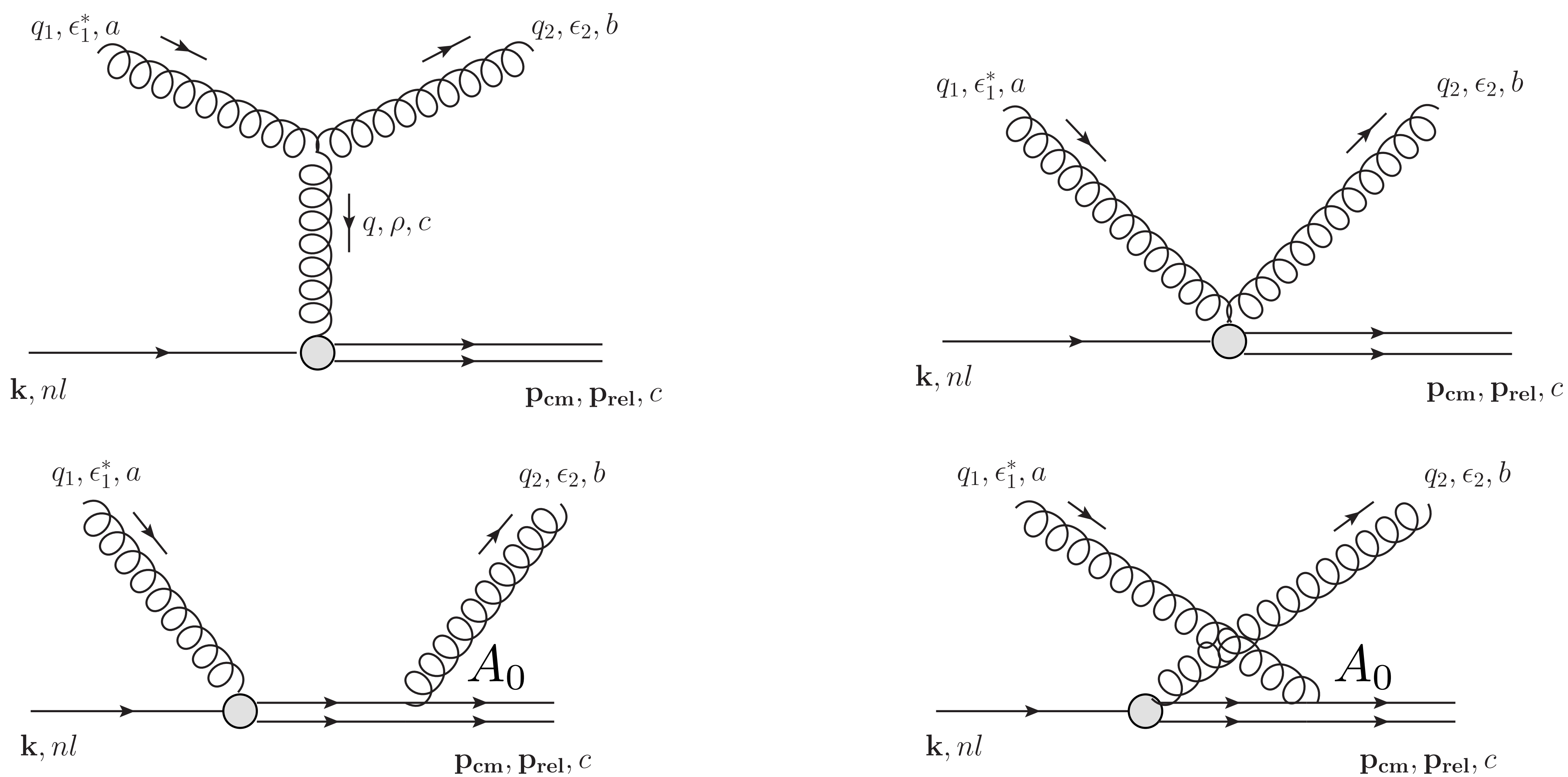
$$\Big[\frac{p_1 p_2 + \boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{\boldsymbol{q}^2} + \frac{2(q^0)^2 (p_1 p_2 - \boldsymbol{p}_1 \cdot \hat{q} \cdot \boldsymbol{p}_2 \cdot \hat{q})}{((q^0)^2 - \boldsymbol{q}^2 + i\epsilon)^2} \Big]$$

Infrared singularity

Infrared safe: finite binding energy

Soft safe: finite binding energy
Collinear divergent

Backup: NLO Amplitudes: Gluon



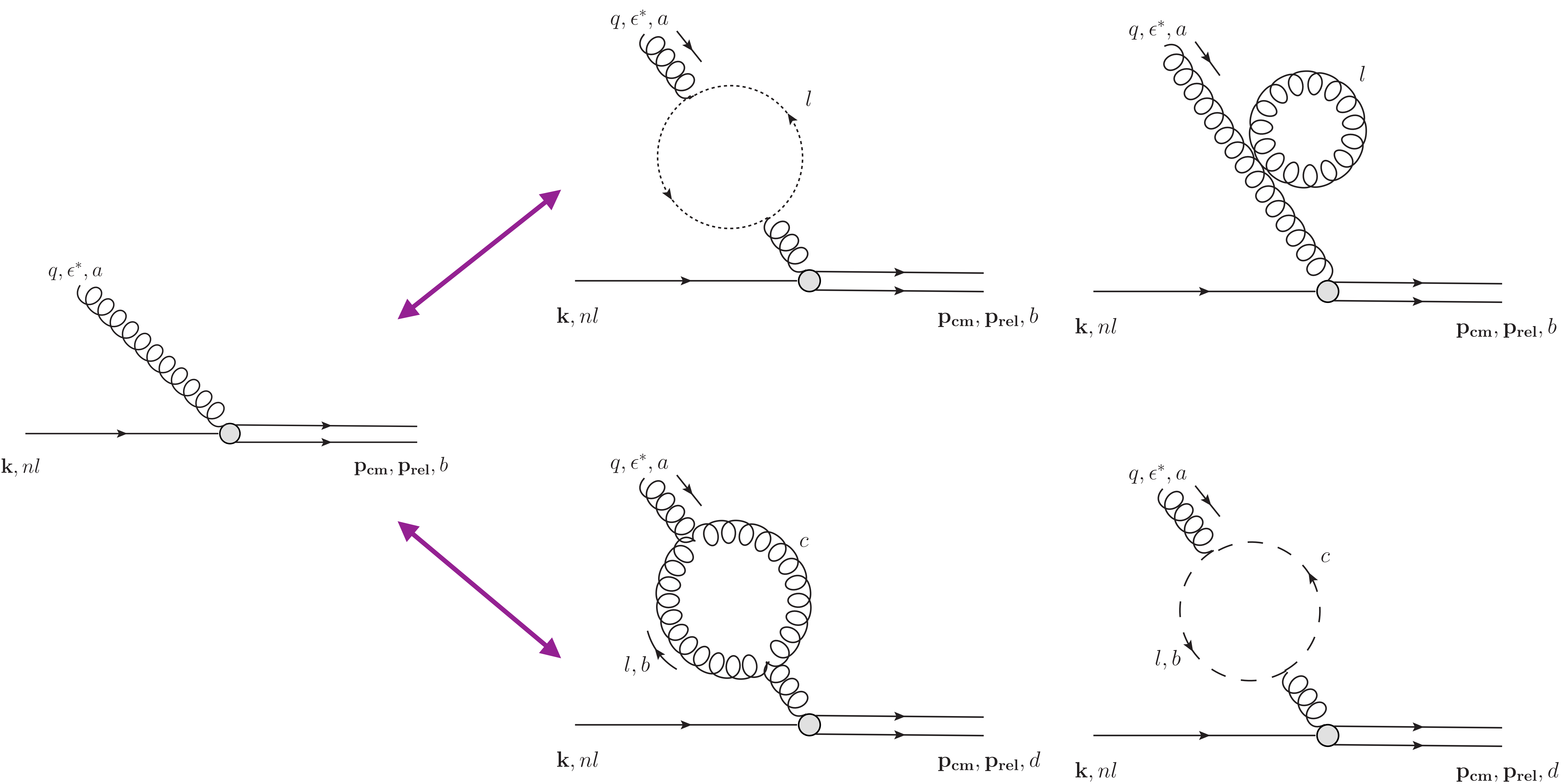
Ward identity $$i(q_1)_\mu \sum \mathcal{M}^{\mu\nu} (\epsilon_2)_\nu = 0 + \mathcal{O}(v^2)$$

Choose Coulomb gauge, only first one contributes

Same collinear divergence as in NLO w/ light quark

Backup: Collinear Divergence Cancellation: Interference

$$n_B(p_1 - p_2)(n_F(p_1) - n_F(p_2)) = -n_F(p_1)(1 - n_F(p_2))$$



$$n_B(q_1 - q_2)(n_B(q_1) - n_B(q_2)) = -n_B(q_1)(1 + n_B(q_2))$$

Backup: Collinear Divergence Cancellation