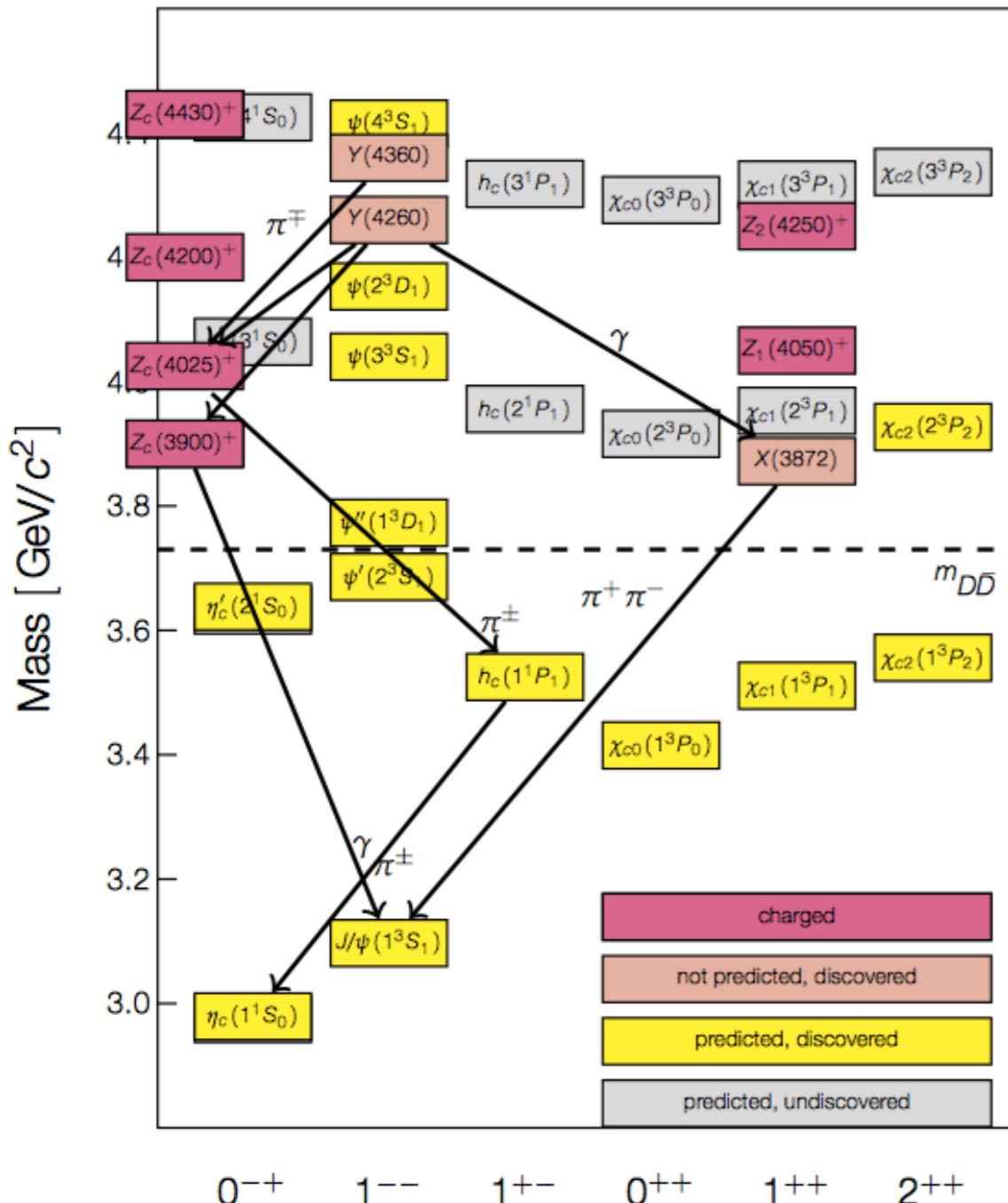




# $\chi(3872)$ as a four-quark state in a Dyson-Schwinger/Bethe-Salpeter approach

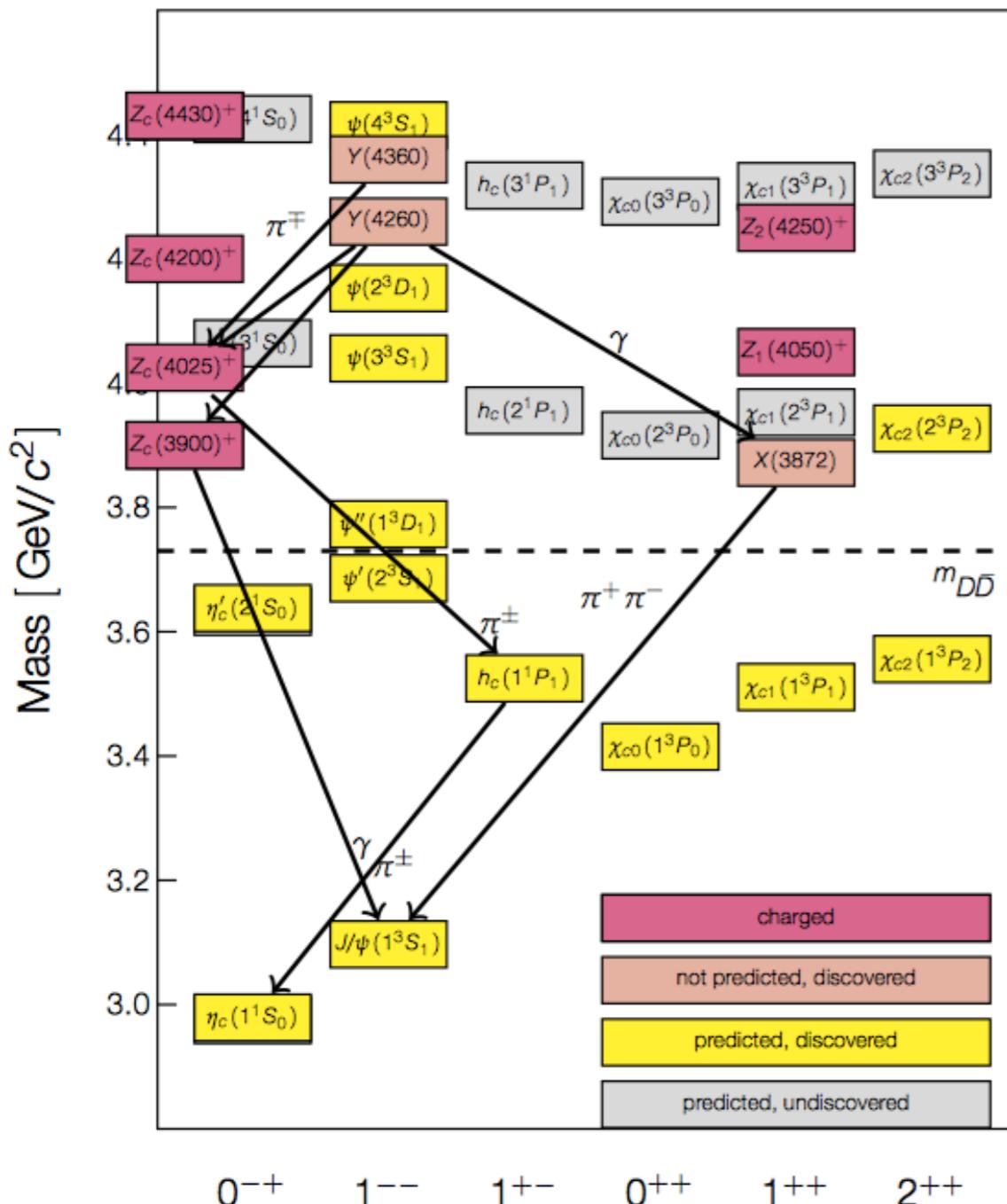
Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

# Tetraquark candidates with $c\bar{q}q\bar{c}$ -content



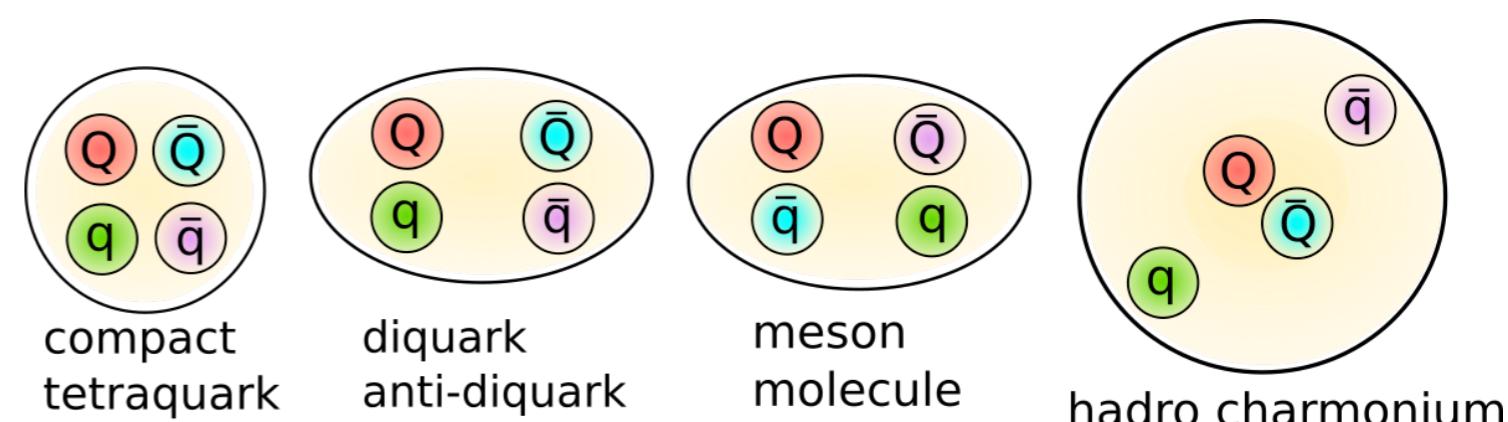
Many new unexpected states found: Belle, BABAR, BES, LHCb ...

# Tetraquark candidates with $c\bar{q}\bar{q}c$ -content



Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Internal structure ??



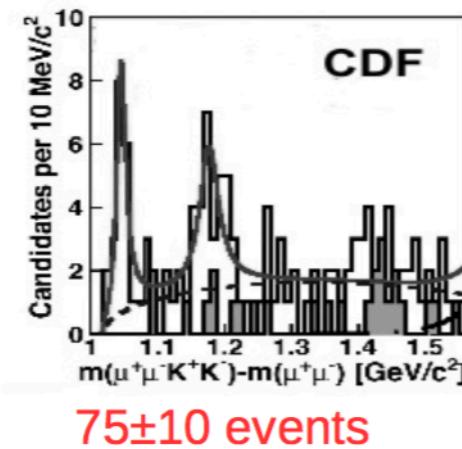
Related to details of underlying QCD forces between quarks and gluons

# Tetraquark candidates with $cs\bar{s}\bar{c}$ -content

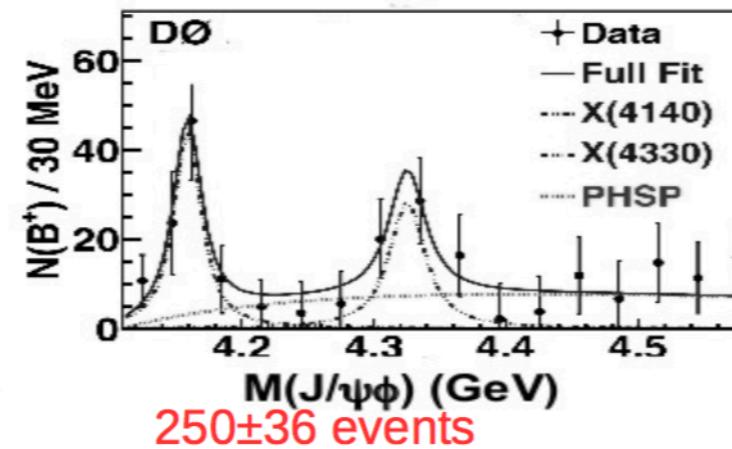
## Looking for exotic structures with $\bar{c}c\bar{s}s$ quarks

### THE INTRIGUING CASE OF THE X(4140)

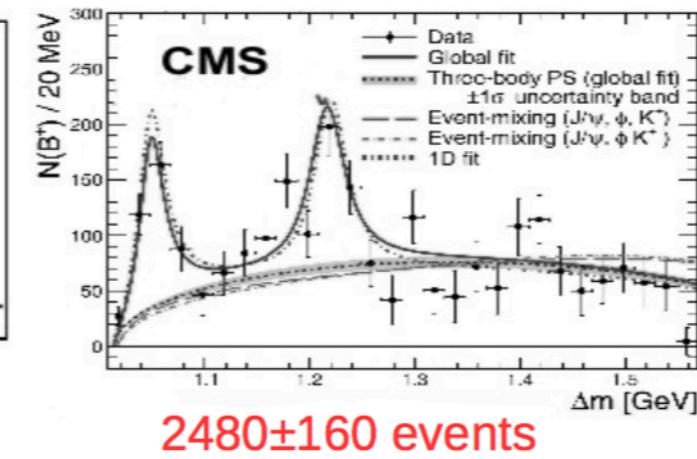
- The X(4140) was observed in the invariant mass system of  $J/\psi KK$  ( $\phi \rightarrow K^+K^-$ )
- The X(4140) can be considered the strange counterpart of the X(3872)
- Is the X(4140) a real particle?



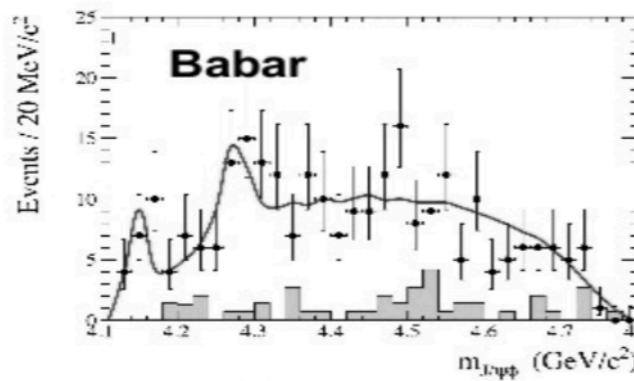
$75 \pm 10$  events



$250 \pm 36$  events

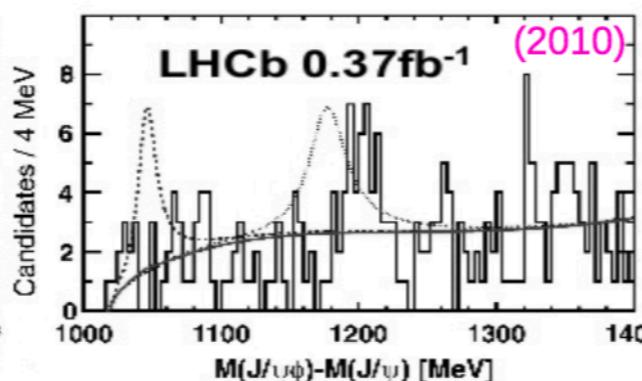


$2480 \pm 160$  events

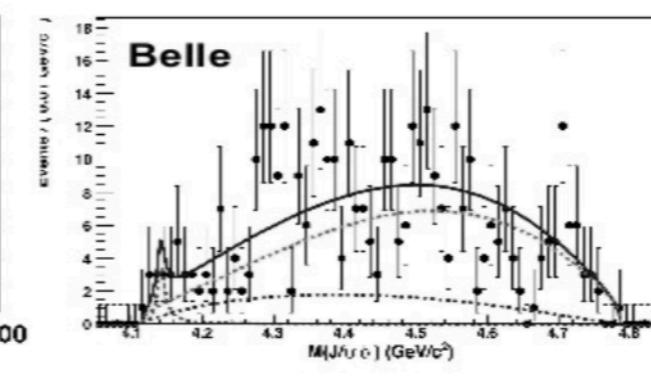


$189 \pm 14$  events

Phys. Rev. D91 (2015) 012003



$346 \pm 20$  events

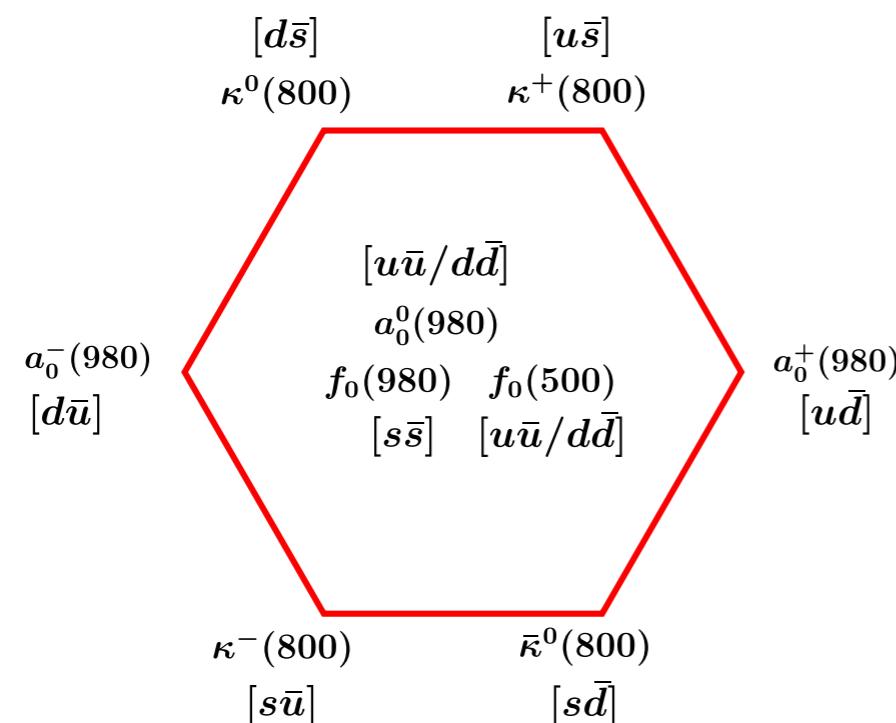


$325 \pm 21$  events



# Tetraquark candidates with $qq\bar{q}\bar{q}$ -content

Light scalar mesons:

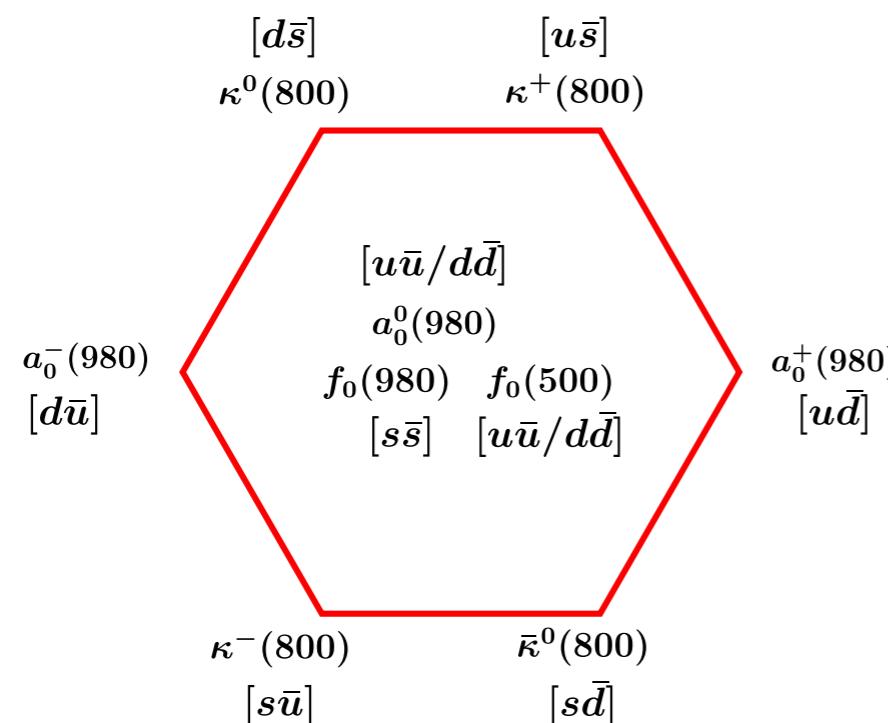


wrong level ordering

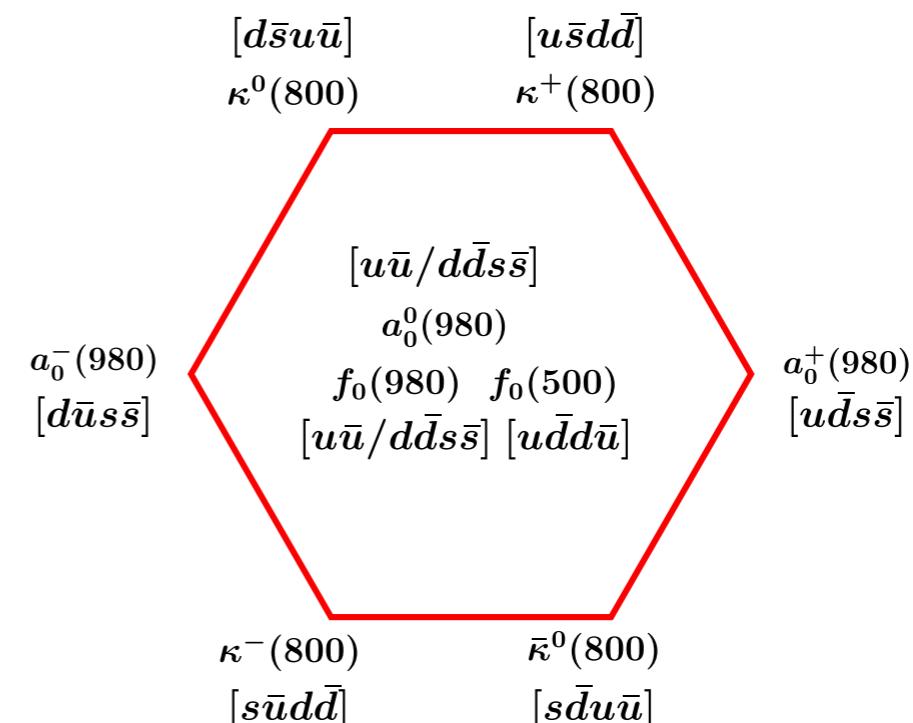
R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

# Tetraquark candidates with $qq\bar{q}\bar{q}$ -content

Light scalar mesons:



wrong level ordering



correct level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

# Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

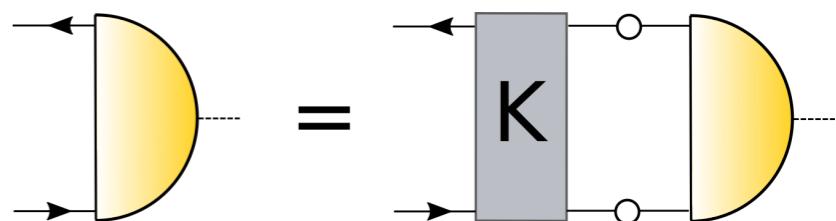
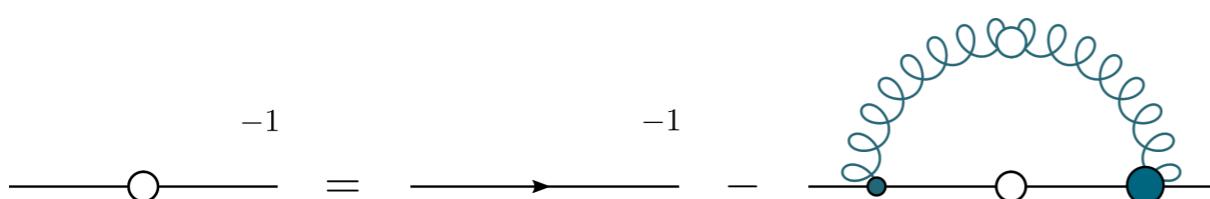
Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Key elements: quark propagator and interaction kernels

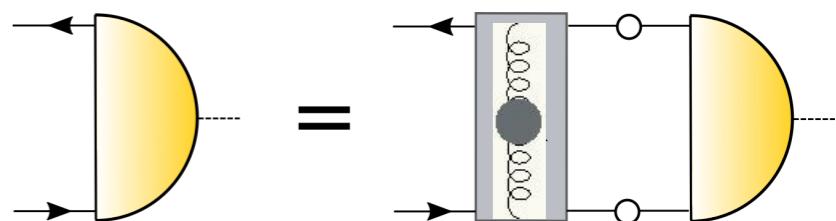
$$[S(p)]^{-1} = [-i\cancel{p} + \textcolor{green}{M}(p^2)]/Z_f(p^2)$$

## Rainbow-Ladder (RL):



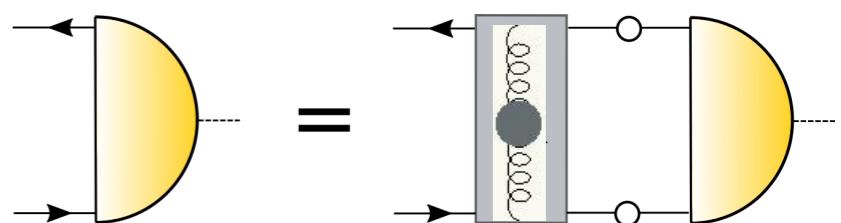
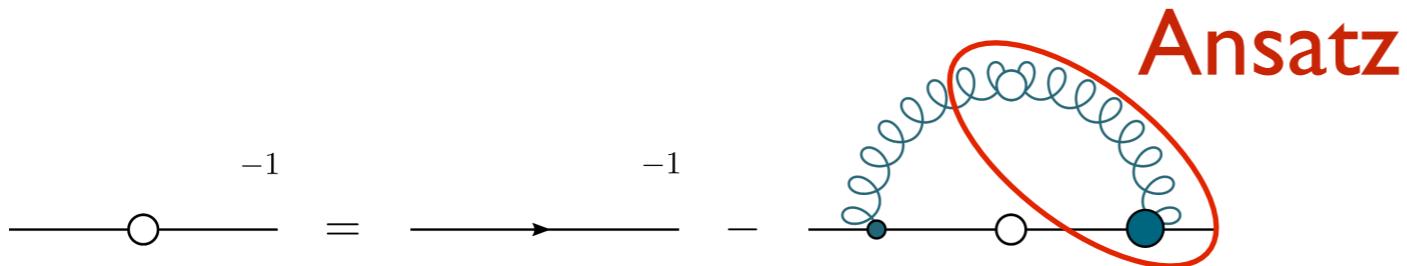
$$[S(p)]^{-1} = [-i\cancel{p} + \textcolor{green}{M}(p^2)]/Z_f(p^2)$$

## Rainbow-Ladder (RL):

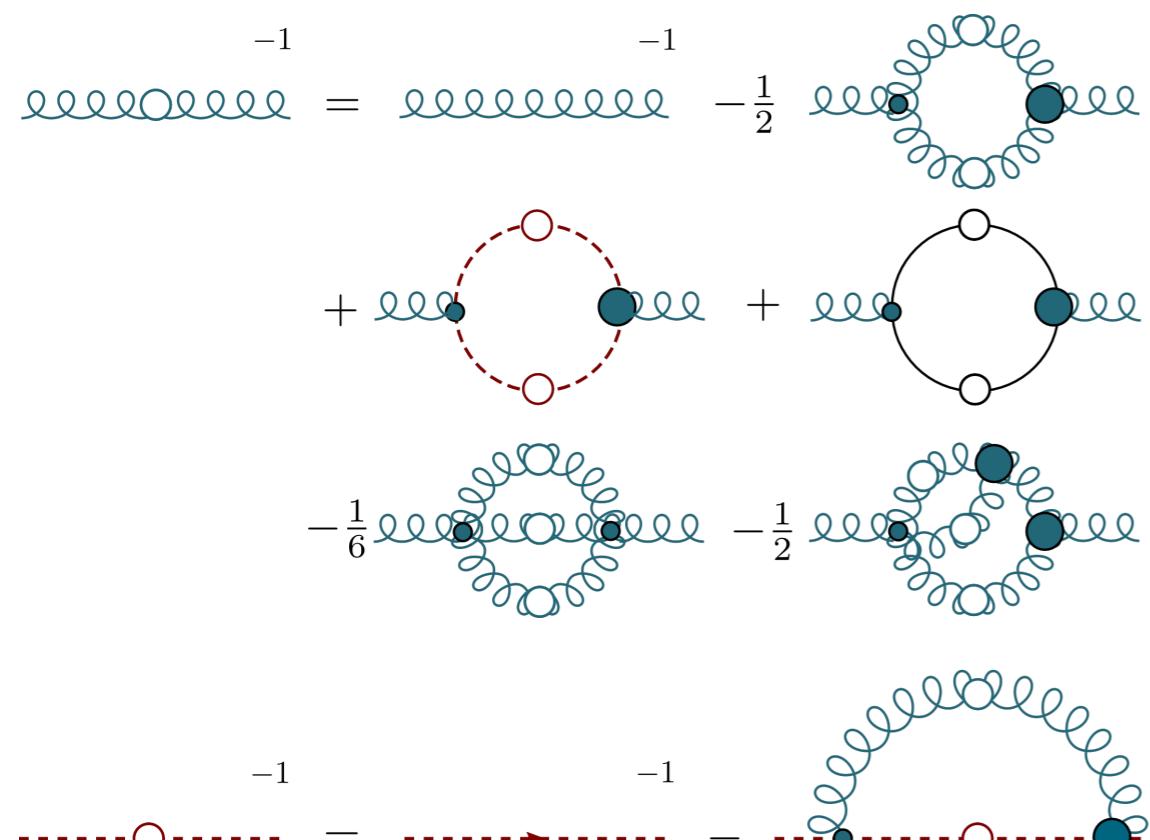


$$[S(p)]^{-1} = [-i\cancel{p} + M(p^2)]/Z_f(p^2)$$

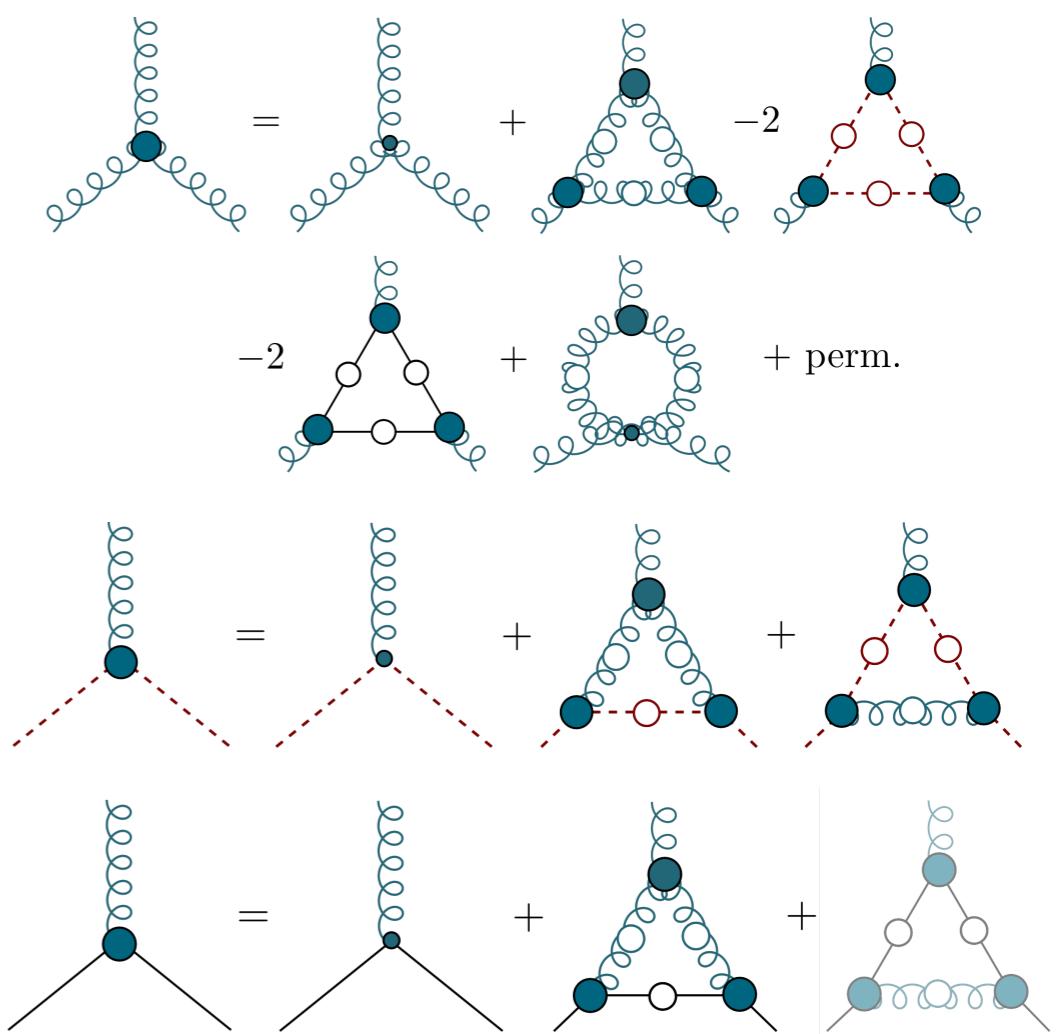
## Rainbow-Ladder (RL):



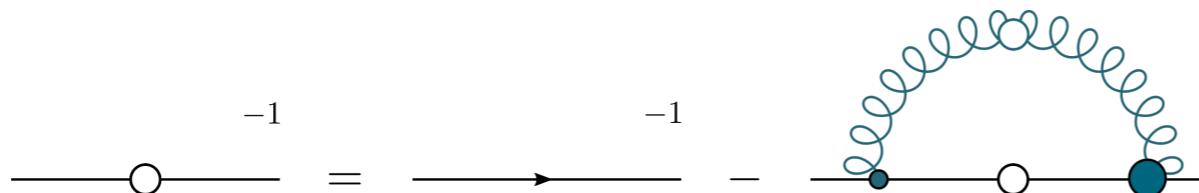
## Beyond the rainbow (BRL):



Williams, CF, Heupel, PRD 93 (2016) 034026  
CF, Williams, PRL 103 (2009) 122001

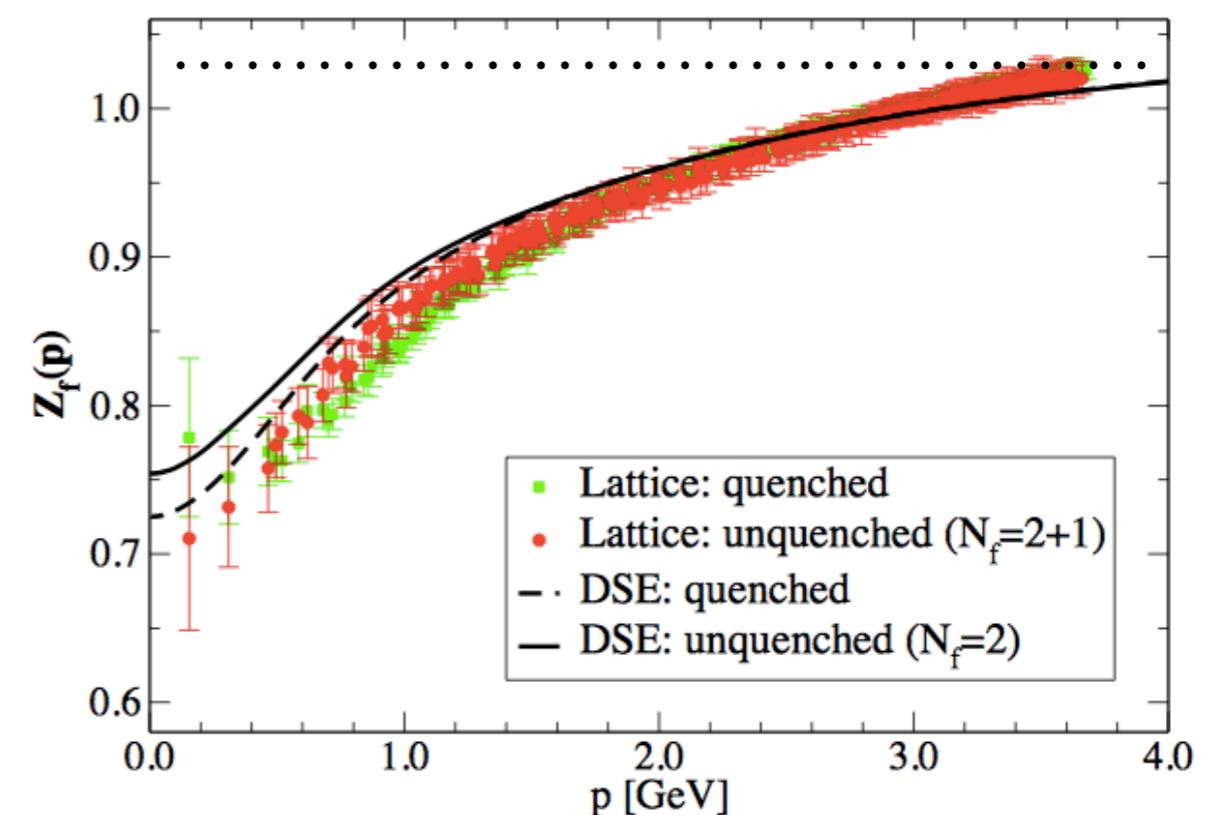
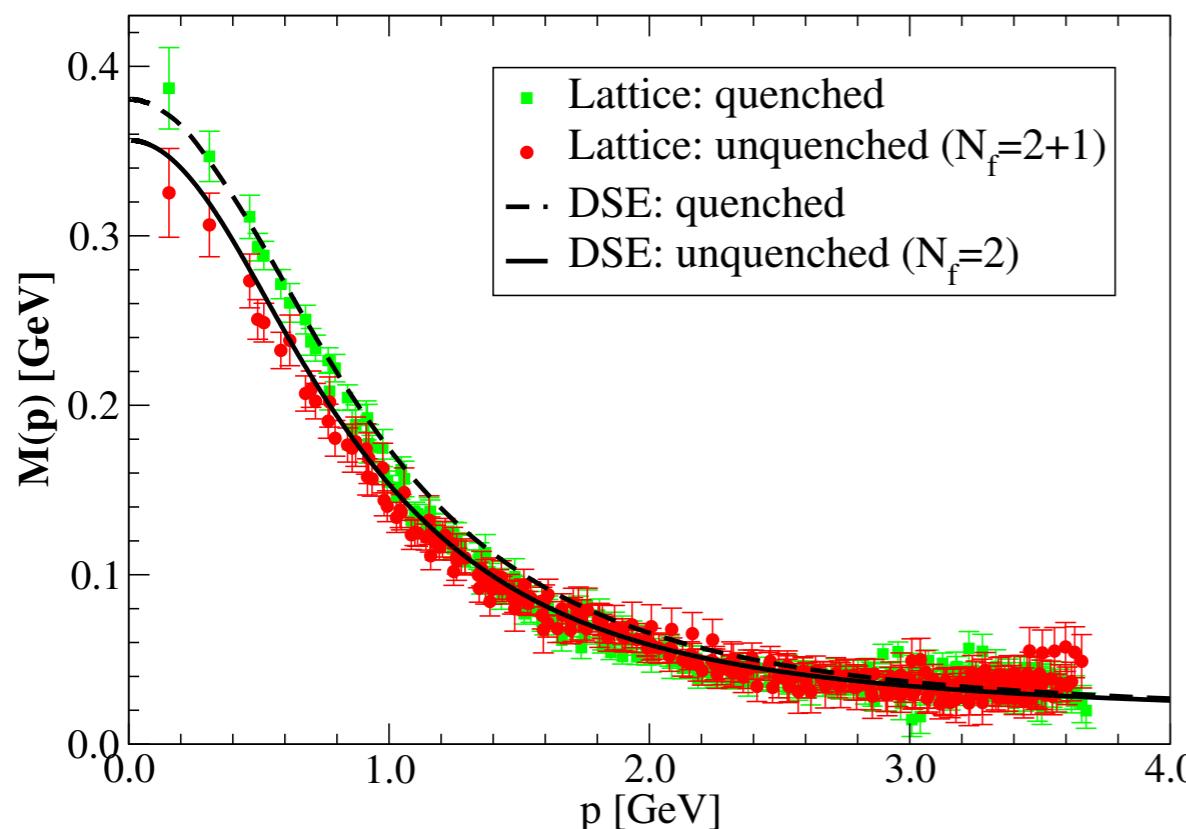


# Quarks: mass from interaction

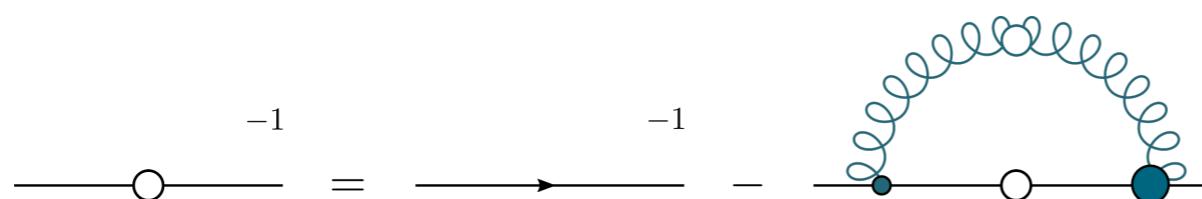


$$S(p) = Z_f(p^2) \frac{-ip + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47  
Williams, CF, Heupel, PRD 93 (2016) 034026  
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

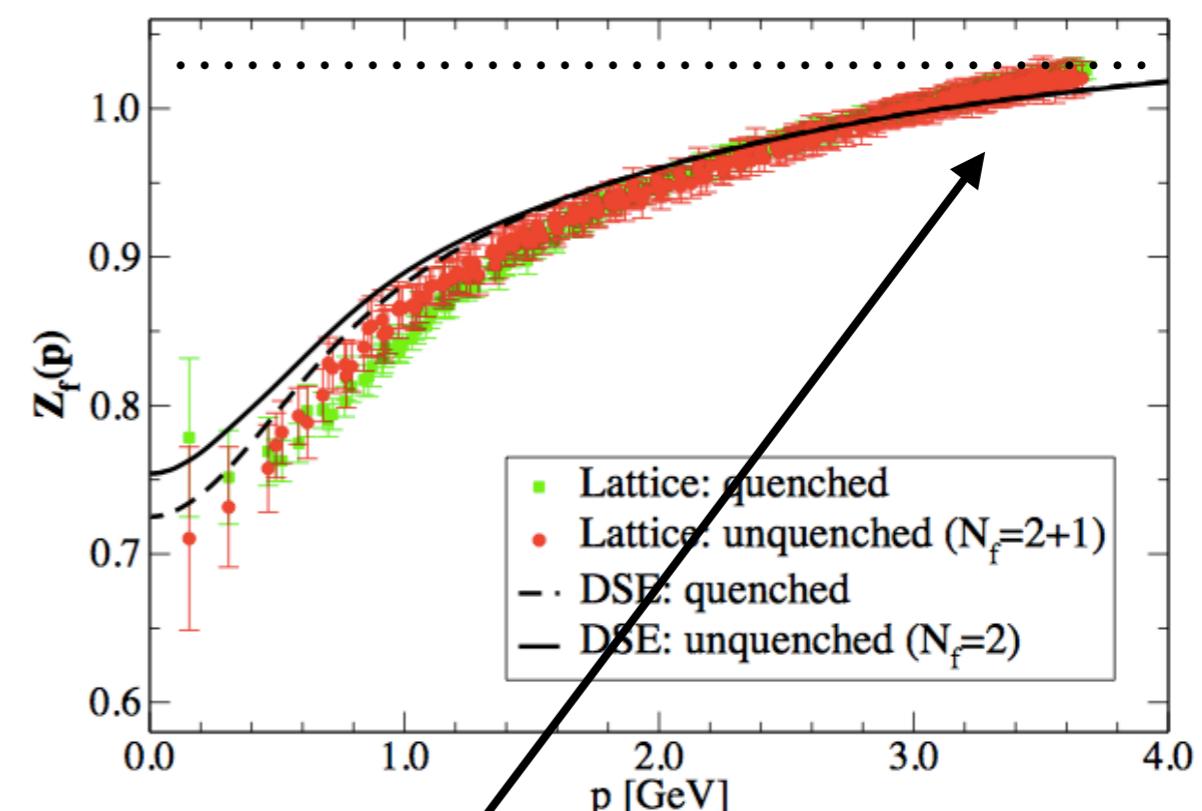
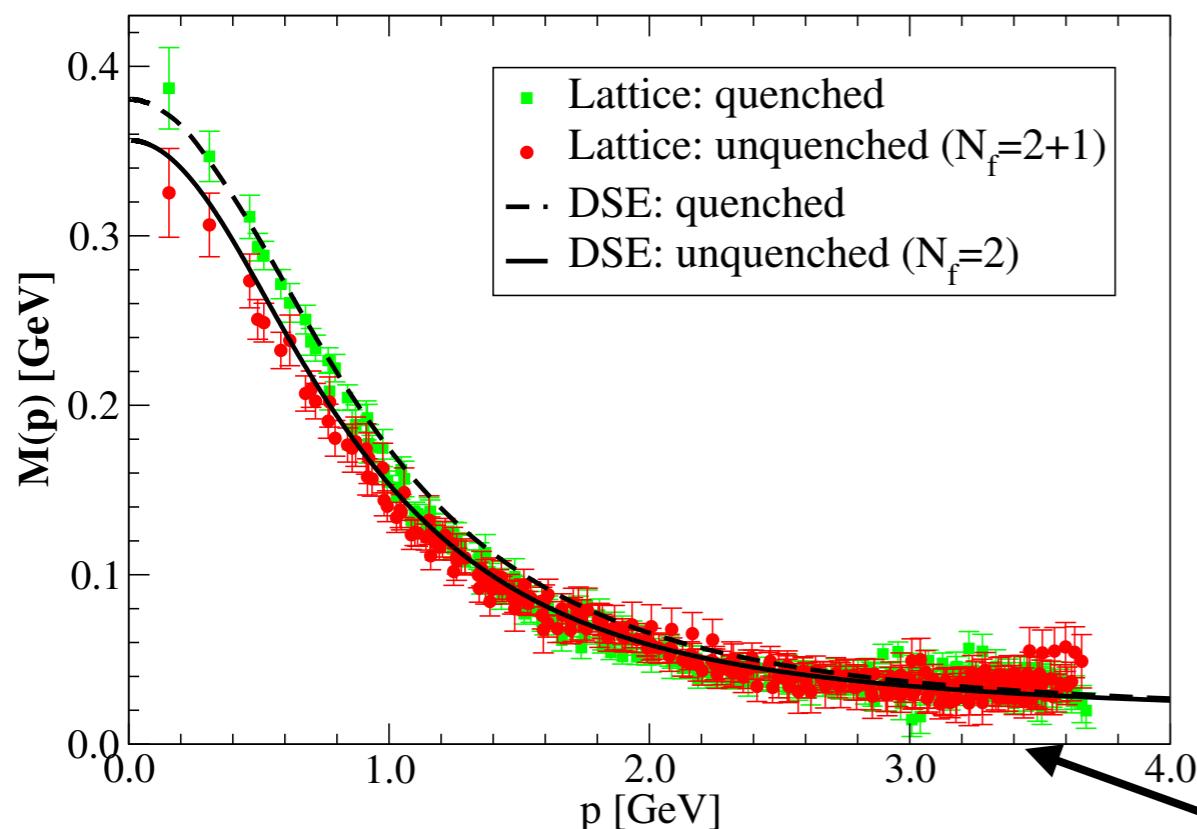


# Quarks: mass from interaction



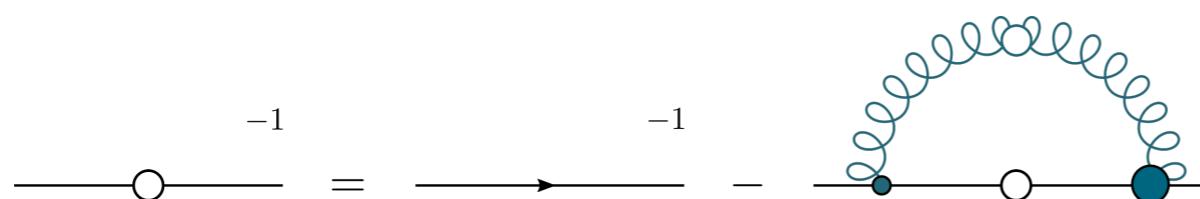
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DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47  
 Williams, CF, Heupel, PRD 93 (2016) 034026  
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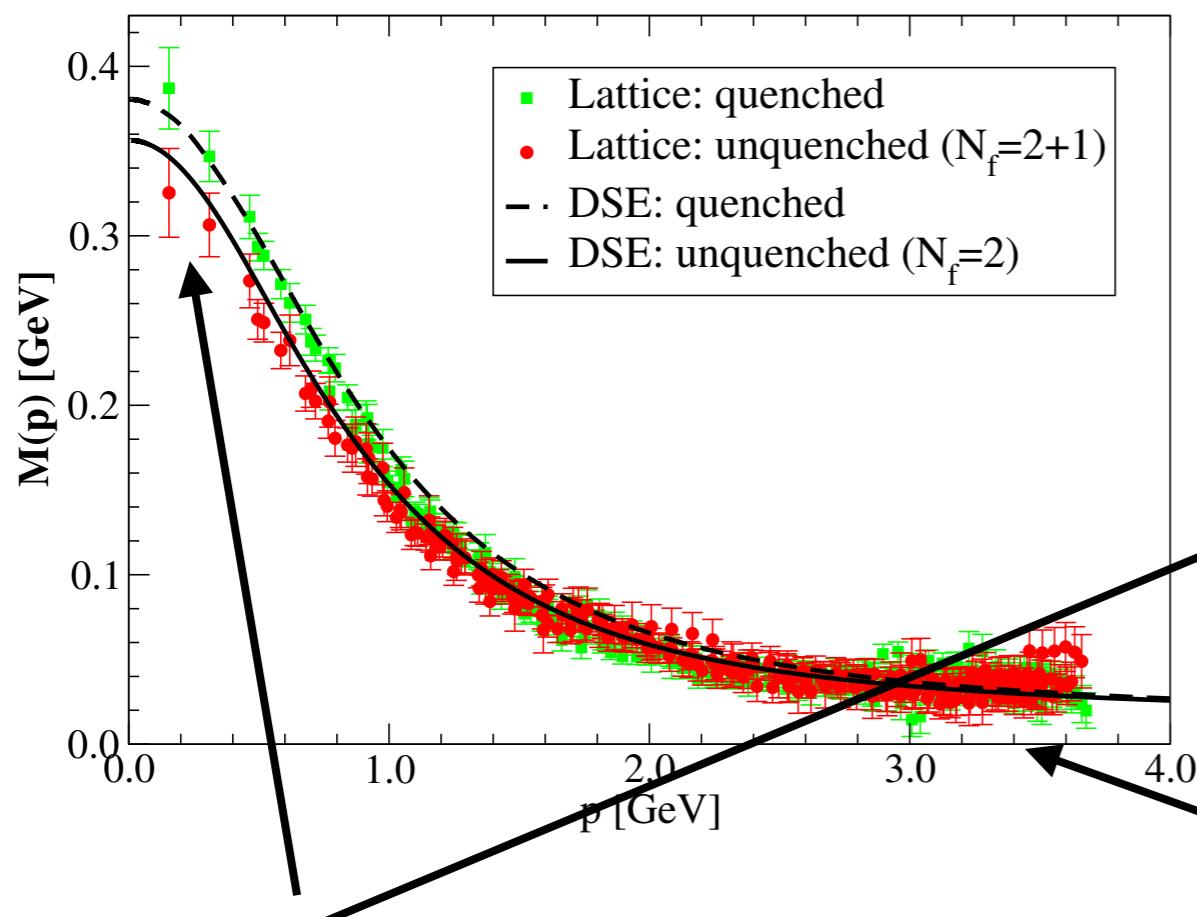
‘current quark’:  
 - small mass; non-composite

# Quarks: mass from interaction

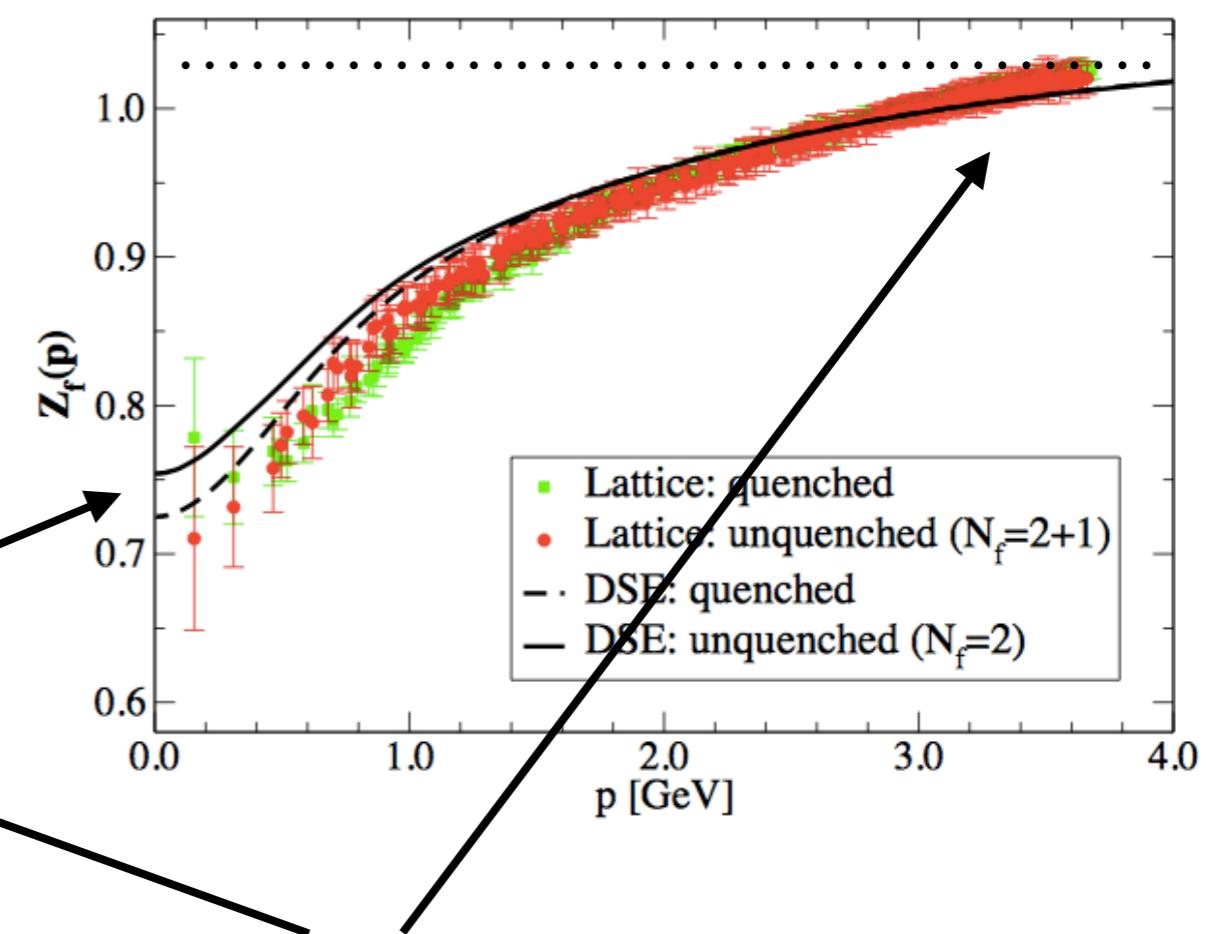


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DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47  
 Williams, CF, Heupel, PRD 93 (2016) 034026  
 Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

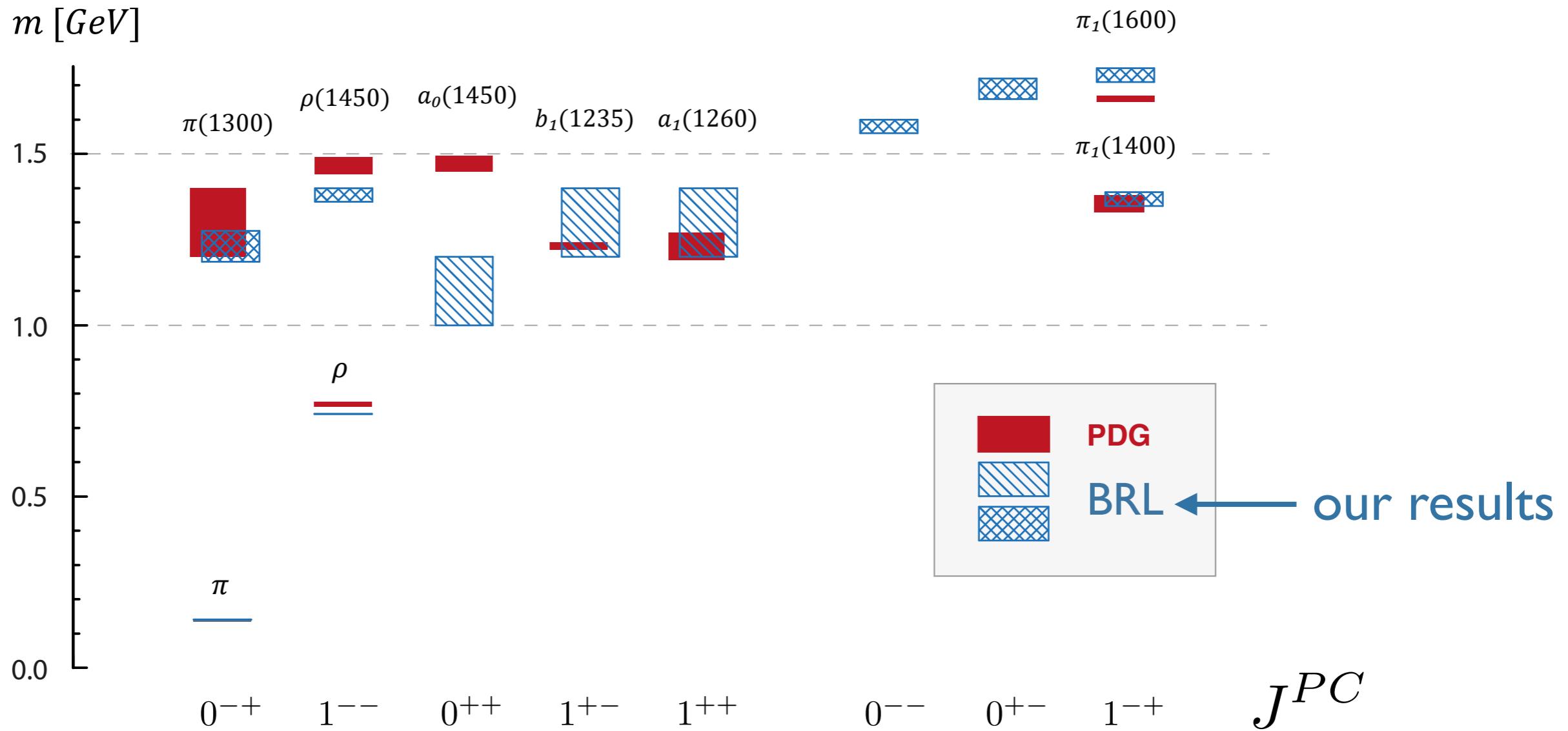


**'constituent quark':**  
**large mass; very composite**



**'current quark':**  
**- small mass; non-composite**

# Light meson spectrum



Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- scalars  $0^{++}$  are special .... see later
- For pseudoscalar and vector ground states: RL equally good

# Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

Two-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Three- and four-body interactions

# Tetraquarks from the four-body equation

Exact equation:

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 - \text{Diagram}_3 + \text{Diagram}_4 + \text{Diagram}_5 + \text{perm.}$$

The equation shows a central yellow circle representing a tetraquark state, equal to a sum of terms. The first term is a single yellow circle. The second term is a yellow circle with a blue square interaction box attached to its top-left vertex. The third term is a yellow circle with a blue square interaction box attached to its bottom-left vertex. The fourth term is a yellow circle with two blue square interaction boxes attached to its top-left and bottom-left vertices. The fifth term is a yellow circle with three blue square interaction boxes attached to its top-left, bottom-left, and top-right vertices. A red diagonal slash is drawn through the fourth and fifth terms, indicating they are subtracted from the sum. The label '+ perm.' indicates that the order of the boxes matters.

Two-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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Two-body interactions

Three- and four-body interactions

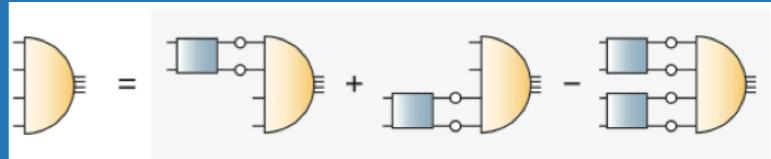
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

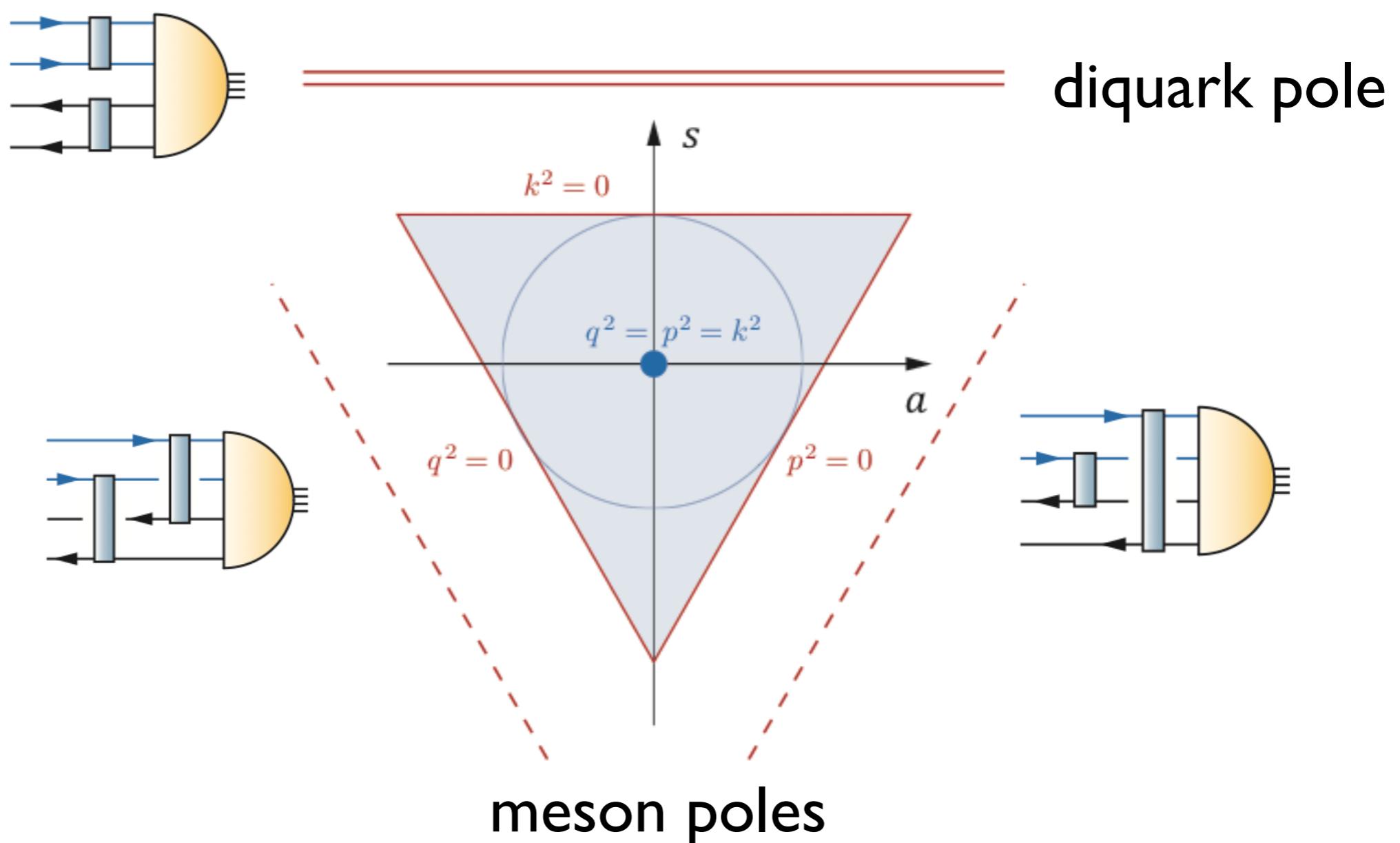
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Two-body interactions: allow for internal clustering
- use RL-approximation

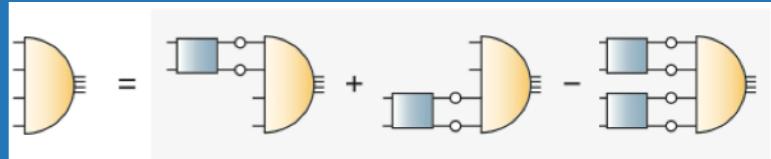
# Four-body equation: permutations



- Singlet:  $S_0 = (p^2 + q^2 + k^2)/4$       p,q,k: relative momenta
- Doublet:  $s \sim p^2 + q^2 - 2k^2$   
 $a \sim q^2 - p^2$



# Four-body equation: permutations

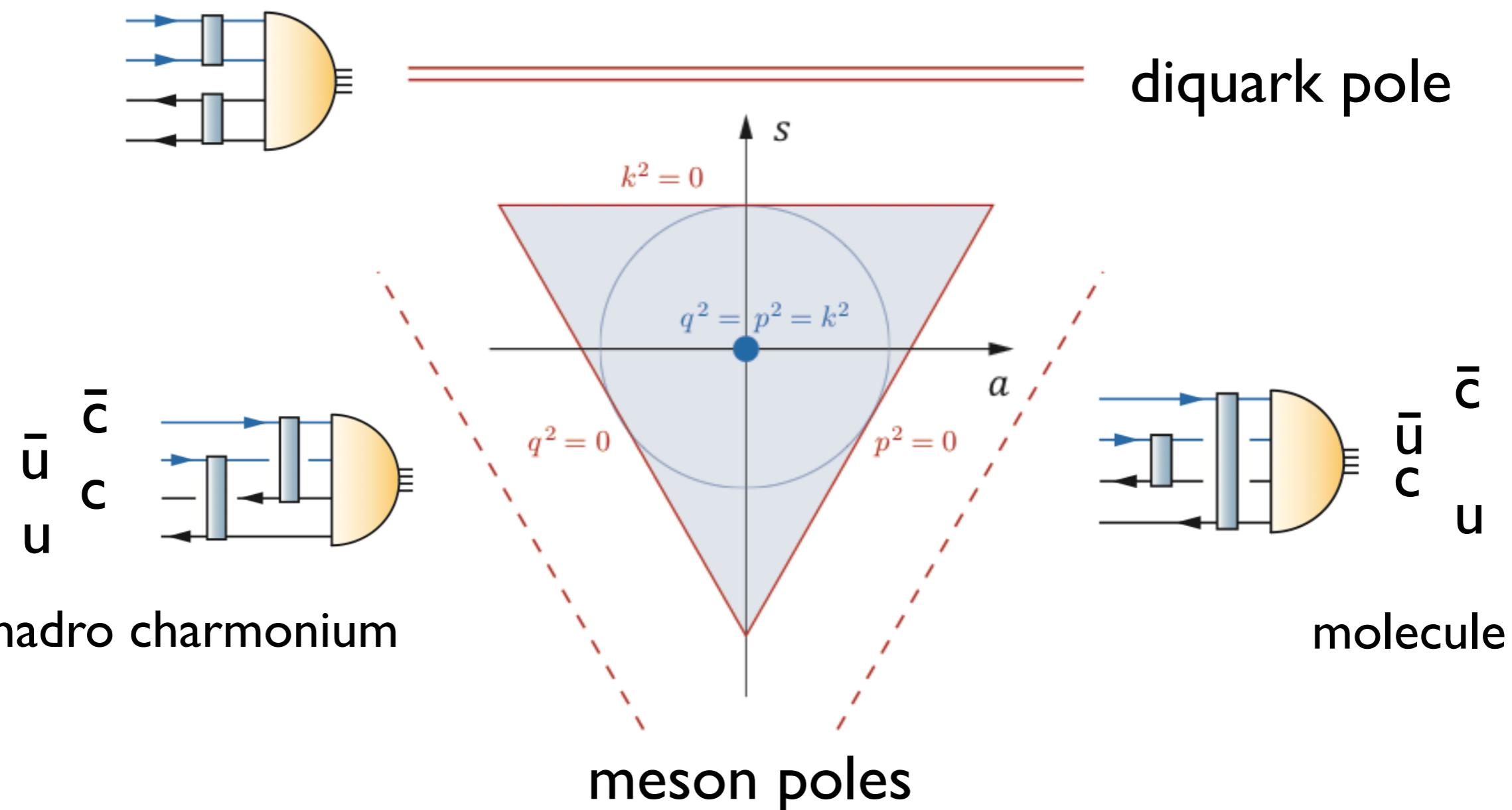


- Singlet:  $S_0 = (p^2 + q^2 + k^2)/4$

p,q,k: relative momenta

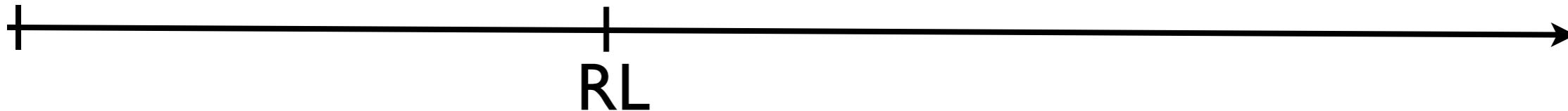
- Doublet:  $s \sim p^2 + q^2 - 2k^2$

$$a \sim q^2 - p^2$$

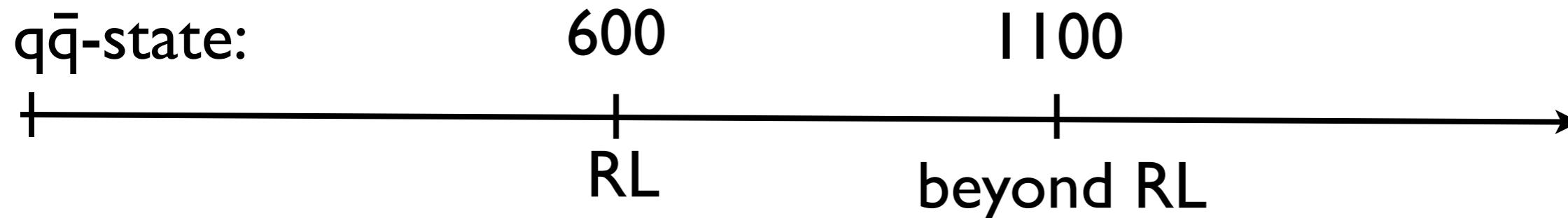


# Bound state vs resonance: light scalars

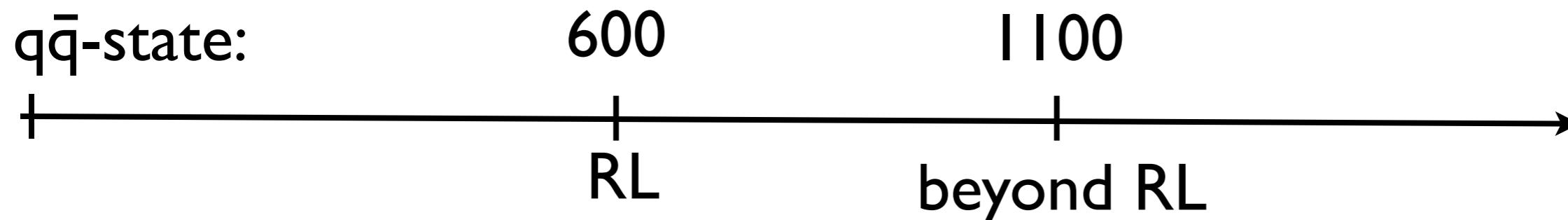
$q\bar{q}$ -state:



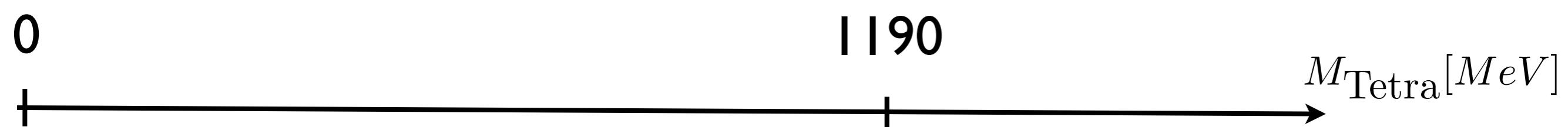
# Bound state vs resonance: light scalars



# Bound state vs resonance: light scalars



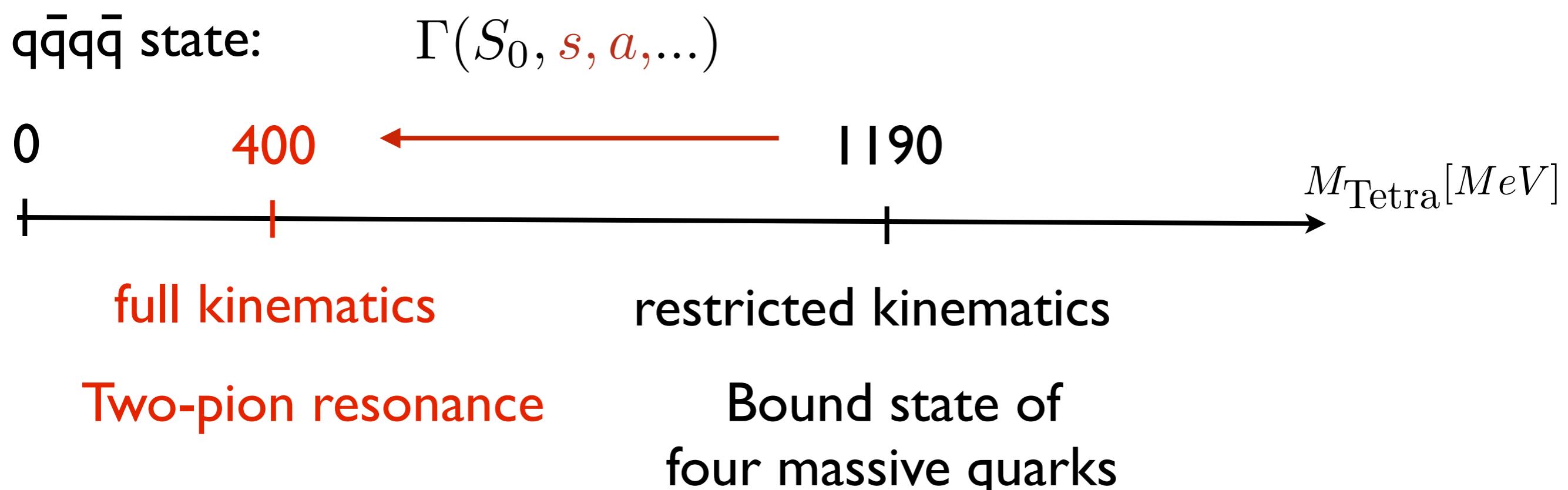
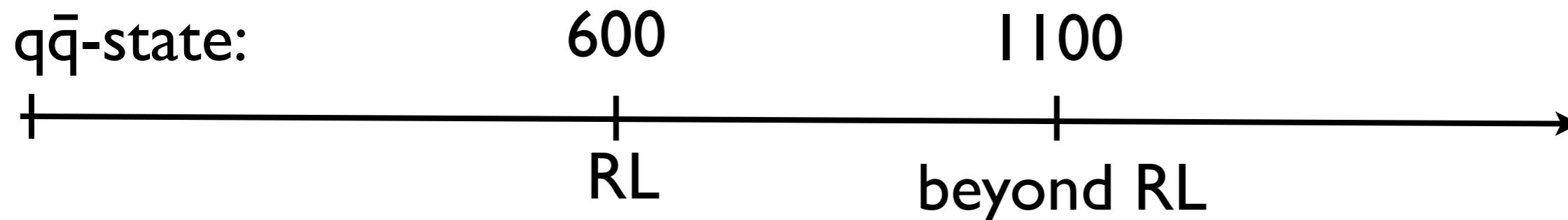
q $\bar{q}q\bar{q}$  state:  $\Gamma(S_0, \cancel{s}, \cancel{a}, \dots)$



restricted kinematics

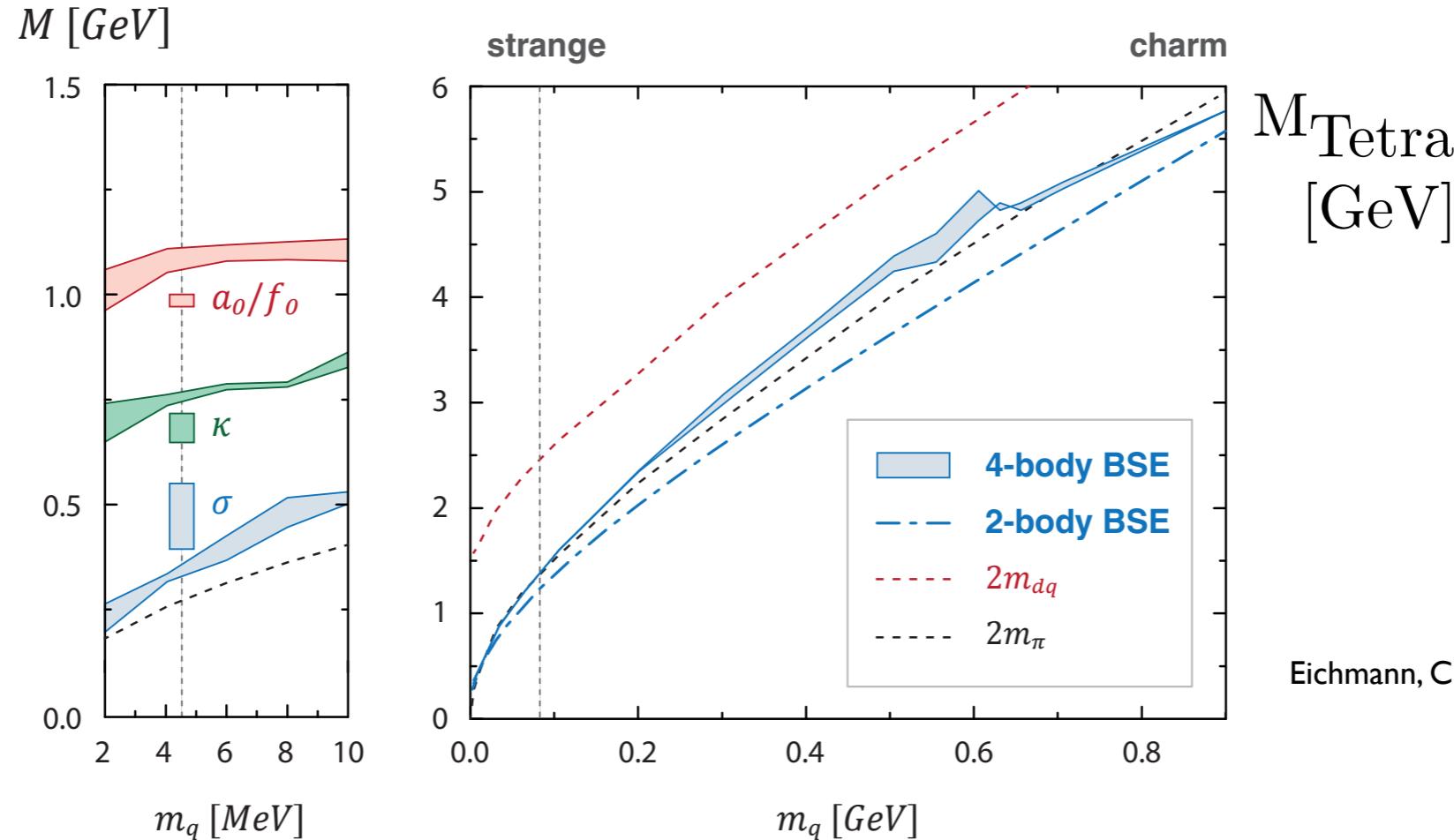
Bound state of  
four massive quarks

# Bound state vs resonance: light scalars



→ identify with  $f_0(500)$  (' $\sigma$ -meson')

# Mass evolution of tetraquark

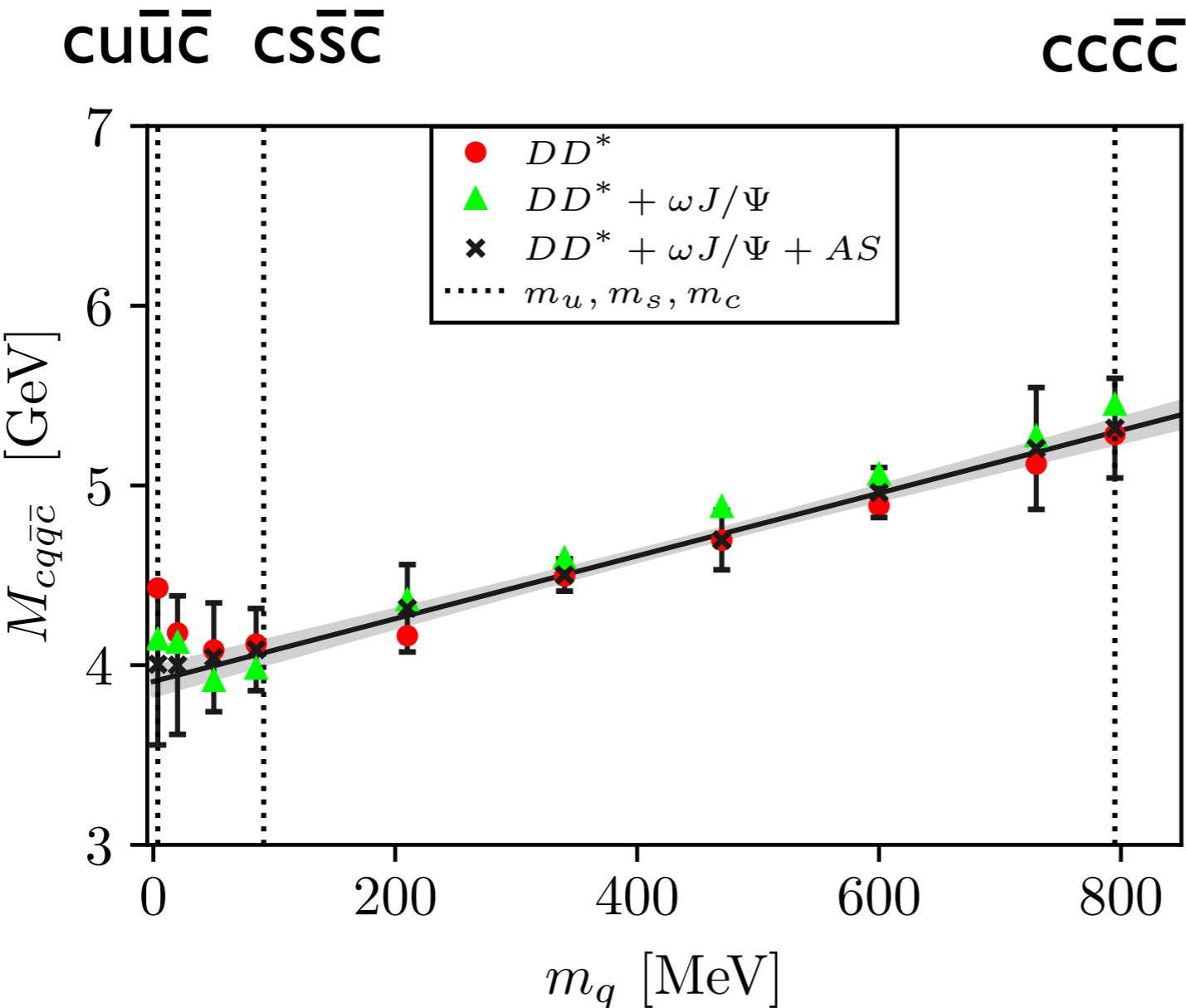
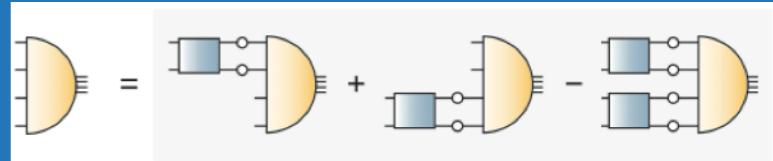


Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large  $m_q$
- Dynamical decision: **meson clusters, not diquarks**
- Results:
  - $m_\sigma \sim 350$  MeV
  - $m_\kappa \sim 750$  MeV
  - $m_{a_0, f_0} \sim 1080$  MeV
- $m_{ss\bar{s}\bar{s}} \sim 1.5$  GeV
- $m_{cc\bar{c}\bar{c}} \sim 5.7$  GeV

qualitatively similar to two-body framework

Heupel, Eichmann, CF, PLB 718 (2012) 545-549



$m_c$  fixed  
 $m_q$  varied

●  $DD^*$  dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74) \text{ MeV} \longrightarrow X(3872)$$

$$M_{1^{++}}^{cs\bar{s}\bar{c}} = 4068(61) \text{ MeV} \longrightarrow \text{relation to } X(4140) ?$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

# Summary

Internal dynamics very important !!

- Four-quarks states (so far) dominated by meson-meson clusters (diquarks are almost irrelevant)
- Dynamical description of  $\sigma$ :  $\pi\text{-}\pi$  resonance

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Dynamical description of  $X(3872)$ :  $DD^*$  dominated
- Dynamical description of  $cc\bar{s}\bar{c}$ :  $D_sD_s^*$  dominated

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

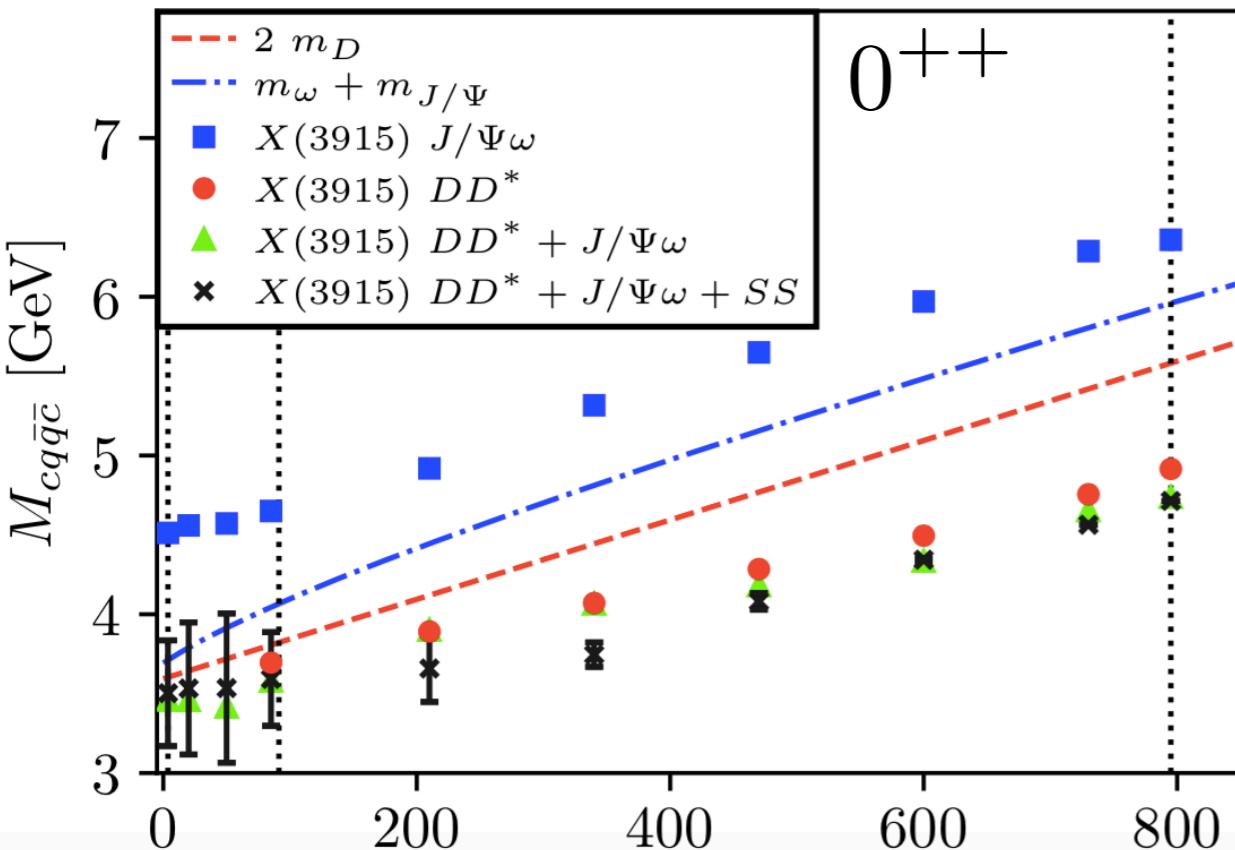
## Outlook:

- preliminary results for other channels:  $0^{++}, 1^{+-}$

# Backup Slides

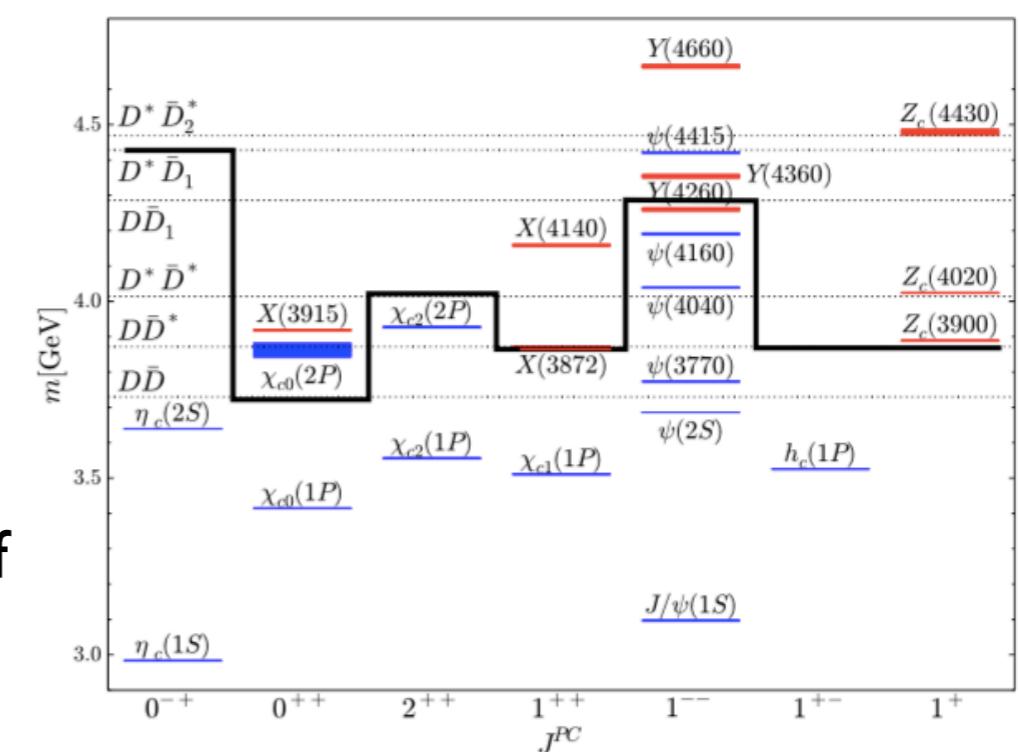
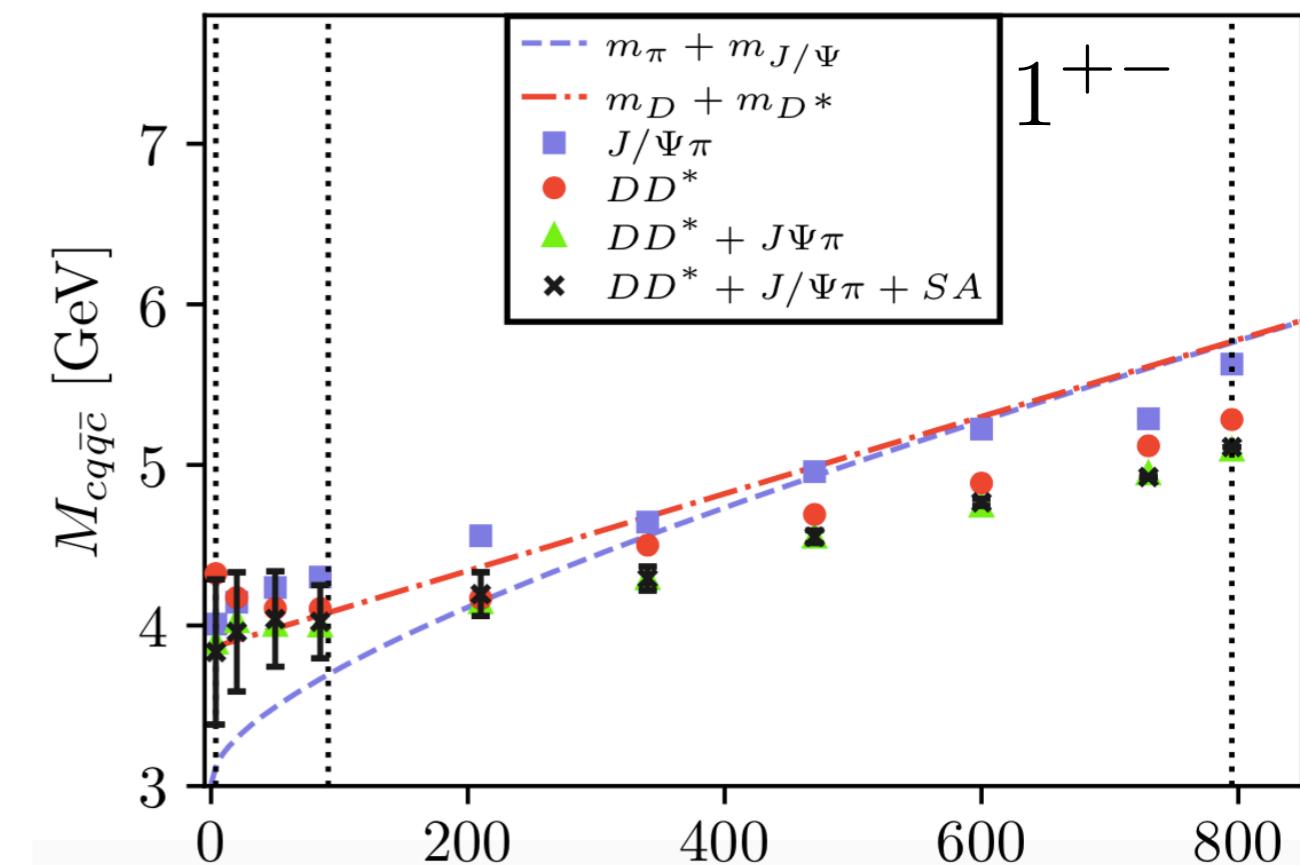
# Scalar $0^{++}$ and axial vector $1^{+-}$ channels

**PRELIMINARY !!**

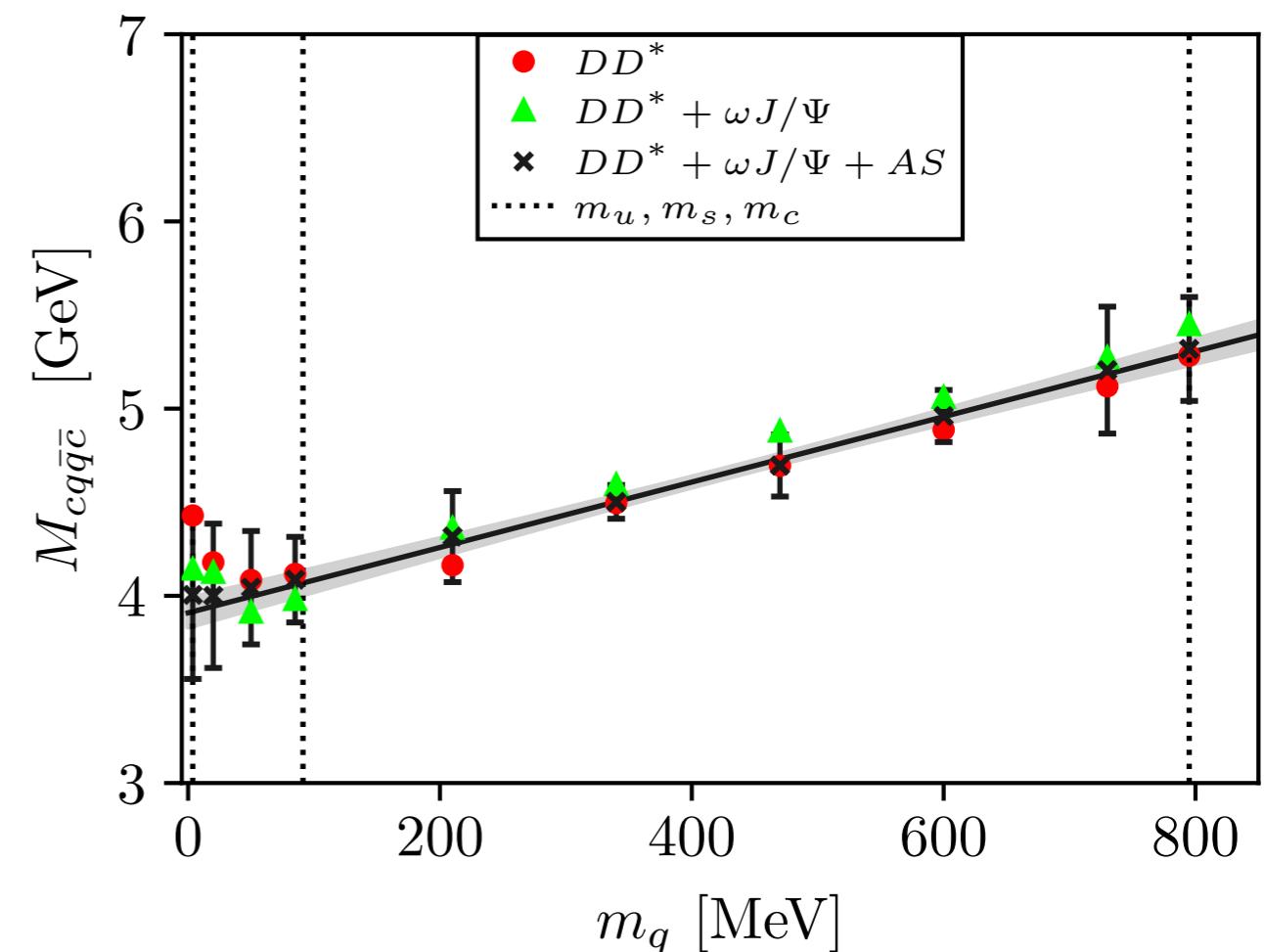
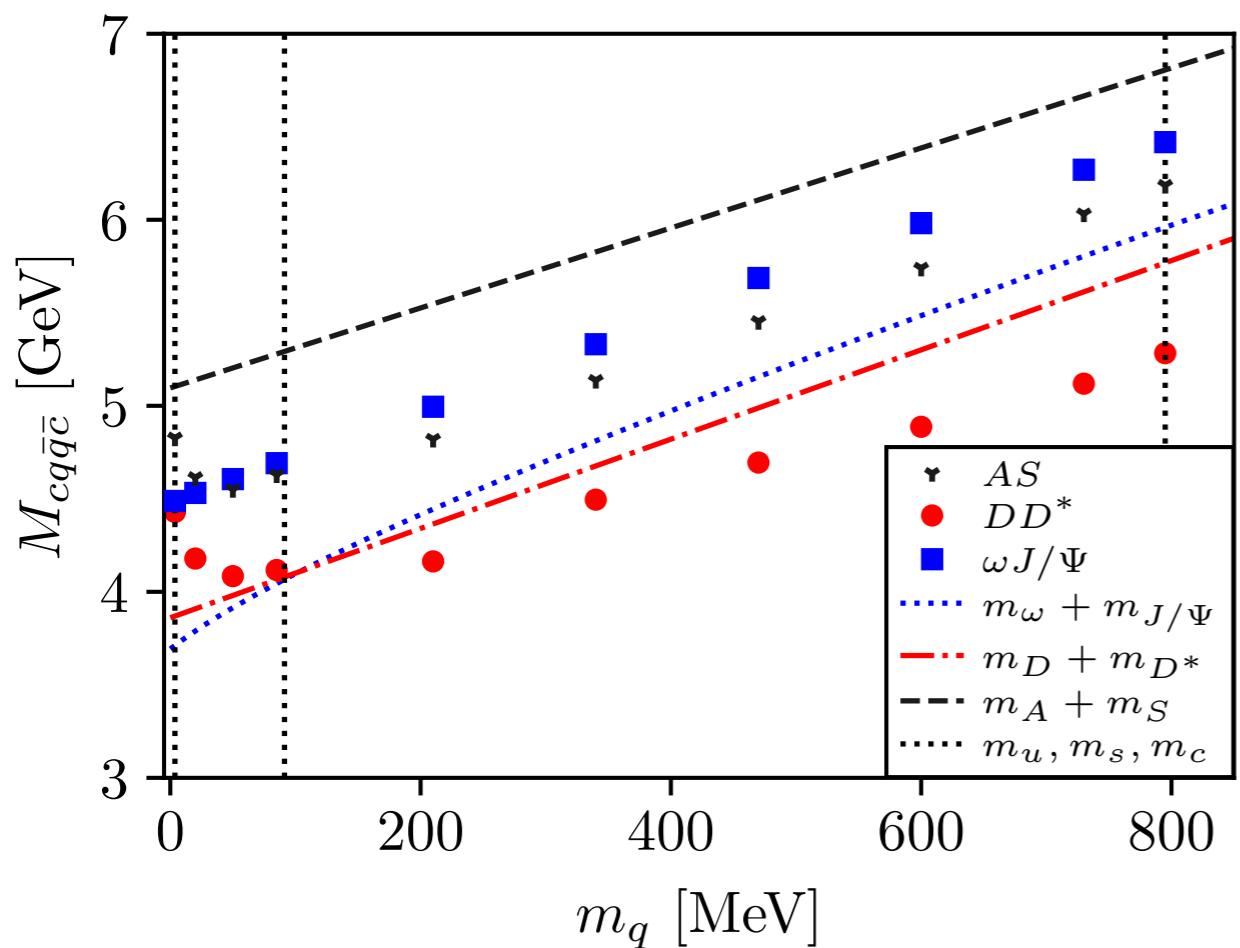


Wallbott, Eichmann and CF, in preparation

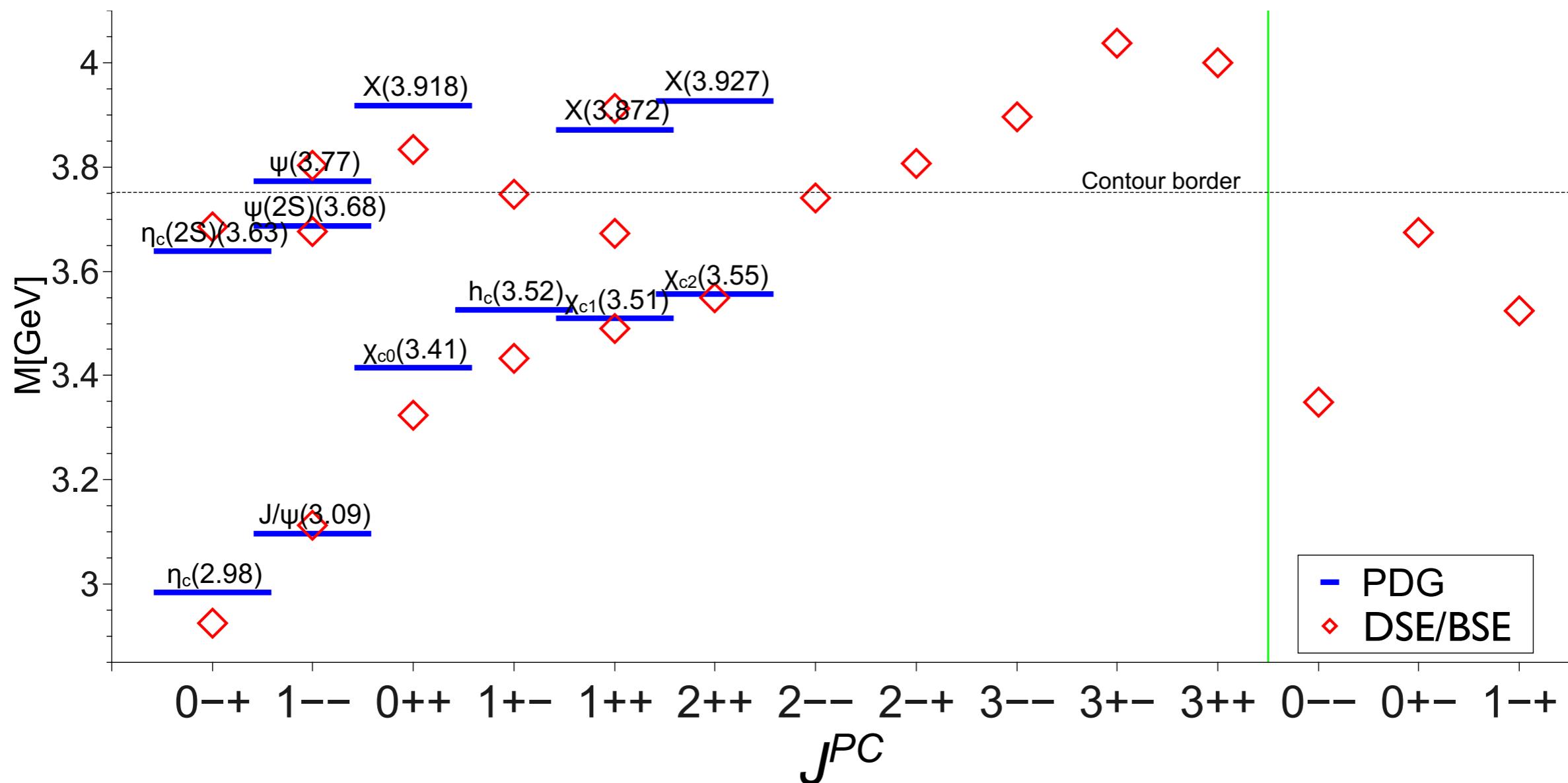
mass pattern matches molecule picture of  
Cleven et al. PRD 92 (2015) 014005:



# $\mathbf{X(3872)}$ and $\mathbf{X(4140)}$



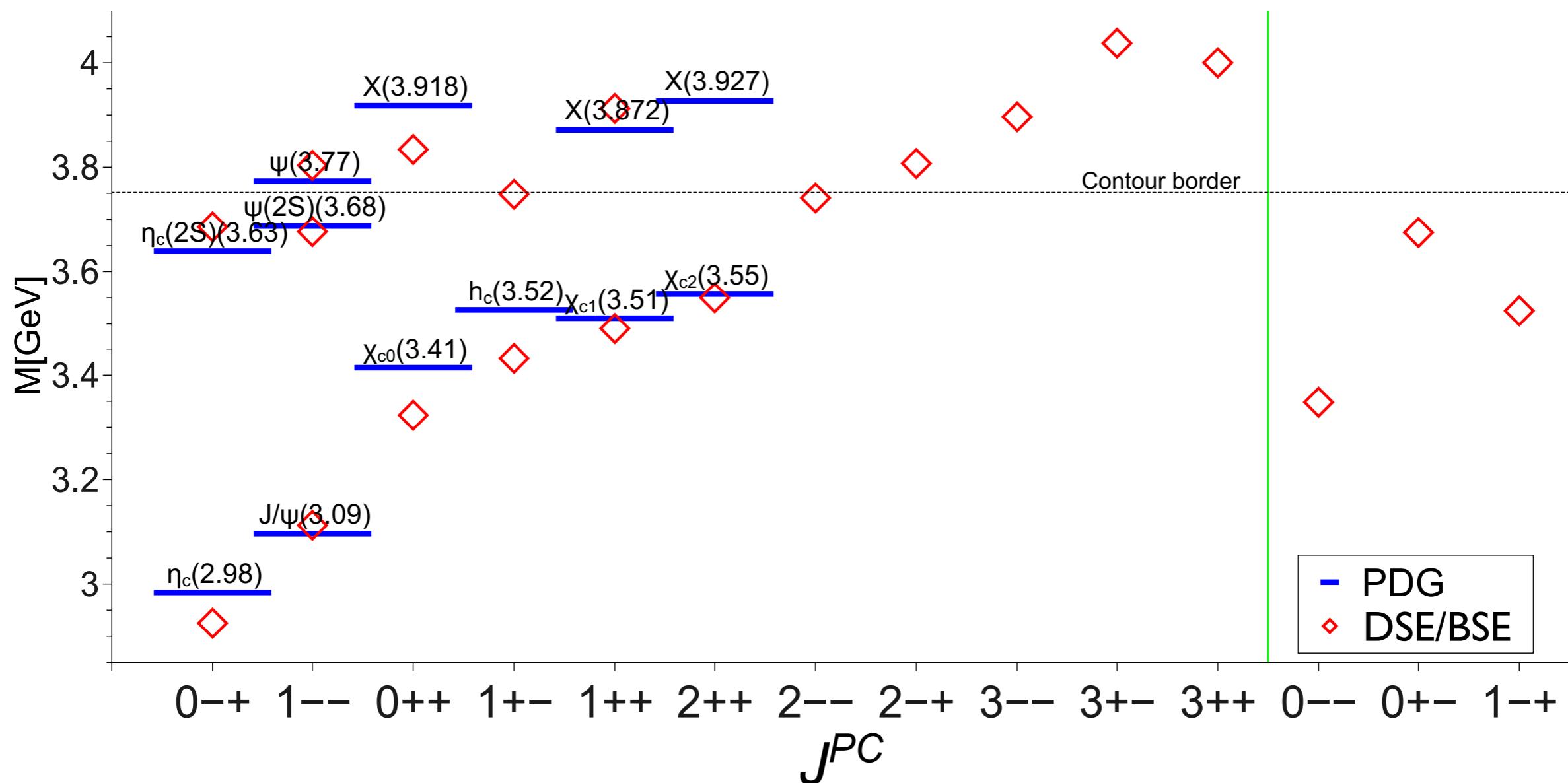
# Charmonium spectrum



CF, Kubrak, Williams, EPJA 51 (2015)  
Hilger et al. PRD 91 (2015)

- good channels:  $1--, 2^{++}, 3^{--}, \dots$
- acceptable channels:  $0^{-+}, 1^{+-}$
- deficiencies in other channels: missing spin-structure

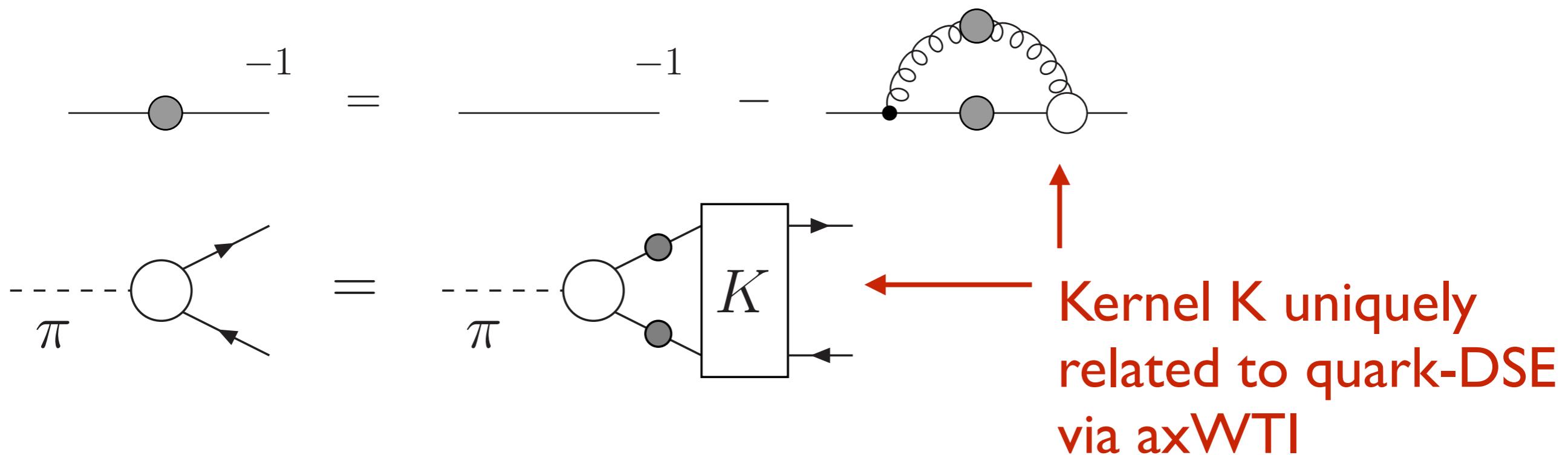
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- acceptable channels:  $0^{-+}, 1^{+-}$
- deficiencies in other channels: missing spin-structure

CF, Kubrak, Williams, EPJA 51 (2015)  
Hilger et al. PRD 91 (2015)

# Theoretical Tools II: DSEs and BSEs



→ Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex