Tetraquark mixing framework to explain the two light-meson nonets

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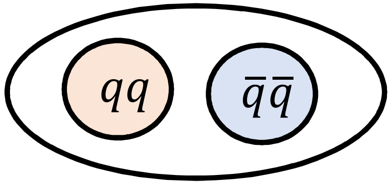
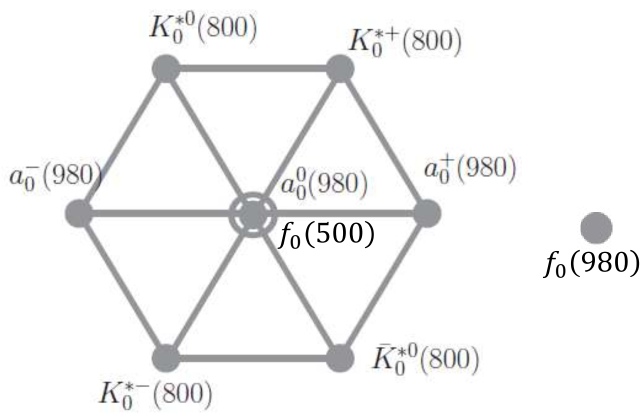
Tetraquark



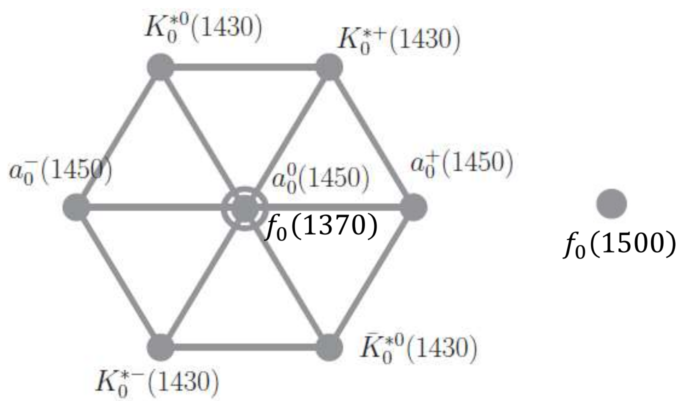
In this work,

- we explore tetraquark possibility in the light meson system.
- To be specific, we reexamine the diquark-antidiquark model by Jaffe and motivate tetraquark mixing framework for the resonances in the $J^P = 0^+$ channel.
- We introduce two types of tetraquark and their mixing to explain two nonets in PDG,

Light nonet
$f_0(500), f_0(980), K_0^*(800), a_0(980)$



Heavy nonet
$f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$



Diquark-antidiquark model by Jaffe

Jaffe model

According to diquark-antidiquark model [Jaffe 1977]

- Tetraquarks can be constructed by combining diquark(qq) and antidiquark($\bar{q}\bar{q}$), $qq\bar{q}\bar{q}$, ($q = u, d, s$), while assuming all the quarks are in an S-wave.
- In this construction, the spin-0 diquark with $qq \in J = 0,\ \bar{3}_c, \bar{3}_f,$ is used as a building block for tetraquarks
  – because this is the most compact object among all the possible diquarks.

⟨qq structure⟩

$Spin$	Color	Flavor	$\langle V_{CS} \rangle$	Type
0	$\bar{3}_c$	$\bar{3}_f$	-2	Attractive
1	6_c	$\bar{3}_f$	$-1/3$	Attractive
1	$\bar{3}_c$	6_f	$2/3$	Repulsive
0	6_c	6_f	1	Repulsive

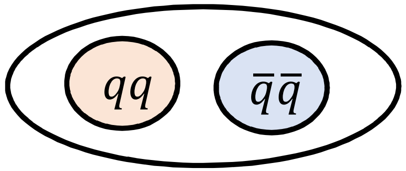
Possible diquarks allowed by Pauli principle.
$\langle V_{CS} \rangle$ is given in a certain unit.

Hyperfine color-spin interaction

$$V_{CS} \propto -\sum_{i \neq j} \lambda_i \cdot \lambda_j J_i \cdot J_j$$

λ_i: Gell-Mann matrix for color
J_i: spin

Tetraquark



Jaffe model

$qq\bar{q}\bar{q}$ from the spin-0 diquark

$$[qq \in (J_{12}=0, \bar{3}_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J_{34}=0, 3_c, 3_f)]$$

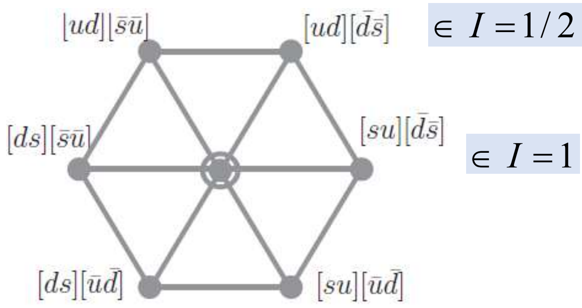
Spin: $[J_{12}=0]\otimes[J_{34}=0] = [J=0] \quad \Longrightarrow \quad |J, J_{12}, J_{34}\rangle = |000\rangle$

Color: $\bar{3}_c \otimes 3_c \Rightarrow 1_c$, i.e., $|1_c, \bar{3}_c, 3_c\rangle$

Flavor: forming a nonet, $\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$

Flavor nonet

$\mathbf{8_f}$



at the center

$\frac{1}{\sqrt{6}}\{2[ud][\bar{u}\bar{d}] - [ds][\bar{d}\bar{s}] - [su][\bar{s}\bar{u}]\}$

$\frac{1}{\sqrt{2}}\{[su][\bar{s}\bar{u}] - [ds][\bar{d}\bar{s}]\}$

$\mathbf{1_f}$

$\frac{1}{\sqrt{3}}\{[ds][\bar{d}\bar{s}] + [su][\bar{s}\bar{u}] + [ud][\bar{u}\bar{d}]\} \in I = 0$

Notation: $[ud] = \frac{1}{\sqrt{2}}(ud - du)$, etc.

Characteristics of Jaffe's tetraquarks

1. Spin and parity are $J^P = 0^+$.
2. Possible isospins are $I = 0, \frac{1}{2}, 1$.
3. The $I_z = 0$ members have $C = +$.
4. The mass ordering among the octet members,
   $(I=1) > \left(I=\frac{1}{2}\right) > (I=0)$,
   ex) $M([su][\bar{d}\bar{s}]) > M([su][\bar{u}\bar{d}])$.

※ Recall, a nonet from two-quark system ($q\bar{q}$) has the opposite mass ordering.

The lowest-lying resonances in $J^{P(C)} = 0^{+(+)}$: $K_0^*(800)$, $a_0(980)$, $f_0(500)$, $f_0(980)$, could be the candidates.
$\Rightarrow$We call these light nonet.

The 2nd tetraquark type

our update

Another tetraquark can be constructed by the spin-1 diquark

because this spin-1 diquark also forms a bound state even though it is less attractive than the spin-0 diquark.

$\langle qq$ structure$\rangle$

$Spin$	Color	Flavor	$\langle V_{CS} \rangle$
0	$\bar{3}_c$	$\bar{3}_f$	-2
1	6_c	$\bar{3}_f$	$-1/3$
1	$\bar{3}_c$	6_f	$2/3$
0	6_c	6_f	1

$qq\bar{q}\bar{q}$ from the spin-1 diquark in the $J^P = 0^+$ channel

Spin: $[J_{12} = 1] \otimes [J_{34} = 1] \Rightarrow [J = 0] \quad \Longrightarrow |J, J_{12}, J_{34}\rangle = |011\rangle$

Color: $6_c \otimes \bar{6}_c \Rightarrow 1_c$, i.e., $|1_c, 6_c, \bar{6}_c\rangle$

Flavor: $\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$ form the same nonet in flavor !

What about $|111\rangle, |211\rangle$?
$\Rightarrow$ see the additional slides

- This 2nd tetraquark is more compact than the 1st tetraquark from the spin-0 diquark (later!).
- But this 2nd tetraquark requires another nonet to be found in PDG.

Yes ! PDG has another nonet to support our approach.

our update

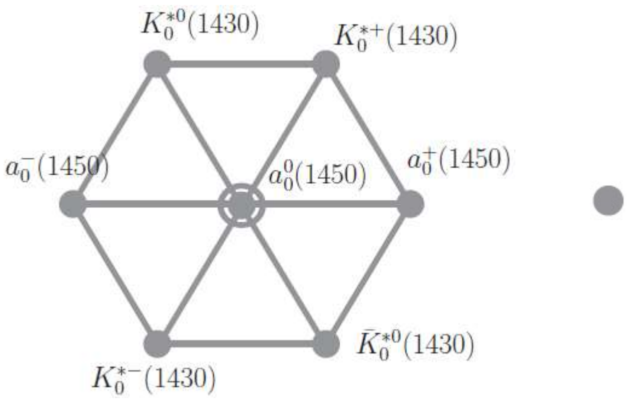
Heavy nonet in $J^P = 0^+$ (our selection)

- A similar nonet can be selected from higher resonances, $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$
  - GMO relation within $\sim 6\%$, $M^2[a_0(1450)] + 3M^2[f_0(1370)] \approx 4M^2[K_0^*(1430)]$.
  - $f_0(1500)$ is the heaviest.
- They have the anticipated isospins, $I = 0, \frac{1}{2}, 1$.
- Their mass ordering, though marginal, still holds here, $M[a_0(1450)] > M[K_0^*(1430)]$ with $\Delta M \sim 50$ MeV, $M[K_0^*(1430)] \gtrsim M[f_0(1370)]$.

Name	I	J^{PC}	M(MeV)	Γ(MeV)
$f_0(1370)$	0	0^{++}	1200-1500	200-500
$f_0(1500)$	0	0^{++}	1505	109
$K_0^*(1430)$	1/2	0^+	1425	270
$a_0(1450)$	1	0^{++}	1474	265
$f_0(1710)$	0	0^{++}	1723	139
$f_0(2020)$	0	0^{++}	1992	442
$f_0(2100)$	0	0^{++}	2101	224
$f_0(2200)$	0	0^{++}	2189	238
$f_0(2330)$	0	0^{++}	2314	144
$K_0^*(1950)$	1/2	0^+	1945	201

Well separated in mass

$J^{P(C)} = 0^{+(+)}$ with higher masses



Two states in $I = 0 \Longrightarrow f_0(1370), f_0(1500)$.

Heavy nonet could be the 2nd candidate for the tetraquark !

Tetraquark mixing framework

(Our proposal)

Two tetraquark types differ by the spin and color configurations.

$|000\rangle_{\bar{3}_c,3_c} \Longrightarrow |000\rangle$ $|011\rangle_{6_c,\bar{6}_c} \Longrightarrow |011\rangle$

- Both form flavor nonet ($8_f \oplus 1_f$).

Two nonets in PDG which satisfy the tetraquark characteristics.

Light nonet (Jaffe's nonet)	Heavy nonet (additional selection by us)
$f_0(500), f_0(980), K_0^*(800), a_0(980)$	$f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

- The huge mass gap between the two, $\gtrsim 500$ MeV.

What kind of correspondence between the two ?

$\boxed{\text{Two tetraquark types} \Longleftrightarrow \text{Two nonets in PDG}}$

A crucial observation is that

- the two tetraquark types, $|000\rangle$, $|011\rangle$, in each isospin channel, mix through the color-spin interaction !

$$V_{CS} \propto \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

λ_i: Gell-Mann matrix for color
J_i: spin
m_i: constant quark mass

– The mixing terms are nonzero, $\langle 011|V_{CS}|000\rangle \neq 0$.
– $\langle V_{CS}\rangle$ forms a 2x2 matrix in $|000\rangle, |011\rangle$, constituting the hyperfine mass matrix.

The upshot is that

- physical resonances, the two nonets in PDG, can be identified by the eigenstates that diagonalize the 2x2 matrix in each isospin channel [$\sum m_q$, $\langle V_{CE}\rangle$ in the Hamiltonian are also diagonal in these eigenstates].
  ☞ The two nonets in PDG are superposition of $|000\rangle, |011\rangle$.

- The mixing is found to be strong so it can explain the large mass gap between the two nonets (later!).

This is our tetraquark mixing framework for the two nonets in $J^P = 0^+$.

☞ We look for its phenomenological signatures from experimental observables such as mass or decay property !

digression

In literature, there are other models to explain the two nonets.

- Two-quark picture with $\ell = 1$ or its variants.
- Mixing of two-quark and four-quark
- Meson molecular picture

But they have some limitation (see the additional slides).

testing ground

- Our testing ground for the tetraquark mixing framework is the two nonets.

Isospin	Light nonet	Heavy nonet
$I=1$	$a_0(980)$	$a_0(1450)$
$I=1/2$	$K_0^*(800)$	$K_0^*(1430)$
$I=0$	$f_0(500)$	$f_0(1370)$
	$f_0(980)$	$f_0(1500)$

Flavor mixing between $|\mathbf{8}_f\rangle_{I=0}, |\mathbf{1}_f\rangle_{I=0}$ is considered through SSC, IMC, RCF.

- First, we calculate $\langle V_{CS}\rangle$ in each isospin channel because its diagonalization leads to the physical states identified as the two nonets.

$$V_{CS} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

One surprising result: The light nonet has more probability to stay in $|011\rangle$ rather than in $|000\rangle$.

Hyperfine mass matrix in the $I = 1$ channel [corresponding to $a_0(980)$, $a_0(1450)$].

- Diagonalization leads to the physical hyperfine masses

$\langle V_{CS}\rangle$	$\|000\rangle$	$\|011\rangle$
$\|000\rangle$	-173.9	-222.3
$\|011\rangle$	-222.3	-331.5

$\longrightarrow$

$\langle V_{CS}\rangle$	$\|0_A^{a_0}\rangle$	$\|0_B^{a_0}\rangle$
$\|0_A^{a_0}\rangle$	-16.8	0.0
$\|0_B^{a_0}\rangle$	0.0	-488.5

and eigenstates corresponding to $a_0(980)$, $a_0(1450)$,

$$|0_A^{a_0}\rangle = -0.817|000\rangle + 0.577|011\rangle \quad \Rightarrow |a_0(1450)\rangle$$

$$|0_B^{a_0}\rangle = 0.577|000\rangle + 0.817|011\rangle \quad \Rightarrow |a_0(980)\rangle\ .$$

This identification follows from $\langle 0_A^{a_0}|V_{CS}|0_A^{a_0}\rangle > \langle 0_B^{a_0}|V_{CS}|0_B^{a_0}\rangle$

As advertised,

- The **strong mixing** causes **large separation** in hyperfine masses [$\Delta\langle V_{CS}\rangle \approx 500$ MeV].
  ☞ can explain the large mass gap (500 MeV or so) between the two nonets!

- $|011\rangle$ is more compact, $\langle 000|V_{CS}|000\rangle > \langle 011|V_{CS}|011\rangle$.
  ☞ $a_0(980)$ has more probability to stay in $|011\rangle$ rather than in $|000\rangle$.
  $\Longrightarrow$ Surprising(!) but this is also supported by recent QCDSR, arXiv:1904.12311 [H.Lee, C.Kim, H.Kim]

☞ The similar result is obtained for the other members in the light nonet.

Supporting signatures from mixing par. and hyperfine masses

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Including other members, the mixing formulas are collectively written as

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle, \quad \text{for } f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle, \quad \text{for } f_0(500), f_0(980), K_0^*(800), a_0(980)$$

Mixing parameters, α, β — Hyperfine mass $\langle V_{CS}\rangle$

Isospin	α	β	Light	$\langle V_{CS}\rangle$	Heavy	$\langle V_{CS}\rangle$
$I = 1$	0.8167	0.5770	$a_0(980)$	-488.5	$a_0(1450)$	-16.8
$I = 1/2$	0.8130	0.5822	$K_0^*(800)$	-592.7	$K_0^*(1430)$	-26.9
$I = 0$ (RCF)	0.8136	0.5814	$f_0(500)$	-667.5	$f_0(1370)$	-29.2
$I = 0$ (RCF)	0.8157	0.5784	$f_0(980)$	-535.1	$f_0(1500)$	-20.1

~100 ~75 ~10

close to the 8_f member

Supporting signatures

1. α, β are almost independent of isospin
   $\Rightarrow$ Support our identification of the two nonets in PDG as flavor nonet.
2. Hyperfine mass ordering can explain the mass ordering in the light nonet.
   Ex) $M[a_0(980)] - M[K_0^*(800)] \approx 300 \Longleftarrow \Delta m_q \approx 170, \Delta\langle V_{CS}\rangle \approx 100$ MeV
   $M[K_0^*(800)] - M[f_0(500)] \approx 200 \Longleftarrow \Delta m_q \approx 100, \Delta\langle V_{CS}\rangle \approx 75$ MeV
3. For the heavy nonet, the hyperfine ordering goes away [$\Delta\langle V_{CS}\rangle \lesssim 10$ MeV], which can partially explain the marginal mass splitting, $M[a_0(1450)] - M[K_0^*(1430)] \approx 50$ MeV.

signature 4

4. Our tetraquarks satisfy the mass splitting formula

$\Delta M \approx \Delta\langle V_{CS}\rangle$

The mass difference between the two nonets in each isospin channel can be approximated by the hyperfine mass splitting because $\Delta(\sum m_q) \approx 0,\ \Delta\langle V_{CE}\rangle \approx 0$.

So this formula should work for the experimental mass splitting.

For $I = 1$

Heavy nonet	Light nonet	ΔM_{exp} (M e V)	$\Delta\langle V_{CS}\rangle = \langle V_{CS}\rangle_{\mathrm{HN}} - \langle V_{CS}\rangle_{\mathrm{LN}}$		
			SSC	IMC	RCF
$a_0(1450)$	$a_0(980)$	**494**	**471.7**	-	-

☞$\Delta\langle V_{CS}\rangle$ agrees with the experimental mass spitting, ΔM_{exp}.

equal when $m_u = m_d$

For $I = 0, 1/2$

Heavy nonet	Light nonet	ΔM_{exp} (M e V)	SSC	IMC	RCF
$f_0(1500)$	$f_0(980)$	**515**	**541.7**	471.7	515
$f_0(1370)$	$f_0(500)$	875	611.7	681.7	638.3
$K_0^*(1430)$	$K_0^*(800)$	743	565.8	-	-

M_{exp} is broad or not fixed well

- The $I = 0$ results do not depend much on how the flavor mixing is implemented.
- For the last two lines, precise agreement is not anticipated as the participating resonances are either broad or their masses are poorly known.

This shows, the huge gap (500 MeV or so) between the two nonets is basically generated by the strong mixing between $|000\rangle, |011\rangle$.

Signatures from fall-apart modes of our tetraquarks

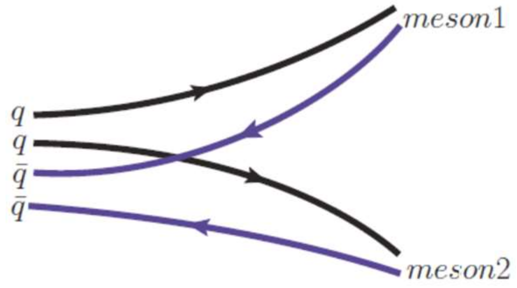
Fall-apart modes

Tetraquarks can decay into two mesons through fall-apart mechanism.

Two categories

- PS-PS mode, PP
- V-V mode, VV

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$



$qq\bar{q}\bar{q}$ fall-apart decay

(24)

$$q_1 q_2 \bar{q}^3 \bar{q}^4 \Rightarrow [(8_c)_1 \otimes (8_c)_{24}]_{1_c} \oplus \boxed{[(1_c)_1 \otimes (1_c)_{24}]_{1_c}}$$

(13)

two-meson modes

	Heavy nonet	Light nonet
PP	$-\frac{\alpha}{2\sqrt{3}} + \frac{\beta}{\sqrt{2}}$	$\frac{\beta}{2\sqrt{3}} + \frac{\alpha}{\sqrt{2}}$
VV	$-\frac{\alpha}{2} - \frac{\beta}{\sqrt{6}}$	$\frac{\beta}{2} - \frac{\alpha}{\sqrt{6}}$

Coefficient of each decay channel represents relative strength of fall-apart mode.
Due to the relative signs, we find that

- PP mode: suppressed in heavy nonet but enhanced in the light nonet.
- VV mode: enhanced in heavy nonet but suppressed in the light nonet.

opposite trend!

PP modes from $a_0(980)$, $a_0(1450)$

Coupling strengths up to an overall constant

	$a_0^+(1450)$	$a_0^+(980)$
$\bar{K}^0K^+$	$-\frac{\alpha}{2\sqrt{3}}+\frac{\beta}{\sqrt{2}}=0.1722$	$\frac{\beta}{2\sqrt{3}}+\frac{\alpha}{\sqrt{2}}=0.7441$
$\eta\pi^+$	$-\frac{\alpha}{3\sqrt{2}}+\frac{\beta}{\sqrt{3}}=0.1406$	$\frac{\beta}{3\sqrt{2}}+\frac{\alpha}{\sqrt{3}}=0.6076$
$\eta'\pi^+$	$\frac{\alpha}{6}-\frac{\beta}{\sqrt{6}}=-0.0994$	$\boxed{-\frac{\beta}{6}-\frac{\alpha}{\sqrt{6}}=-0.4296}$

kinematically not allowed

- The relative enhancement factor is about 'four' !
- Can be tested by the ratios of partial widths .
  ☞The ratios eliminate unknown dependence on the overall constant.
- Note, the partial widths depend on kinematical factors as well as the coupling strengths.

The agreement is quite good !

Based on expt. analysis

	Theory	Bugg	PDG
$\frac{\Gamma[a_0(980)\to\pi\eta]}{\Gamma[a_0(1450)\to\pi\eta]}$	2.51–2.54	2.53	2.93–3.9
$\frac{\Gamma[a_0(980)\to K\bar{K}]}{\Gamma[a_0(1450)\to K\bar{K}]}$	0.52–0.89	0.62	0.61–0.81

Bugg: PRD78,074023(2008)

- The relative coupling strength of the heavy nonet is strongly enhanced by a factor ~15 !

- But most channels are not accessible experimentally due to kinematical constraint, $M(\text{mother}) < M_1 + M_2(\text{daughters})$.

VV modes from the two nonets

	Mode	$a_0^+(980)$	$a_0^+(1450)$	Ratio
$I = 1$	$\bar{K}^{*0}K^{*+}$	-0.0449	-0.6439	14.33
	$\phi\rho^+$	0.0449	0.6439	
	Mode	$K_0^{*+}(800)$	$K_0^{*+}(1430)$	Ratio
$I = 1/2$	ρ^+K^{*0}	-0.0408	-0.6442	15.78
	ρ^0K^{*+}	-0.0289	-0.4555	
	ωK^{*+}	0.0289	0.4555	
	Mode	$f_0(500)$	$f_0(1370)$	Ratio
$I = 0\ (\sim\mathbf{8}_f)$	$\rho^0\rho^0$	0.0185	0.2869	15.54
	$\bar{K}^{*0}K^{*0}$	-0.0133	-0.2069	
	$\phi\omega$	0.0188	0.2927	
	$\omega\omega$	-0.0185	-0.2869	
	Mode	$f_0(980)$	$f_0(1500)$	Ratio
$I = 0\ (\sim\mathbf{1}_f)$	$\rho^0\rho^0$	0.0100	0.1463	14.70
	$\bar{K}^{*0}K^{*0}$	0.0276	0.4057	
	$\phi\omega$	-0.0390	-0.5737	
	$\omega\omega$	-0.0100	-0.1463	

Any supporting clue (?)

$f_0(1370) \longrightarrow \rho\rho$, $f_0(1500) \longrightarrow \rho\rho$

- Barely satisfy the kinematical constraint, $M[f_0(1370)], M[f_0(1500)] \geq 2M_\rho$~1551 MeV only through high tail of the decay widths.

- Even so, PDG shows some branch ratios, which can support the enhancement in the VV mode from the heavy nonet.
  – $f_0(1370) \longrightarrow \rho\rho$ is listed as 'dominant' in the 4π mode.
  – $f_0(1500) \longrightarrow \rho\rho$ is seen in the 4π mode.

Summary

- We propose a tetraquark mixing framework to explain the two nonets,
  Light nonet: $f_0(500), f_0(980), K_0^*(800), a_0(980)$
  Heavy nonet: $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$
  Clues for the tetraquark nonets: quantum numbers (I, J^{PC}), GMO relation, the mass ordering

- According to this, the two nonets are the mixture of two tetraquark types,

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

$$\alpha \approx \sqrt{2/3}, \beta \approx \sqrt{1/3}$$

  Here α, β are obtained by diagonalizing the color-spin interaction.

- The inequality, $\alpha > \beta$, suggests that the light nonet has more probability to stay in $|011\rangle$ rather than in $|000\rangle$ (!!).

  ☞This surprising result is supported by QCD sum rules also !

Other signatures to support this tetrquark mixing framework

- The mixing parameters are almost independent of isospin
  - ✓ our mixing formulas generate the flavor nonets consistently with the two nonets in PDG.
- Our hyperfine masses, $\langle V_{CS} \rangle$, can explain
  - ✓ the mass splitting in the light nonet,
  - ✓ partially the marginal mass splitting in the heavy nonet.
- Hyperfine mass splitting agrees with the mass splitting between the two nonets (500 MeV or so), $\Delta M \approx \Delta \langle V_{CS} \rangle$.

- The mixing framework provides distinct signatures from fall-apart modes.
  - ✓ PP modes: suppressed in heavy nonet but enhanced in the light nonet.
    The PP signature has been tested relatively well from the ratios of partial widths, $a_0(980), a_0(1450) \Longrightarrow K\bar{K}, \eta\pi$.
  - ✓ VV modes: enhanced in heavy nonet but suppressed in the light nonet.
    The VV signature has some supporting hints from $f_0(1370) \longrightarrow \rho\rho$, $f_0(1500) \longrightarrow \rho\rho$.

Our work provides a new view on tetraquarks, especially how they are realized in the light meson system, i.e., through ``mixing framework''.

Thank you for your attention !

Additional slides

for some questions

digression

One question

- The spin-1 diquark scenario requires additional nonets to be found in $J^P = 1^{+-}, 2^{++}$ corresponding to the configurations

$$|111\rangle_{6_c,\bar{6}_c} \qquad |211\rangle_{6_c,\bar{6}_c}$$

※ One can prove that C-parity is negative for $J = 1$, positive for $J = 2$.

Are there such nonets in PDG ? My answer is 'Maybe'.

- There are lots of resonances to choose but the candidate selection is not definite.

Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$h_1(1170)$	0	1+-	1170.0	360
$b_1(1235)$	1	1+-	1229.5	142
$h_1(1380)$	?	1+-	1386.0	91
$h_1(1595)$	0	1+-	1594.0	384
$K_1(1270)$	1/2	1+	1272.0	90
$K_1(1400)$	1/2	1+	1403.0	172
$K_1(1650)$	1/2	1+	1650.0	150

$J^{P(C)} = 1^{+(-)}$ resonances

- Highlighted members can be selected but with some ambiguity,
  - unknown isospin of $h_1(1380)$,
  - the mass ordering, slightly violated, $M[b_1(1235)] < M[K_1(1270)]$

Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$f_2(1270)$	0	2++	1275.1	185.1
$a_2(1320)$	1	2++	1318.3	105
$f_2(1430)$	0	2++	1430.0	?
$f'_2(1525)$	0	2++	1525.0	73
$f_2(1565)$	0	2++	1562.0	134
$f_2(1640)$	0	2++	1639.0	99
$a_2(1700)$	1	2++	1732.0	194
$f_2(1810)$	0	2++	1815.0	197
$f_2(1910)$	0	2++	1903.0	196
$f_2(1950)$	0	2++	1944.0	472
$f_2(2010)$	0	2++	2011.0	202
$f_2(2150)$	0	2++	2157.0	152
$f_2(2300)$	0	2++	2300.0	149
$f_2(2340)$	0	2++	2345.0	322
$K_2^*(1430)$	1/2	2+	1425.0	98.5
$K_2^*(1980)$	1/2	2+	1973.0	373

$J^{P(C)} = 2^{+(+)}$ resonances

digression

The selection is ambiguous

- maybe due to further mixings with additional tetraquarks constructed by other diquarks, and possible contamination from two-quark component with $\ell = 1$.
- This ambiguity does not mean that $|111\rangle$, $|211\rangle$ do not exist.
  $\Rightarrow$ It simply says that the candidates do not stand out in a well-separated entity.
  $\Rightarrow$ It does not rule out our mixing framework in the 0^+ channel.

Other models to explain the two nonets

with some limitations.

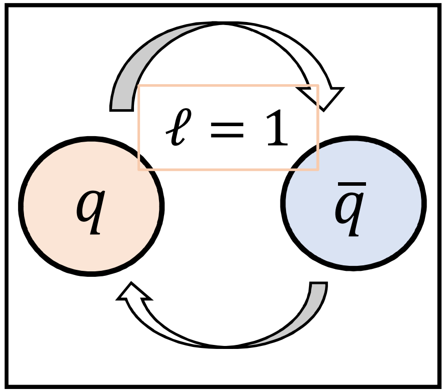
The two-quark picture($q\bar{q}$) with $\ell = 1$

- can make nonets also with $J^P = 0^+$.
- Does this picture explain the two nonets in PDG ? My answer is `No'

$$q\bar{q}: (S = 0,1)\otimes(\ell = 1) \Longrightarrow J = 0,1,2$$

Total J	Configuration	# of confs.
$J = 0$	$(S = 1, \ell = 1)$	one
$J = 1$	$(S = 0, \ell = 1), (S = 1, \ell = 1)$	two
$J = 2$	$(S = 1, \ell = 1)$	one

- This picture has only one configuration in $J^P = 0^+$, not enough to explain the two nonets in $J^P = 0^+$.

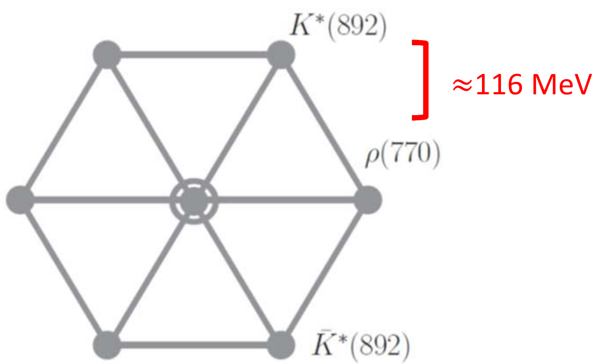


Alternatively, one may view

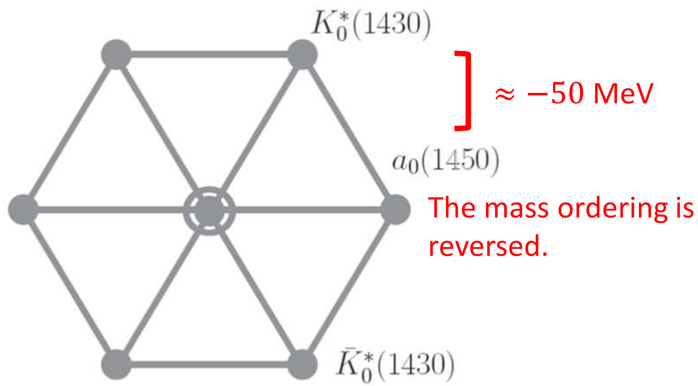
- the heavy nonet by $q\bar{q}$ ($\ell = 1$) and the light nonet by $qq\bar{q}\bar{q}$

- The heavy nonet must have the configuration
  $(S = 1$, vector nonet$) \otimes (\ell = 1) \Longrightarrow J = 0$
  $\Rightarrow$ orbital excitations of the vector mesons, ρ, ω, K^*, ϕ.

- In this picture, the spin-orbit (SO) can make the heavy nonet 'heavier' than the vector nonet.



$\Longrightarrow$
$(\ell = 1)$



- To reproduce the reversed gap (≈ -50 MeV), SO must have strong isospin dependence, strong enough to flip the mass ordering established by the quark masses. ☞ This picture is not realistic !

Mixing model

Mixture of $q\bar{q}$, $qq\bar{q}\bar{q}$

One may view the two nonets as a mixture of $q\bar{q}$ $(\ell = 1)$ and $qq\bar{q}\bar{q}$?
Black et.al, PRD 59(1999)

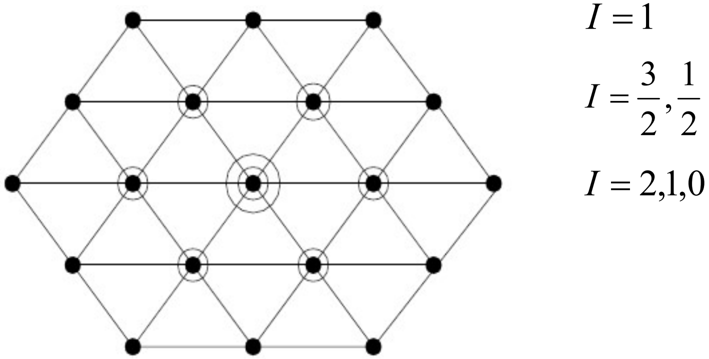
- Black et.al introduce the effective fields corresponding to $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets, and make SU(3) invariant Lagrangian among them.
- As pointed by Maiani et.al. EPJC50(2007), the required mixing seems too large given the fact that very different configurations are involved.

- In particular, $q\bar{q}(\ell = 1)$, $qq\bar{q}\bar{q}$ do not mix under the color-spin interaction !
  $\langle q\bar{q}|qq\bar{q}\bar{q}\rangle = 0$, $\langle q\bar{q}|V_{CS}|qq\bar{q}\bar{q}\rangle = 0$.
- It is hard to establish such a mixing from well-known quark-quark interactions.

One may view the two nonets as meson-meson bound states.

- Since mesons are colorless, this model suggests shallow bound states.
  ex) $f_0(980) \sim K\bar{K}$ since $M[f_0(980)] \sim 2M_K$.
  But it is hard to view $f_0(500)$ as a shallow bound state of $\pi\pi$.

- Since the lowest-lying mesons form a nonet in flavor, the meson-meson states can form diverse multiplets including the 27-plet
$$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$$
  $\Rightarrow$ PDG does not support this picture. (ex. no 0^+ resonances with $I = 2$.)



$\mathbf{27}_f$

But note Jido et.al(PRL2006)