Positronium on the Light-front

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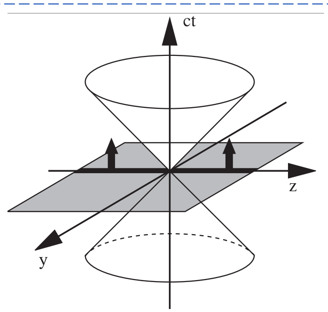
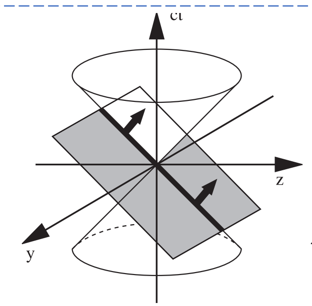
Outline

- Light-front Dynamics & **Basis Light-front Quantization**
- Why positronium
- Nonperturbative **renormalization**
- Numerical Results
- Summary and Outlook

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- not just a coordinate transformation.
- a new theory in a different quantization.

Why go to light front?

- Frame independent wavefunction
- Simple vacuum structure
- Boost invariant
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^{-}|\beta\rangle = P^{-}_{\beta}|\beta\rangle$$

  - P^{-}: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^{-}_{β}: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_{\perp})$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

More talks about Light-front dynamics & basis light-front quantization:

- Shaoyang Jia, “Valence structures of light and strange mesons from the basis light-front quantization framework”, 20/pm
- Jiangshan Lan, “On light mesons Parton distribution functions from basis light front quantization”, 18/pm
- Siqi Xu, “three dimension imaging of proton from BLFQ”, 20/am
- Sreeraj Nair, “Quark Wigner distributions Using Light-front Wave Functions”, 20am
- Xingbo Zhao, “Basis Lightfront Approach to Hadron Structure”, 20/pm
- Chandan Mondal, “Wigner distribution and spin structure of pion from light front holographic QCD” , 20/pm

Why Positronium

Positronium is a test bed for

- Relativistic bound state structure beyond leading Fock-sector
- Basis Light-front Quantization on first-principle of QED, esp., nonperturbative renormalization procedure
- Connection with one-photon-exchange effective theory

[Wiecki, et al, 2015]

Light-front QED Hamiltonian

- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$

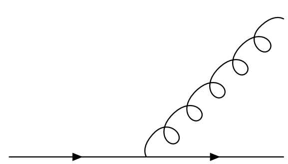
- Light-front QED Hamiltonian from standard Legendre transformation

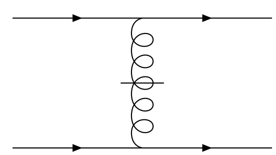
$$P^- = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \; F^{\mu+}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

Light-cone gauge: $(A^+ = 0)$

$$= \int \mathrm{d}^2x^\perp \mathrm{d}x^- \; \underbrace{\frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2A^j}_{\text{kinetic energy terms}}$$

$$+ \underbrace{ej^\mu A_\mu}_{\text{vertex interaction}} + \underbrace{\frac{e^2}{2}j^+\frac{1}{(i\partial^+)^2}j^+}_{\text{instantaneous photon interaction}}$$

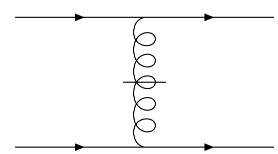
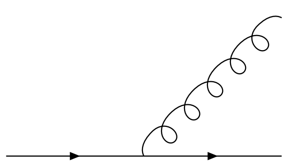
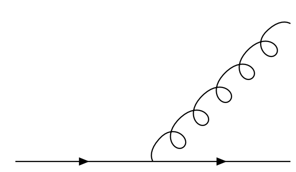




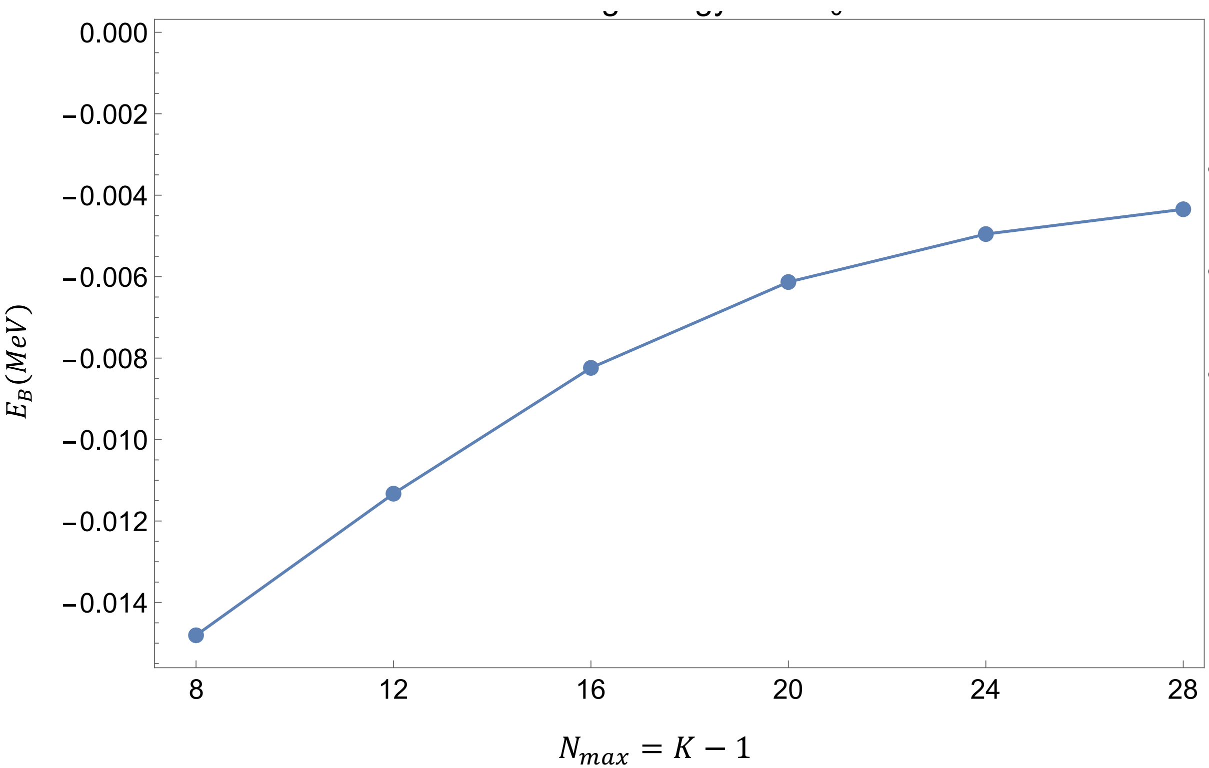
Interaction Part Of Hamiltonian

$|\mathbf{Ps}\rangle = a|e\bar{e}\rangle + b|e\bar{e}\gamma\rangle + \mathrm{c}|\gamma\rangle + \mathrm{d}|e\bar{e}e\bar{e}\rangle + \ldots$

m_e=1.0MeV $\alpha = 0.3$

H_{int}	$\vert e\bar{e}\rangle$	$\vert e\bar{e}\gamma\rangle$
$\langle e\bar{e}\vert$		
$\langle e\bar{e}\gamma\vert$		0

Ground State Binding Energy



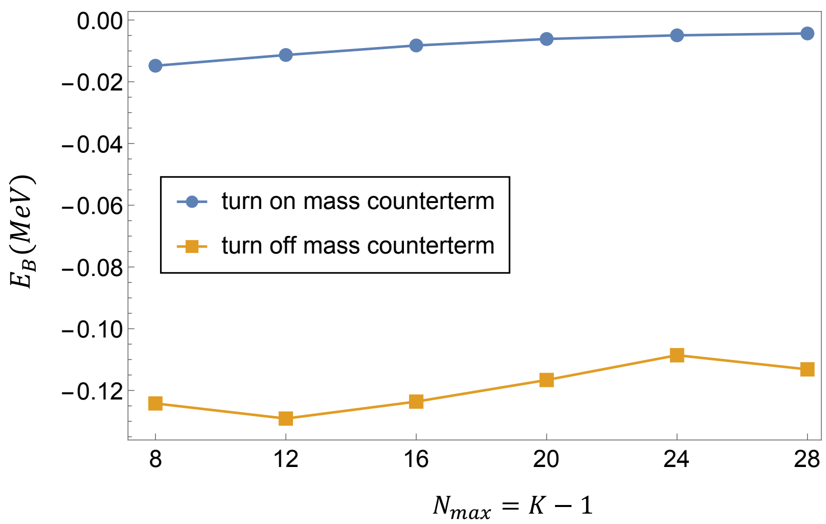
$$E_B = M_P - 2M_e$$

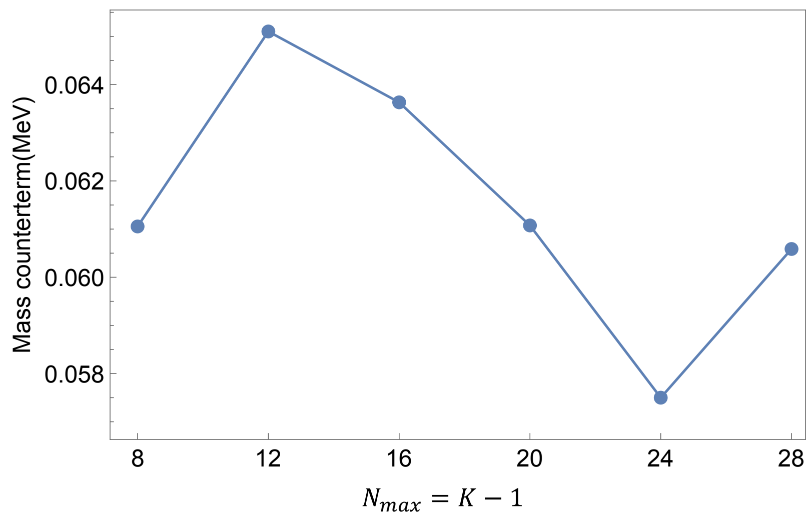
- E_B: binding energy of positronium
- M_P: Invariant mass of positronium
- M_e: Invariant mass of free electron

Binding energy looks convergent. nontrivial

[Kaiyu Fu et al, in preparation]

Mass Renormalization





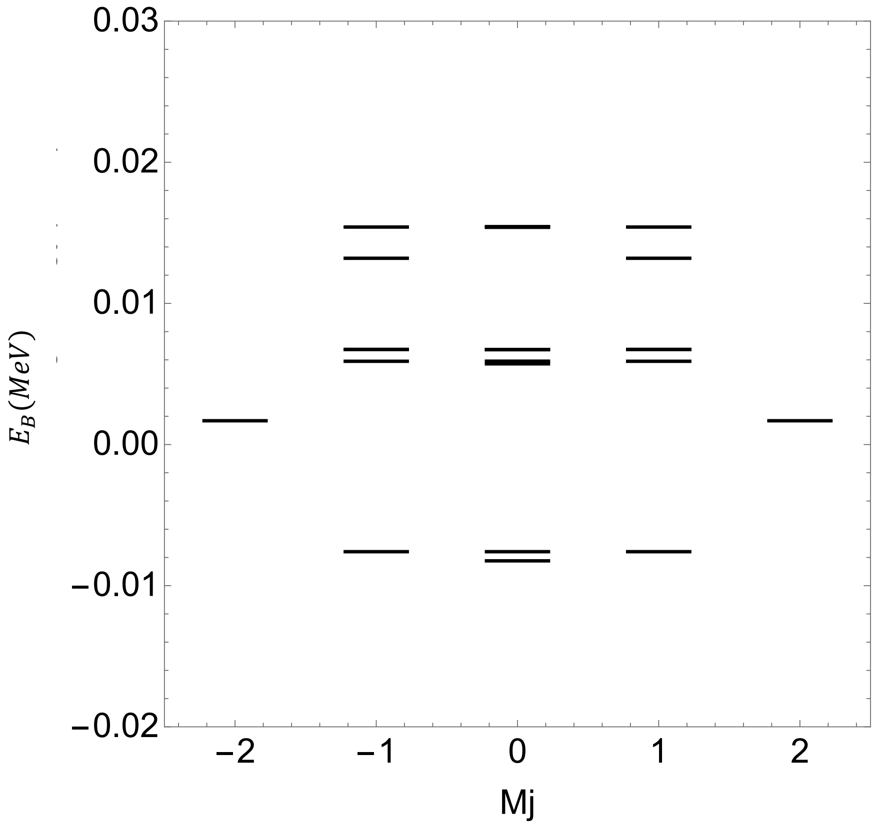
- Mass renormalization is performed on the level single physical electron
- Mass counterterm is determined by fitting single electron mass
- Plug the physical electron and positron into the positronium.

Mass counterterm is much larger than E_B

[Kaiyu Fu et al, in preparation]

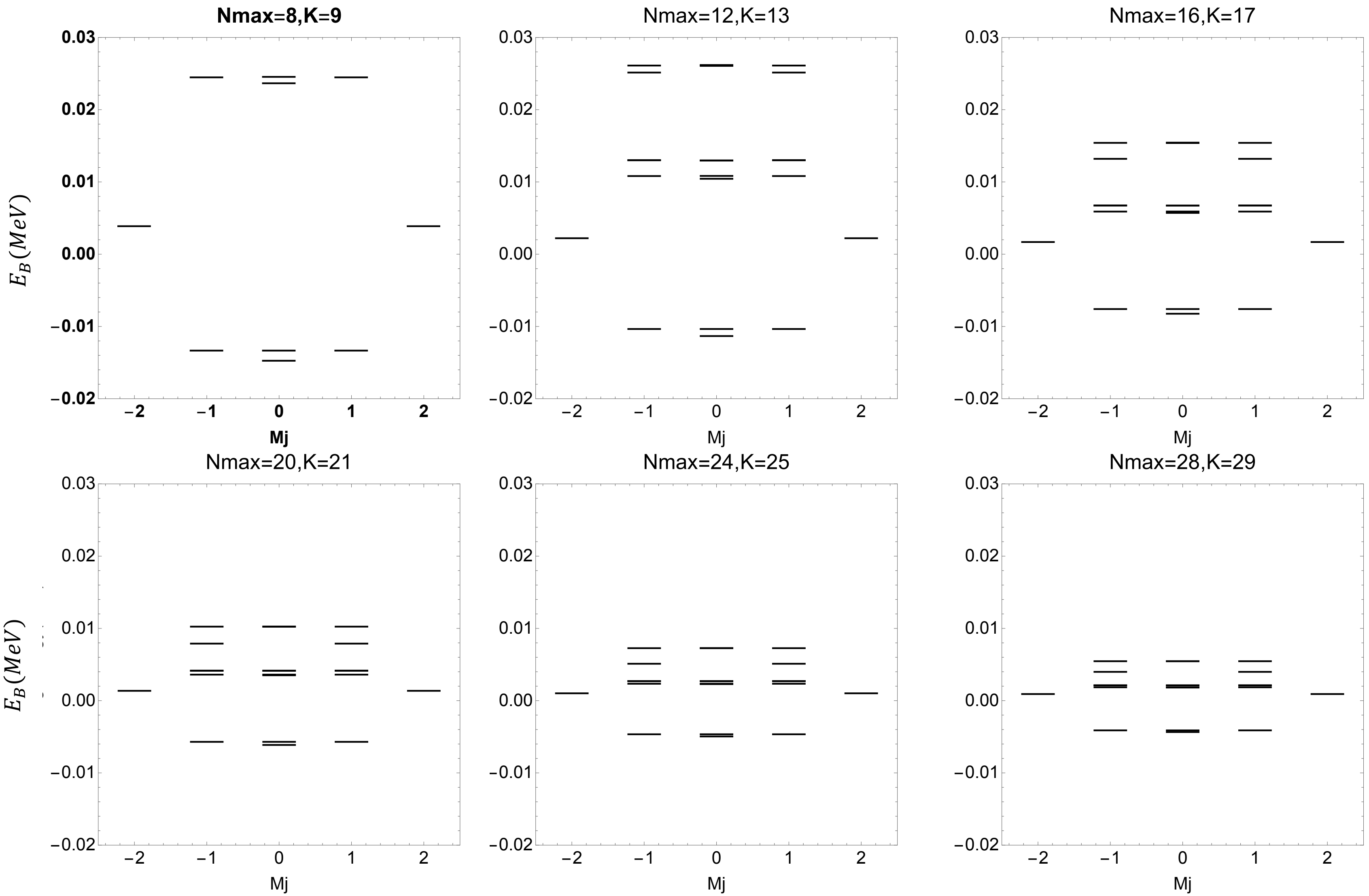
Energy spectrum

Nmax=16, K=17



lowest 8 states of Mj=0 : parity and charge conjugation parity agree with hydrogen atom.

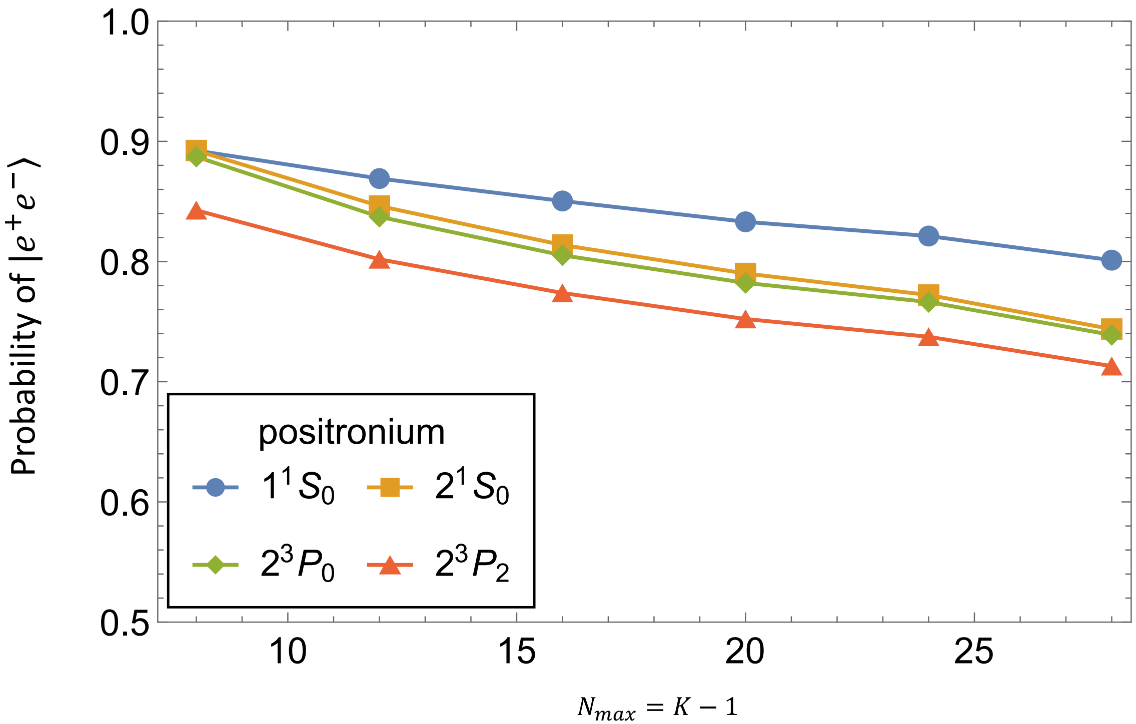
[Kaiyu Fu et al, in preparation]



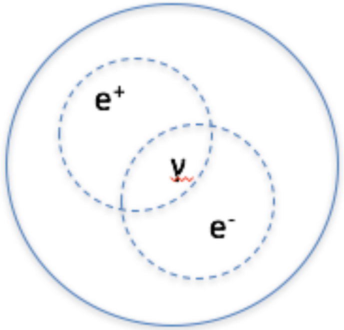
Rotational symmetry is restoring

[Kaiyu Fu et al, in preparation]

Probability Of $|e^+e^-\rangle$



- Interaction mediated through photon.
- Finite probability to find photon



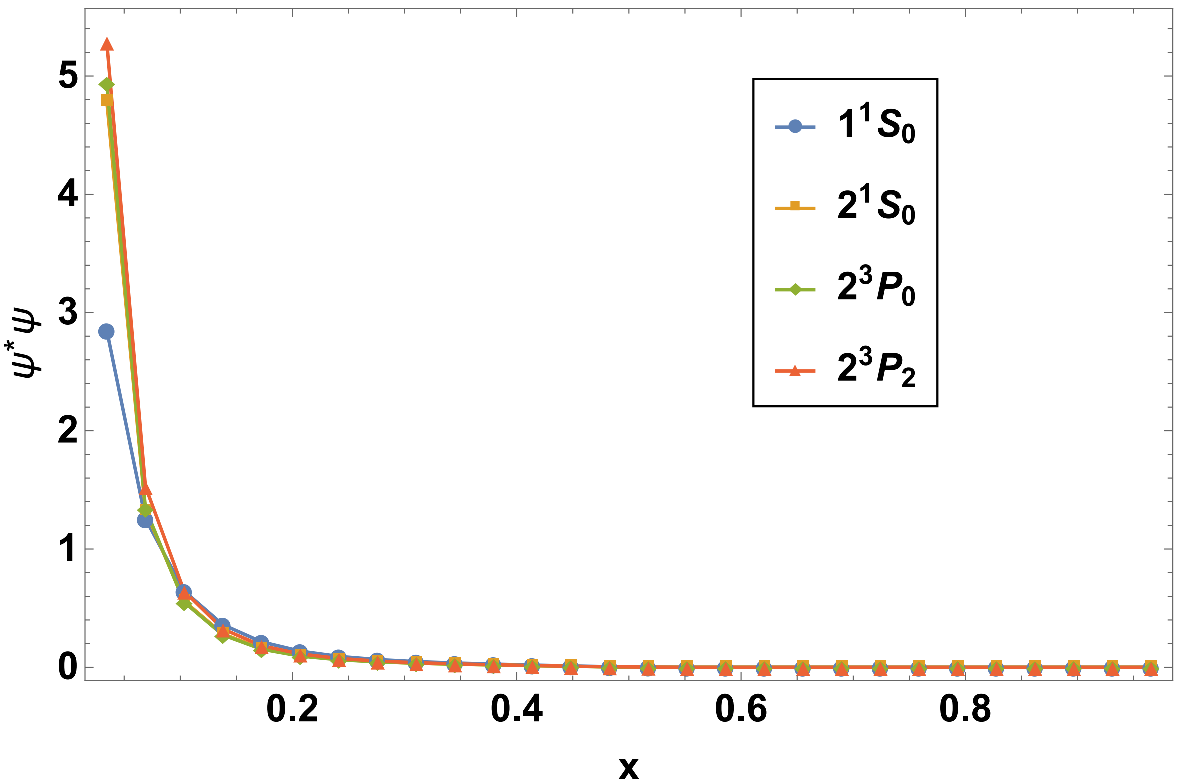
$1 -$ probability of $|e^+e^-\rangle$: the probability to find photon

Excited states have larger $|e\bar{e}\gamma\rangle$ component

[Kaiyu Fu et al, in preparation]

Photon Distribution In Positronium

Nmax=28, K=29



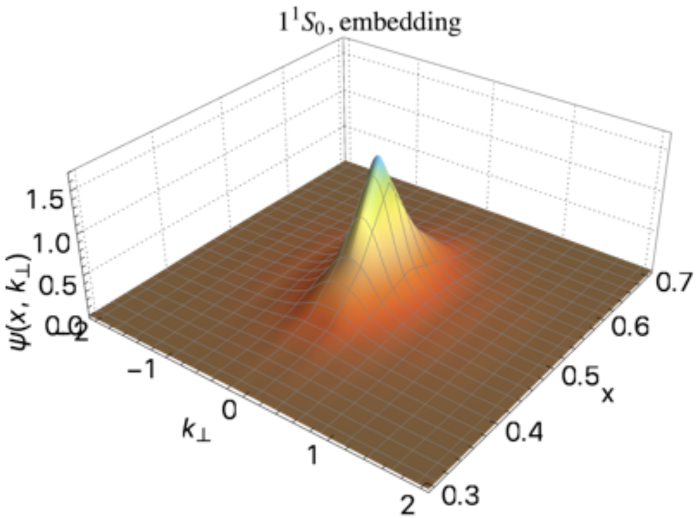
[Kaiyu Fu et al, in preparation]

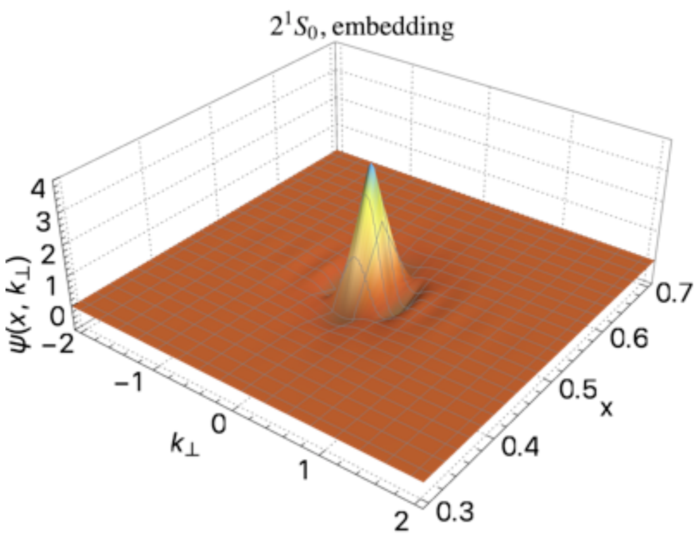
- In excited states photons have larger probability at small-x region
- Photon is massless, so peak is at small-x region

Wavefunction

This work

Nmax=20,K=21,MJ=0

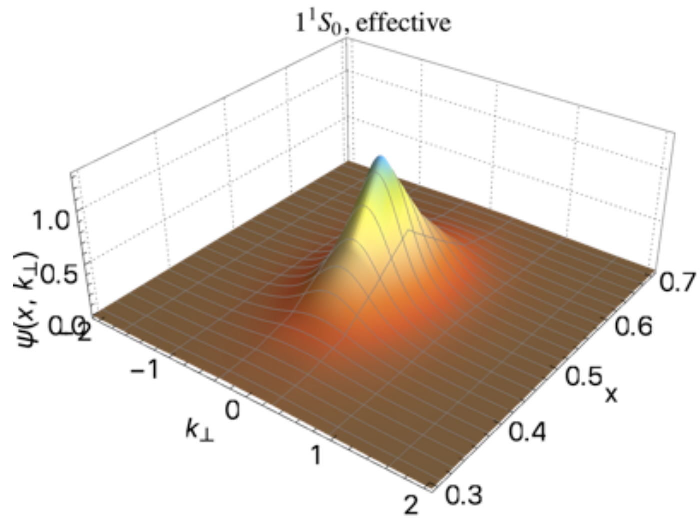


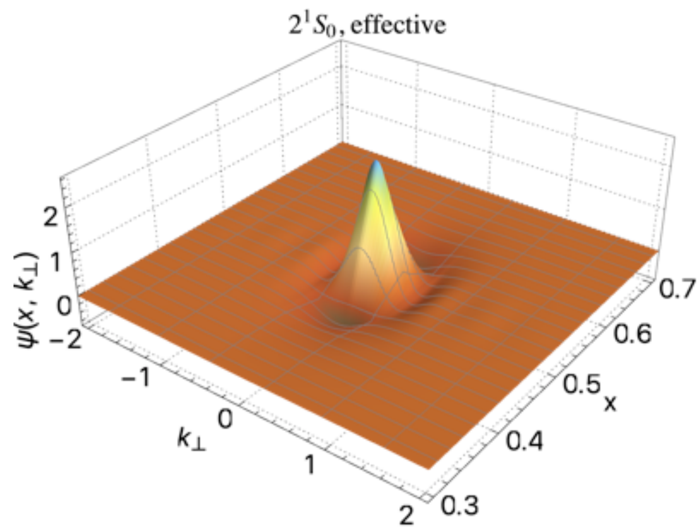


[Kaiyu Fu et al, in preparation]

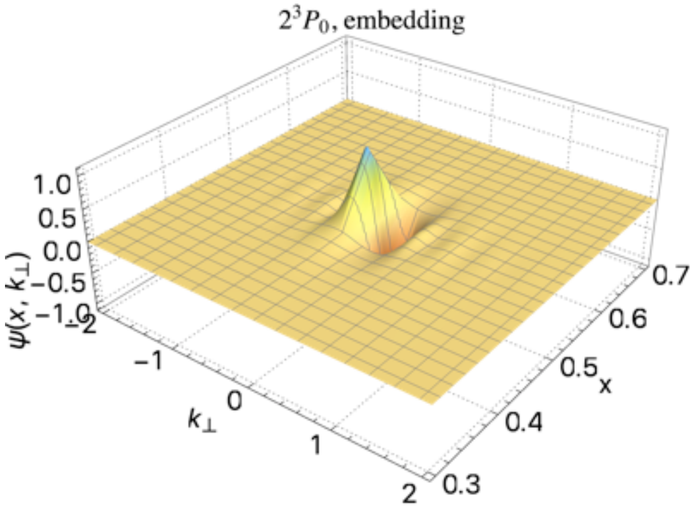
The effective one-photon-exchange

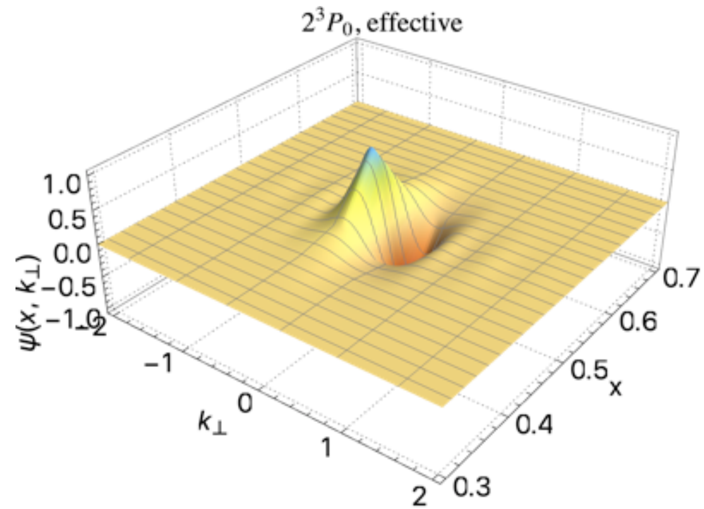
Nmax=20,K=19,MJ=0

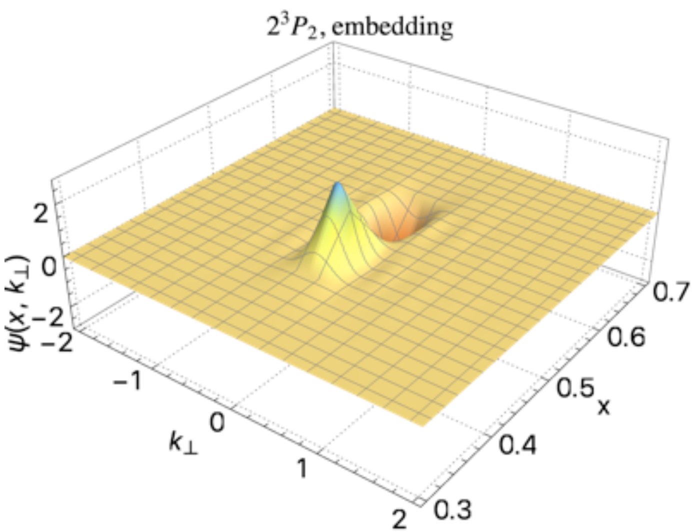


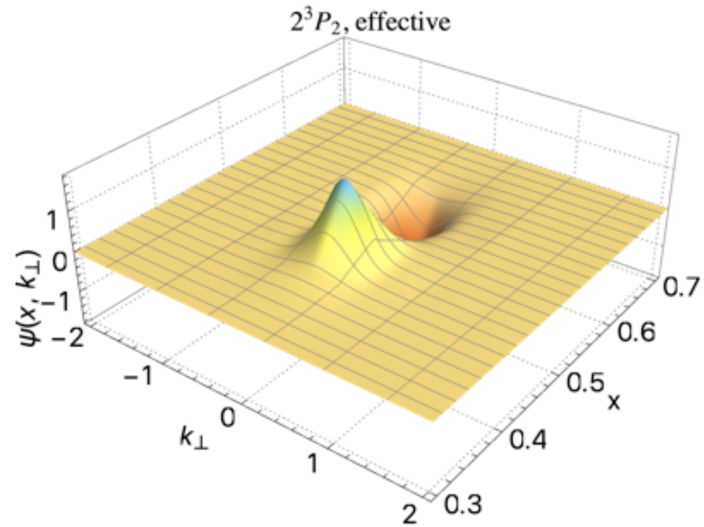


[Wiecki, et al, 2015]









[Kaiyu Fu et al, in preparation]

[Wiecki, et al, 2015]

Conclusions

- Solve positronium system based on first-principle of QED
- Direct access to photon content
- Consistent access to both spectrum and structure
- Rotation Symmetry is restoring as basis size increase
- Mass renormalization is performed on the level of electron
- Wave function and energy spectrum for low-lying states reasonably agree with those from the effective one-photon-exchange approach
- The convergence of physical results looks promising

Outlook

- Further convergence study

- More observables: PDF, GPD, TMD, GTMD,

Wignar distribution, double parton distribution function…

- Heavy quarkonium & Light meson systems

- Exotic hadron states