

Interactions between two heavy mesons within chiral effective field theory

Zhan-Wei LIU

Lanzhou University



Collaborators:

Ning LI, Xiang LIU, Bo WANG, Hao XU, Shi-Lin ZHU

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Introduction

Hadron spectrum and interactions

Hadron-hadron interactions are important for the hadron spectrum

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- threshold effects

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Deuteron: bound state of proton and neutron

P_c states reported at LHCb recently; $Z_b(10610)$, $Z_b(10650)$

Hadron spectrum and interactions

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$D_s(2317)$: contribution of DK continuum

- molecular states

Deuteron: bound state of proton and neutron

P_c states reported at LHCb recently; $Z_b(10610)$, $Z_b(10650)$

- other exotic states

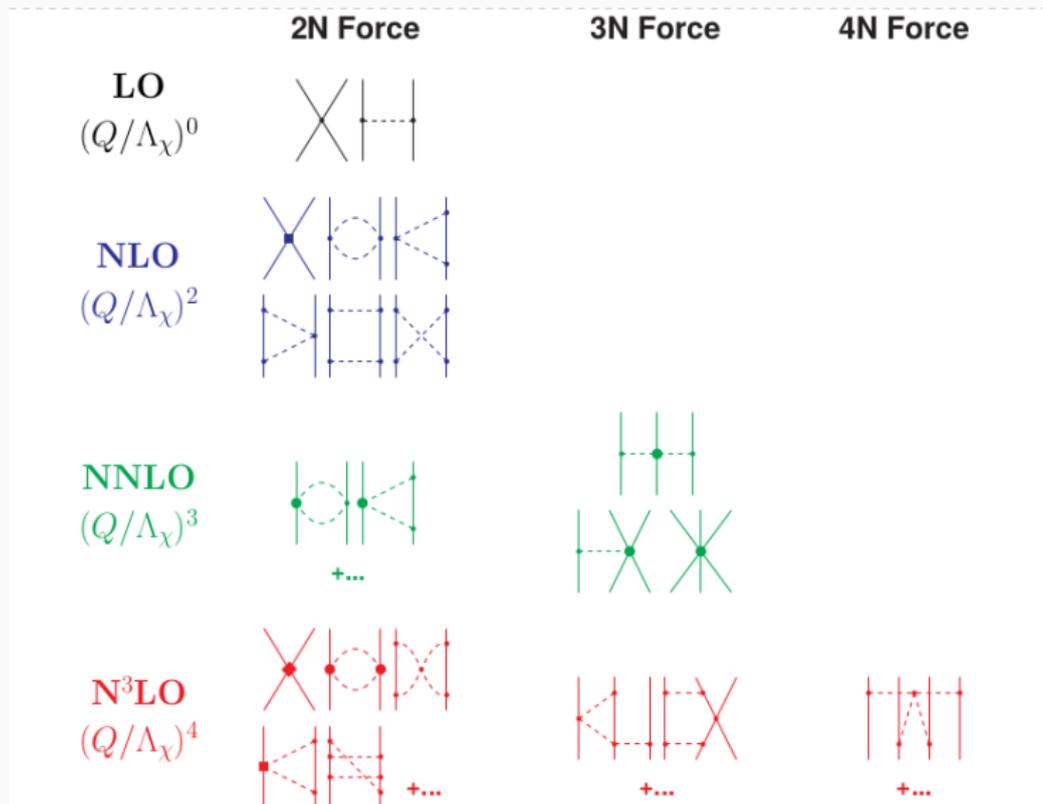
X, Y, Z states, debate with different interpretations:

molecules? tetraquark? ordinary charmonium? two diquark? kinetic effects?

Study of Interactions within chiral perturbation theory (ChPT)

- ChPT with respect on symmetries of QCD
- Power counting
 - NOT in power series: $\alpha_s, \alpha_s^2, \alpha_s^3, \dots$
 - expanded with small momentum
 - systematically study, order by order, error controlled
 - check of standard model
- Natural extension
 - 2-body force, 3-body force,...
- Wide applications

Nucleon-nucleon interaction



ChPT with heavy hadrons involved

- Dealing systems with light mesons

ChPT results can be expanded as power series of

$$m_\phi/\Lambda_\chi, q/\Lambda_\chi, \dots$$

- Power Counting Breaking (PCB) in systems with heavy hadrons involved

large masses of heavy hadrons make q^μ is never small again

power counting can be **recovered** with the help of residual momentum \tilde{q}^μ

$$\tilde{q}^\mu = q^\mu - m(1, \vec{0}).$$

Solutions for systems with one heavy hadron

- **Heavy hadron effective field theory (EFT)**

nonrelativistic reduction at Lagrangian level, breaking of analyticity.

Simple and still correct if not analytically extending results too far away

- **Infrared regularization**

relativistic Lagrangian, drop PCB terms at regularization

good power counting and analyticity

- **Extended on-mass-shell scheme**

relativistic Lagrangian, drop PCB terms at final results

good power counting and analyticity

Results with three different schemes will be same if

- being summarized at ALL orders, or
- the mass of heavy hadron becomes infinitely large.

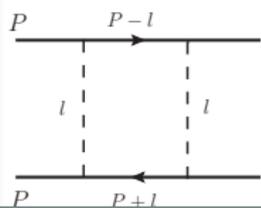
ChPT with few hadrons involved—new trouble

The amplitude of following 2-Particle-Reducible diagram contains ¹

$$\mathcal{I} \equiv i \int d^0 \rho \frac{i}{\rho + P^0 - \vec{P}^2/(2m_N) + i\epsilon} \frac{i}{-\rho + P^0 - \vec{P}^2/(2m_N) + i\epsilon} \approx \frac{-\pi}{\vec{P}^2/(2m_N) + i\epsilon} \quad (1)$$

- naïve power counting scheme $\rightarrow \mathcal{I} \sim O(1/|\vec{P}|)$
- eq. (1) $\rightarrow \mathcal{I} \sim O(m_N/|\vec{P}|^2)$

\mathcal{I} is actually enhanced by a large factor $m_N/|\vec{P}|$.



Solid line for nucleon, dashed line for pion.

(P represents the residual momentum)

Box Diagram.

¹we have not listed the parts preserving power counting

Weinberg scheme

- not directly calculate physical observables with perturbation theory
- systematically study effective potentials first (without 2PR contribution)
- solve the dynamical equation to get the physical observables (equivalent to recover the 2PR contributions)

Effective potentials between two heavy mesons

With Heavy Meson EFT, we study the systems made up of

- DD
- D^*D
- D^*D^*

Similar for $B^{(*)}B^{(*)}$ and corresponding anti-meson pair system.

We have not studied systems like $D\bar{D}$ because there exist annihilation effects.

- **Leading order vertex**

contact terms: $D^{(*)}D^{(*)}D^{(*)}D^{(*)}$ vertex

$D^{(*)}D^{(*)}\pi$, $D^{(*)}D^{(*)}\pi\pi$ vertex

- **Next-to-leading order vertex**

they absorb divergences, provide finite higher-order corrections

$$\begin{aligned}\mathcal{L}_{4H}^{(0)} &= D_a \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + D_b \text{Tr} [H\gamma_\mu \gamma_5 \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \bar{H}] \\ &\quad + E_a \text{Tr} [H\gamma_\mu \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \lambda_a \bar{H}] + E_b \text{Tr} [H\gamma_\mu \gamma_5 \lambda^a \bar{H}] \text{Tr} [H\gamma^\mu \gamma_5 \lambda_a \bar{H}],\end{aligned}$$

$$\mathcal{L}_{H\phi}^{(1)} = -\langle (iv \cdot \partial H) \bar{H} \rangle - \langle H v \cdot \Gamma \bar{H} \rangle + g \langle H \psi \gamma_5 \bar{H} \rangle - \frac{1}{8} \Delta \langle H \sigma^{\mu\nu} \bar{H} \sigma_{\mu\nu} \rangle,$$

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$$\begin{aligned}\mathcal{L}_{4H}^{(2)} &= D_a^h \text{Tr} [H\gamma_\mu \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] \text{Tr} (\chi_+) + \dots \\ &+ D_a^d \text{Tr} [H\gamma_\mu \tilde{\chi}_+ \bar{H}] \text{Tr} [H\gamma^\mu \bar{H}] + \dots \\ &+ D_1^g \text{Tr} [(D^\mu H)\gamma_\mu \gamma_5 (D^\nu \bar{H})] \text{Tr} [H\gamma_\nu \gamma_5 \bar{H}] + \dots\end{aligned}$$

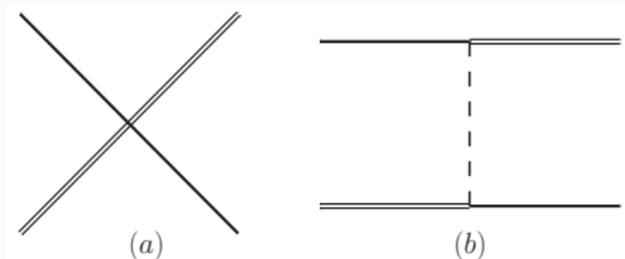
Diagrams

- **Leading order**

contact, one-pion exchange

- **Next-to-leading order**

two-pion exchange, renormalization to $D^{(*)}D^{(*)}\pi$ coupling, loop corrections to contact term, tree diagrams with NL vertice



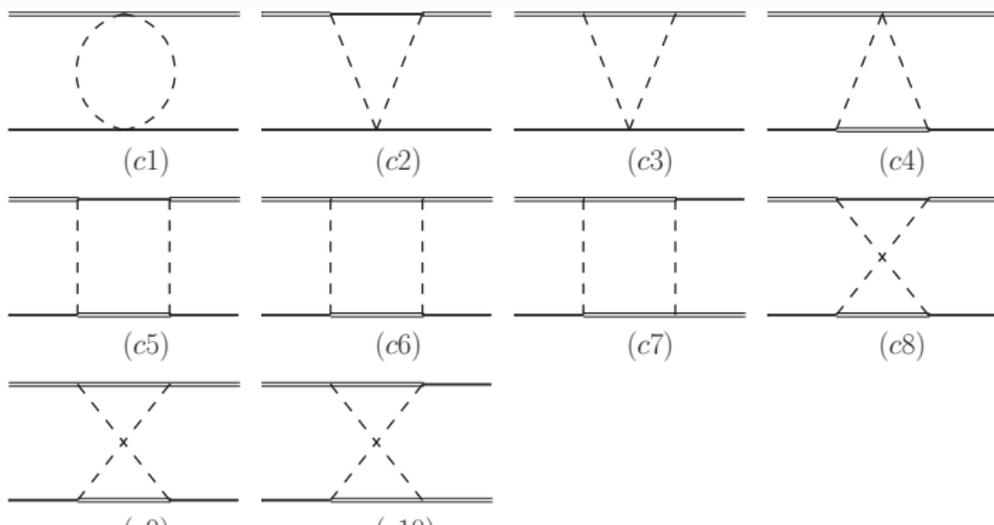
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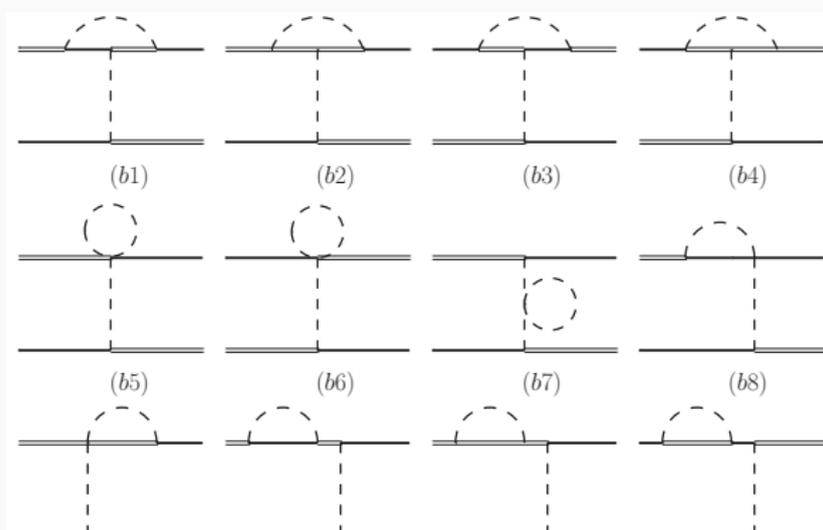
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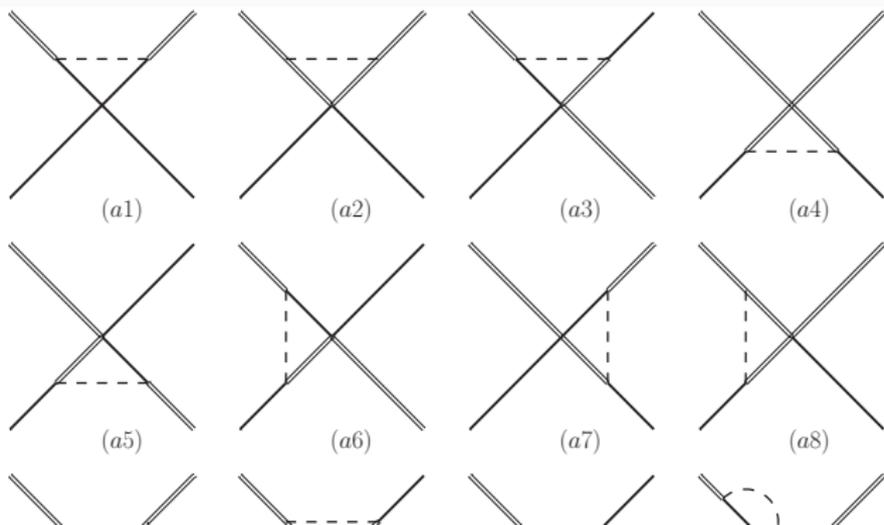
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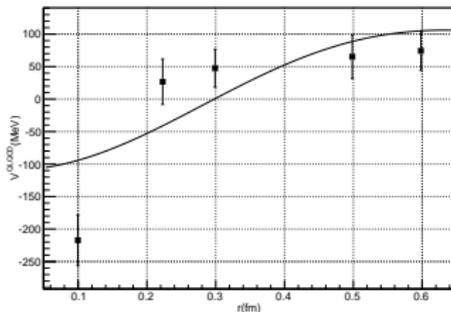
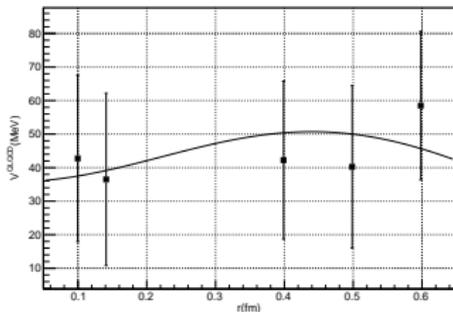


Determination of low-energy constants

- fit to experimental data
- first principle of QCD
- fit to data of Lattice QCD
- phenomenological models

Determination of low-energy constants

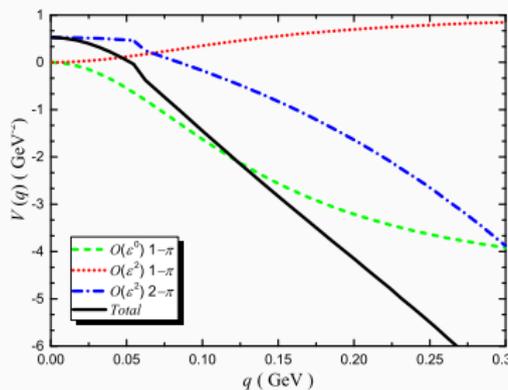
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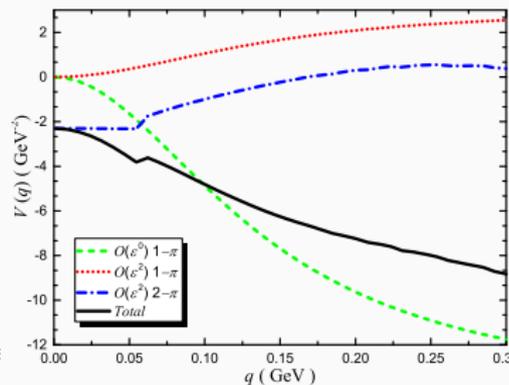
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Effective potentials in momentum space



$DD^* : l=1$



$DD^* : l=0$

Possible molecular states

Search for new states

- Potentials \rightarrow partial waves, dynamical equation (momentum space)
 - \rightarrow T matrices \rightarrow poles
- Potentials \rightarrow Fourier transform, dynamical equation (coordinate space)
 - \rightarrow eigenvalues of bound states for different partial waves

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Taking DD^* as an example

- $l = 0$: bound state with around $E = -21_{-38}^{+19}$ MeV.
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- Comparison with one-boson-exchange model

Li, Sun, Liu, Zhu, PRD88(2013), 114008; $l = 0$: $-43 \sim -5$ MeV;

Liu, Wu, Valderrama, Xie, Geng, PRD99(2019), 094018; $l = 0$: -3_{-15}^{+4} .

$\rho/\omega/\dots$ contribution is covered not only by the two-pion-exchange part but also by contact terms.

We use similar approaches to study the system of $\bar{B}^{(*)}\bar{B}^{(*)}$ in S wave

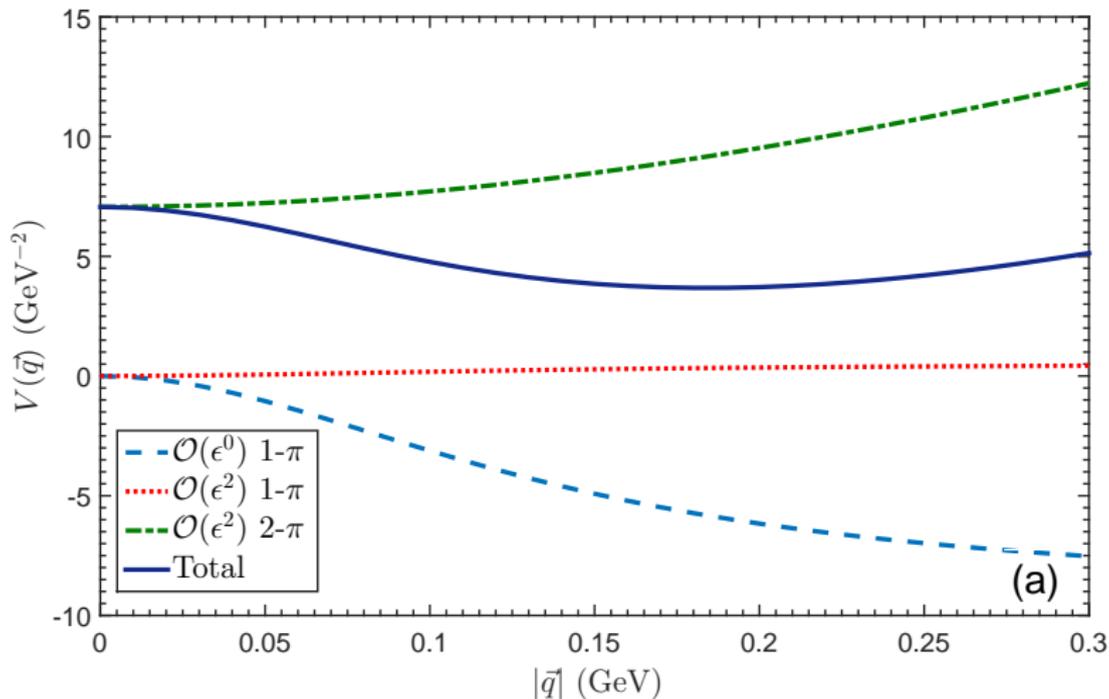
- $\bar{B}\bar{B}$: $I(J^P) = 1(0^+)$
- $\bar{B}\bar{B}^*$: $I(J^P) = 1(1^+)$, $I(J^P) = 0(1^+)$
- $\bar{B}^*\bar{B}^*$: $I(J^P) = 1(0^+)$, $I(J^P) = 1(2^+)$, $I(J^P) = 0(1^+)$

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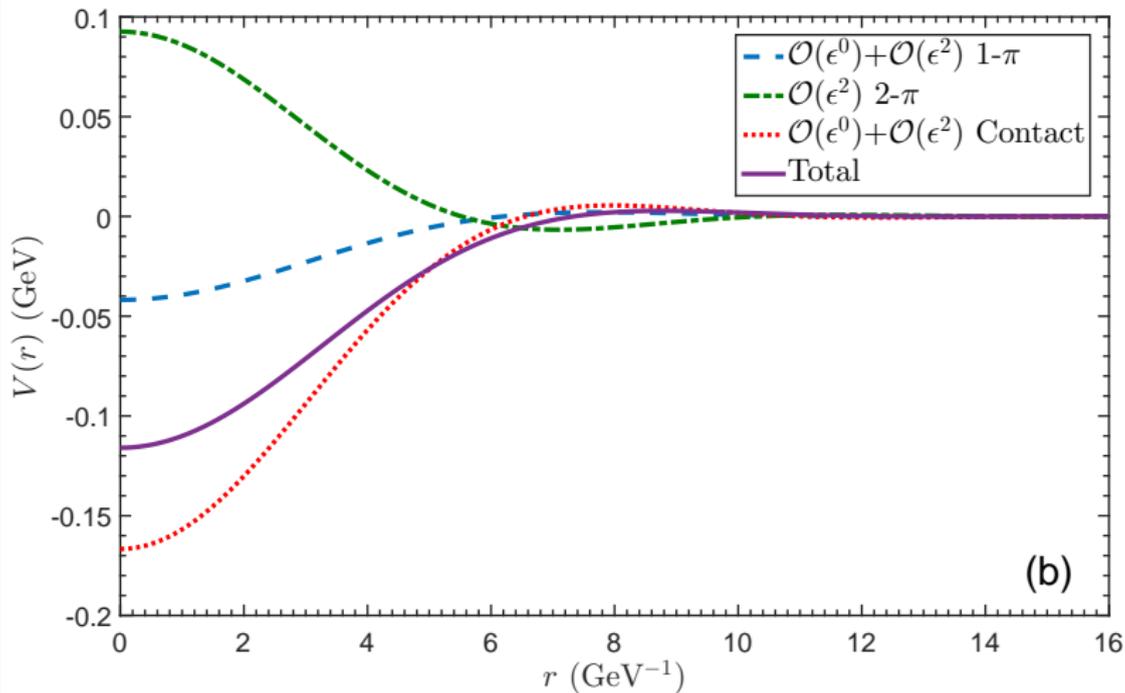
Example:

potentials for $\bar{B}^* \bar{B}^*$ with $I(J^P) = 0(1^+)$ in momentum space



Example:

potentials for $\bar{B}^* \bar{B}^*$ with $I(J^P) = 0(1^+)$ in coordinate space



Results for $\bar{B}^{(*)}\bar{B}^{(*)}$ systems

we find two bound states in the channels of $0(1^+) \bar{B}\bar{B}^*$ and $\bar{B}^*\bar{B}^*$

- binding energies: $\Delta E_{\bar{B}\bar{B}^*} \simeq -12.6_{-12.9}^{+9.2}$ MeV, $\Delta E_{\bar{B}^*\bar{B}^*} \simeq -23.8_{-21.5}^{+16.3}$ MeV
masses: $m_{\bar{B}\bar{B}^*} \simeq 10591.4_{-12.9}^{+9.2}$ MeV, $m_{\bar{B}^*\bar{B}^*} \simeq 10625.5_{-21.5}^{+16.3}$ MeV
- strong decays are forbidden because of phase space
they can be searched in $\bar{B}\bar{B}\gamma$ or $\bar{B}\bar{B}\gamma\gamma$

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Bound states $[bb\bar{q}\bar{q}]$ with $I(J^P) = 0(1^+)$ also existed in other framework

Eichten, Quigg, PRL.119(2017),202002;
Karlner, Rosner, PRL.119(2017)202001;
Bicudo, Scheunert, Wagner, PRD95(2017)034502;
Wang, Acta Phys.Polon.B49(2018)1781;
Park, Noh, Lee, Nucl.Phys.A983(2019)1-19;
Liu, Wu, Valderrama, Xie, Geng, PRD99(2019),094018;
Francis, Hudspith, Lewis, Maltman, PRL.118(2017)142001;
Junnarkar, Mathur, Padmanath, PRD99(2019)034507;
Leskovec, Meinel, Pflaumer, Wagner, PRD100(2019),014503

Uncertainty of low-energy constants

Table 1: The binding energies of $0(1^+) \bar{B}\bar{B}^*$ and $\bar{B}^*\bar{B}^*$ states obtained with different strategies in units of MeV.

Binding energy	No $\mathcal{O}(\epsilon^2)$ LECs	Strategy A	Strategy B
$\Delta E_{\bar{B}\bar{B}^*}$	$-12.6^{+9.2}_{-12.9}$	$-10.4^{+7.2}_{-9.7}$	$-15.9^{+9.7}_{-12.7}$
$\Delta E_{\bar{B}^*\bar{B}^*}$	$-23.8^{+16.3}_{-21.5}$	$-20.1^{+14.5}_{-20.0}$	$-28.2^{+18.6}_{-23.6}$

Summary

We have studied the potentials upto one-loop level between two heavy mesons within chiral perturbation theory.

By solving the Schrodinger equations, we found some bound states in some channels.

With the wavefunctions obtained, we can further study other properties of these new states.

Thanks!

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This report is mainly based on the following articles

- Phys.Rev. D99 (2019) no.3, 036007
- Phys.Rev. D99 (2019) no.1, 014027
- Phys.Rev. D89 (2014) no.7, 074015