Wigner Distributions and Spin Structure of Pion in Light Front Holographic QCD

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Overview

The holographic light-front Schrödinger Equation

Brodsky & de Téramond (PRL, 2009) | Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 2015)

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- $\zeta^2 = x(1-x)b_\perp^2$: key light-front variable for bound states.
- $x = k^+/P^+$: light-front momentum fraction of quark.
- $b_\perp$: transverse distance between quark and antiquark.
- U_{eff} : effective potential.

- No quantum loops (no Λ_{QCD})
- Massless quarks ($m_f \to 0$)

} semiclassical approximation

A unique confinement potential

Brodsky, Teramond & Dosch, Phys. Lett. B 729, (2014)

$$U_{\rm eff}(\zeta) = \overbrace{\kappa^4\zeta^2}^{\text{Conformal SB}} + \overbrace{2\kappa^2(J-1)}^{\text{AdS/QCD}}$$

- Mass scale κ emerges as the strength of unique harmonic oscillator potential.
- AdS/QCD mapping : $\zeta \leftrightarrow z_5$ (z_5 is $5^{\rm th}$ dimension of AdS) gives the spin term $2\kappa^2(J-1)$ in potential.

Meson light-front wavefunction :

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} f(x) e^{iL\varphi}$$

- Fixes $f(x) = \sqrt{x(1-x)}$ in wavefunction.

Restoring dependence on quark masses

- In momentum space, 'complete' invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2}{x(1-x)} \to \frac{\mathbf{k}_\perp^2 + m_f^2}{x(1-x)}$$

Brodsky, de Téramond, Subnuclear Series Proc. (2009)

- Pseudoscalar meson wavefunction becomes

$$\Psi^P(x,\zeta^2) = \mathcal{N}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\overbrace{\exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]}^{\text{mass term}}$$

- m_f are effective quark masses (because of coupling of valence sector to higher Fock sectors).

Restoring dependence on quark spins

- In semiclassical approximation :

$$\Psi^P(x,\zeta^2,\lambda,\lambda') = \Psi^P(x,\zeta^2) \times S_{\lambda,\lambda'}^{\text{non dynamical}}$$

with

$$S_{\lambda,\lambda'}^{\text{non dynamical}} = \lambda\delta_{\lambda,-\lambda'}$$

- Normalization

$$\sum_{\lambda,\lambda'} \int \mathrm{d}^2\mathbf{b}\mathrm{d}x |\Psi^\pi(x,\zeta^2,\lambda,\lambda')|^2 = 1$$

Dynamical spin effects

M. Ahmady, CM, R. Sandapen, PRD 98 (2018)

$$\Psi(x,\mathbf{k}_\perp) \to \Psi(x,\mathbf{k}_\perp,\lambda,\lambda') = \Psi(x,\mathbf{k}_\perp) \times S_{\lambda\lambda'}(x,\mathbf{k}_\perp)$$

- For pseudoscalar meson, we use

$$S^{\rm dynamical}_{\lambda\lambda'}(x,\mathbf{k}_\perp) = \frac{\bar{v}_{\lambda'}(x,\mathbf{k}_\perp)}{\sqrt{1-x}} \left[\overbrace{\frac{M_P^2}{2P^+}\gamma^+\gamma^5}^{\text{from } (P\cdot\gamma)\gamma^5} + B M_P \gamma^5 \right] \frac{u_\lambda(x,\mathbf{k}_\perp)}{\sqrt{x}}$$

$$S^{\rm dynamical}_{\lambda\lambda'}(x,\mathbf{k}_\perp)|_{B=0} \to S^{\rm non\ dynamical}_{\lambda\lambda'} = \frac{1}{\sqrt{2}}\lambda\delta_{\lambda,-\lambda'}$$

- We call B the dynamical spin parameter:
  1. $B \neq 0$: dynamical spin effects.
  2. $B \gg 1$: maximal dynamical spin effects.

Charge radii

M. Ahmady, CM, R. Sandapen, PRD 98 (2018)

- universal AdS/QCD scale: $\kappa = (523 \pm 24)$ MeV.
- quark masses: $[m_{u/d}, m_s] = ([330, 500] \pm 30)$ MeV.

$$\sqrt{\langle r_P^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x \mathrm{d}^2 \mathbf{b}_\perp [b_\perp (1-x)]^2 |\Psi_P(x, \mathbf{b}_\perp)|^2 \right]^{1/2}$$

- $\pi^\pm$: maximal ($B \gg 1$) dynamical spin preferred
- $K^\pm$: no ($B = 0$) dynamical spin preferred

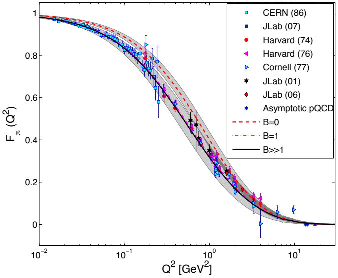
P	B	$\sqrt{\langle r_P^2 \rangle}_{\mathrm{Th.}}$ [fm]	$\sqrt{\langle r_P^2 \rangle}_{\mathrm{Exp.}}$ [fm]
$\pi^\pm$	0	0.543 ± 0.022	
	1	0.673 ± 0.034	0.672 ± 0.008
	$\gg 1$	0.684 ± 0.035	
$K^\pm$	0	0.615 ± 0.038	
	1	0.778 ± 0.065	0.560 ± 0.031
	$\gg 1$	0.815 ± 0.070	

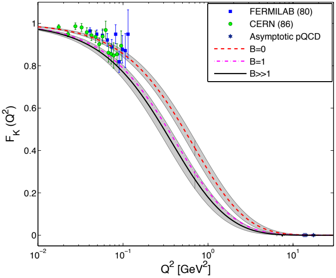
Spacelike EM form factors

M. Ahmady, CM, R. Sandapen, PRD 98 (2018)

Drell & Yan (PRL, 70); West (PRL, 70)

$$F_{\rm EM}(Q^2) = 2\pi \int \mathrm{d}x \mathrm{d}b_\perp \; b_\perp \; J_0[(1-x)b_\perp Q] \; |\Psi^P(x,\mathbf{b}_\perp)|^2$$





$\pi^{\pm}$: maximal ($B \gg 1$) dynamical spin. $K^{\pm}$: no ($B = 0$) dynamical spin.

Decay constants

M. Ahmady, CM, R. Sandapen, PRD 98 (2018)

$$\langle 0|\bar{\Psi}\gamma^\mu\gamma_5\Psi|P\rangle = f_P p^\mu$$

$$f_P = 2\sqrt{\frac{N_c}{\pi}}\int \mathrm{d}x\{((x(1-x)M_\pi^2) + Bm_f M_\pi\}\frac{\Psi^\pi(x,\zeta)}{x(1-x)}\Big|_{\zeta=0}$$

- $\pi^\pm$: maximal ($B \gg 1$) dynamical spin preferred
- $K^\pm$: no ($B = 0$) dynamical spin preferred

P	B	$f_P^{\text{Th.}}$ [MeV]	$f_P^{\text{Exp.}}$ [MeV]
$\pi^\pm$	0	162 ± 8	$130 \pm 0.04 \pm 0.2$
	1	138 ± 5	
	$\gg 1$	135 ± 6	
$K^\pm$	0	156 ± 8	156 ± 0.5
	1	142 ± 7	
	$\gg 1$	135 ± 6	

$\pi \to \gamma$ transition form factor

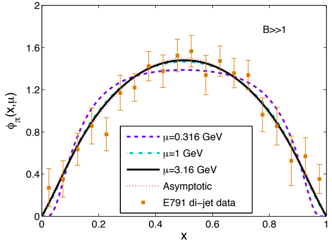
M. Ahmady, CM, R. Sandapen, PRD 98 (2018)

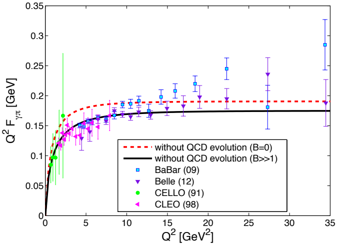
Lepage & Brodsky, PRD (1980)

$$F_{\gamma P}(Q^2) = \frac{\sqrt{2}}{3} f_P \int_0^1 \mathrm{d}x \frac{\overbrace{\varphi_P(x, xQ)}^{\text{meson DA}}}{Q^2 x}$$

- Meson DA is obtained from holographic wavefunction
- Maximal ($B \gg 1$) dynamical spin preferred for pion

talked by M. Ding (17 Aug) & R. Khepani (17 Aug)





Wigner distributions

talked by D. Sokhan (19 Aug) & S. Nair (20 Aug)

Unpolarized quark : $\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp) = \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp)$

- Defined as :

$$\rho^{[\Gamma]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, S) \equiv \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp} \overbrace{\hat{W}^{[\Gamma]}(\boldsymbol{\Delta}_\perp, \mathbf{k}_\perp, x, S)}^{\text{Correlator}} .$$

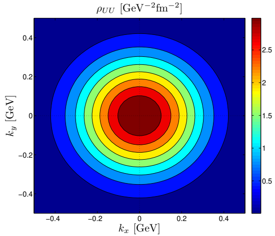
- Generalized correlator :

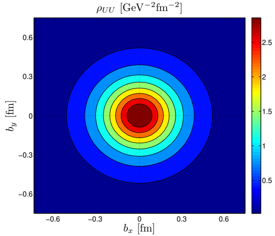
$$\hat{W}^{[\Gamma]}(\Delta_\perp, \mathbf{k}_\perp, x; S) = \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik\cdot z}$$

$$\left\langle M(P''; S) \middle| \bar{\psi}\left(-\frac{z}{2}\right)\Gamma \mathcal{W}_{[-\frac{z}{2},\frac{z}{2}]}\psi\left(\frac{z}{2}\right) \middle| M(P'; S)\right\rangle\Big|_{z^+=0}$$

Drell-Yan frame ($\Delta^+ = 0$)

- $k_y = 0.3$ GeV (in $b_\perp$ space)
  $b_y = 0.3$ fm (in $k_\perp$ space)





Ahmady, CM, Sandapen, Zhao : work in progress

Wigner distributions

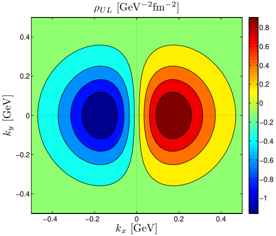
Ahmady, CM, Sandapen, Zhao : work in progress

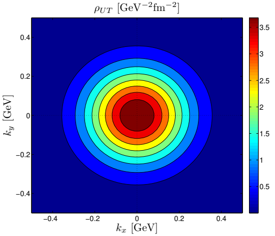
Longitudinally polarized :

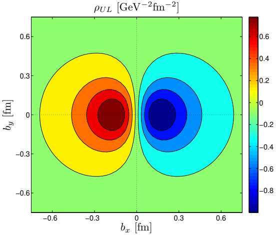
$$\rho_{UL}(\mathbf{b}_\perp, \mathbf{k}_\perp) = \rho^{[\gamma^+\gamma_5]}(\mathbf{b}_\perp, \mathbf{k}_\perp)$$

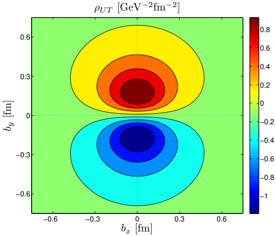
Transversely polarized :

$$\rho^j_{UT}(\mathbf{b}_\perp, \mathbf{k}_\perp) = \rho^{[\sigma^{+j}\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp)$$









Transverse spin density

Ahmady, CM, Sandapen, Zhao : work in progress

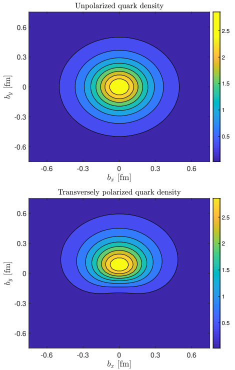
- Defined as

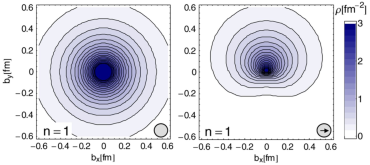
$$\rho^n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp)$$
$$= \frac{1}{2}\left[A^\pi_{n0}(b^2_\perp) - \frac{s^i_\perp \epsilon^{ij}\, b^j_\perp}{m_\pi} B^{\pi\,\prime}_{Tn0}(b^2_\perp) \right],$$

- The moments of GPDs:

$$\int_0^1 dx\, x^{n-1} H^\pi(x,\ b^2_\perp) = A^\pi_{n0}(b^2_\perp)\,,$$
$$\int_0^1 dx\, x^{n-1} E^\pi_T(x,\ b^2_\perp) = B^\pi_{Tn0}(b^2_\perp)\,.$$

- Reasonable agreement with Lattice QCD

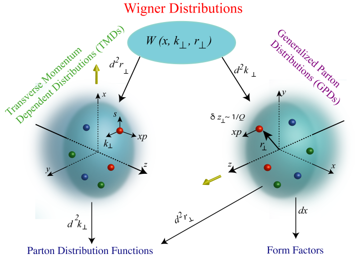




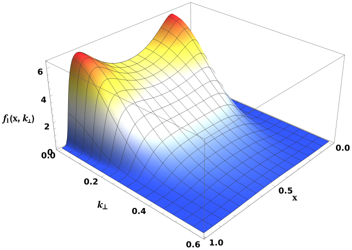
Lattice: D. Brommel *et. al.*, PRL 101, 122001 (2008)

TMDs

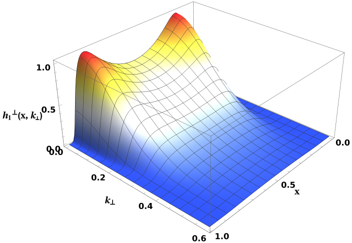
M. Ahmady, CM, R. Sandapen, arXiv:1907.06561 (2019)



Unpolarized TMD



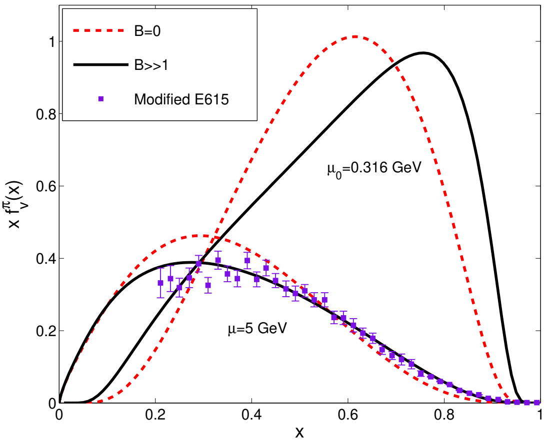
Boer-Mulders TMD



- Boer-Mulders function generated by the perturbative kernel in FSI
- No dynamical spin ($B = 0$) $\rightarrow$ BM function vanishes

PDF

M. Ahmady, CM, R. Sandapen, PRD 98 (2018)



Maximal ($B \gg 1$) dynamical spin preferred for pion

Conclusions

- Dynamical spin effects are maximal in pion but negligible in kaon.
- Dynamical spin in pion leads to an excellent description of a wide range of data: the decay constant, charge radius, spacelike EM and transition form factors, PDA, PDF...
- Wigner distributions for different quarks polarization have been discussed.
- Transverse spin structure is consistent with Lattice QCD prediction.

Thank you for your attention

Massless pseudoscalars

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- with

$$U_{\mathrm{eff}}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

- gives

$$M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J-1)$$

- Predicts Regge trajectories with massless ground states ($n = L = J = 0$).
- Identify with the Goldstone bosons (π, K, η) of spontaneous chiral symmetry breaking in QCD.

Fixing the fundamental AdS/QCD scale

Regge slopes global fits

Reference	Fit	κ[MeV]
Brodsky et al. [Phys. Rep. 15]	Mesons	540–590
Brodsky et al. [IJMD, 16]	Mesons & Baryons	523

Universal $\kappa \sim 500$ MeV successfully predicts

- Diffractive ρ production Forshaw & Sandapen (PRL, 2012)
- Diffractive ϕ production Ahmady, Sandapen, Sharma (PRD, 2016)
- Λ_{QCD} to 5-loops in $\overline{\mathrm{MS}}$ Brodsky, Deur, de Téramond (J.Phys, 2017)