Lattice results on dibaryons and baryon-baryon interactions

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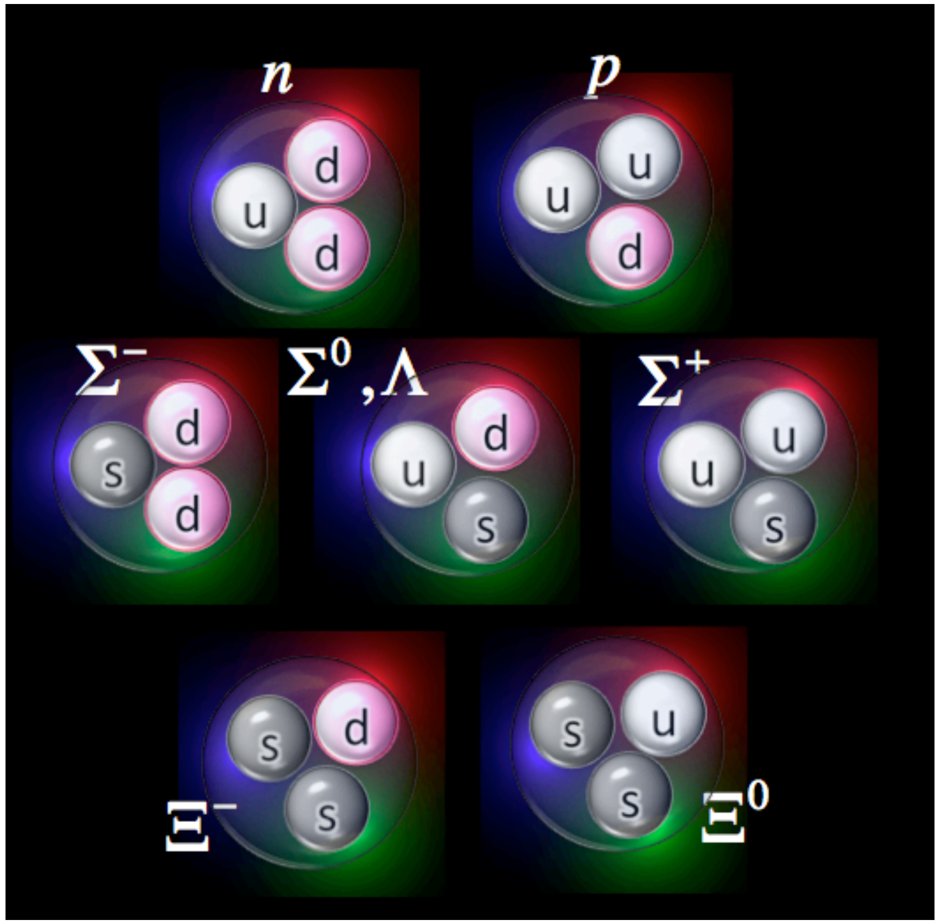
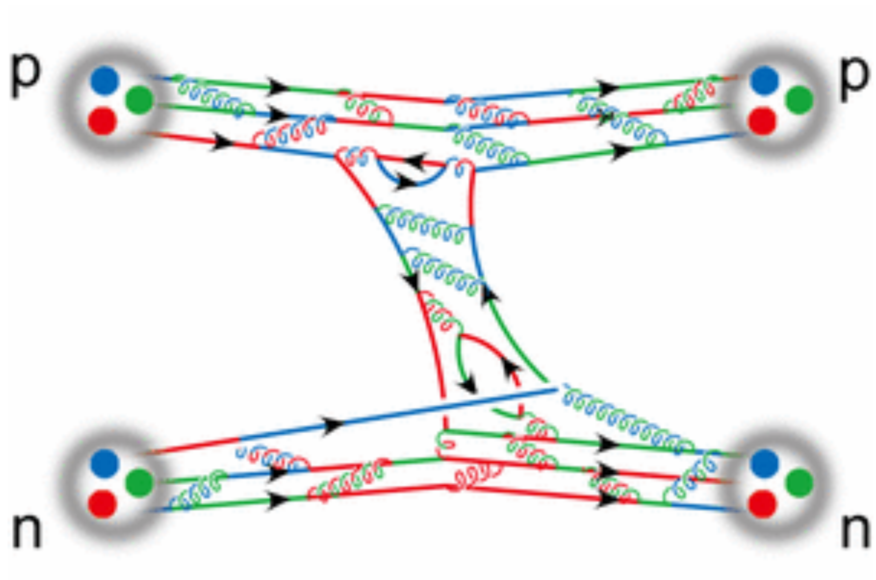
(Lattice) QCD : theory for quarks and gluons → Hadrons



Hadron interactions in lattice QCD

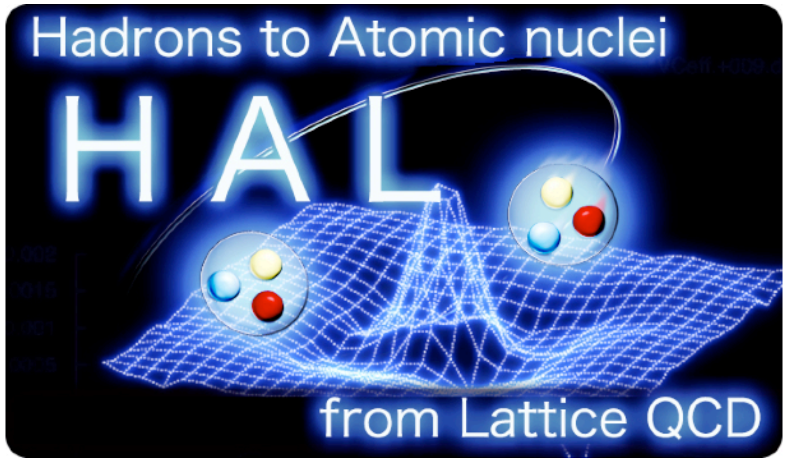


Two methods

Finite volume method: successful for meson-meson interactions

J. Dudek: previous talk

Potential method: successful for baryon-baryon interactions



Plan of my talk

I. HAL QCD potential method

II. Dibaryons

1. at physical pion mass
2. at heavier pion masses

III. Summary

I. HAL QCD potential method

N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001,
"The Nuclear Force from Lattice QCD"

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys. 123 (2010) 89-128,
" Theoretical Foundation of the Nuclear Force in QCD and its applications to Central and Tensor Forces in Quenched Lattice QCD Simulations "

HAL QCD Collaboration (S. Aoki *et al.* ,), PTEP 2012 (2012) 01A105,
"Lattice QCD approach to Nuclear Physics"

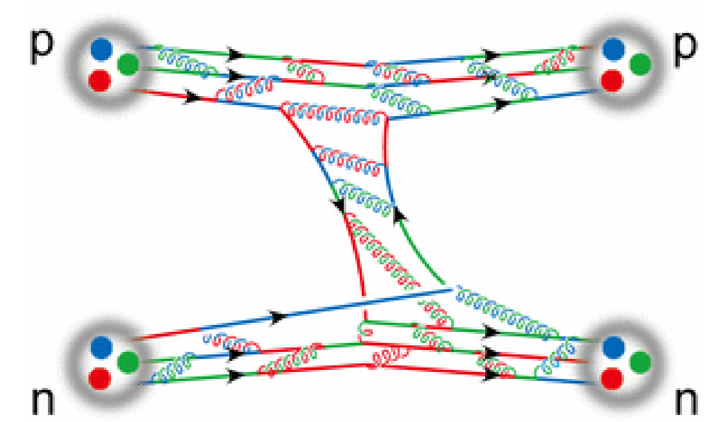
Our strategy in lattice QCD

Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

QCD eigen-state



$NN \longrightarrow NN$ only elastic scattering

energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi$

Asymptotic behavior in the center of mass (CM)

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{lm}(\Omega_{\mathbf{r}}) \qquad r = |\mathbf{r}| \to \infty$$

scattering phase shift (phase of the S-matrix by unitarity) in QCD.

Step 2 define the energy-independent "potential" with derivatives from these NBS wave functions as

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = V(\mathbf{x},\nabla)\varphi_{\mathbf{k}}(\mathbf{x})$$

for $^\forall\mathbf{k}$ with $W_k \le W_{\rm th}$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

For NN

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_1\cdot\sigma_2) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_1\cdot\mathbf{x})(\sigma_2\cdot\mathbf{x}) - (\sigma_1\cdot\sigma_2)$ spins

By construction

potential $V(\mathbf{x},\nabla)$ is faithful to QCD phase shift $\delta_l(k)$.

Remark

No non-relativistic approximation in CM

$$-\Box - m^2 = (W_k/2)^2 + \nabla^2 - m^2 = k^2 + \nabla^2$$

Step 3 Determination of local terms order by order

Leading Order potential $V_0^{\rm LO}(r) := V_0(r) + V_\sigma(r)(\sigma_1\cdot\sigma_2) + V_T(r)S_{12}$

One $\varphi_{\mathbf{k}}(\mathbf{x})$ → $V_0^{\rm LO}(r;\varphi_{\mathbf{k}}) = \dfrac{[\epsilon_{\mathbf{k}} - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$ LO approximation

Another $\varphi_{\mathbf{q}}(\mathbf{x})$ → $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) = \dfrac{[\epsilon_{\mathbf{q}} - H_0]\,\varphi_{\mathbf{q}}(\mathbf{x})}{\varphi_{\mathbf{q}}(\mathbf{x})}$ LO approximation

If $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) \simeq V_0^{\rm LO}(r;\varphi_{\mathbf{k}})$ → **LO approximation is good at** $|\mathbf{k}| \le |\mathbf{p}| \le |\mathbf{q}|$

If $V_0^{\rm LO}(r;\varphi_{\mathbf{q}}) \ne V_0^{\rm LO}(r;\varphi_{\mathbf{k}})$ → **NLO term can be determined from**

$$[\epsilon_{\mathbf{k}} - H_0)]\,\varphi_{\mathbf{k}} = \left[V_0^{\rm NLO}(r) + V_1^{\rm NLO}(r)\mathbf{L}\cdot\mathbf{S}\right]\varphi_{\mathbf{k}}$$
$$[\epsilon_{\mathbf{q}} - H_0)]\,\varphi_{\mathbf{q}} = \left[V_0^{\rm NLO}(r) + V_1^{\rm NLO}(r)\mathbf{L}\cdot\mathbf{S}\right]\varphi_{\mathbf{q}}$$

Demonstration

Separable potential $U(\vec{x},\vec{y}) = wv(\vec{x})v(\vec{y})$ $v(\vec{x}) = e^{-\mu x}, \quad x := |\vec{x}|$

highly non-local

L=0 wave function

R: IR cut-off

$$\psi_k^0(x) = \frac{e^{i\delta(k)}}{kx}\left[\sin(kx+\delta(k)) - \sin\delta(k) e^{-\mu x}\left(1 + x\frac{\mu^2+k^2)}{2\mu}\right)\right] \qquad x \le R$$

$$= C\frac{e^{i\delta(k)}}{kx}\sin(kx+\delta_R(k)) \qquad x > R$$

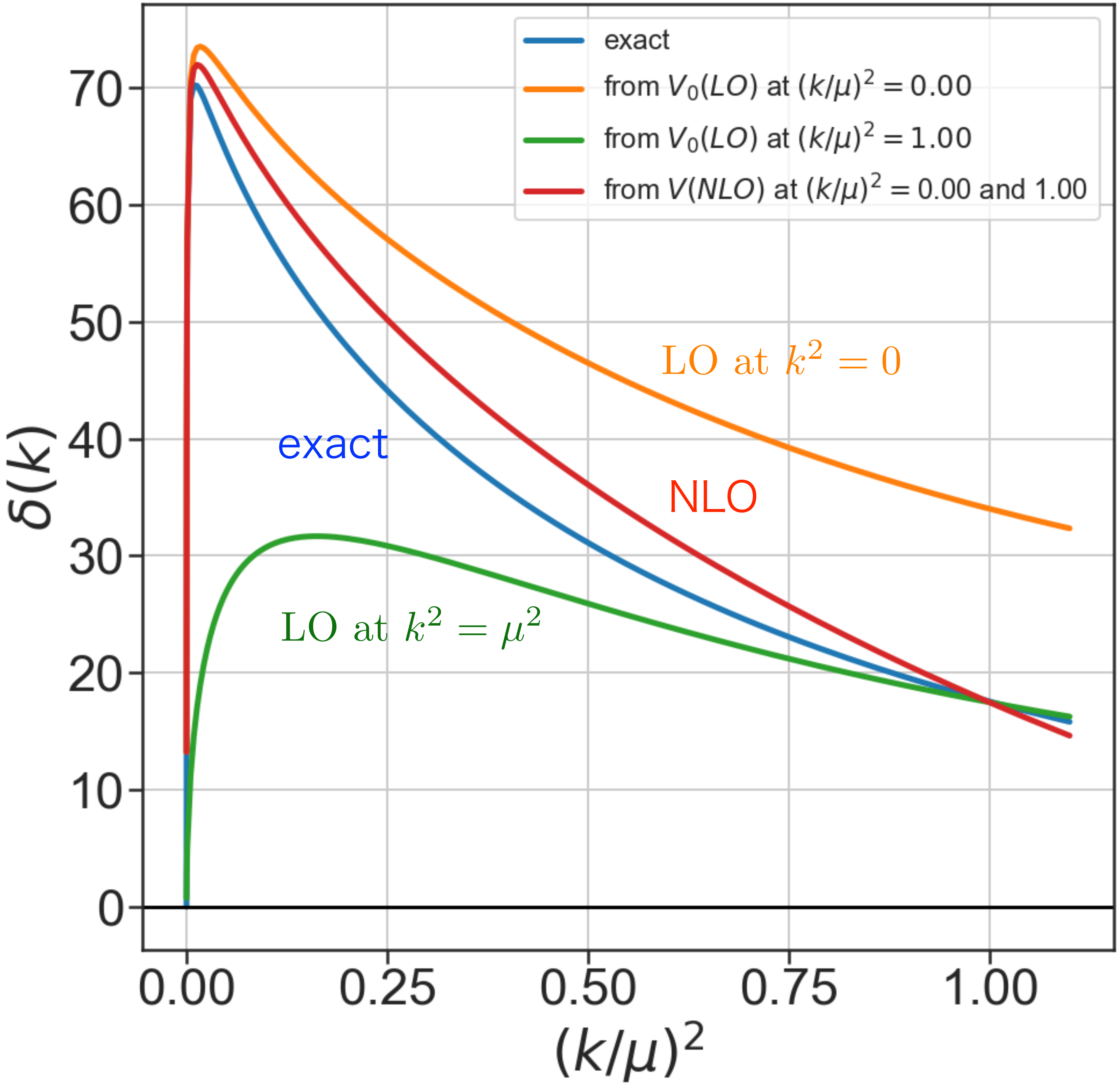
phase shift $\delta_R(k)$ is exactly calculable.

separable potential $U(\vec{x},\vec{y})$

LO potential $V_0^{\mathrm{LO}}(r)$ from $k^2 = 0$ or $k^2 = \mu^2$

NLO potential $V_0^{\mathrm{NLO}}(r) + V_1^{\mathrm{NLO}}(r)\nabla^2$

$$\omega/\mu^4 = -0.017, m/\mu = 3.30, R\mu = 2.5$$

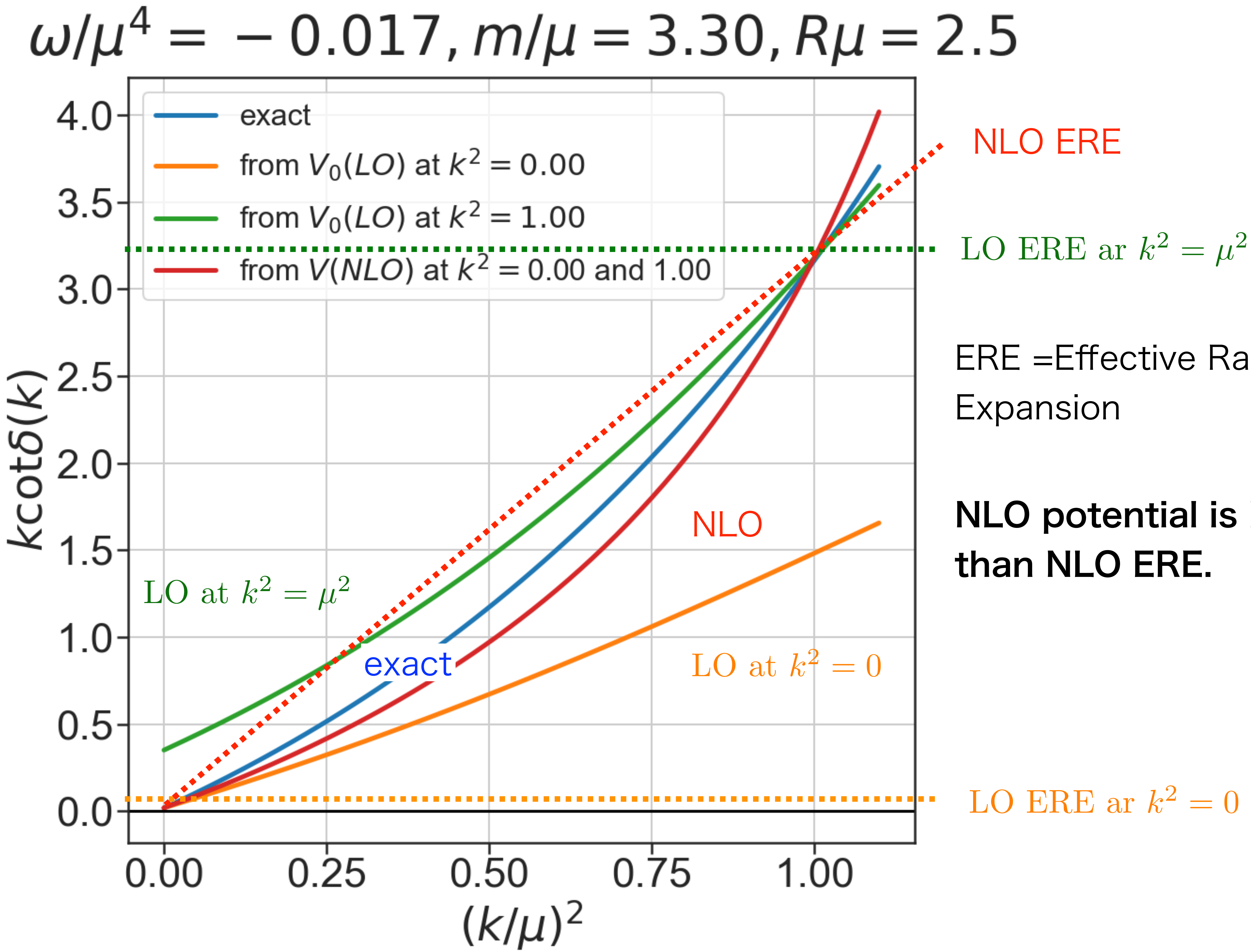


$$U(\vec{x}, \vec{y}) = wv(\vec{x})v(\vec{y})$$

$$v(\vec{x}) = e^{-\mu x}, \quad x := |\vec{x}|$$

NLO potential reproduces the exact phase shift rather well.

$k\cot(\delta_0(k))$



ERE =Effective Range Expansion

NLO potential is better than NLO ERE.

Step 3' Improved extraction of potentials

Ishii et al. (HALQCD), PLB712(2012) 437

Normalized 4-pt function

$$R(\mathbf{r},t) \equiv F(\mathbf{r},t)/G_N(t)^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t} + \cdots \qquad \Delta W_n = W_n - 2m_N$$

4-pt function $F(\mathbf{r}, t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\overline{\mathcal{J}}(t_0)|0\rangle$

NN creation op.

Master equation

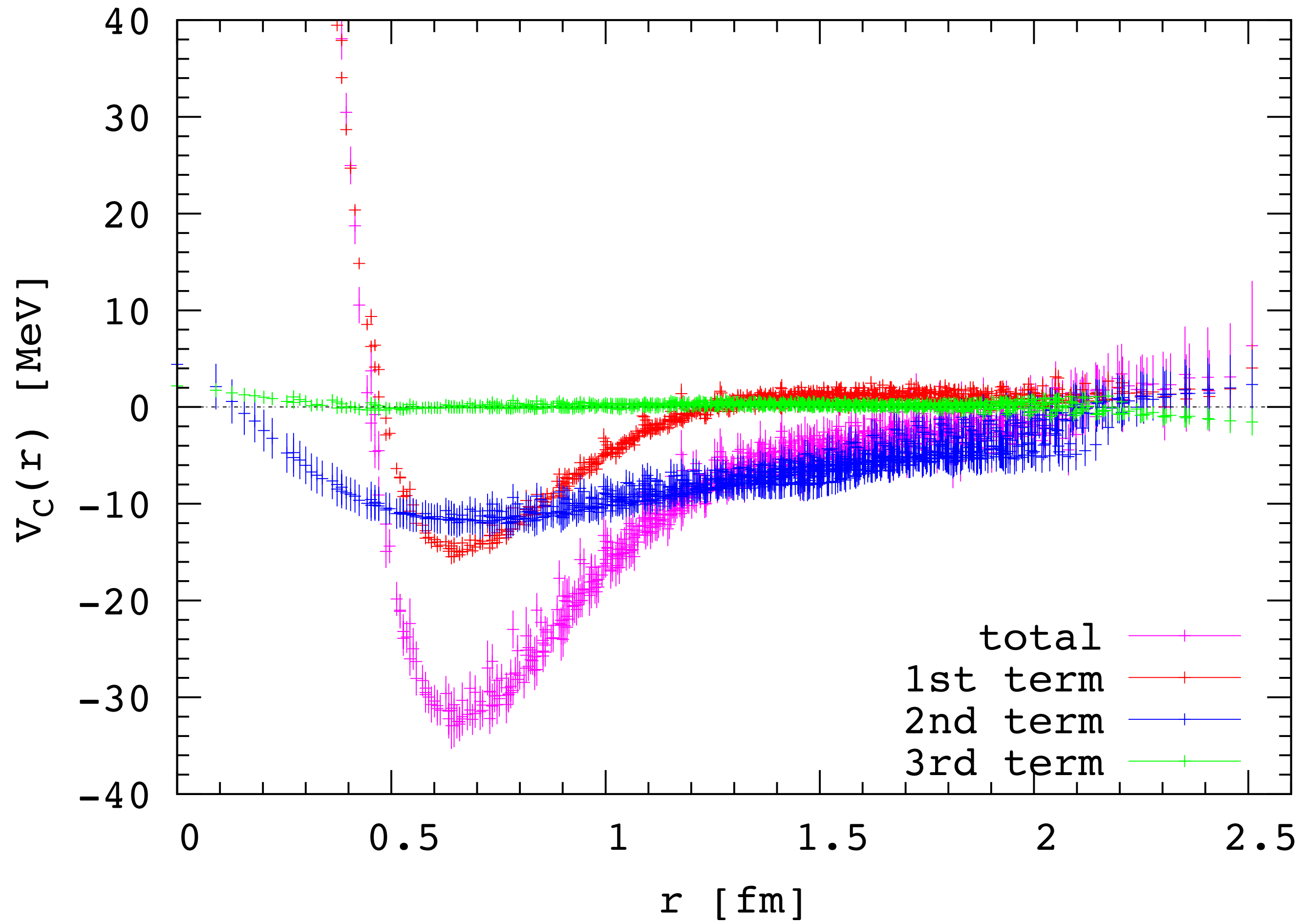
Potential

$$\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N^2}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t) = V(\mathbf{r},\nabla)R(\mathbf{r},t) + \cdots = V_0^{\mathrm{LO}}(r)R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd

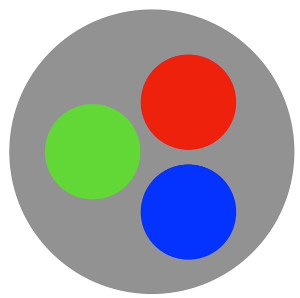
remaining t-dependence of the potential

1. Inelastic contributions
2. Higher order terms in the derivative expansion



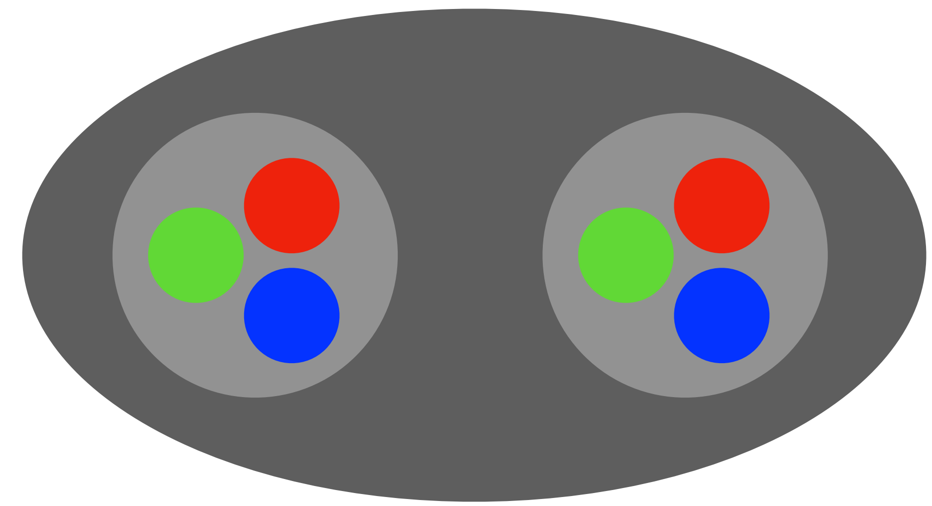
II. Dibaryons

Baryon (B=1)



Proton, Neutron,
Lambda, Omega,…

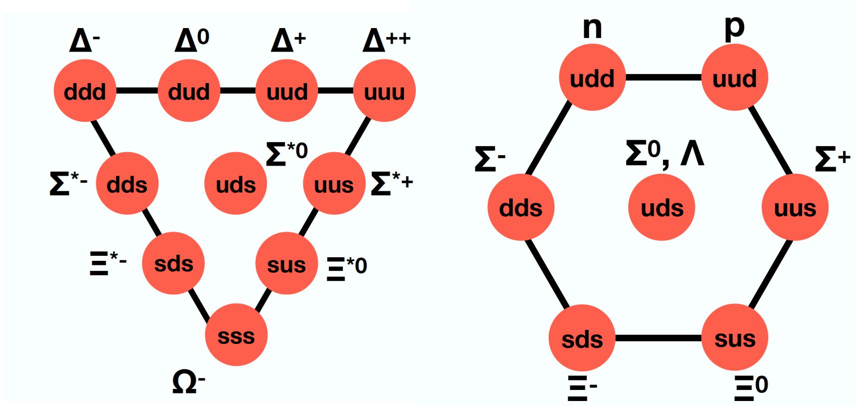
Dibaryon (B=2)



Deuteron
observed in 1930s
+ d*(2380) resonance

Dibaryon = two baryon bound state or resonance

SU(3) classification for Dibaryon candidates (B=2)



Jaffe (1977)
H-dibaryon(J=0)

1) octet-octet system

$$8 \otimes 8 = 27 \oplus 8_s \oplus \boxed{1} \oplus \boxed{\overline{10}} \oplus 10 \oplus 8_a$$

Deuteron(J=1)

2) decuplet-octet system

NΩ system and NΔ system (J=2)

$$10 \otimes 8 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Goldman et al (1987)
Dyson, Xuong (1964)

3) decuplet-decuplet system

$$10 \otimes 10 = \boxed{28} \oplus 27 \oplus 35 \oplus \boxed{\overline{10}}$$

$d^*(2380)$ resonance

ΩΩ system (J=0)
Zhang et al(1997)

ΔΔ system (J=3)

Dyson, Xuong (1964)
Kamae, Fujita(1977)
Oka, Yazaki(1980)

1. Physical pion mass

Lattice QCD at (almost) physical pion mass

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

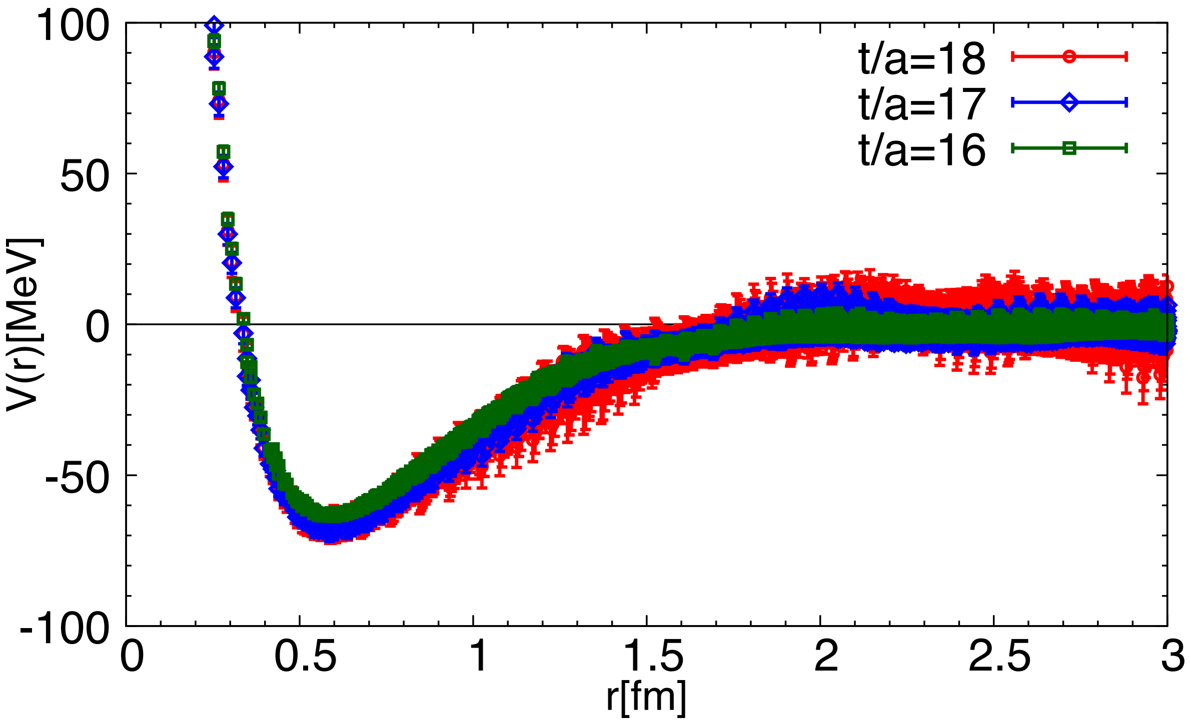
K-computer [10PFlops]

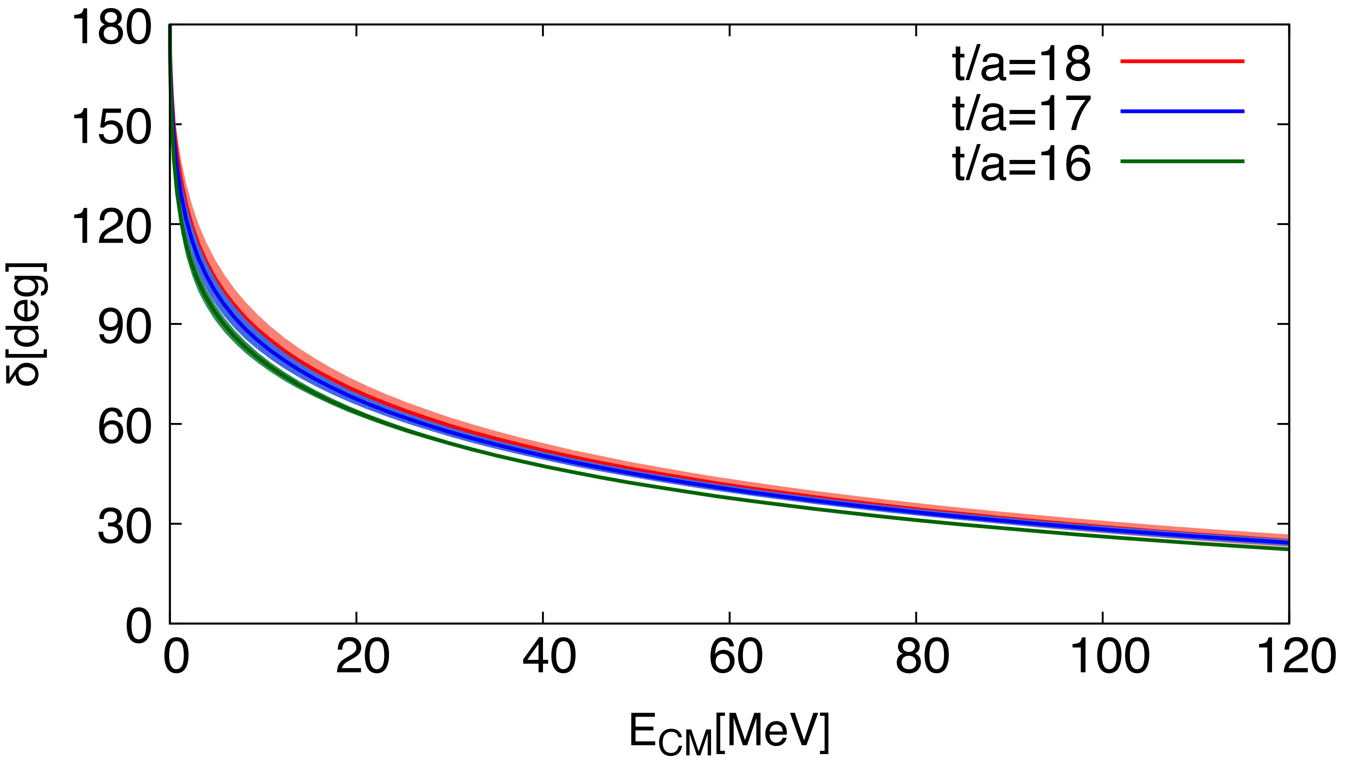
$$\Omega^- \Omega^-$$

(ssssss)

S. Gongyo et al., Phys. Rev. Lett. 120 (2018) 212001.

$\Omega\Omega(^1S_0)$ potential → phase shift





A similar structure to NN

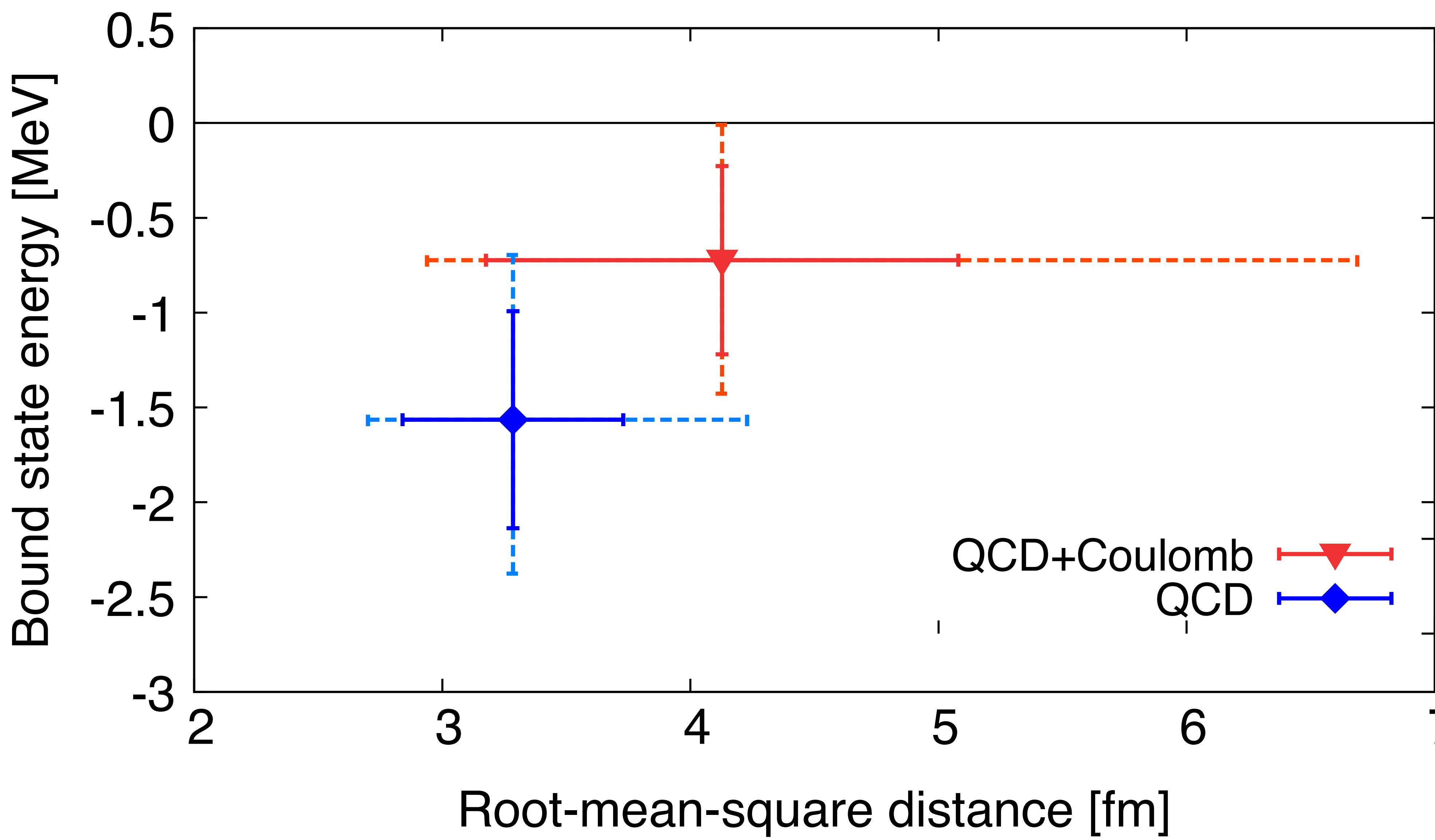
Strong attraction

repulsive behavior

→ bound state

Binding energy

$$H = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\rm LQCD}(r) + \frac{\alpha}{r}$$



$$(B_{\Omega\Omega}^{\rm (QCD)}, B_{\Omega\Omega}^{\rm (QCD+Coulomb)}) = (1.6(6){\rm MeV}, 0.7(5){\rm MeV})$$

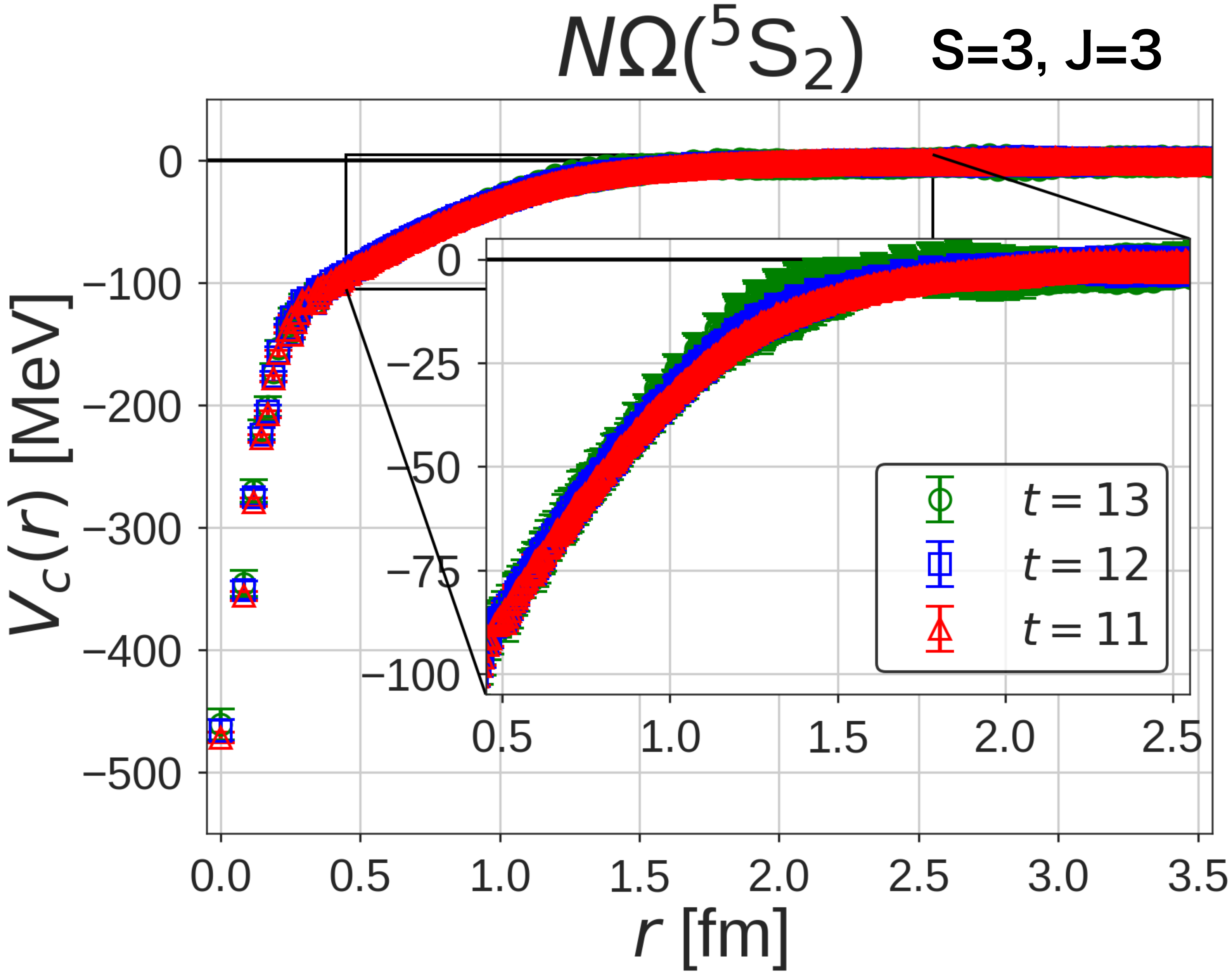
The most strange (sss sss) dibaryon ?

A candidate for the second bound dibaryon.

$N\Omega^-$

T. Iritani et al., PLB 792 (2019) 284 (arXiv: 1810.03416)

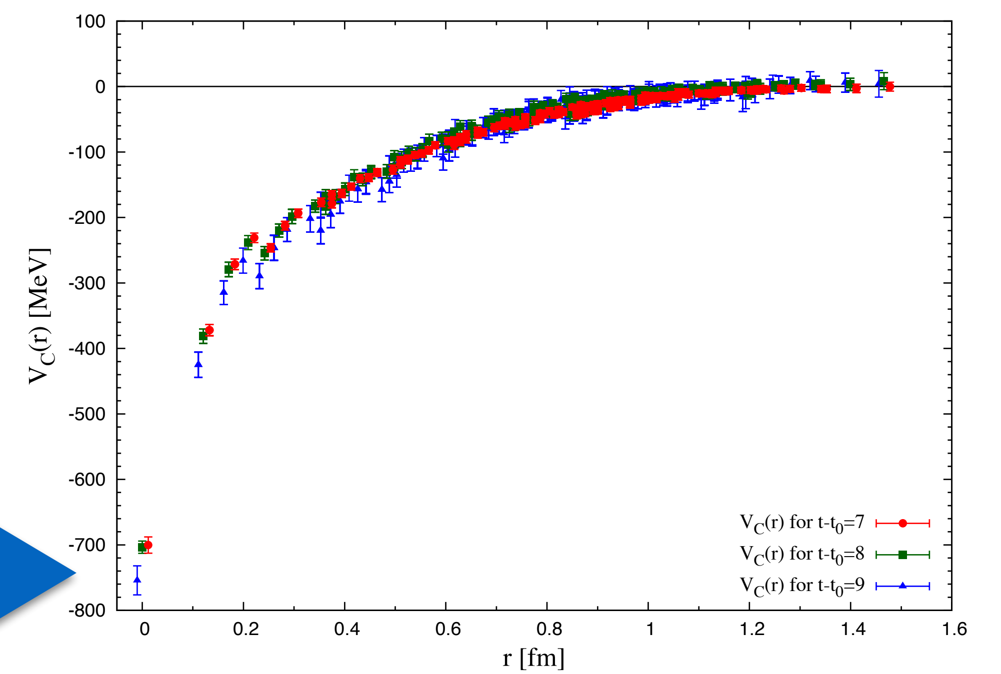
$N\Omega$ potential in $^5S_2$ channel



* attractive potential without repulsive core

* long range attraction

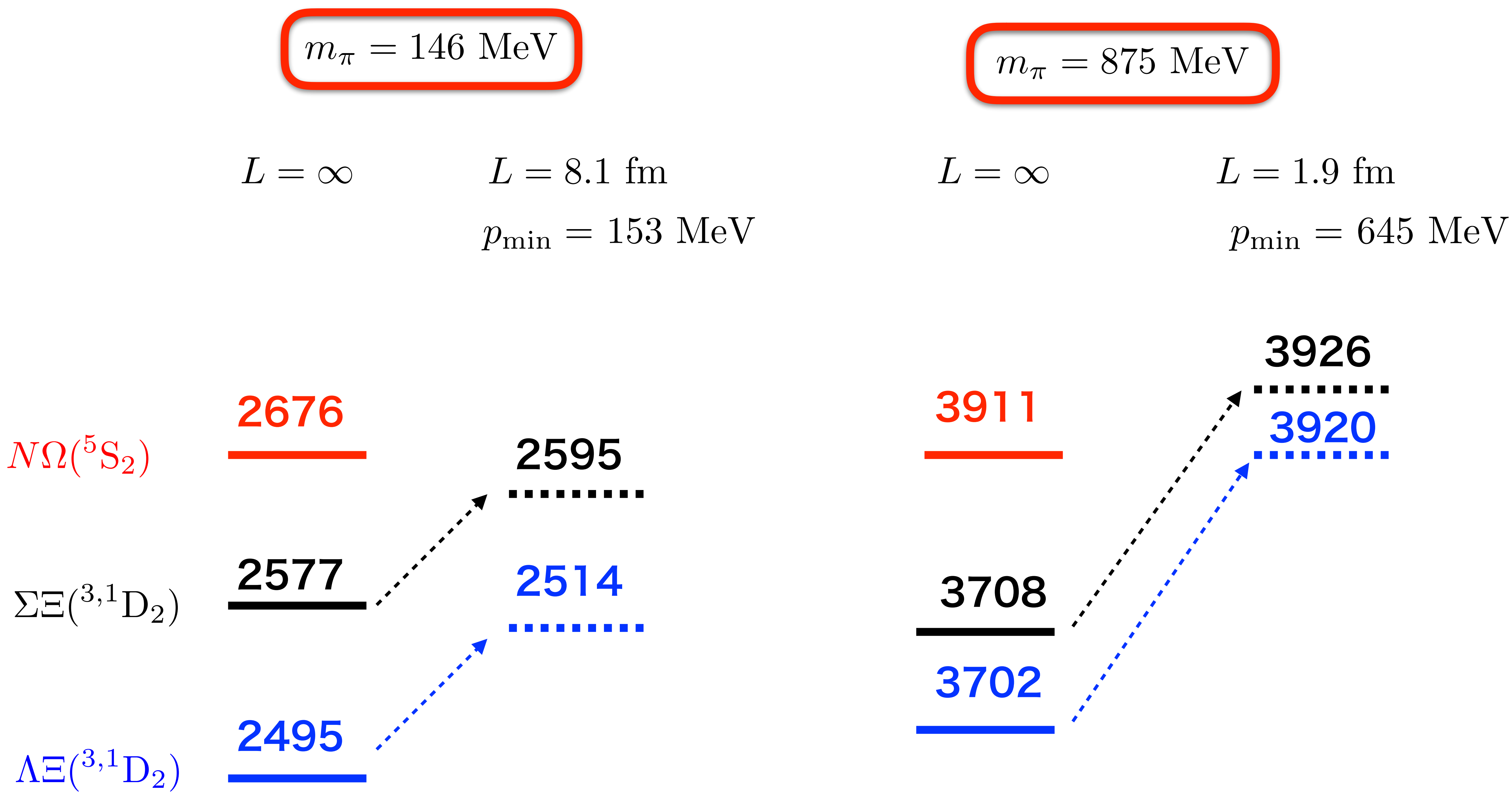
Etminan et al., NPA928(2014)89



qualitatively the same at $m_\pi \simeq 875$ MeV

B.E. = 18.9(5.0)(+12.1)(-1.8) MeV

Remark

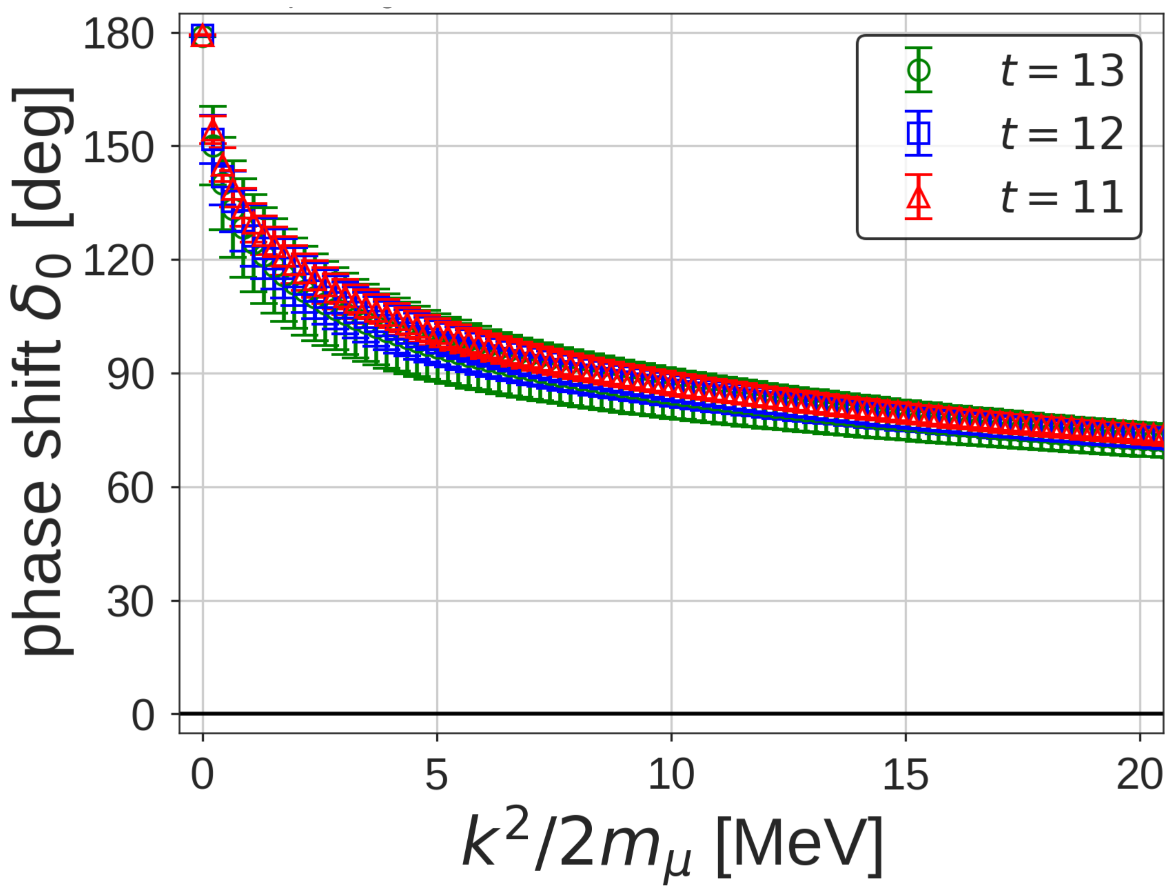


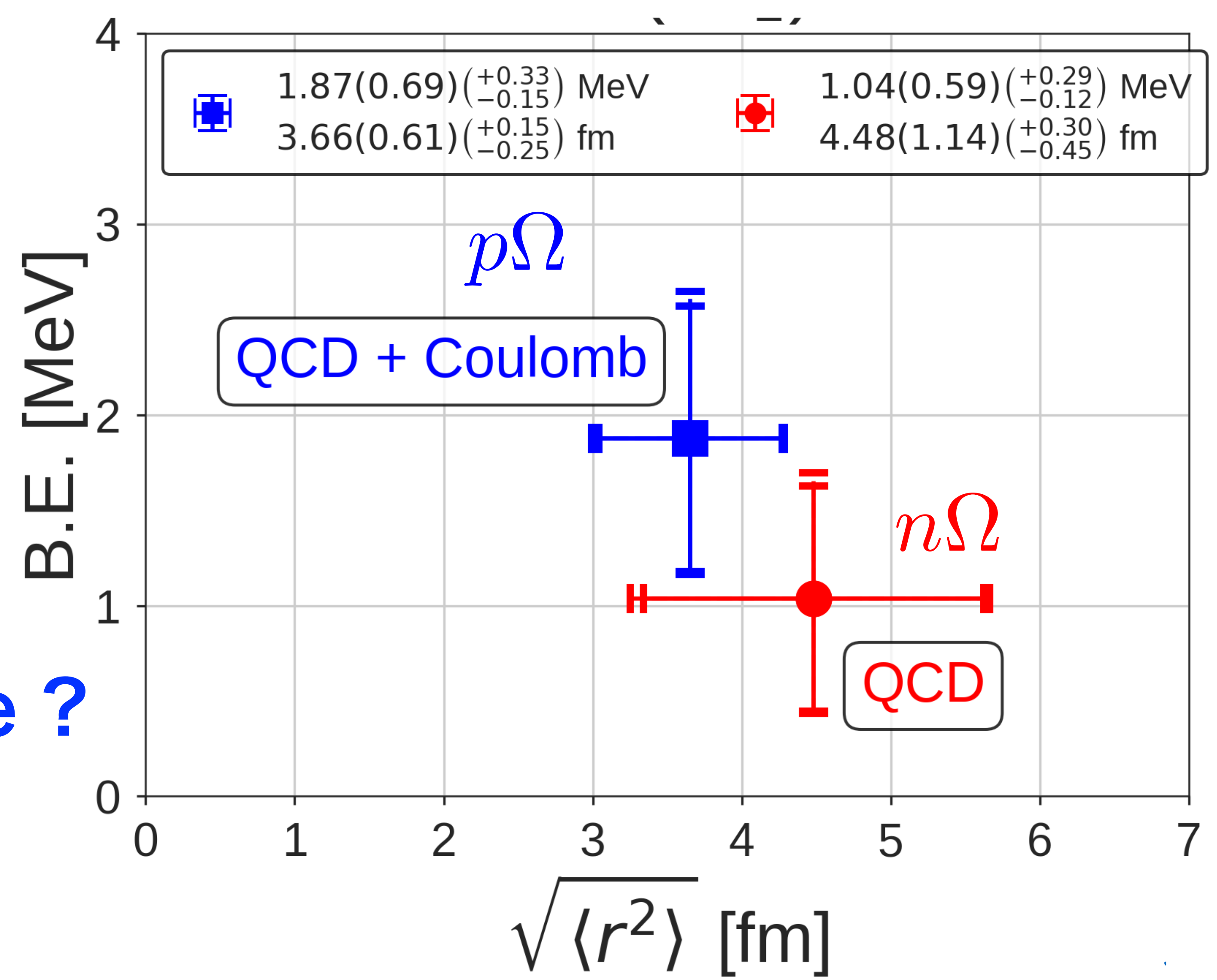
* Single channel analysis only.

* Assume small couplings to D-waves, supported by weak t-dep.

* Coupled channel analysis in the future

Phase shift and binding energy



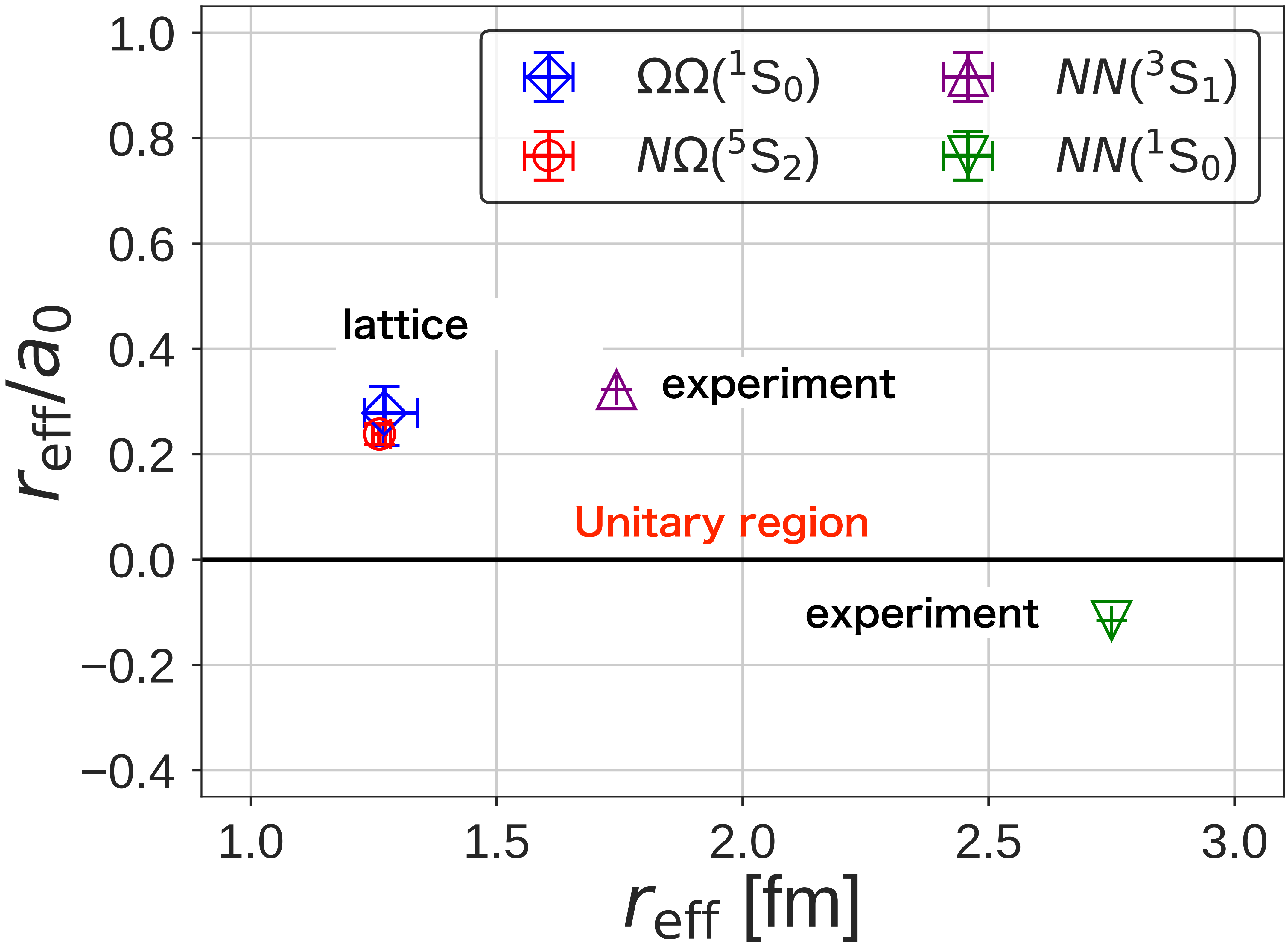


New dibaryon resonance ?

Comparison

$$\frac{r_{\mathrm{eff}}}{a_0} \text{ vs } r_{\mathrm{eff}}$$

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{r_{\mathrm{eff}}}{2} k^2 + \cdots$$



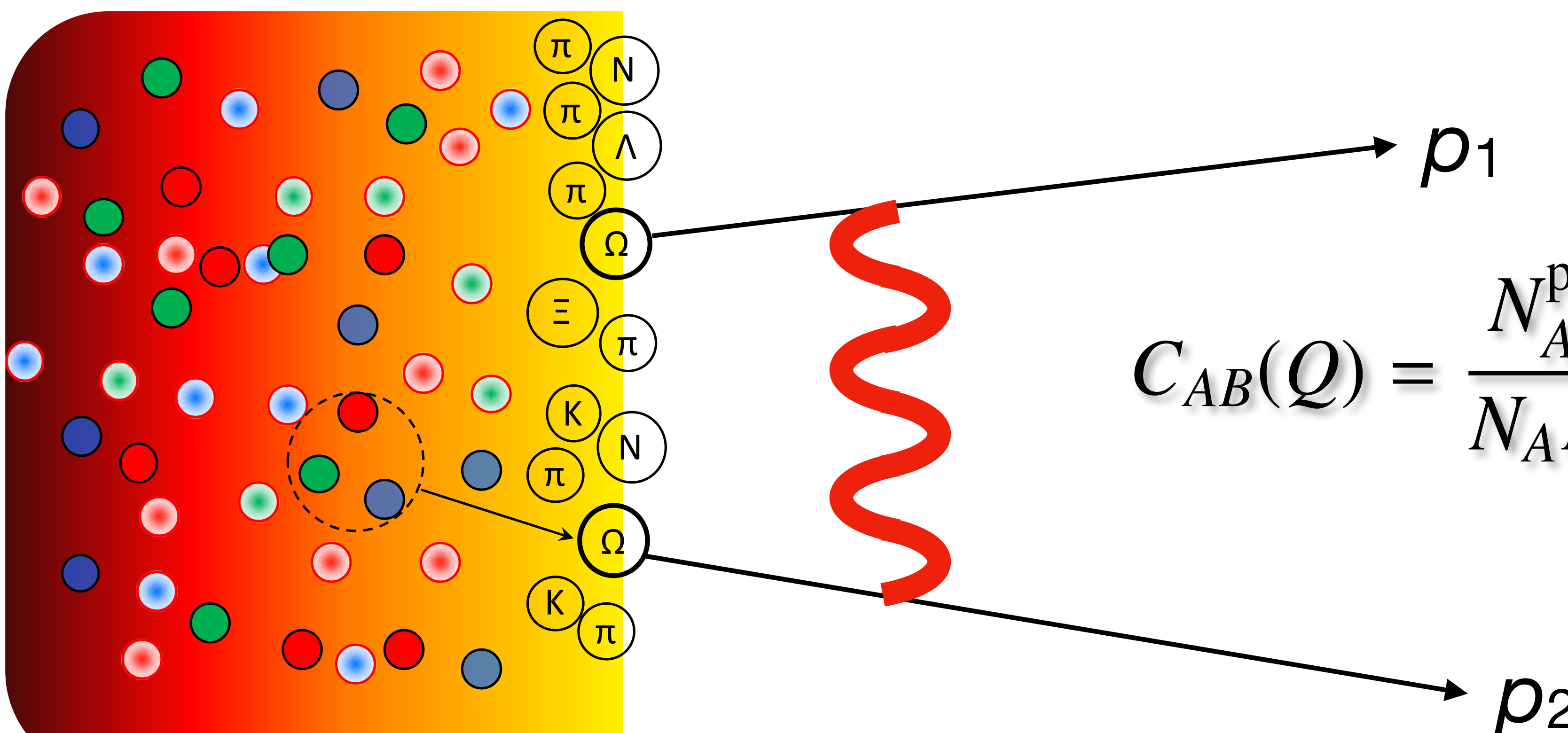
universal ?

How can we confirm ?

Measurement of two-baryon correlation at RHIC & LHC

STAR Coll., Phys. Lett. B790 (2019) 490 "NΩ correlation in Au+Au"
ALICE Coll., arXiv:1905.07209 "ΛΛ correlation in p+p, p+Pb"
ALICE Coll., arXiv:1904.12198 "NXi correlation in p+p, p+Pb"

two-baryon interaction ⇔ two-baryon correlation



$$C_{AB}(Q) = \frac{N_{AB}^{\rm pair}(Q)}{N_A N_B(Q)} = \begin{cases} 1 \\ \text{others} \end{cases}$$

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2} P\right)^2}$$

K. Morita et al., PRC94(2016)031901 "NΩ correlation from HAL pot"
K. Morita et al., NPA967(2017)856 "NXi correlation from HAL pot."
K. Morita et al., arXiv:1908.05414 "NΩ & ΩΩ correlations from HAL pot."

$$N_{AB}(Q) = \int \frac{d^3p_A}{E_A}\frac{d^3p_B}{E_B} N_{AB}(\mathbf{p}_A, \mathbf{p}_B)\delta(Q - \sqrt{-q^2})$$

# of hadron pairs

$$N_{AB}(\mathbf{p}_A, \mathbf{p}_B) \simeq \int d^4x d^4y \underbrace{S_A(x, \mathbf{p}_A)S_B(y, \mathbf{p}_B)}_{\text{source}} \underbrace{|\Psi(x, y, \mathbf{p}_A, \mathbf{p}_B)|^2}_{\text{2-body wave function}}$$

source

2-body wave function
from
the interaction potential

If the source is approximately known,
one can test hadron interactions using the above formula.

Proton-Ω correlation in RHIC

STAR Coll. ,Phys. Lett. B790(2019)490

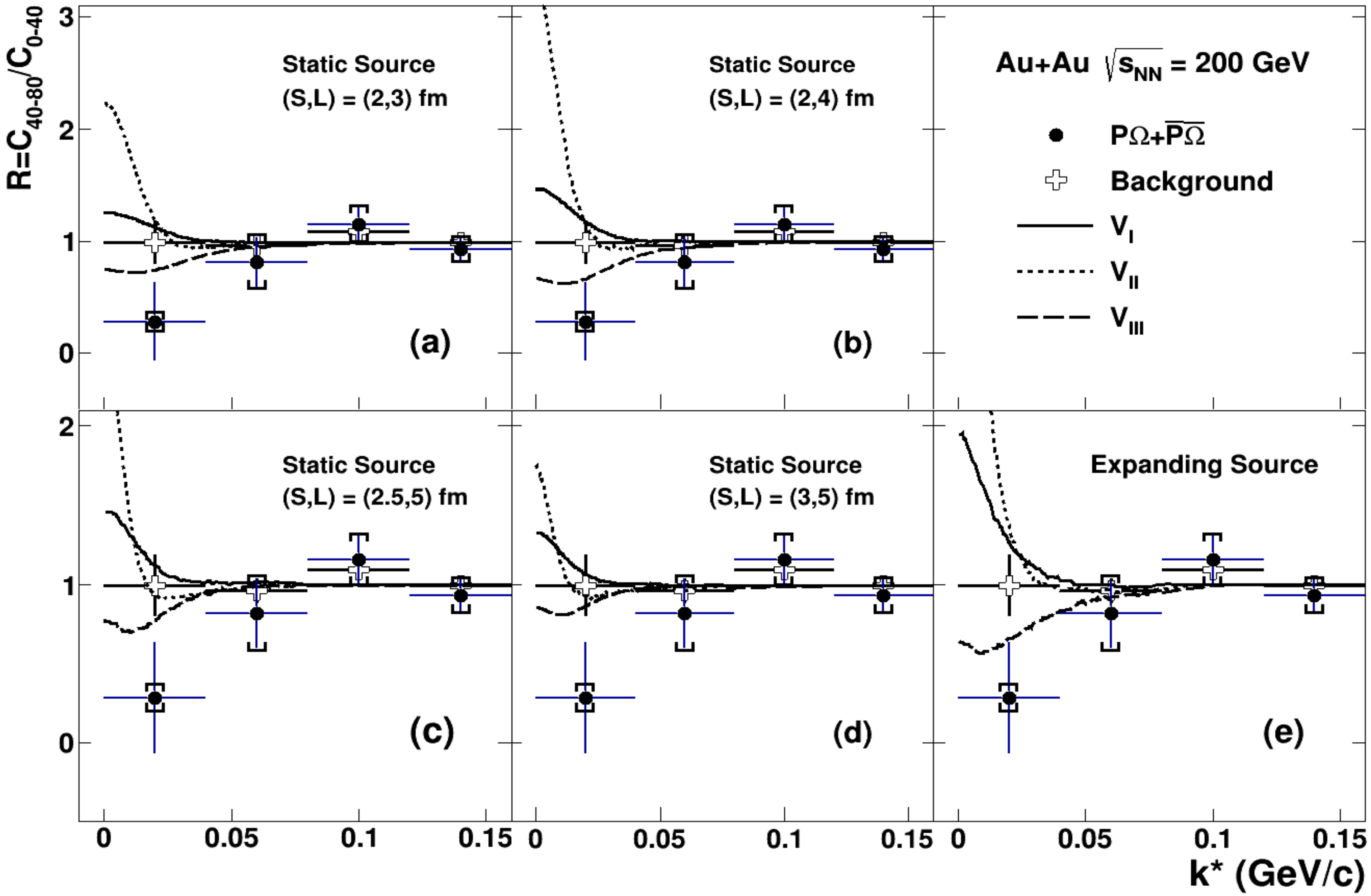
ratio of small to large systems

$\mathrm{Au} + \mathrm{Au}$

centrality

40-80% (small)

0-40% (large)



Morita et al., PRC94(2016)031901

V_I : unbound

V_{II} : $E_B = 6.3$ MeV

V_{III} : $E_B = 26.9$ MeV

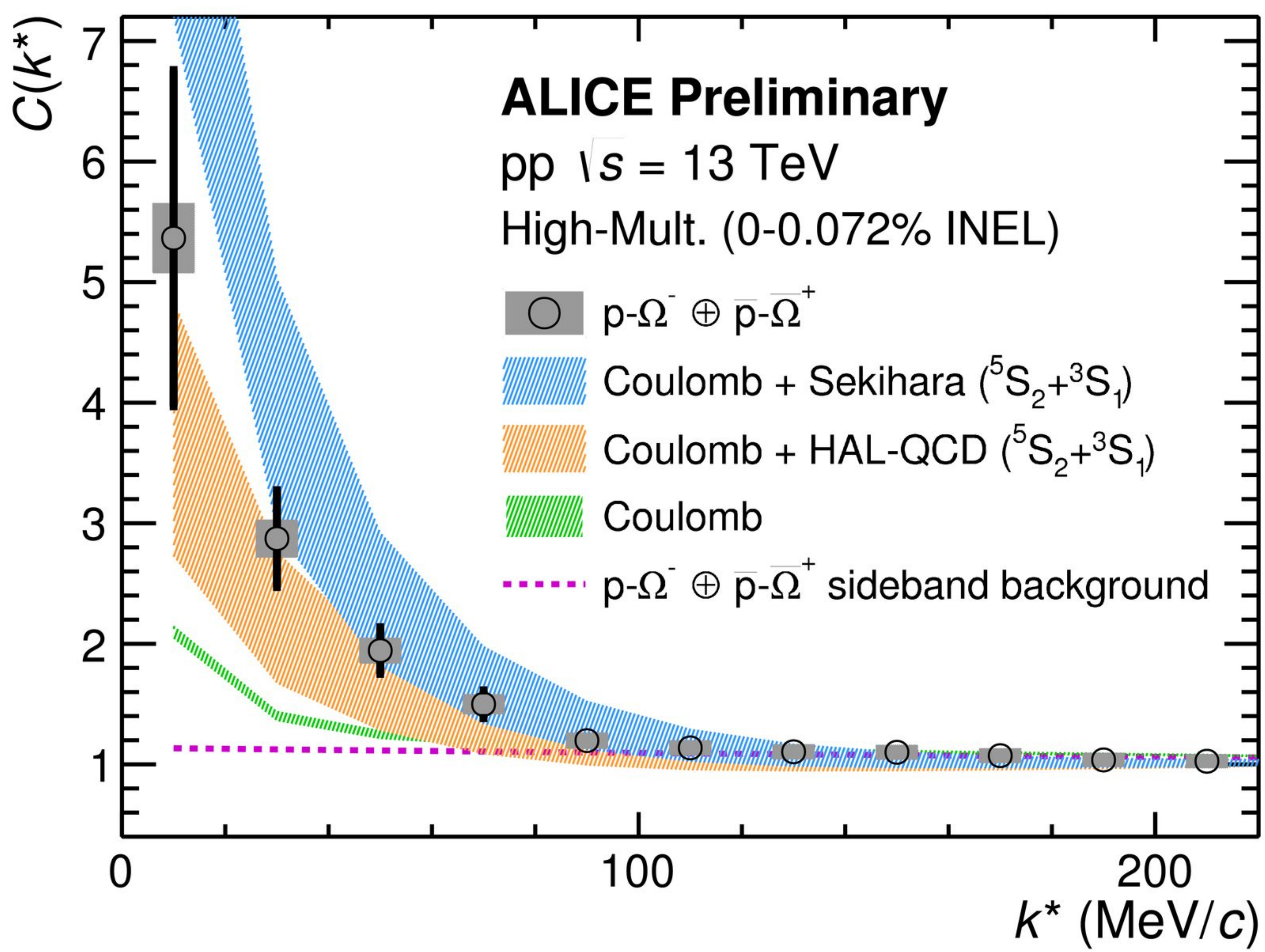
potential at $m_\pi = 875$ MeV

Data at $k^* < 40$ MeV favor V_{III}.

One can also use p+p data (LHC).

Oton Vazquez Doce (ALICE), talk in Session 7 on Aug.18

$p\Omega$ correlations



*HAL QCD potential at physical pion seems consistent with data.
*Need more accurate potential/data for a further confirmation.

$\Omega\Omega$ in near future ?

2. Heavier pion masses

$\Delta\Delta$ system with $J = 3$

S. Gongyo et al., in preparation.

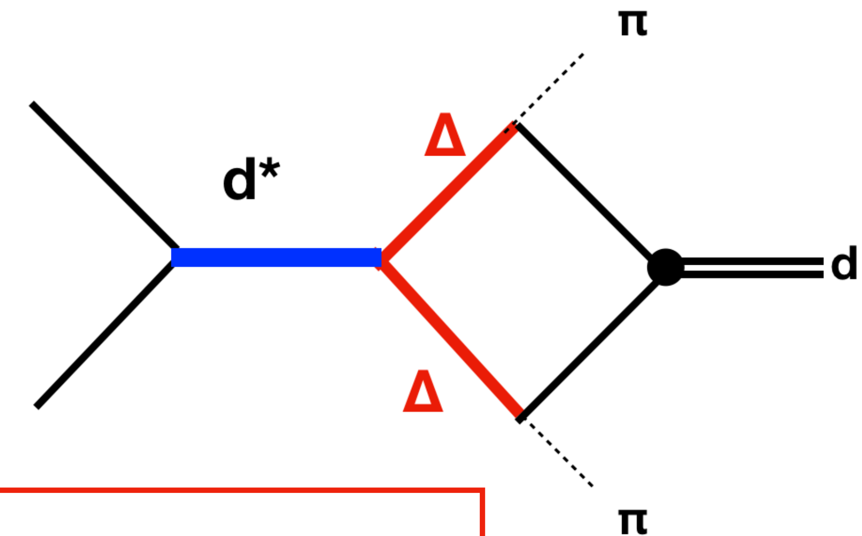
d*(2380) resonance

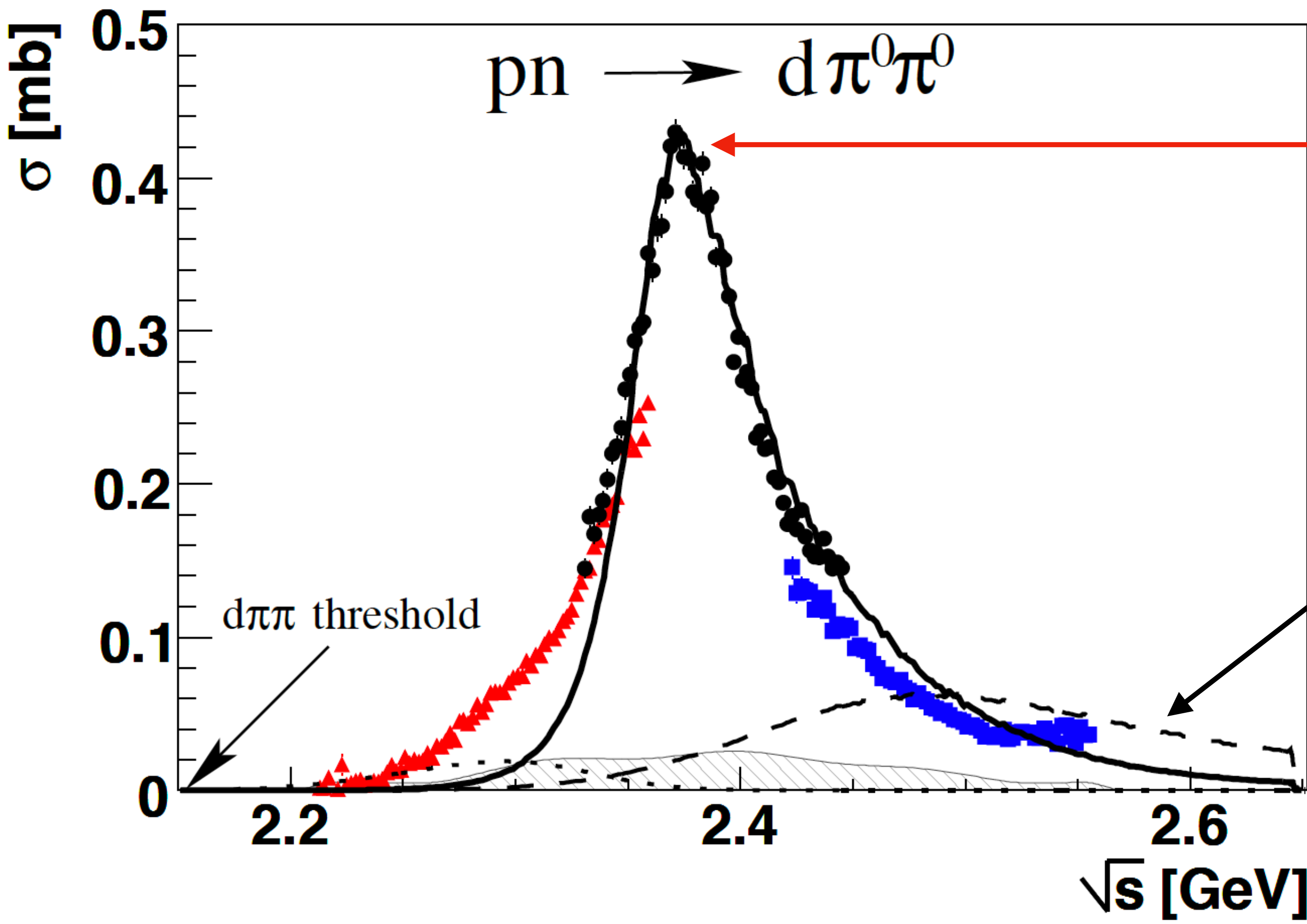
WASA@COSY, PRL 106, 242302 (2011)

d* (2380) observed by WASA@COSY col.

$$p + n(d) \rightarrow d + \pi^0 + \pi^0(+p_{\mathrm{spectator}})$$

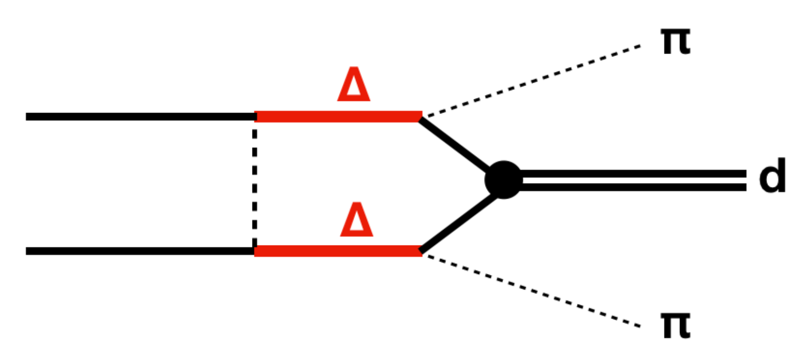
m~ 2.38 GeV, Γ ~ 70 MeV, $J^{\pi} = 3^+$, I=0





d* resonance
m~2.38 GeV
Γ~70 MeV

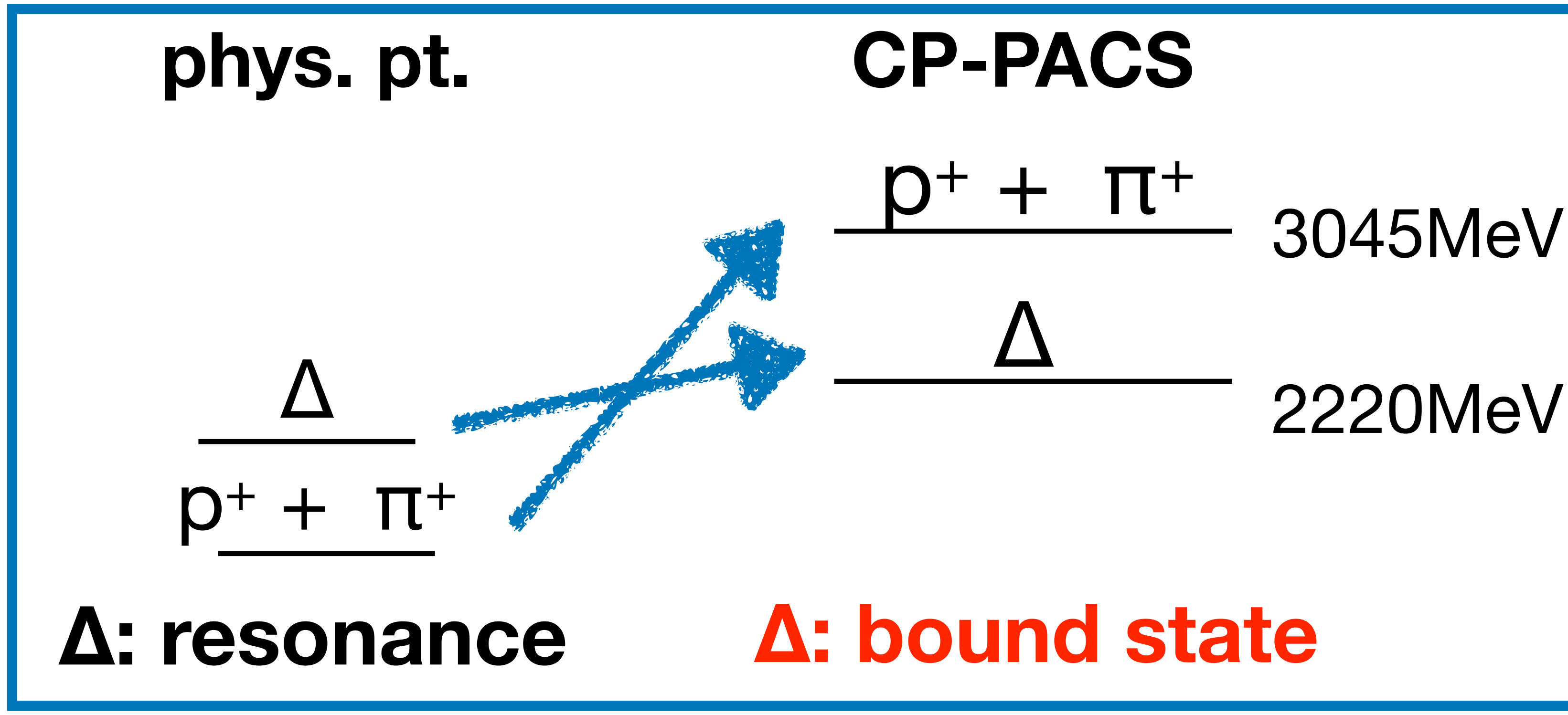
ΔΔ contributions

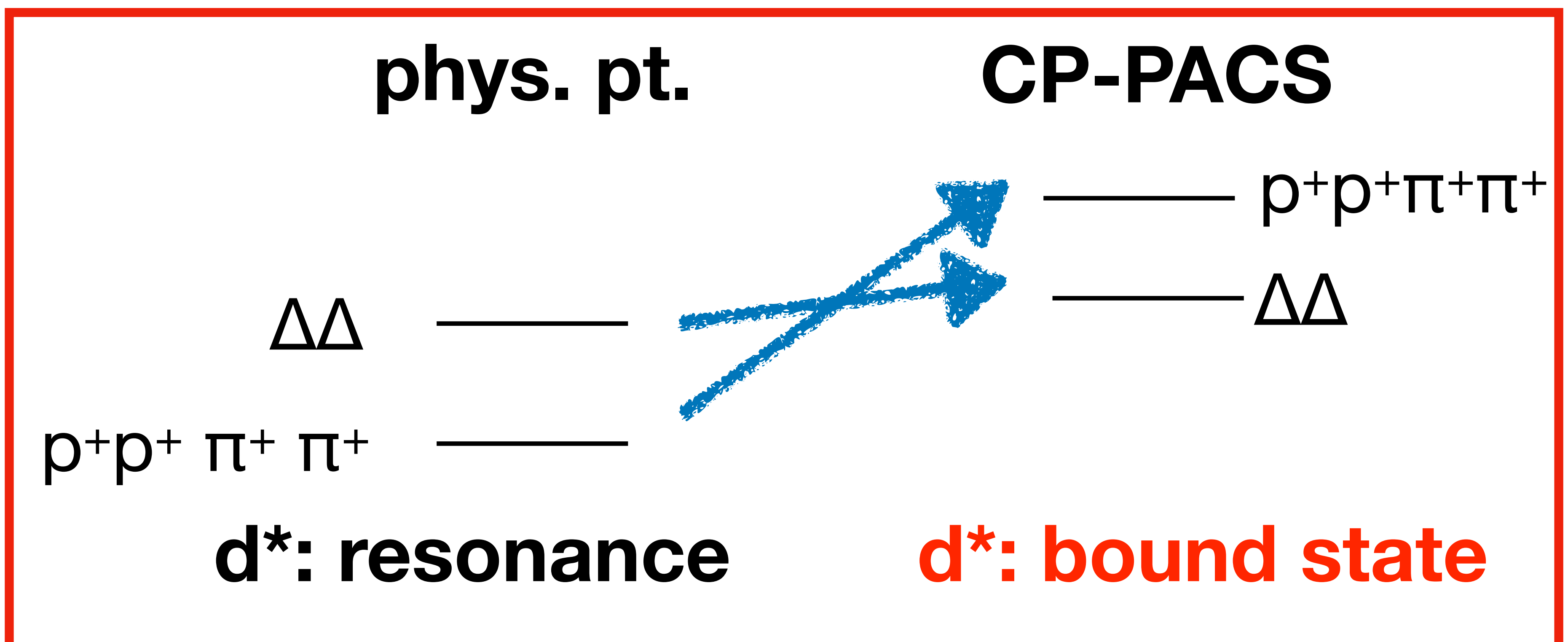


3-flavor full QCD in the SU(3) limit

CP-PACS Conf. $L = 1.93\ \mathrm{fm}$

	[MeV]
m_{ps}	1015
m_{oct}	2030
m_{dec}	2220

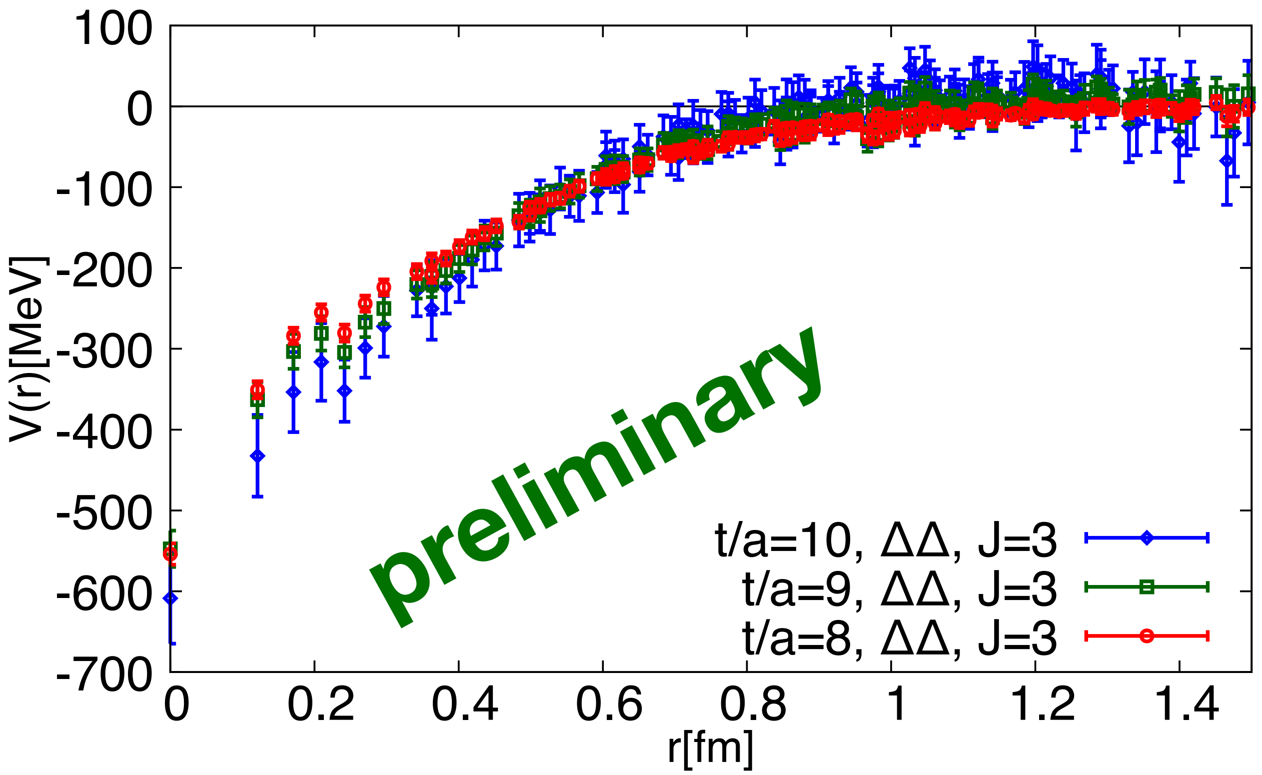


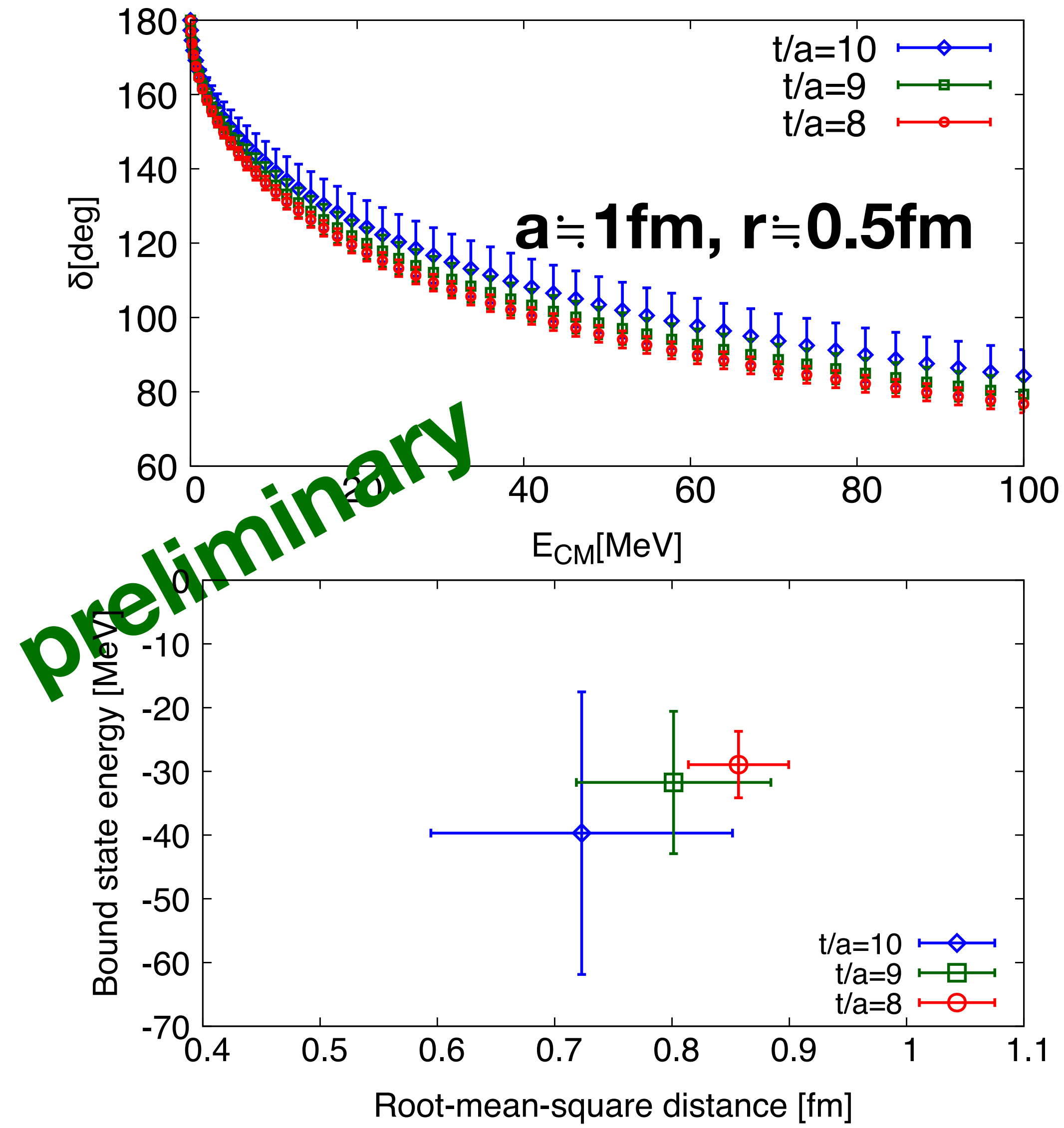


$\overline{10}$ potential in decuplet-decuplet system

ΔΔ in $J^p(I)$ =$3^+(0)$

$m_\Delta \simeq 2225\text{MeV}$





We assume that
decay to NN($^3D_3$) is neglected

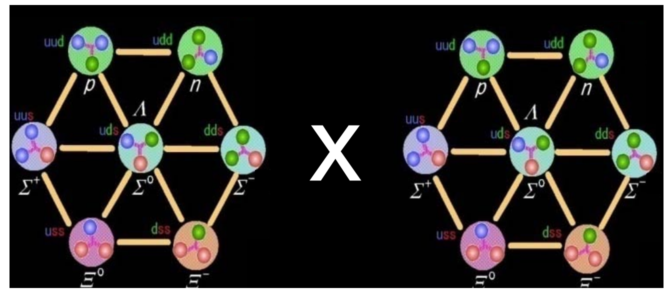
- In short range, there is no repulsive core
- Deep bound state is found

d* is supported from lattice QCD

H-dibaryon

Baryon potential in the flavor SU(3) limit

$m_u = m_d = m_s$



two octet baryons

$$8 \otimes 8 = \underbrace{27 \oplus 8_s \oplus 1}_{\text{Symmetric}} \oplus \underbrace{\overline{10} \oplus 10 \oplus 8_a}_{\text{Anti-symmetric}}$$

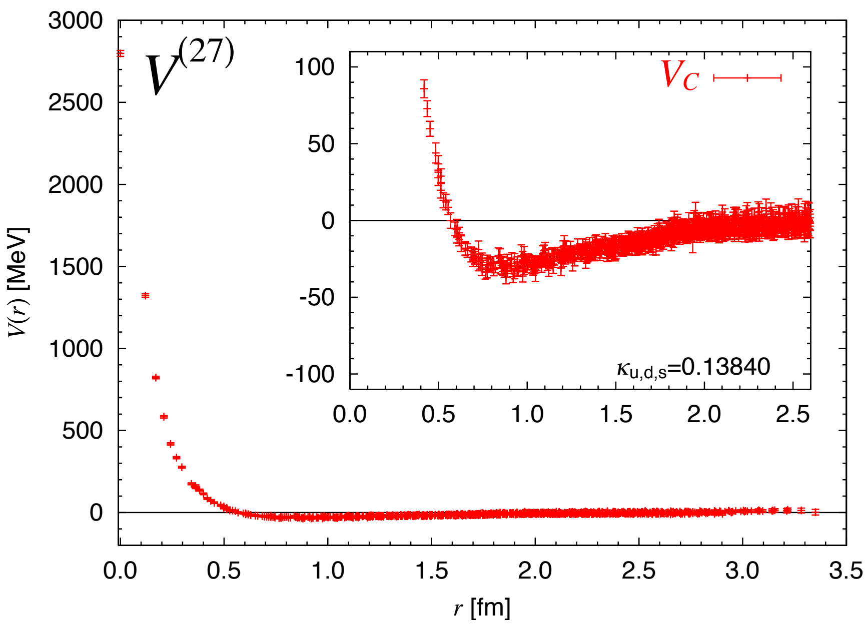
6 independent potentials in flavor-basis

$$V^{(27)}(r), V^{(8_s)}(r), V^{(1)}(r) \longleftarrow {}^1S_0$$

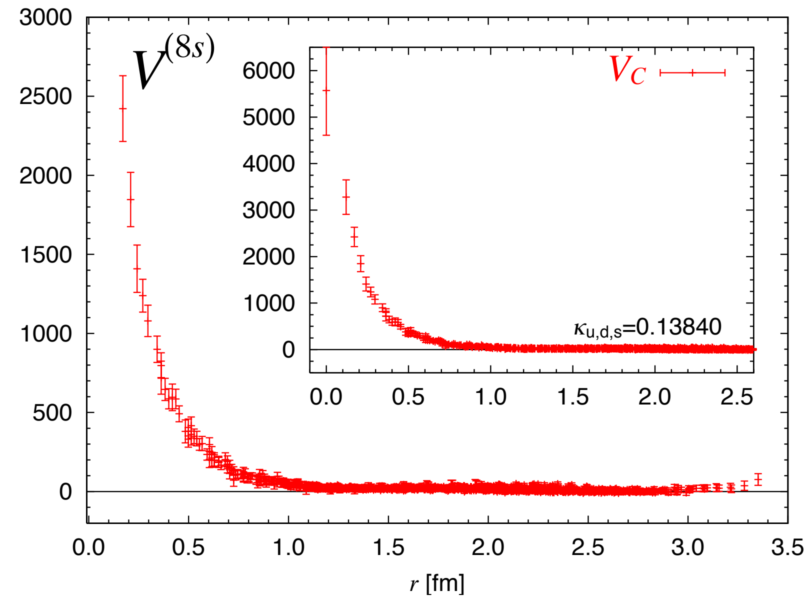
$$V^{(\overline{10})}(r), V^{(10)}(r), V^{(8_a)}(r) \longleftarrow {}^3S_1$$

Flavor dependences of BB interactions

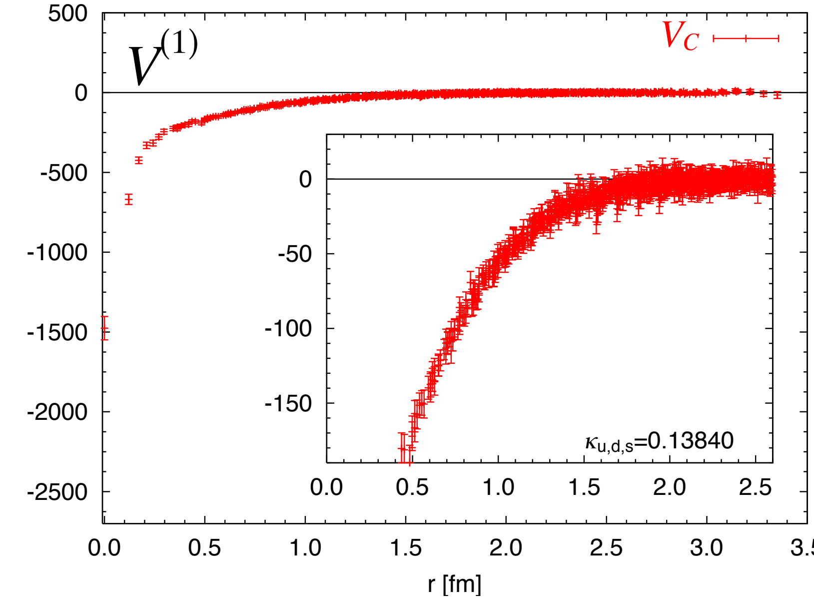
$L \simeq 4\ \mathrm{fm}, \quad m_\pi \simeq 470\ \mathrm{MeV}$



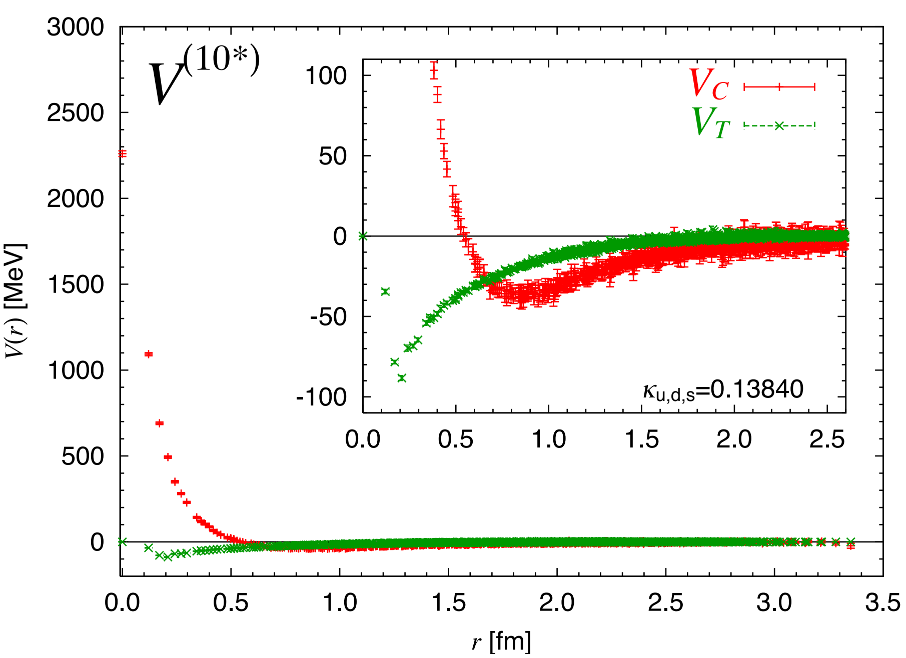
same as NN



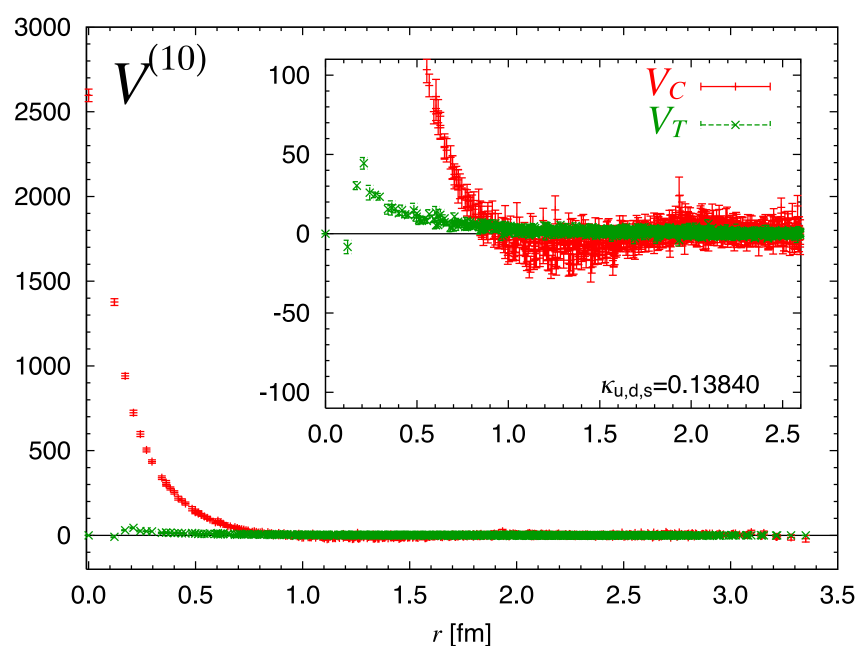
8s: strong repulsive core. repulsion only.



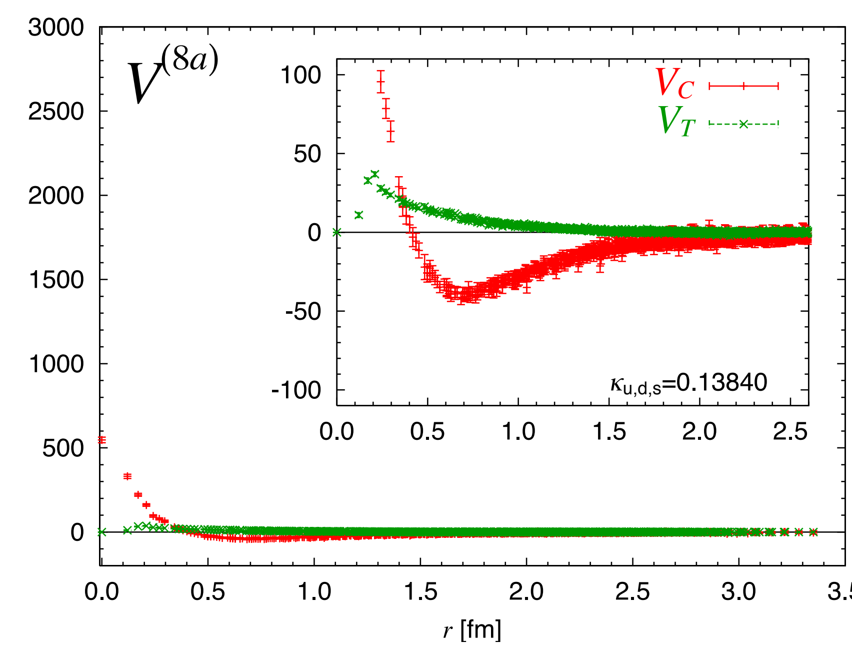
1: attractive instead of repulsive core ! attraction only . H-dibaryon.



same as NN



10: strong repulsive core. weak attraction.



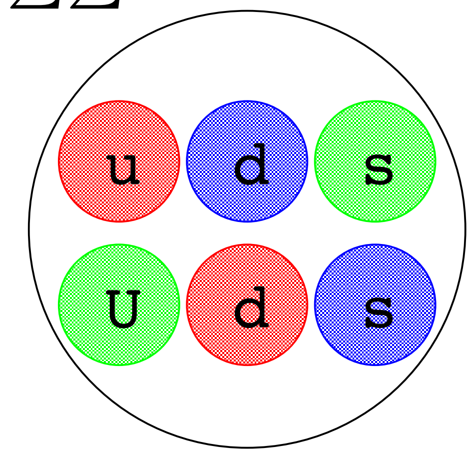
8a: weak repulsive core.
strong attraction.

Force for the singlet is attractive at all distances. Bound state ?

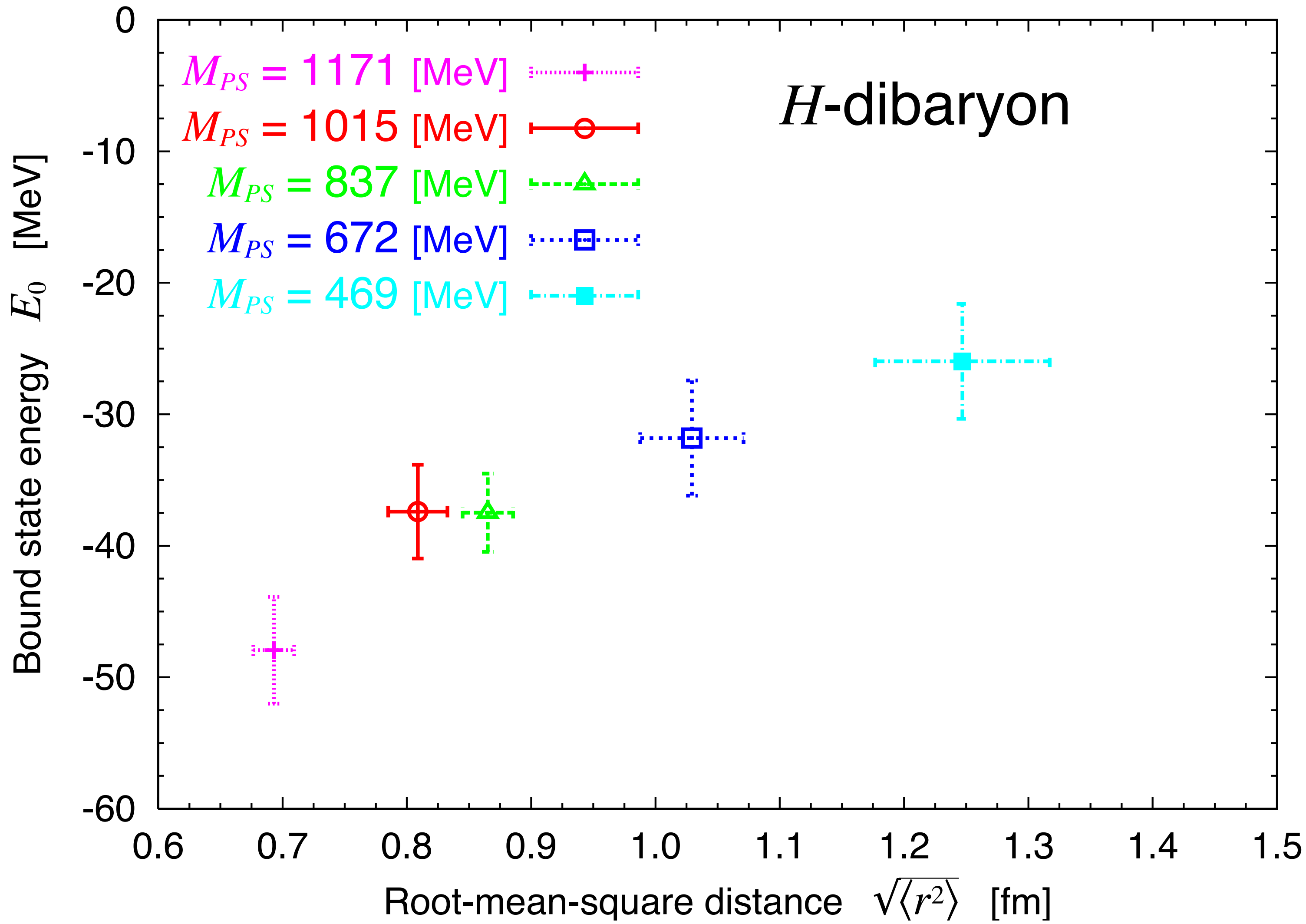
Attractive potential
in the flavor singlet channel

possibility of a bound state (H-dibaryon)

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$



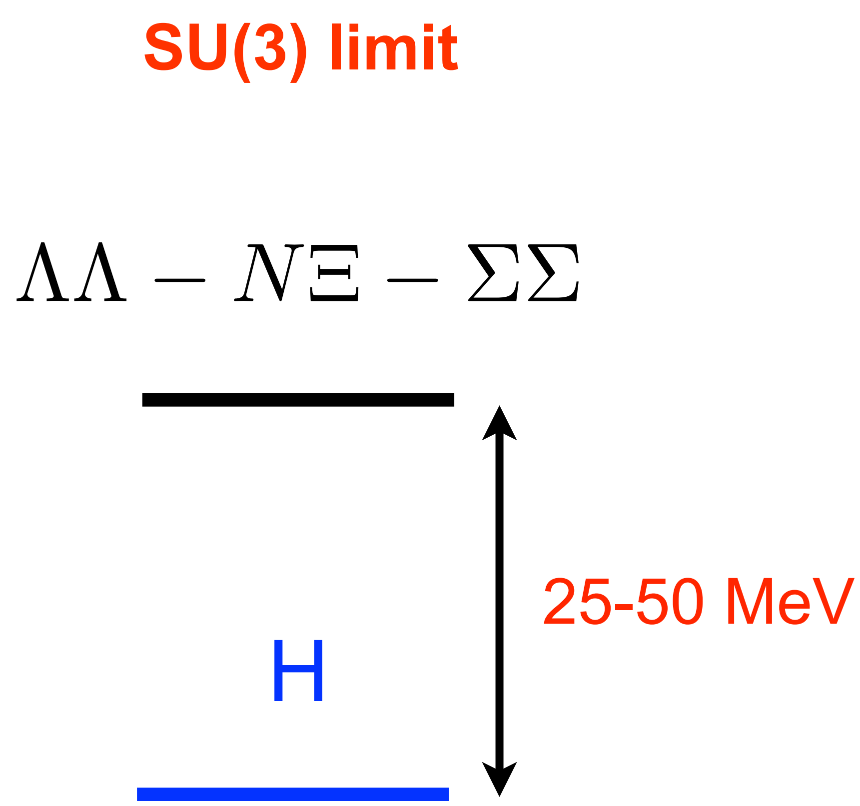
Inoue et al. (HAL QCD Coll.), PRL106(2011)162002

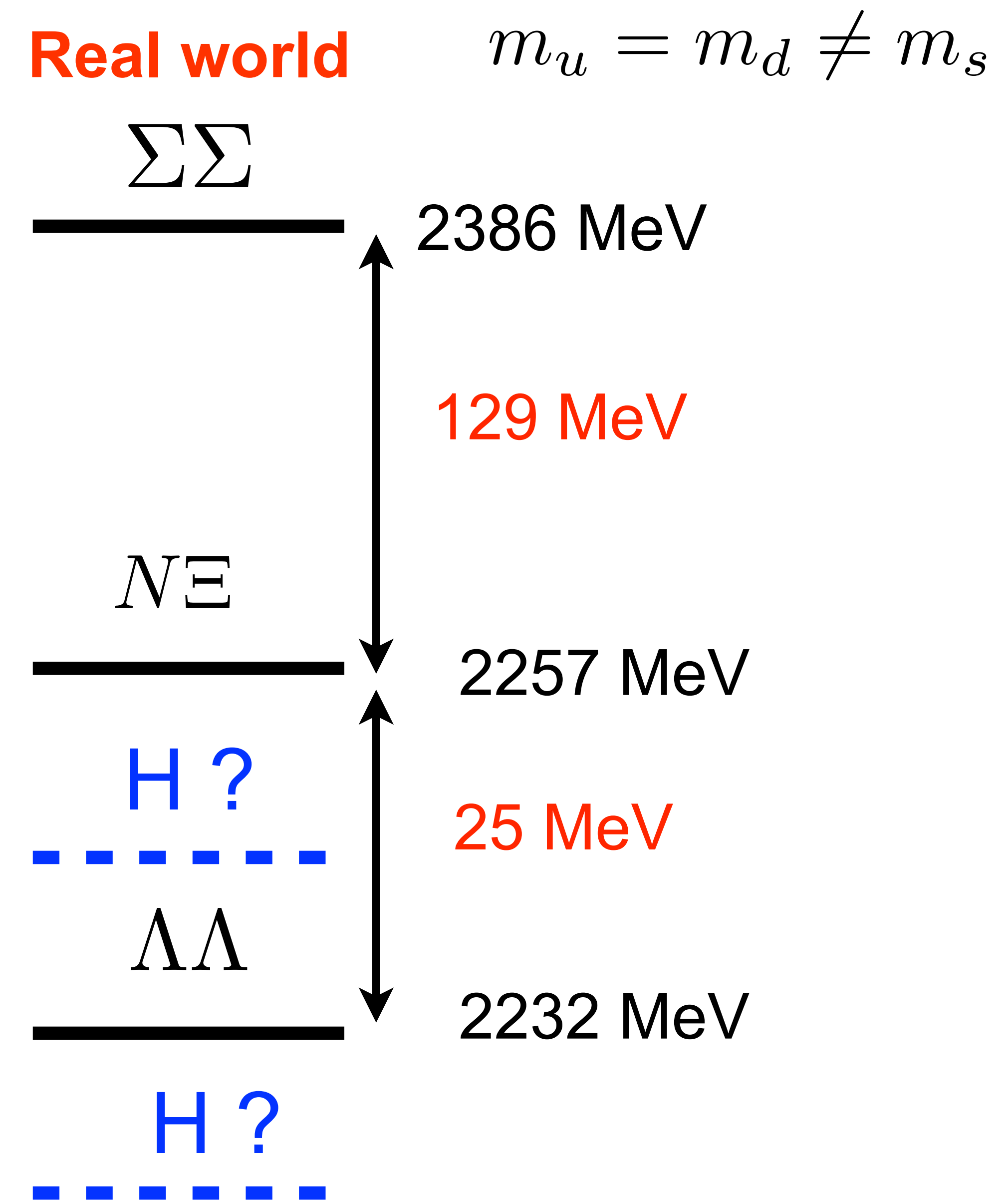


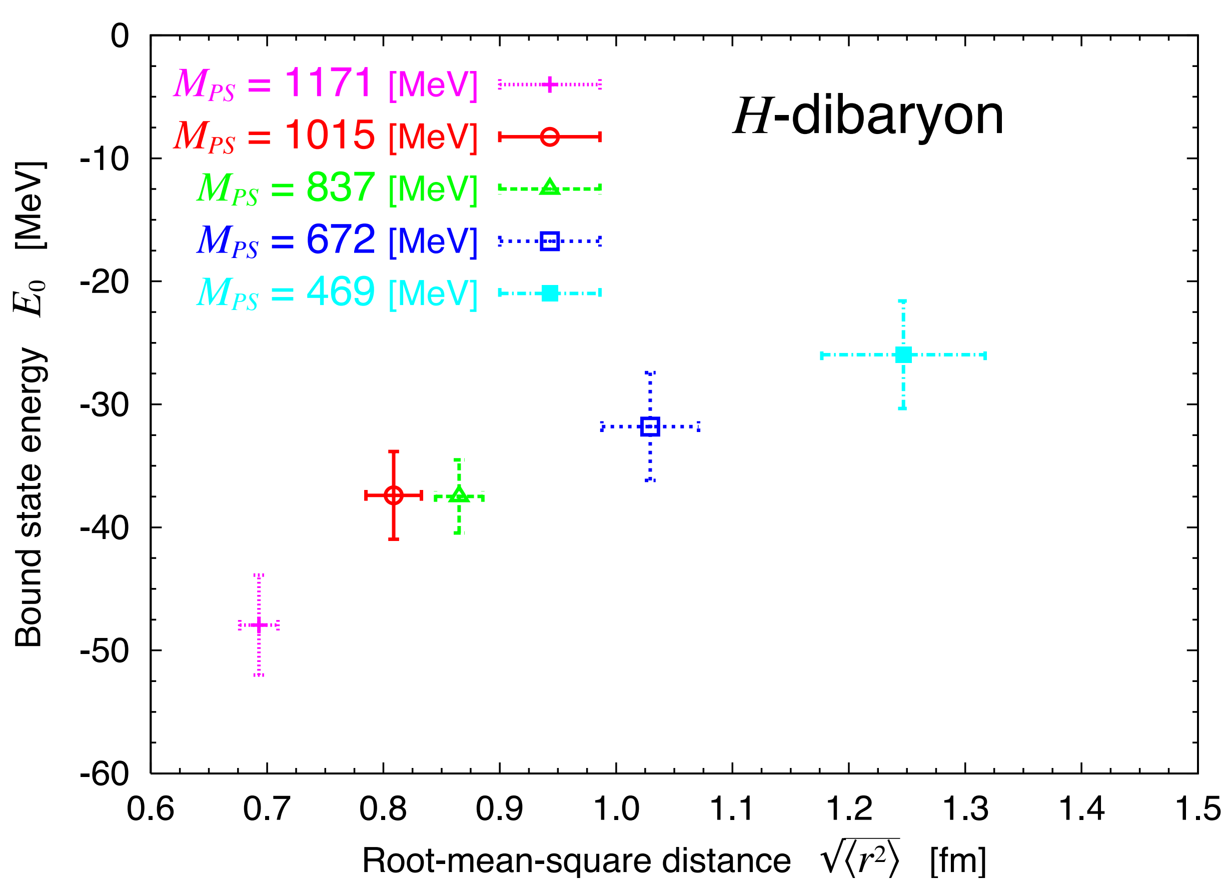
An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?

H-dibaryon with the flavor SU(3) breaking







Coupled channel HAL QCD potential

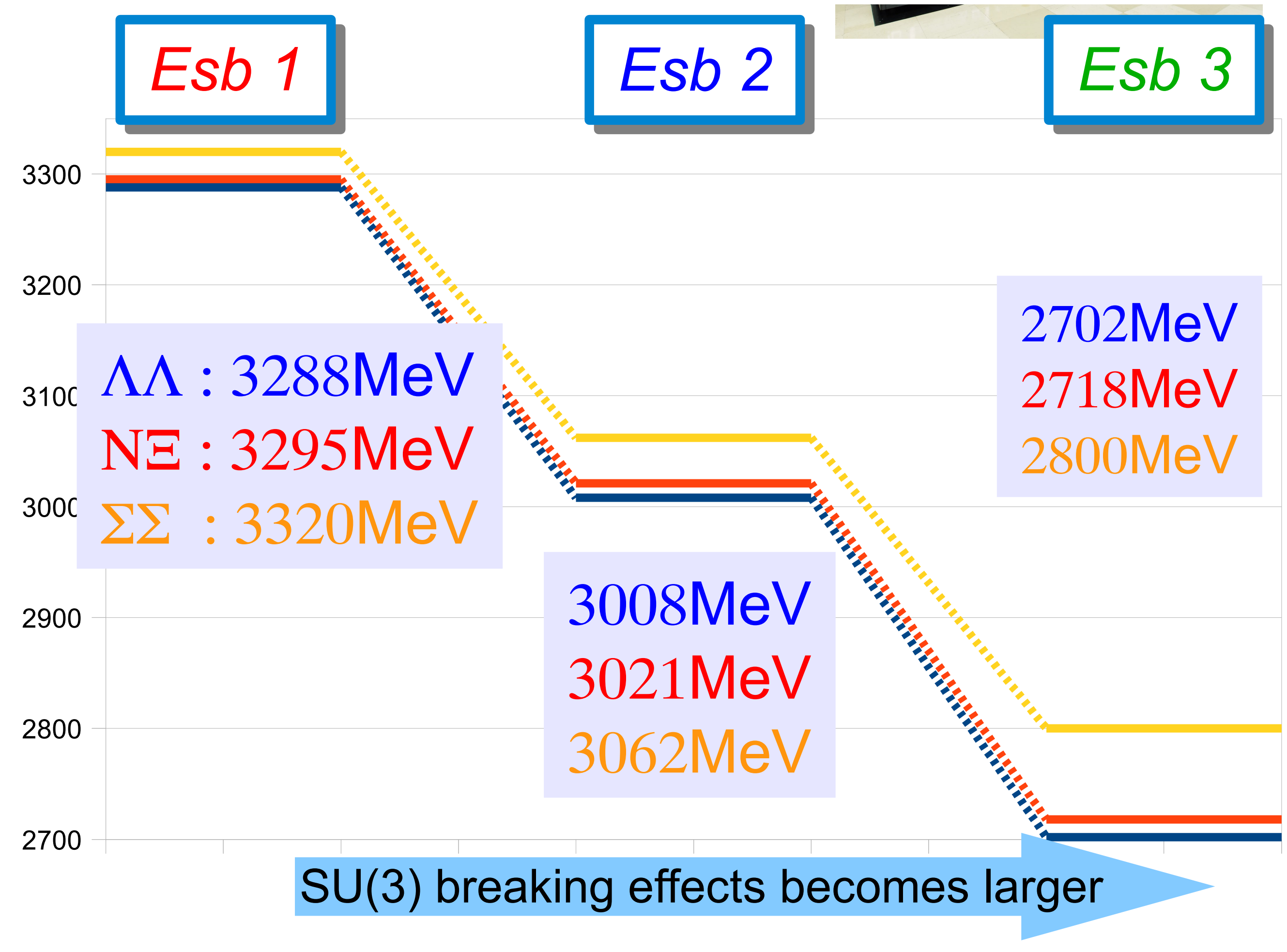
Sasaki for HAL QCD Collaboration

Gauge ensembles

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

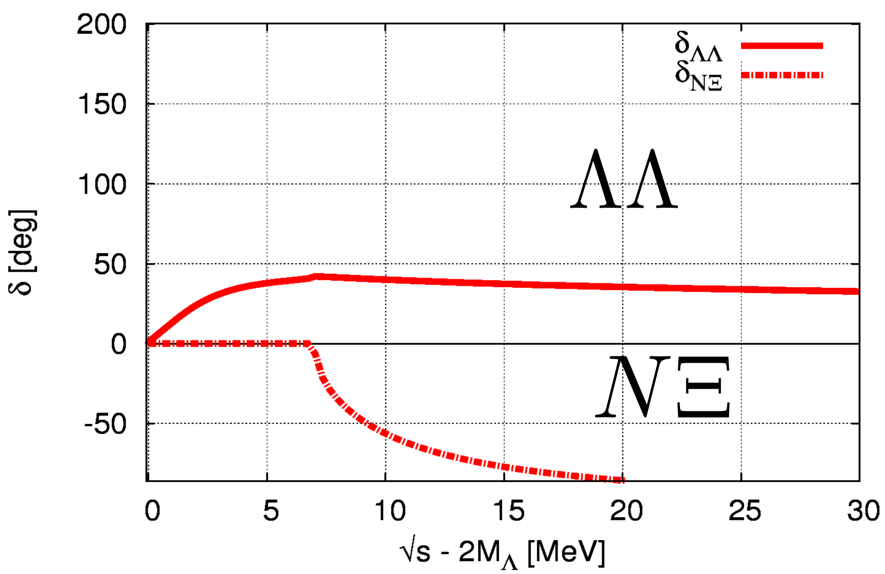
u,d quark masses lighter →

thresholds



$\Lambda\Lambda$ and $N\Xi$ phase shift

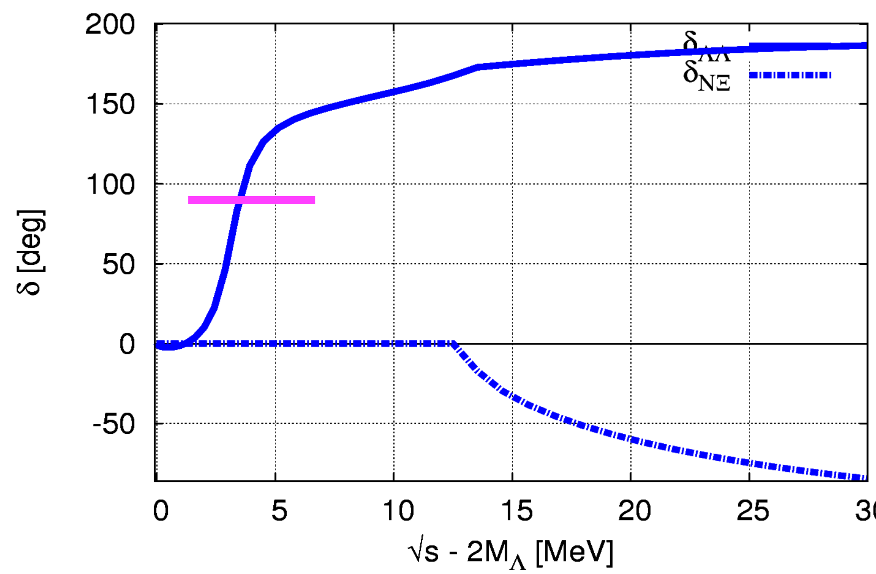
Esb1 : mπ= 701 MeV



Bound H-dibaryon

coupled to NΞ

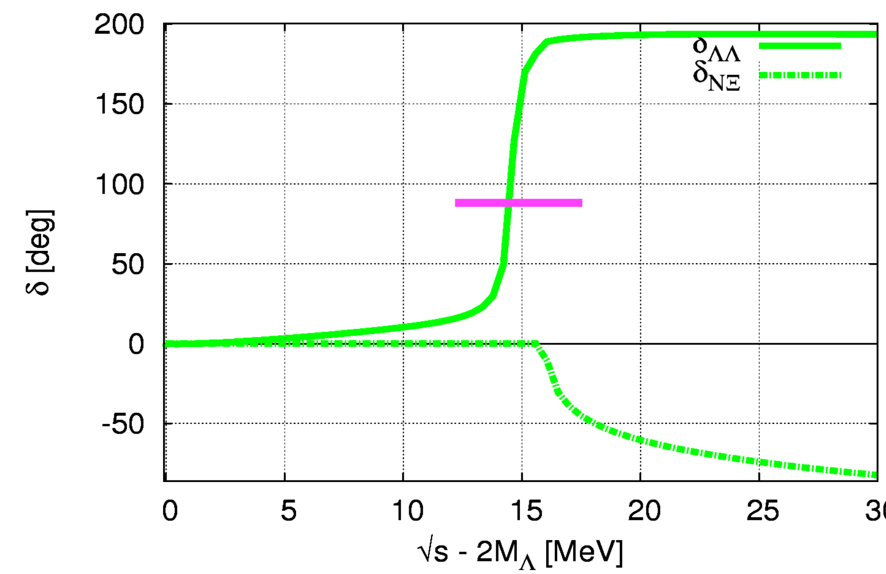
Esb2 : mπ= 570 MeV



H as ΛΛ resonance

H as bound NΞ

Esb3 : mπ= 411 MeV



H as ΛΛ resonance

H as bound NΞ

This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ ? Need simulation at physical point. (work in progress)

Physically, it is essential that H-dibaryon is a bound state in the flavor SU(3) limit.

III. Summary

- The HAL QCD Potential is a very powerful tool to investigate baryon interactions.
- Dibaryons.
    - Omega-Omega : shallow bound state at physical pion mass
    - N-Omega: dibaryon resonance at physical pion mass ?
    - confirmation by 2-particle correlations in future
    - bound $\Delta\Delta$ at flavor SU(3) limit: support d*(2380)
    - bound H dibaryon at flavor SU(3) limit: physical pion mass ?
- Other applications (rho & sigma resonances, heavy baryons, Tetra quark, Penta quark, 3 body forces)

Thank you for your attention !

HAL QCD Collaboration

Hadrons to Atomic nuclei
HAL
from Lattice QCD

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